We consider a model of meson, based on the Poincaré-covariant quark model with analytical solution of the mass spectrum. We investigate possible limitations of model parameters, implied from the lepton decay constants as well as from the mass spectrum of light mesons.
1 Introduction

The investigation of bound states of particles is one of the effective methods to study the properties and dynamics of the interaction of particles. This method is widely used in various areas of nuclear physics and physics of elementary particles. In studies of properties of quarks, of which the mesons and baryons consist, this method has important value, as the quarks are not observed as free states. Today, the electroweak decays of hadrons enable us to measure the parameters of the Standard Model (SM), and also these decays are the tool of exploration of effects of new physics i.e. physics beyond the SM. In particular, the hadronic decays allow determining the elements of a mass matrix, as well as angles of mixing. The leptonic decays of pseudoscalar mesons in a model with two charged Higgs bosons become sensitive to masses of these bosons [?]. Information about the structure of hadrons are required for these investigations and it is important to have the description of properties of hadrons in the framework the relativistic models of bound states.

Many different descriptions of relativistic bound systems have been developed and even a brief survey of the vast literature on this subject goes far beyond the scope of this paper. In the present work, for the description of bound states, we use a Poincaré-covariant model of hadrons. The basis of this model is a constituent quark model and Relativistic Hamiltonian Dynamics (RHD)[?].

The aim of this work is to present combined description of lepton decay constants of pseudoscalar meson and Regge trajectories of light meson in the Poincaré covariant quark model, based on the point form of the RHD. We consider a simple model with an analytical solution of the mass spectrum and investigate the possible limitations of model parameters, implied from the lepton decay constants as well as the mass spectrum of light mesons.

2 Bound quark–antiquark of a state in the RHD

In the quark model the mesons represent a system consisting of a quark and an antiquark. In the framework of RHD, the interaction, which is determined by the generators of the Poincaré group \( \hat{P}_\mu \) and \( \hat{M}^{\mu\nu} \) is introduced as follows. The construction of generators for a system of interacting particles starts from the generators of an appropriate system composed out of noninteracting particles (further we shall note such operators without ”hat”), and then add interaction so that the obtained generators also satisfy the commutation relations of Poincaré group. Unlike the case of a usual nonrelativistic quantum mechanics, in the relativistic case it is necessary to add interaction \( \hat{U} \) in more than one generator to satisfy the algebra of the Poincaré group. Dirac [?] has shown that there is no unambiguous separation of generators into the dynamic set (generators containing the interaction \( \hat{U} \)) and a kinematic set. There are three versions of separation on dynamic and kinematic sets (so-called RHD forms): the point, instant form and dynamics on light front. In all three forms the interaction contains mass operator \( \hat{M} \) i.e. \( \hat{M} = M_0 + \hat{U} \), where \( M_0 \) is an effective mass of a system of noninteracting particles. In an instant form the interaction enters also in the operator of a boost \( \hat{N} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03}) \), which makes wave functions of mesons Lorentz -noninvariant. In the dynamics on light front the interaction is contained with components \( J_1, J_2 \) of the operator of an angular momentum \( \hat{J} = \hat{J} \) (\( \hat{J} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12}) \)),
that results in violation of a rotational covariance. In a point form RHD 4-velocities of bound
and noninteracting systems are equal, i.e.

\[ \dot{V} = \dot{V}_{12}, \quad V = P/M, \quad V_{12} = P_{12}/M_0, \] (1)

where \( P \) and \( P_{12} \) are the 4-momenta of the bound and free from the interactions of particle
system. In all forms of the RHD 4-momentum of a bound system \( P \) and total momentum of
free particles are not equal, i.e. \( P \neq P_{12} \).

Let’s consider in the context of RHD, a bound state with momentum \( P \), mass \( M \), spin \( J \) and
it’s projection \( \mu \) consisting of two particles. Let these particles have the following characteristics:
momenta \( p_1 \) and \( p_2 \), masses \( m_1 \), \( m_2 \), spins \( s_1 \) and \( s_2 \), projection of spins \( \lambda_1 \) and \( \lambda_2 \). The
construction of a bound two-particle state includes the following stage [?]:

1. Definition of the two-particle Hilbert space as the tensor product of the one-particle spaces
and of the appropriate basis:

\[ |p_1 \lambda_1 \rangle |p_2 \lambda_2 \rangle \equiv |m_1 s_1 ; p_1 \lambda_1 \rangle \otimes |m_2 s_2 ; p_2 \lambda_2 \rangle \] (2)

with the normalization

\[ \langle p_1' \lambda_1' | p_2' \lambda_2' | p_1 \lambda_1 \rangle | p_2 \lambda_2 \rangle = \delta_{\lambda_1' \lambda_1} \delta_{\lambda_2' \lambda_2} \delta(p_1' - p_1) \delta(p_2' - p_2). \]

2. The Clebsch-Gordon coefficients of the Poincare group are constructed and are used
to reduce the two-particle representation of a Poincare group to linear superposition (direct
integral) of irreducible representations. As result we obtain the basis:

\[ |P_{12}, \mu, |J, M_0(k)\rangle, (ls); |m_1 s_1 ; m_2 s_2 \rangle = \sum_{ls} \sum_{\lambda_1 \lambda_2} \int \frac{d\bar{k}}{\pi} \sqrt{\frac{\omega_{m_1}(\bar{p}_1) \omega_{m_2}(\bar{p}_2)}{\omega_{m_1}(\bar{k}) \omega_{m_2}(\bar{k}) \omega_{M_0}(\bar{P}_{12})}} \sum_{n \lambda} \omega_{n \lambda} \langle s_1 \nu_1, s_2 \nu_2 | s \lambda \rangle \langle lm, s \lambda | J \mu \rangle Y_{lm} \left( \frac{\bar{k}}{\bar{k}} \right) \]

\[ D^{1/2}_{\lambda_1 \nu_1}(\bar{n}(p_1, P_{12})) D^{1/2}_{\lambda_2 \nu_2}(\bar{n}(p_2, P_{12})) |p_1 \lambda_1 \rangle | p_2 \lambda_2 \rangle, \] (3)

We used the following notations:

\[ P_{12} = p_1 + p_2 \]
is the total momentum of a free system, whereas

\[ \bar{k} = \bar{p}_1 + \frac{\bar{P}_{12}}{M_0} \left( \frac{\bar{P}_{12} \cdot \bar{p}_1}{\omega_{M_0}(\bar{P}_{12}) + M_0} + \omega_{m_1}(\bar{p}_1) \right), \quad \bar{k} = \bar{k}/|\bar{k}| \]
is the relative momentum of two particles, and \( \langle s_1 \nu_1, s_2 \nu_2 | s \lambda \rangle, \langle lm, s \lambda | J \mu \rangle \) are Clebsch-Gordon
coefficients of SU(2)-group. The function \( Y_{lm} \left( \frac{\bar{k}}{\bar{k}} \right) \) is spherical harmonics and \( D^{1/2}(\bar{n}) = 1 - i\bar{n} \cdot \sigma/\sqrt{1 + \bar{n}^2} \) is the D-function of Wigner rotation, which is determined by the vector-parameter
\[ n(p_1, p_2) = \frac{\vec{u}_1 \times \vec{u}_2}{(1 - \vec{u}_1 \cdot \vec{u}_2)} \] with \( \vec{u} = \vec{p}/(\omega_m(\vec{p}) + m) \). Apart from that, the following reductions are used:

\[
\begin{align*}
M_0 &= M_0(k) = \omega_{m_1}(\vec{k}) + \omega_{m_2}(\vec{k}), \\
\omega_m(\vec{k}) &= \sqrt{k^2 + m^2}, k \equiv |\vec{k}|.
\end{align*}
\]  

The following step consists of adding the interaction \( \hat{U} \) to the mass operator of a non-interacting system:

\( \hat{M} = M_0 + \hat{U} \).

If the operator \( \hat{U} \) satisfies the conditions:

\[
\begin{align*}
\hat{M} &= \hat{M}^\dagger, \quad M > 0, \\
[\hat{P}_{12}, \hat{U}]_- &= [i\sqrt{\hat{P}_{12}}, \hat{U}]_- = [\hat{J}, \hat{U}]_- = 0
\end{align*}
\]
then a similar set of interacting particles will satisfy the same commutation relations as the set of non-interacting system.

The problem of eigenvalues of the mass of a bound system can be expressed in three equivalent forms in the Hilbert space\[?\]:

\[
\begin{align*}
\hat{M} | \Psi \rangle &\equiv (M_0 + \hat{U}) | \Psi \rangle = M | \Psi \rangle, \\
M_0^2 + \hat{W}_U | \Psi \rangle &= M^2 | \Psi \rangle, \hat{W}_U = \hat{M}^2 - M_0^2, \\
(k^2 + \hat{W}) | \Psi \rangle &= \eta | \Psi \rangle, \hat{W} = \frac{1}{4} \left[ (\hat{M}^2 - M_0^2) + (m_1^2 - m_2^2) \ast \left( \frac{1}{M^2} - \frac{1}{M_0^2} \right) \right],
\end{align*}
\]

where \( M \) and \( \eta \) are connected:

\[
M^2 = 2\eta + m_1^2 + m_2^2 + 2\sqrt{\eta(\eta + m_1^2 + m_2^2)} + m_1^2m_2^2.
\]  

The solution of the problem (??) will allow to find wave functions, which determine the vertex of the transition from free from interaction to a bound system of particles. In the point form this wave function is determined as follows:

\[
\left\langle \hat{V}_{12}, J, \mu, k, (ls) \left| \hat{V}, J', \mu', M \right. \right\rangle = \delta_{JJ'}\delta_{\mu\mu'}\delta(\hat{V} - \hat{V}_{12})\Psi^{J\mu}(k l s).
\]  

Wave function generally satisfies the integro-differential equation, which follows from Egs.(??) and (??):

\[
\sum_{k' l' s'} \int_0^\infty <k l s \| \hat{W}^{Jd} \| k' l' s'> \Psi^J(k' l' s'; M)k'^2dk' +
\]

\[
+k^2\Psi^J(k l s) = \eta\Psi^J(k l s)
\]  

with the reduced matrix element of operator \( \hat{W} \):

\[
\left\langle \hat{V}_{12}, J, \mu, k, (ls) \left| \hat{W} \left| \hat{V}_{12}', J', \mu', k', (l's') \right. \right\rangle =
\]

\[
\delta_{JJ'}\delta_{\mu\mu'}\delta(\hat{V}_1 - \hat{V}_{12}') \left\langle k, (ls) \left| \hat{W}^{Jd} \right| \right. k', (l's') \right\rangle.
\]  

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The wave functions are normalized:

\[ N_c \sum_{l,s} \int_0^\infty dk \ k^2 |\Psi^J(kls)|^2 = 1, \quad (11) \]

where \( N_c \) is the number of quark colors. Thus in the point form the meson state is defined as state of on-shell quark and antiquark with the meson wave function \( \Psi^J(kls) \)

\[
|\bar{P}, J, \mu, M\rangle = \frac{M}{\omega_M(\bar{P})} \sum_{l,s} \int d^3k \ \frac{\omega_m(\bar{p}_1) \omega_m(\bar{p}_2)}{\omega_m(\bar{k})} \ Psi^J(kls) \sum_{m_\lambda} \sum_{\nu_1\nu_2} \langle s_1\nu_1, s_2\nu_2 | s\lambda \rangle \langle l\mu, s\lambda | J\mu \rangle Y_{l\mu} (\bar{k}) \\
D^{1/2}_{\lambda_1\nu_1} (\bar{n} (p_1, P_{12})) D^{1/2}_{\lambda_2\nu_2} (\bar{n} (p_2, P_{12})) |p_1\lambda_1 | p_2\lambda_2 \rangle. \quad (12)
\]

Let's mark, equation (??) for a wave function is a similar radial equation in a quantum mechanics (only the impulse representation).

3 Regge trajectories of mesons in Poincaré-covariant quark model

In this section we apply the formalism developed above to calculate the mass spectra of mesons containing \( u, d \) and \( s \) quarks. To choose the appropriate interquark potential we use the well-known experimental fact that light hadrons populate approximately linear Regge trajectories, i.e. \( M^2 \approx \beta l + \text{const} \), with the same slope \( \beta \approx 1.2 \text{ GeV}^2 \), for all trajectories (see, for example, [?]). We take the effective model potential \( \bar{W} \) in the oscillator form with spin-spin interaction \( a_s (\vec{s}_1 \vec{s}_2) \)

\[
\bar{W}(r) = W_0 \delta (r) + \beta_{rQ}^4 r^2 + a_s (\vec{s}_1 \vec{s}_2) \quad (13)
\]

where \( W_0, a_s \) and \( \beta_{rQ} \) are free parameters.

Using standard relations for the coordinate operator, orbital momentum and spin operators we reduce the integro-differential equation of RHD (??) to the ordinary quantum-mechanical radial equation with the oscillator potential (only impulse representation)

\[
\left[ \frac{\partial^2}{\partial k^2} + \frac{2}{k} \frac{\partial}{\partial k} - \frac{l(l+1)}{k^2} - \frac{k^2}{\beta_{rQ}^4} \right] \Psi(kls) = \frac{\bar{W}_0 - \eta}{\beta_{rQ}^4} \Psi(kls),
\]

where \( \bar{W}_0 = W_0 + a_s \left( -\delta_{s0} \frac{3}{4} + \delta_{s1} \frac{1}{4} \right) \).

The eigenfunctions of Eq. (??) are

\[
\Psi(kls) = N_{nd} \exp \left( -\frac{k^2}{2\beta_{rQ}^2} \right) \left( \frac{k}{\beta_{rQ}} \right)^l \frac{\Gamma(n + l + 3/2)}{\Gamma(n + 1)} F \left( -n, l + \frac{3}{2}, \frac{k^2}{\beta_{rQ}^2} \right) \quad (15)
\]

with

\[
N_{nd} = \frac{\beta_{rQ}^{l-3/2} \Gamma(n + l + 3/2)}{\Gamma(l + 3/2) \Gamma(n + 1)},
\]

5
where \( n, l = 0, 1, 2, \ldots \), \( F(a, b, z) \) is the hypergeometric function, \( \Gamma(n) \) is the Gamma function. Note that the wave function of the ground state \( (n, l = 0) \) has the oscillator form, which is used in many relativistic models of hadrons:

\[
\Psi(kls) \equiv \Psi(k, \beta_{qQ}) = 2/ \left( \beta_{qQ}^{3/2} \pi^{1/4} \right) \exp \left( -\frac{k^2}{2 \beta_{qQ}^2} \right).
\]  

Quantization condition is defined by

\[
\eta = W_0 + 2\beta_{qQ}^2 (2n + l + 3/2) + a_s \left( -\delta_{s0} \frac{3}{4} + \delta_{s1} \frac{1}{4} \right).
\]  

The spectra of mesons, composed of quarks with equal masses \( (m_q = m_Q \equiv m) \) are given by:

\[
M_{qQ}^2 (l) = 4 \left( m^2 + W_0 + a_s \left( -\delta_{s0} \frac{3}{4} + \delta_{s1} \frac{1}{4} \right) \right) + 8\beta_{qQ}^2 \left( 2n + l + \frac{3}{2} \right).
\]  

Thus we reproduce the linear dependence of \( M^2(l) \) in the framework of the two-body relativistic equation (??).

Now we determine possible limitations on parameters of the bound systems with equal quark masses \( (u - d \text{ and } s - s \text{ states}) \), which are implied from meson Regge trajectories.

The parameters \( W_0 \) and \( \beta_{uu} \) have been found from fitting the \( \rho \)-Regge trajectories (see Fig.1):

\[
m_u^2 + W_0 + a_s \frac{1}{4} = -0.28962 \pm 0.0264 \text{ GeV}^2, \beta_{uu} = 0.3818 \pm 0.0116 \text{ GeV}.
\]  

If we assume that \( \beta_{uu} = \beta_{ud} = \beta_{dd} \), we obtain that the differences of the squared masses of spin-singlet and spin-triplet for \( u - d \) systems are determined by \( M_{S=1}^2 (l = 0) - M_{S=0}^2 (l = 0) = 4a_s \).

Using that \( M_{\rho}^2 - M_{\pi}^2 = 0.5711 \text{ GeV}^2 \) we see that parameter of spin-spin interaction have value:

\[
a_s = 0.14275 \pm 0.00025 \text{ GeV}^2.
\]  

Experimentally, the differences of the squared masses of corresponding spin-singlet and spin-triplet quarkonium states, which contain at least one light quark, weakly depend from quark masses. For example

\[
M_{\rho}^2 - M_{\pi}^2 = 0.5711 \text{ GeV}^2, M_{K^*}^2 - M_{K}^2 = 0.55 \text{ GeV}^2, M_{D^*}^2 - M_{D}^2 = 0.55 \text{ GeV}^2.
\]

Therefore we suggest that parameter \( a_s \) does not depend from masses of quarks. Thus we have that

\[
m_u^2 + W_0 = -0.3253 \pm 0.0264 \text{ GeV}^2.
\]  

There are eight meson Regge trajectories populated by \( u - d \) bound states (for each isospin \( I \) and angular momenta \( J = l+1, J = l, J = l-1 \) and total spin \( S \) of \( qQ \) system-\( S = 0, 1 \) ). Some of the trajectories are plotted in Fig.2 using parameters (??)-(??). We observe that all experimental data are in good agreement with the spectrum given by Eq.(??) for \( S = 1 \) (Fig.2a-2b). As in the case of bound states with \( S = 0 \), the agreement between our theoretical predictions and the existing experimental data is not good (see Fig.2c-2d). Such deviations can be explained by the absence of tensor spin-dependent terms, short-distance term of the potential and octet-singlet mixing for the \( \eta \)-meson trajectory.
If we shall assume "ideal" mixing for $\phi$ meson i.e. the mesons, which correspond to $\phi$-meson Regge trajectory (see Fig.3), consist of only s-quarks, from linear fit (22), we obtain

$$\beta_{ss} = 0.400077 \pm 0.00841 \text{ GeV},$$
$$m_s^2 + W_0 = -0.26486 \pm 0.02013 \text{ GeV}.$$

We can also find a ratio between masses of quarks. It is easy to verify, that $m_s$ and $m_u$ have the relation

$$m_s^2 - m_u^2 = 0.06044 \pm 0.0332 \text{ GeV}^2.$$  (23)

Thus, using Regge trajectories of mesons containing quarks with equal masses we have the following limitations of model parameters:

$$a_s = 0.14275 \pm 0.00025 \text{ GeV}^2,$$
$$\beta_{uu} = \beta_{ud} = \beta_{dd} = 0.3818 \pm 0.0115 \text{ GeV},$$
$$m_u^2 + W_0 = -0.3253 \pm 0.0264 \text{ GeV}^2,$$
$$m_s^2 + W_0 = -0.26486 \pm 0.02013 \text{ GeV}^2.$$  (24)

As is easy to see, our method for bound systems with different quark masses does not require a special procedure to solve the main equation of RHD (22), as was pointed out in [?] (introduction of additional parameter). When the masses of the quark and antiquark are different, using Eq.(22) $M_{QQ}^2$ can be written

$$M_{QQ}^2 (l) = \frac{2\eta + m_q^2 + m_Q^2 + 2\sqrt{\eta(\eta + m_q^2 + m_Q^2)} + m_q^2 m_Q^2}{(2\eta + m_q^2 + m_Q^2) \sqrt{1 - \left( \frac{m_q^2 - m_Q^2}{2\eta + m_q^2 + m_Q^2} \right)^2}}$$  (25)

where the quantization condition of the $\eta$ is defined by Eq.(22). The dependence $M_{QQ}^2 (l)$ is also linear if we assume that in Eq.(22)

$$\left( \frac{m_q^2 - m_Q^2}{2\eta + m_q^2 + m_Q^2} \right)^2 \ll 1.$$

However, from the analysis the Regge of trajectories, we cannot fix masses of the quarks. Therefore the additional experimental data are necessary for further analysis. We shall consider the limitations that we can obtain from alternative experimental data. Then we shall again pass to the construction of the Regge trajectories of mesons with strange quark.

Concluding this section, we remark that, the analytical solution of a main equation of the RHD can be used as zero approximation for solving the problem with more realistic potentials.

4 Leptonic decays in a Poincaré-covariant model of mesons

In this section we shall consider possible constraints on the parameters of the model, which can be obtained from leptonic decays of pseudoscalar mesons (pion, kaon). In the SM the width of purely leptonic decays of charged mesons $P^+(Q\bar{q}) \rightarrow l^+\nu_l$ has the expression:

$$\Gamma_{SM} (P^+ \rightarrow l^+\nu_l) = \frac{G_F^2}{8\pi} |V_{Ql}|^2 f_p^2 m_l^2 M_P \left( 1 - \frac{m_l^2}{M_P^2} \right)^2,$$  (26)
where \( G_F \) is a Fermi constant and \( m_l, M_P \) are the masses of the charged lepton \( l^+ \) and of the pseudoscalar meson \( P(Q\bar{q}) \), respectively. The leptonic decay constant \( f_p \) of a pseudoscalar meson \( P(Q\bar{q}) \) is defined by the matrix element

\[
\langle 0 | J^\mu_\mu(0) | P, M_P \rangle = i (1/2\pi)^{3/2} \frac{1}{\sqrt{2\omega_{M_P}(P)}} \epsilon_{\mu} f_p,
\]

(27)

where \( J^\mu_\mu(0) \) is the operator of the meson current.

Using the relativistic impulse approximation and Eqs. (20), (21) we find that in the point form dynamics

\[
f_p(m_q, m_Q, \beta_{qQ}) = N_c/\left(\pi \sqrt{2}\right)(m_q + m_Q) \int_0^\infty dk^2 \Psi(k, \beta_{qQ}) \frac{M_P^2 - (m_q - m_Q)^2}{\omega_{m_q}(k)\omega_{m_Q}(k) M_P^2},
\]

(28)

where \( N_c \) is the number of colors, \( m_q \) and \( m_Q \) are the masses of the quarks. If \( m_q = m_Q \), the leptonic decay constant is defined by

\[
f_p(m_Q, \beta_{qQ}) = \frac{N_c m_Q}{\pi} \int_0^\infty dk^2 \Psi(k, \beta_{qQ}) \frac{1}{\omega_{m_Q}(k) M_P^2}.
\]

(29)

For further calculations we shall use the wave function of oscillator form (22), which is the solution equation of the relativistic bound states with model potential (23). Using the experimental values of the pion and kaon decay constants and the theoretically calculated formula we shall receive the limitations on parameters of the model. The modern experimental data give the following values of constants of decays of a pion and kaon [2] -

\[
f^{\exp}_\pi = 0.1307 \pm 0.00046 \text{ GeV}, \quad f^{\exp}_K = 0.1598 \pm 0.00188 \text{ GeV}.
\]

The \( \pi \)-meson decay constant is defined by

\[
f_\pi(m_u, \beta_{ud}) = \frac{\sqrt{3} m_u}{\pi^{5/4} \Gamma(-1/4)} \left[ 2^{3/4} \Gamma\left(-\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) \frac{1}{\beta_{ud}^2} \right]
\]

(30)

\[
\times \left[ 1 F_1\left(3, 1; \frac{m_u^2}{2\beta_{ud}^2}\right) - 2 \frac{m_u^2}{\beta_{ud}^2} \Gamma\left(-\frac{3}{4}\right) 1 F_1\left(3, 7; \frac{m_u^2}{2\beta_{ud}^2}\right) \right].
\]

For the decay of the \( K \)-meson, which consists of quarks of different mass, the decay constant is set by Eq. (24). To obtain the limitations on the parameters of model, it is necessary to solve the set of equations:

\[
f_\pi(m_u, \beta_{ud}) = f^{\exp}_\pi, \quad f_K(m_u, m_s, \beta_{us}) = f^{\exp}_K.
\]

(31)

For the solution of this system of equations we use the following procedure: we get the limitations on \( m_u, \beta_{ud} \), which follow from the first equation of a system (24). Further let’s assume, that

\[
m_s = a * m_u, \beta_{us} = V * \beta_{ud},
\]

(32)
where $a, V$ are some numbers. The values of $a$ and $V$ are obtained by minimization of deviation

$$|f_K(m_u, \beta_{ud}, a, V) - f_{K_{SP}}|.$$ 

Graphically it means, that the solution points coincide for both experiments on the plane $m_u - \beta_{ud}$. The result of the procedure are displayed in Fig. 4. The coincidence curve is achieved at $a = 1.48, V = 1.236$. If we assume that $\beta_{ud} = 0.329$ GeV (see Fig. 4), we receive

$$cm_u = 0.250\, \text{GeV}, \quad m_s = 0.370\, \text{GeV},$$

$$\beta_{us} = 0.407\, \text{GeV}. \quad (33)$$

This result (33) agrees with the results obtained in the instant form of the RHD for oscillator wave function [?]. In the dynamics of light front for the data of the quark masses and the parameter $\beta$ of the wave function are approximately equal numerically (33) [?]. Now we shall use the value of the parameters $\beta_{ud}$, which we have found from the analysis of the Regge trajectories (see (33))–$\beta_{ud} = 0.3818$ GeV. In this case we obtain, that

$$m_u = 0.216 \pm 0.02\, \text{GeV}, \quad m_s = 0.320 \pm 0.03\, \text{GeV}.$$ 

We shall return to the analysis of the Regge trajectories. When the masses of quarks are fixed we can calculate the remaining parameters of a model potential. So from a ratio (33) we receive for the parameter $W_0$ the following value:

$$W_0 = -0.3720 \pm 0.0264\, \text{GeV}^2.$$ 

Using mass pseudoscalar $K$-meson $m_K = 0.4937$ GeV and equation (33) we obtain, that parameter of a wave function $\beta_{us}$ is equal:

$$\beta_{us} = 0.3925 \pm 0.06\, \text{GeV}.$$ 

Thus for mesons containing $u, d$ and $s$ quark with the interquark potential (33) we have:

$$a_s = 0.14275 \pm 0.00025\, \text{GeV}^2,$$

$$\beta_{uu} = \beta_{ud} = \beta_{dd} = 0.3818 \pm 0.0115\, \text{GeV},$$

$$\beta_{us} = 0.3925 \pm 0.06\, \text{GeV},$$

$$\beta_{ss} = 0.400077 \pm 0.00841\, \text{GeV},$$

$$W_0 = -0.3720 \pm 0.0264\, \text{GeV}^2,$$

$$m_u = m_d = 0.216 \pm 0.02\, \text{GeV},$$

$$m_s = 0.320 \pm 0.03\, \text{GeV}. \quad (34)$$

Let’s remark, that the squared masses of the $u$ and $s$-quarks, obtained from the leptonic decay constants, and from the analysis of the $\phi$-meson Regge trajectories are approximately agreed (see (33) and (34))

$$m^2_s - m^2_u = 0.056 \pm 0.021\, \text{GeV}^2 - \text{leptonic decays},$$

$$m^2_s - m^2_u = 0.06044 \pm 0.0332\, \text{GeV}^2 - \text{Regge trajectories}. \quad (34)$$
The evaluations of parameter $V = \beta_{us}/\beta_{ud}$ are agreed little a bit worse

$$V = 1.236 \pm 0.020 \text{ - leptonic decays},$$

$$V = 1.02 \pm 0.30 \text{ - Regge trajectories.}$$

We compute the strange meson Regge trajectories using Eqs.(??) and (??) (see Fig.5). The model describes two Regge trajectories with $S = 1$ and one with $S = 0$ in quite a satisfactory way (see Fig.5a, 5b, 5d). But for the $K^*$-meson trajectory the agreement between the model and the experimental data is not good (see Fig.5c).

Thus, in the framework of the model of the mesons based on a point form of the RHD with seven parameters we have a satisfactory description of decay constants of pseudoscalar mesons and their masses. Also the model potential, which solves the equation of the bound two-particle relativistic system with an adequate accuracy reproduces the Regge trajectories of light mesons. But a realistic potential (one boson exchange + nonperturbative part of potential+⋯) is necessary for a more satisfactory description of the spectrum of masses.

5 Acknowledgments

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References

Figure 1: $\rho$-meson trajectory
Figure 2: (a) $\omega$-meson Regge trajectory, (b) $a_0$-meson Regge trajectory, (c) $\pi$-meson Regge trajectory, (d) $\eta$-meson Regge trajectory

Figure 3: $\phi$-meson trajectory
Figure 4: Allowed regions for leptonic decays
Figure 5: $K$-meson trajectories
$I=1/2, L=J, S=0$

$d)$

$K_2(1580)$

$K_1(1270)$

$M^2, \text{GeV}^2$

$L$
I=1/2, L=J+1, S=1

\[ M^2, \text{GeV}^2 \]

\[ K_0^*(1430) \]
\[ K^*(1680) \]
\[ K^*(1980) \]
$I=1/2, \ L=J, \ S=1$

(b)

$K_1(1400)$

$K_2(1770)$

$K_3(2324)$

$K_5^*(2500)$
I = 1/2, L = J - 1, S = 1

(a)

$M^2, \text{GeV}^2$

$K^*_2(1430)$
$K^*_3(1780)$
$K^*_4(2045)$
$K^*_3(2380)$

$L$
Region for $\pi$ decay

Region for $K$ decay

$a=1.48 \ V=1.236$
$I=0$, $L=J-1$, $S=1$

$M^2$, GeV$^2$

$L$

- $\phi(1020)$
- $f_2(1525)$
- $\phi_3(1850)$
- $f_j(2220)$
I=0, L=J, S=0

\( M^2, \text{GeV}^2 \)

- \( \eta \)
- \( \eta_2(1980) \)
- \( h_1(1170) \)
I=1, L=J, S=0

(c)

$\pi_2(1670)$

$b_1(1235)$

$\pi$

$M^2, \text{GeV}^2$

$L$
I = 1, L = J + 1, S = 1

(b)

$\rho_{3}(2250)$

$\rho(1700)$

$a_{0}(1320)$
\[ I=0, \ L=J-1, \ S=1 \]

\[ \text{M}^2, \text{GeV}^2 \]

\[ (a) \]

\[ f_6(2510) \]

\[ f_4(2050) \]

\[ \omega(1670) \]

\[ f_2(1270) \]

\[ \omega(782) \]

\[ L \]
$I=1, L=J-1, S=1$