Dynamical Relaxation of the CP Phases in Next-to-Minimal Supersymmetry

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Abstract

After promoting the phases of the soft masses to dynamical fields corresponding to Goldstone bosons of spontaneously broken global symmetries in the supersymmetry breaking sector, the next-to-minimal supersymmetric model is found to solve the $\mu$ problem and the strong CP problem simultaneously with an invisible axion. The domain wall problem persists in the form of axionic domain formation. Relaxation dynamics of the physical CP-violating phases is determined only by the short-distance physics and their relaxation values are not necessarily close to the CP-conserving points. Consequently, the solution of the supersymmetric CP problem may require heavy enough superpartners and nonminimal flavor structures, where the latter may be also relevant for avoiding the formation of axionic domain walls.
The next-to-minimal supersymmetric standard model (NMSSM), having no gauge extension compared to the minimal model (MSSM), has an extended Higgs sector spanned by the opposite hypercharge Higgs-doublet superfields $\tilde{H}_u$, $\tilde{H}_d$ and a gauge singlet superfield $\tilde{S}$ [1]. The primary motivation for extending the Higgs sector of the minimal model (MSSM) by a singlet is to generate the Higgsino mass term $\tilde{H}_u \cdot \tilde{H}_d$ radiatively via the vacuum expectation value (VEV) of the singlet. The coupling of this singlet field to the Higgs doublets cures the hierarchy problem or the 'μ-problem' [2] of the minimal model through the superpotential

$$W = h_s \tilde{S} \tilde{H}_u \cdot \tilde{H}_d + \frac{1}{3} k_s \tilde{S}^3 + h_u \tilde{Q} \cdot \tilde{H}_u \tilde{U}^c + h_d \tilde{Q} \cdot \tilde{H}_d \tilde{D}^c + h_e \tilde{L} \cdot \tilde{H}_d \tilde{E}^c$$

with the usual notation for quark superfields, $\tilde{Q}, \tilde{U}, \tilde{D}$, lepton superfields $\tilde{L}, \tilde{E}$, and the Yukawa couplings, $h_s, k_s, h_u, h_d, h_e$. This superpotential contains no dimensionfull parameters so that it is conformal invariant. Since all couplings are inherently cubic, beyond gauge symmetries, it has a global continuous $R$-symmetry, denoted henceforth by $U(1)_R$. The minimal supersymmetric model, whose superpotential is obtained by letting $k_s \rightarrow 0$ and $h_s \tilde{S} \rightarrow \mu$ in $W$ above, does not share these properties. Its superpotential depends on a mass parameter, $\mu$, respecting all symmetries and this parameter can be anywhere between the weak scale and the Planck scale; hence, the hierarchy problem. In the conformal limit, $\mu \rightarrow 0$, the MSSM superpotential possesses two global symmetries: a Peccei-Quinn symmetry $U(1)_Q$ and a continuous $R$-symmetry $U(1)_R$.

Adding the most general soft supersymmetry breaking terms to the $F$-- and $D$-- term contributions, part of the NMSSM Lagrangian sensitive to phases of the Yukawa couplings and soft masses is given by

$$-\mathcal{L}_{NMSSM} \supset \left( h_s k_s^* \tilde{S}^* \tilde{H}_u \cdot \tilde{H}_d + \text{H.c.} \right) + \left( \frac{1}{2} m_\lambda \lambda \bar{\lambda} + \text{H.c.} \right) + \left( A_s \tilde{S} \tilde{H}_u \cdot \tilde{H}_d + \text{H.c.} \right) + \left( \frac{1}{3} A_k \tilde{S}^3 + \text{H.c.} \right) + \left( A_u \tilde{Q} \cdot \tilde{H}_u \tilde{U}^c + A_d \tilde{Q} \cdot \tilde{H}_d \tilde{D}^c + A_e \tilde{L} \cdot \tilde{H}_d \tilde{E}^c + \text{H.c.} \right)$$

where the mass quadratics of the scalars fields relevant for symmetry breaking are not shown. In this expression, the first term follows from the $F$--term contribution and it depends on the Higgs Yukawa couplings $h_s$ and $k_s$. The remaining operators are all soft supersymmetry breaking terms that bring about a number of mass parameters: $m_\lambda$ ( $\tilde{\lambda} = \tilde{g}, \tilde{W}^{3,+,-}, \tilde{B}$ ) and several triscalar couplings, $A_s, A_k, A_u, A_d, A_e$. To preserve the generality of the discussion, these triscalar couplings are all taken nonuniversal without any reference to the corresponding Yukawa couplings. A more conventional form where they appear multiplied by the Yukawa couplings can be obtained after a simple redefinition.

In general, all the Yukawa couplings and soft masses in (2) are complex quantities. The Yukawa couplings follow from the symmetric part of the theory and their phase structure remains undetermined. The phases of the soft terms, on the other hand, depends on details of the SUSY breaking sector and they can be predicted in specific models such as dynamical supersymmetry breaking scenarios. Assuming the existence of an appropriate set of parameters satisfying phenomenological constraints [3,4] without breaking charge
and/or color, at tree approximation, the vacuum energy is determined by the Higgs fields. As is clear from (2), the vacuum energy depends only on three phases

\[ \phi_F = \text{Arg}[h_s k_s^*] + \text{Arg}[v_u v_d] - 2\text{Arg}[v_s], \]
\[ \phi_S = \text{Arg}[A_s] + \text{Arg}[v_u v_s] + \text{Arg}[v_s], \]
\[ \phi_K = \text{Arg}[A_k] + 3\text{Arg}[v_s]. \]

where the neutral components of the Higgs doublets and the singlet field develop the vacuum expectation values (VEV’s) \( \langle H_u^0 \rangle = v_u, \langle H_d^0 \rangle = v_d \) and \( \langle S \rangle = v_s \). The minimization of the vacuum energy requires the cancellation of all tadpoles including the ones in the pseudoscalar directions. The latter impose the following constraints on the phases in (3)

\[ |h_s||k_s| \sin \phi_F = -\frac{|A_s|}{|v_s|} \sin \phi_S = \frac{|A_k||v_s|}{3|v_u v_d|} \sin \phi_K \]

leaving only a single, independent, global phase, say \( \phi_K \), which contributes to the vacuum energy by \( (1/3)|A_k||v_s|^3 \cos \phi_K \). Consequently, in the NMSSM the CP symmetry is explicitly violated at tree level by this unremovable phase whose implications for Higgs sector and electric dipole moments (EDM’s) of neutron and electron have been analyzed in [5]. In complete contradiction with NMSSM, however, in the minimal model, obtained by letting \( k_s \rightarrow 0, \ h_s S \rightarrow \mu \) and \( A_s S \rightarrow m^2_{12} \) in (2) above, there is no possibility for CP violation at the tree approximation as the minimization conditions there force \( \text{Arg}[m^2_{12} v_u v_d] \) to vanish identically. This can also be seen from (4) which yields \( \sin \phi_S = \sin (\text{Arg}[m^2_{12} v_u v_d]) = 0 \), leaving \( \phi_K \) and \( \phi_F \) irrelevant, in the MSSM limit. As a result, unlike the minimal model where explicit CP violation can be induced only through the loop effects via the phases of \( \mu \) and \( A_{u,d,e} \) [6], in the NMSSM there is a potential source for CP violation already at tree level [5].

Though NMSSM is a viable model for various particle physics applications [3–5], it is a disfavoured model from the cosmological point of view. As mentioned before, the NMSSM superpotential (1) has a global continuous \( R \)-symmetry, \( U(1)_R \). This symmetry, however, is explicitly broken down to its \( Z_3 \) subgroup by the soft supersymmetry breaking terms in (2) due to Hermitian conjugation. This discrete symmetry is spontaneously broken during the phase transition associated with electroweak symmetry breaking in the early universe, and necessarily, there arise topologically distinct degenerate domains. The walls separating these domains are a cosmological disaster [7,8] as their surface energy would dominate the energy density of the universe unless the symmetry breaking scale is below a few MeV [9]. There have been several suggestions to sidestep the domain wall problem including, for example, breaking the degeneracy of the vacua [7], or allowing for nonrenormalizable interactions that break \( Z_3 \) symmetry [10], or imposing \( Z_2 \) symmetry on the nonrenormalizable interactions [11] to cure the hierarchy problem [12], or using the symmetry non–storation at high temperature [13] to show that the domain–wall problem may not exist at all [14].

The above-mentioned points constitute a summary of the particle physics and cosmological aspects of the next-to-minimal supersymmetry. In Sec. II, the NMSSM will be discussed in a completely new perspective by promoting the phases in the soft terms to dynamical fields transforming nontrivially under the \( U(1)_R \) symmetry of the superpotential. This will be seen to lead a solution for the strong CP problem with an invisible axion. However, it will be seen that the domain wall problem persists in the form of an axionic
domain formation problem. In Sec. III radiative corrections to the vacuum energy are computed, and dynamical relaxation of the CP phases is considered. Here the dominance of the short-distance physics and existence of finite CP violation are particularly emphasized. Sec. V is devoted to a discussion of the MSSM limit where it is also shown that the CP-violating relaxation point of the NMSSM phases is, in fact, the phase lifted by the $U(1)_{PQ}$ symmetry arising in the vanishing cubic singlet coupling in the superpotential. Sec. VI concludes the work.

II. DYNAMICAL PHASES AND DOMAIN WALLS

It is with the soft terms that the supersymmetry and gauge symmetry are broken in the visible sector. Additionally, their nonvanishing phases lead to a violation of the CP symmetry thereby contributing to CP-violating observables such as EDM’s and neutral meson mixings. A true characterization of these CP-violating phases depends on the short-distance structure of the theory. For instance, in string theory, CP is a gauge symmetry and it is broken spontaneously together with supersymmetry; hence, the short-distance theory is a CP-conserving one [15]. Alternatively, the short-distance physics may not respect CP symmetry at all in which case there can be explicit CP-breaking operators. In any case, the source of explicit CP violation in the long-distance theory should follow from the short-distance physics; therefore, it is convenient to identify the CP-violating phases with the Goldstone bosons of some global symmetries spontaneously broken together with supergravity.

Depending on the mechanism that transmits the SUSY breaking to the visible sector the SUSY breaking scale, $M_{SUSY}$, could be anywhere between just above the weak scale to the Planck scale. In what follows, for both definiteness and phenomenological viability, all short-distance characterization of the model will be based on the supergravity breaking, where there is a renormalizable hidden sector in which SUSY is broken in the flat spacetime limit but transmitted to the visible sector via gravitational strength interactions. In this case the scalar fields in the hidden sector have VEV’s at the intermediate scale: $M_{SUSY} \sim \sqrt{m_{3/2}} M_P$. In general, there is no a priori reason to forbid some global symmetries in the SUSY breaking sector which are spontaneously broken together with supersymmetry around $M_{SUSY}$. Then the soft masses in the low energy effective theory below $M_{SUSY}$ will have nonvanishing phases corresponding to the Goldstone bosons of those spontaneously broken global symmetries. If scale of SUSY breaking is high enough (e.g., at the intermediate scale) the Goldstone bosons decouple and there remains no observable effects at low energies. It is clear that the low energy theory will have the same global symmetries existing before the SUSY breaking provided that these symmetries are now realized nonlinearly or in Goldstone mode. Indeed, nonlinear realization of the short-distance global symmetries by long-distance physics plays essential role in the invisible majoron models [16] associated with spontaneously broken lepton number in the standard model, and in the relaxation of the CP phases in moduli dominated string-inspired supergravity models [17] associated with the internal axion. Specifically, that the soft terms can realize a given global symmetry in Goldstone mode has been used in [18] to interpret the supersymmetric flavour problem and in [19] to show the relaxation of the CP phases in the MSSM to CP conserving points. The latter corresponds to a generalization of the Peccei–Quinn mechanism for the relaxation of the QCD vacuum
angle [20] and it will be closely followed in computing the vacuum expectation values of the dynamical phases in the NMSSM.

Obviously, if the soft terms in (2) vanish the NMSSM Lagrangian possesses the $U(1)_R$ symmetry mentioned before. Alternatively, the Lagrangian can assume the same global symmetry if the soft masses realize it in Goldstone mode, that is, if their phases are dynamical variables transforming under $U(1)_R$. Assuming that the theory above $M_{SUSY}$ protects appropriate global symmetries [21,16,17,19], one can impose certain transformation rules on the soft terms so as to restore the $Z_2$ symmetry in (2) to the continuous symmetry of the superpotential

$$m, A_s, A_k, A_u, A_d, A_e \rightarrow e^{-iR_W \alpha} m, A_s, A_k, A_u, A_d, A_e$$

where $\alpha$ is the $U(1)_R$ rotation angle. Pertinent to a global $R$–symmetry [22] fermionic and bosonic components of chiral and vector superfields transform differently, in particular, the Grassmann coordinate, $\theta$, has charge $R_\theta = R_Y = R_W / 2$, where $R_W$ is the $R$–charge of the superpotential (1) satisfying $R_W = 3R_S = R_S + R_{H_u} + R_{H_d} = R_Q + R_{H_u} + R_U = R_Q + R_{H_d} + R_D = R_L + R_{H_d} + R_E$. Clearly, it is impossible to assign vanishing charges to both Higgs doublets so that $U(1)_R$ is spontaneously broken together with the gauge symmetries. Moreover, fermion superfields cannot be assigned $R$–charges like $R_Q = -R_U$ or $R_Q = -R_D$ or $R_L = -R_E$; hence, $U(1)_R$ group has necessarily a quantum mechanical anomaly with respect to both QCD and QED.

After identifying the dynamical phases of the soft parameters with the Goldstone bosons of some spontaneously broken global symmetries in SUSY breaking sector, it is now necessary to determine the fate of the nonlinearly–realized $U(1)_R$ symmetry. First of all, $R$–invariance restricts the total number of physical phases to be one less compared to the phases of the soft parameters, that is, any physical quantity computed in the NMSSM admits a $U(1)_R$ rephasing that eliminates one of the phases. Therefore, without loss of generality one can, for example, take the gaugino masses real so that phases of the triscalar couplings become the physical SUSY CP-violating phases. Adopting this convention, it is clear that the dynamical phase of the gaugino mass, $\text{Arg}[m_{\lambda}] \equiv G_\lambda(x)/M_{SUSY}$, cannot receive a potential from the visible sector interactions. If there are no explicit breaking terms coming from short distances, or equivalently, if the $U(1)_R$ symmetry is respected by all interactions up to the SUSY breaking scale then $G_\lambda(x)$ possesses a strictly flat potential. Therefore, this field would be a strictly massless pseudoscalar were not it for the long–distance effects that modify the picture. As noted before the $U(1)_R$ symmetry is broken spontaneously by the Higgs VEV’s and explicitly by the QCD instanton effects. With such a breaking scheme, the similarity between the procedure applied here and KSVZ [23] or DFSZ [24,25] axion models is manifest. Indeed, the $U(1)_R$ invariance here replaces the $U(1)_{PQ}$ symmetry in the axion models. The properties of the $U(1)_R$ Goldstone boson are then fixed by the SUSY breaking scale to leading order [25,26]: $G_R(x) \approx G_\lambda(x) + \mathcal{O}(m_3/2/M_{SUSY}) \times \Im m[H^0_u, H^0_d, S]$ and $f_R \approx M_{SUSY} + \mathcal{O}(m^2/2/M_{SUSY})$ where $f_R$ is the axion decay constant. The gluinos and quarks contribute to both color current and axial $U(1)_R$ current

$$j^a_{R} = R_{H_u} \bar{u} \gamma^\mu \gamma^5 u + R_{H_d} \bar{d} \gamma^\mu \gamma^5 d + R_{\lambda} \bar{g}^a \gamma^\mu \gamma^5 g^a,$$

so that the $U(1)_R$ has a quantum mechanical anomaly with respect to QCD: $\partial_\mu j^a_{R} = \frac{\alpha_s}{4\pi} R_{\lambda} G_{\mu}^a G^{a,\mu}$ where $R_{\lambda}$ is the anomaly coefficient. The Goldstone boson $G_R(x)$ receives a potential from this anomaly, and its VEV is given by $V_G = -(M_{SUSY} \theta_{QCD})/R_{\lambda}$.
This particular relation relaxes the effective QCD vacuum angle to zero whereby solving the strong CP problem. Besides, the same anomaly–induced potential generates a finite mass for $G_R(x)$ [26,25]

$$m_R \sim \frac{m_\pi f_\pi}{M_{SUSY}}$$

(7)

turning it to an invisible axion for laboratory experiments as long as $M_{SUSY}$ is above the weak scale. In fact, this axion can be well in the axion window allowed by the astrophysical and cosmological bounds [27] if $M_{SUSY}$ refers to an intermediate scale [21]. The QED anomaly of $U(1)_R$ symmetry, on the other hand, determines the lifetime of the axion $\tau(G_R \to 2\gamma) \approx \tau(\pi^0 \to 2\gamma)(m_\pi/m_R)^5$ [25].

As a result, promoting the phases of the soft masses to dynamical fields transforming nontrivially under $U(1)_R$ symmetry leads one naturally to a solution for the strong CP problem with an invisible axion [24,25]. Moreover, the $Z_3$ symmetry of the NMSSM Lagrangian (2), whose spontaneous breaking generates the domain walls, is now embedded into this continuous $R$–symmetry so that the domain wall problem may seem sidestepped. However, as was shown long ago in [28], the instanton effects cannot break the global symmetry completely, that is, there remains a residual discrete symmetry which is spontaneously broken during the electroweak symmetry breaking [28,29]. Indeed, independent of the specific $R$–charge assignments (5), for a $U(1)_R$ transformation by $\alpha = 2\pi n$, the gauge invariant operators $S$ and $H_u \cdot H_d$ transform as

$$S \to e^{i\pi \frac{R_S}{A}} S, \quad H_u \cdot H_d \to e^{i\pi \frac{(R_{H_u} + R_{H_d})}{A}} H_u \cdot H_d,$$

(8)

where $R_S/A = (R_{H_u} + R_{H_d})/2A = 2/3$ with $A = R_\lambda$ being the anomaly coefficient. Therefore, the QCD instanton effects leave a $Z_3$ symmetry unbroken so that one is back to the domain wall problem of the original NMSSM. In the literature there have been several proposals for avoiding the axionic domain walls, and they have all been based on either embedding the discrete symmetry into the center of the GUT group [30] or using additional flavour structures with a wider particle spectrum [31]. In the present situation, which is nothing but a DFSZ–type axion model, there is always a domain wall problem according to the rather general statements of [28,29].

Before concluding this section, it is useful to make a comparison with the MSSM: There arise two global symmetries $U(1)_{PQ}$ and $U(1)_R$ after promoting the phases of the soft masses and the $\mu$ parameter to dynamical variables [19]. These two global symmetries can be used to eliminate two of $\text{Arg}[\mu]$, $\text{Arg}[m_{12}^2]$, $\text{Arg}[m_\lambda]$ and $\text{Arg}[A_{u,d,e}]$. The remaining phases are the physical CP–violating phases. One combination of these two global symmetries is anomalous with respect to QCD and it is broken by the instanton effects [19] with a remnant $Z_3$ symmetry which is spontaneously broken by the electroweak breaking as in the NMSSM. The other combination is anomaly–free and remains unbroken unless there are some short–distance effects that break it explicitly.

In summary, after generalizing the phases of the soft terms to dynamical variables, there appears an anomalous, global, continuous $R$–symmetry in the NMSSM Lagrangian. Due to both electroweak breaking and nonperturbative QCD effects this global symmetry is broken in such a way that

- The QCD vacuum angle relaxes to its natural value leading to a solution for the strong CP problem,
• The resulting axion acquires an anomaly–induced mass which is well in the *axion window*,
• The domain wall problem persists in the form of axionic domain formation.

One finally notes that it is essentially the *R* character of the global symmetry that admits NMSSM as the observable sector otherwise the short–distance description can require more singlet fields if one insists on solving *µ* problem and strong CP problem simultaneously [32].

### III. RELAXATION OF THE CP PHASES IN THE NMSSM

The relaxation dynamics of the soft CP phases have been studied in [19] in the MSSM context. In the case of the minimal model the short–distance theory, before SUSY breakdown, is similar to the DFSZ axion models [24,25]. In what follows, the relaxation dynamics of the CP phases in the NMSSM will be analyzed on similar lines with [19]. As summarized in the Introduction, the NMSSM has a light singlet in addition to the weak doublets, and this can affect the relaxation dynamics. At tree approximation, the vacuum manifold is described by the Higgs fields $S$, $H_u$, $H_d$ and the dynamical phases of the soft SUSY breaking masses, $A_s$ and $A_k$. Cancellation of the tadpoles forces the phases to satisfy the relations (4) leaving a single independent phase without any specific condition on its proximity to a CP conserving point. After promoting the phases of the soft masses to dynamical fields with transformation properties in (5), the tree level vacuum energy (2) should be minimized with respect to Higgs fields as well as the dynamical phases of $A_s$ and $A_k$. This then gives

$$\langle \phi_s \rangle = \langle \phi_K \rangle = 0, \quad (\text{Mod} = \pi),$$

and necessarily $\phi_F \equiv 0$ due to (4). Therefore, once the soft phases become dynamical variables the sources of CP violation in the tree level potential disappear completely. These minimization conditions on the phases, however, no longer requires individual dynamical or non–dynamical phases to be close to CP conserving points. Indeed, one has

$$\text{Arg}[SH_u \cdot H_d] = \langle \text{Arg}[A_s^*] \rangle, \quad \text{Arg}[S^0] = \langle \text{Arg}[A_k^*] \rangle, \quad \langle \text{Arg}[A_s A_k^*] \rangle = \theta_s - \theta_k,$$

which are valid (Mod = $\pi$), and $\theta_s = \text{Arg}[h_s]$ and $\theta_k = \text{Arg}[k_s]$ are non–dynamical phases of the Higgs Yukawa couplings. The minimization of the tree–level vacuum energy thus fixes the phases of the Higgs fields in terms of the VEV’s of the dynamical triscalar phases. Moreover, the relative phase between $\langle A_s \rangle$ and $\langle A_k \rangle$ equals the relative phase between the Yukawa couplings. One particularly notices that, in the conventional form of the trilinear couplings for which $A_k \rightarrow k_s A_k$ and $A_s \rightarrow h_s A_s$, the relative phase between $\langle A_s \rangle$ and $\langle A_k \rangle$ vanish (Mod $\pi$).

Although the phases of the soft masses are now dynamical variables corresponding to the Goldstone bosons of some spontaneously broken global symmetries in the SUSY breaking sector, the phases of the Yukawa couplings are of non–dynamical origin and their contribution to CP violating amplitudes should be counted separately. Any symmetries associated to the SUSY CP–violating phases must be broken explicitly by couplings to the visible sector. That is, the short distance–dynamics responsible for their relaxation
process can be obtained by integrating out the visible sector fields. In this sense it is essentially the radiative corrections that determine the VEV of the dynamical phases so that pursuing the tree level vacuum energy further is not necessary. Relaxation dynamics of a particular phase field is determined by the dimensionality of the operator responsible for it. For marginal operators (dimension four) the sensitivity to short–distance physics is logarithmic whereas for non–marginal operators (dimension less than four) the short–distance theory is essential.

When integrating out the observable sector, it is necessary to consider those operators which are invariant under gauge and $U(1)_R$ symmetries but having an explicit dependence on the dynamical phase fields. In listing these operators, not only the spurions $m_\lambda, \cdots, A_f$ but also the Higgs fields are to be included as they acquire VEV’s $\mathcal{O}(m_3/2)$ and influence the long–distance dynamics. The radiative corrections to the vacuum energy are to be computed in the tree-level minimum expressed by (9), in particular, the phase-dependent invariants are formed using (10). In the following, the first two relations in (10) will be used but the third relation will be concluded from the radiative corrections. This will form a consistency check for the long– and short–distance dynamics of the phases. An exhaustive list of the phase–dependent invariants after using the first two relations in (10) is given by

\begin{equation}
A_s A_k^* A_k A_s^* \left( |H_u|^2, |H_d|^2, |S|^2 \right)
A_f A_k^* A_k A_f \left( |H_u|^2, |H_d|^2, |S|^2 \right)
A_f A_f \left( |H_u|^2, |H_d|^2, |S|^2 \right)
\end{equation}

where it is implied that each operator is accompanied by its Hermitian conjugate. To shorten the list of operators, a single sfermion triscalalar coupling $A_f$ is used though the structure can be repeated by using $A_f = A_u, A_d$ or $A_e$, separately. In what follows all calculations will be done with $A_f = A_u$, and relaxation of possible relative phases among $A_u, A_d$ or $A_e$ will not be considered. Since the main interest is in the flavour–conserving CP phases it is convenient to use a universality ansatz for these triscalalar couplings [19].

The way of listing in (11) is such that, except for the last row, all operators in a given row have the same phase after using the relations in (10). Moreover, again excluding the last row, the first operator in each row has mass dimension two, whereas the remaining ones have mass dimension four. The number of phase–dependent invariants in (11) is larger than the independent phases. Indeed, out of four phases coming from $m_\lambda, A_s, A_k$ and $A_f$, there are only three independent ones thanks to the $U(1)_R$ invariance of the NMSSM Lagrangian (2). A set of physical independent phases can be achieved after rephasing an arbitrary physical quantity computed in the NMSSM with, for example, the gaugino mass phase $\text{Arg}[m_A^\lambda]$:

\begin{equation}
\phi_s(x) = \text{Arg}[m_\lambda A_s^*], \quad \phi_k(x) = \text{Arg}[m_\lambda A_k^*], \quad \phi_f(x) = \text{Arg}[m_\lambda A_f^*],
\end{equation}

which is the same set of physical phases used in discussing the $R$–axion in Sec. II. In this basis of the independent phases, the arguments of the rows 1, \cdots, 6 in (11) are given by
respectively. On the other hand, the last row of (11) consists of the operators with respective phases \( \phi_k + \phi_s - \phi_f, \phi_f + \phi_s - \phi_k \) and \( \phi_k + \phi_f - \phi_s \). As a result, phases of all operators in (11) can be written in terms of the three physical CP-violating phases in (12).

The type of the radiative correction to a given operator is determined by its mass dimension. In fact, the basic distinction between the marginal (dimension four) and non-marginal (dimension two operators in (12)) operators concerns their expansion coefficients, that is, while the former can have only dimensionless coefficients with at most logarithmic divergences, the latter are necessarily endowed with mass dimension two coefficients with at most quadratic divergences. Concerning the supergravity breaking, the theory at hand has two mass scales: \( m_{3/2} \sim G_F^{-1/2} \) and \( M_{Pl} \). Therefore, the expansion coefficients for the non-marginal operators could be either at the weak scale \( \sim m_{3/2} \) or at ultra high energy scale \( \sim M_{Pl} \). As a result, on dimensional grounds, the divergent diagrams yield \( \log \left( M_{Pl}^2/m_{3/2}^2 \right) \)–type behaviour for the coefficients of the marginal operators and \( M_{Pl}^{-1} \)–type behaviour for those of the non-marginal operators.

FIG. 1. (a) Sample diagrams producing some of the marginal operators in (11). The dot on the gaugino line shows the gaugino mass insertion.

Depicted in Fig. 1 is a set of sample diagrams generating some of the marginal operators in (11). Here (a), (b), (c) and (d) generate, respectively, \( h_s h_f^* A_f S H_u \cdot H_d \), \( h_s m_\lambda S H_u \cdot H_d \), \( h_s m_\lambda A_s^* S \) and \( h_f m_\lambda A_f^* S \). All the remaining dimension–four operators in (11) can be generated using similar diagrams. For example, the diagram (e) in Fig. 1 generates \( h_t h_s^* m_\lambda A_f^* A_k^* A_s \). It is clear that one would identify these operators with the corresponding marginal ones in (11) if there were no additional phases coming from the Yukawa couplings. Indeed, evaluation of each diagram in Fig. 1 produces a marginal operator belonging to the list in (11); however, there is always a non–dynamical nonvanishing additional phase coming from the associated Yukawa couplings. That the operators generated by the diagrams in Fig. 1 bear a Yukawa pollution will be important
in determining the relaxation points of the dynamical phases. The contribution of the marginal operators in (11) to the vacuum energy can then be estimated roughly as follows

\[ (\Delta V)_{\text{long}} = m_{3/2}^4 \log \left( \frac{M_{\text{Pl}}^2}{m_{3/2}^2} \right) \left\{ c_a \cos (\phi_s - \phi_f + \theta_s - \theta_f) + c_{b,c} \cos (\phi_s + \theta_s) + c_d \cos (\phi_f + \theta_f) + c_e \cos (\phi_f + \phi_k - \phi_s + \theta_f + \theta_k - \theta_s) + \cdots \right\} \]

(13)

where \( c_a, \cdots, c_e \) are, respectively, the weights of the diagrams (a), \( \cdots, (e) \) in Fig. 1, and the ellipses stands for contributions of the diagrams that generate other marginal operators listed in (11). Here the weight factors \( c_i \) consist of the Yukawa and gauge couplings as well as the loop suppression factors. In the contribution of each operator there are additional phases coming from the non-dynamical phases of the Yukawa couplings. In a given diagram there is no further phase shift beyond the ones coming from the Yukawa couplings so that arguments of the cosinus functions are fixed. In writing (13) all Yukawa couplings are taken to be complex without referring to a particular rephasing scheme that can make some of them real. Here one readily observes that the relaxation points, or equivalently, the VEV’s of the dynamical phases depend on the phases of the Yukawa couplings.

It is because of the marginal character of the operators that there is an overall \( \log (M_{\text{Pl}}^2/m_{3/2}^2) \) factor in (13). This log factor states that all these marginal operators are, in fact, insensitive to the short-distance physics compared to its enhanced sensitivity to the long-distance physics. The operators in the last row of (11) arise at higher loop orders than the other marginal operators; therefore, it is essentially the electroweak symmetry breaking that generates \( (\Delta V)_{\text{long}} \). In other words, the Higgs VEV’s generate the dominant contributions to (13), and the corresponding scale of the radiative corrections is necessarily the electroweak scale—a length scale much longer than \( M_{\text{Pl}} \) designating the short-distance end of the model. If the dimension-two operators in (11) were absent then \( (\Delta V)_{\text{long}} \) would be added to the tree level potential (2) to obtain the radiatively-corrected vacuum energy to determine the relaxation points of the dynamical phases together with the masses of the pseudo-Goldstone bosons. In such a case it would be necessary to compute the contributions of all marginal operators in (11); however, as will be seen below the physical dynamical phases \( \phi_s(x), \phi_f(x) \) and \( \phi_k(x) \) do also appear in the expansion coefficients of the non-marginal operators, and their relaxation points are determined essentially by the short-distance physics.

After completing the long-distance part, now there remains the identification of the relevant diagrammatics producing the non-marginal (dimension two) operators in (11). As mentioned before, these operators have necessarily dimension-two expansion coefficients for they are to contribute to the vacuum energy. Depicted in Fig. 2 are the loop diagrams generating the non-marginal operators. In this figure, diagrams (a), \( \cdots, (f) \) generate the first operator in the rows 1, \( \cdots, 6 \) of (11). A close inspection of this figure shows that all these operators are quadratically divergent, and hence, their contribution to the vacuum energy is proportional to \( M_{\text{Pl}}^2 \). For later use, it is convenient to factor out the loop factors of each diagram when estimating their contributions to the vacuum energy:

\[ (\Delta V)_{\text{short}} = \frac{M_{\text{Pl}}^2 m_{3/2}^2}{(4\pi)^6} \left( \ell_f \cos (\phi_f + \theta_f + \delta_f) + \ell_s \cos (\phi_s + \theta_s + \delta_s) \right) \]
where $\bar{C}_i$ shows the weight of the $i$-th diagram in Fig. 2, and $\bar{\delta}_i$ its possible phase shift beyond the ones coming from the Yukawa couplings. One notices that here $\bar{C}_i$ does not include the loop factors; they are functions of only Yukawa and gauge couplings together with other kinematical factors. In (14) there are further phase shifts $\bar{\delta}_i$. Taking these additional phases already relaxed to CP-conserving points may be an underestimation of the short-distance structure of the theory. Indeed, as is clear from the corresponding formulae (2), (13) and (14), contributions of the non-marginal operators are determined solely by the short-distance physics in contrast to long-distance sensitivities of the tree-level vacuum energy and the marginal operators. For this reason, depending on the detailed structure of the theory at short-distances (SUSY breaking scale) there may be additional phases that contribute to phase shifts, $\bar{\delta}_i$. In spite of this possibility, here, as a simplifying assumption, the short distance theory will be assumed to have the same CP conventions as the long-distance theory, and thus, $\bar{\delta}_i = 0, \pi$.

![Diagrams](image)

**Fig. 2.** Diagrams generating the non-marginal operators in (11). The dot on the gaugino line represents the gaugino mass insertion.

As suggested by (14), the diagrams (c) and (e) in Fig. 2 are four-loop ones so that
their contributions to the vacuum energy are suppressed by a loop factor (two orders of magnitude) compared to the others, and thus, these terms will be neglected in minimizing the energy. From (13) and (14) it is easy to see that the scalar potential for each of the phases $\phi_k, \phi_s$ and $\phi_f$ receives contributions from both short- and long-distance physics. As a counter example, one recalls here the MSSM where the marginal operators $m_\lambda \mu (m_{12}^2)^* \text{ and } A_f \mu (m_{12}^2)^*$ are sensitive to only the long-distance physics as in (13); however, the non-marginal operator $m_\lambda A_f^s$ gets contributions from both long- and short-distance physics. In this sense, in the MSSM, VEV’s of the dynamical phases are determined by the long- and short-distance physics for the marginal and non-marginal operators, respectively. This special long-distance sensitivity of the marginal operators in the MSSM, which is induced by $m_{12}^2$, is an anomalous effect [19] compared to the short-distance sensitivities in dynamical squark flavor matrices [18], dynamical Yukawa couplings [33,34], or a dynamical determination of the SUSY breaking scale in no-scale type models [35,36]. In such cases the vacuum energy is quadratically sensitive to the short-distance physics [37] like (14). In this sense the NMSSM is more natural than the MSSM as the vacuum expectation values (relaxation points) of the phases in (12) are determined solely by the short-distance physics. Indeed, the radiatively corrected vacuum energy,

$$V(\phi_f, \phi_s, \phi_k) = V_{\text{tree}} + V_{\text{short}}(\phi_f, \phi_s, \phi_k) + V_{\text{long}}(\phi_f, \phi_s, \phi_k)$$  \hspace{1cm} \text{(15)}$$

is to be minimized against the variations of the dynamical fields $\phi_k(x), \phi_f(x)$ and $\phi_s(x)$. In course of minimization one notices that there is no particular phase that depends only on the long-distance physics; therefore, all tadpole equations are saturated by the short-distance contributions (14) to an accuracy $\mathcal{O}(m_{1/2}^2/M_{Pl}^2)$. The second important point concerns the special form of $(\Delta V)_{\text{short}}$ where the potentials of the phases $\phi_s, \phi_f, \phi_s - \phi_f$ and $\phi_s - \phi_k$ are formed at the same loop order. That is, there is a strong mixing among the non-marginal invariants in (11). Furthermore, one notices that the self-potential of $\phi_k$ is suppressed by one loop factor compared to that of $\phi_s - \phi_k$, signaling another operator hierarchy pattern. As mentioned above, in minimizing the radiatively corrected vacuum energy (15) all phase shifts $\delta_i$ will be assumed to have already relaxed to CP conserving points.

These observations are still not sufficient to start a direct minimization of (15). This is because of the Yukawa pollution in the radiative corrections which requires a physical basis to be chosen. The dependence of the radiative corrections to the Yukawa phases follow from different sources. First, the phase of $h_f$ enters through the corresponding quark loops. However, by rephasing the quark fields their Yukawa couplings can be made real in which case the entire CP violation effects are transferred to the charged-current vertices via the CKM matrix, $V$. In the standard model, CKM matrix is the mere source of CP violation the size of which is characterized by the Jarlskog invariant $J = \text{Im}[V_{ud}V_{ts}^* V_{td}V_{ub}^*]$. The contribution of this parameter to (15) is suppressed by a factor of $\mathcal{O}(\alpha/4\pi)^2 J$ compared to the loop diagrams considered above. Therefore, the possible phase shifts due to the Jarlskog parameter can be neglected compared to the supersymmetric ones, and all fermion Yukawa couplings can be taken real. Contrary to $h_f$, however, the phases of $h_s$ and $k_s$ enter the radiative corrections through the quartic Higgs couplings introduced by the $F$–term contributions in (2). Therefore, the relative phase between $h_s$ and $k_s$ enter the vacuum energy via the scalar potential itself, and in conjunction with the tree-level formulae (4) and (10) it is convenient to keep their phases during the minimization. After these simplifying observations, a direct minimization
of the radiatively corrected vacuum energy (15) gives the following vacuum expectation values for the physical phases (12):

$$\langle \phi_f \rangle = \langle \phi_k \rangle + \theta_k = \langle \phi_s \rangle + \theta_s + \pi$$

(16)

which imposes no condition on the proximity of a particular phase to a CP-conserving point: $\langle \phi_f \rangle, \langle \phi_k \rangle, \langle \phi_s \rangle \neq 0, \pi$. In obtaining this result all $\delta_i$'s are taken to be roughly equal. Secondly, the last two terms in (14) are neglected as they are smaller than the others by two orders of magnitude. A careful look at (16) reveals an important property: The relation $\langle \phi_k \rangle - \langle \phi_s \rangle = \theta_s - \theta_k + \pi$ is nothing but the last equality in (10). While the former follows purely from the short-distance physics, the latter was obtained from the minimization of the tree level vacuum energy (2)—a purely long-distance effect. This harmony between the short- and long-distance results follows from the assumptions about the short distance physics. Indeed, if there were some additional phases at short distances, the phase shifts $\delta_i$ would slide away from the CP-conserving points, and the relations in (16) would be modified accordingly.

The physical Goldstone bosons $G_{f,k,s}(x) \equiv M_{SUSY} \phi_{f,k,s}(x)$, are massive pseudo-scalars with masses

$$m_i^2 \sim M_{Pl}^2 m_{3/2}/M_{SUSY}^2, \ (i = k, s, f)$$

(17)

Therefore, particular short-distance sensitivity of the potentials for $\phi_{f,k,s}(x)$ require the pseudo-Goldstone bosons $G_{f,k,s}(x)$ to have masses right at the intermediate scale. These pseudo-Goldstone bosons have derivative couplings to visible matter suppressed by $1/M_{SUSY}$ so that they are invisible to experiments.

During the entire analysis the triscalar couplings in (2) are written without Yukawa couplings, for convention. If required, one can separate the Yukawa couplings by the replacement, $A_s \rightarrow h_s A_s$, $A_k \rightarrow k_s A_k$ and $A_f \rightarrow h_f A_f$. Then the VEV’s in (16) give

$$\langle \text{Arg}[m_{\lambda} A_f^*] \rangle = \langle \text{Arg}[m_{\lambda} A_k^*] \rangle = \langle \text{Arg}[m_{\lambda} A_s^*] \rangle + \pi,$$

(18)

and in agreement with this, the Yukawa phases on the right-hand side of the third equality in (10) are also cancelled. These last relations are independent of the Yukawa phases and they refer only to the phases of the triscalar couplings relative to the gaugino mass phase. Therefore, the relaxation process refers only to the dynamical phases and the non-dynamical phases of the Yukawa couplings decouple.

Independent of the conventions leading to (16) or (18), the phases appearing in the tree-level vacuum energy (2) relax to CP-conserving points. On the other hand, the marginal operators in the last line of (11) do not relax to CP-conserving points. Indeed, using (16), the phases of the operators $m_{\lambda} A_f A_k^* A_s^*$, $m_{\lambda} A_f A_k A_s^*$ and $m_{\lambda} A_f^* A_k^* A_s$ relax to $\langle \phi_k \rangle - \theta_s - \pi$, $\langle \phi_s \rangle + \theta_k$ and $\langle \phi_s \rangle + \theta_s + \pi$, respectively. Independent of the conventions, even if the dynamical triscalar phases relax (accidentally) to or near CP-conserving points, due to Yukawa pollution, these phases possess nonvanishing $(\text{Mod} = \pi)$ values. Therefore, these inherently long-distance operators potentially violate CP though they appear at higher loop orders compared to other marginals in Fig. 1.

That the minimization of the vacuum energy no longer implies the relaxation of the physical phases to CP-conserving points is important for the phenomenology of the CP-violating phenomena. From the discussions above it follows that an appropriate basis for the physical phases may be obtained by choosing the gaugino masses real and all
triscalar couplings complex. The phases of the triscalar couplings are subject to the relation (16) or (18) depending on the conventions adopted. Therefore, there is a single independent phase, say \( \langle \phi_f \rangle \), which can be arbitrarily away from the CP-conserving points. Also as mentioned before, this result is in contradiction with the MSSM where all CP violating phases relax to the CP conserving points. In this sense there is a finite source for CP violation in the NMSSM Lagrangian. This phase can cause various CP-violating phenomena such as the mixings and decays of the light mesons and Higgs bosons. Concerning the Higgs sector, unlike the predictions of the ordinary NMSSM where there is finite sources for CP violation at the tree-level Higgs potential, relaxation of the dynamical phases wash out the tree-level sources, allowing for CP violation only at the loop level.

IV. THE MSSM LIMIT

To clarify the meaning of the CP-violating relaxation points in (16) or (18) it may be convenient to consider the MSSM limit both algebraically and diagrammatically. This will also be a useful check of the NMSSM predictions above. It is known that neither at the tree level nor at the multi-loop level there exist any source for explicit CP violation in the MSSM [19]. The main difference between the MSSM and NMSSM follows from their symmetries and structure of the superpotentials. With the pure triscalar nature of the soft terms in (2), the scalar potentials of all pseudo-Goldstone bosons turn out to be controlled by the short-distance physics, in particular, there is no invariant that receives a potential only from the long-distance physics—an effect that occurs in the MSSM. Here one particularly notices that unless \( \langle \phi_k \rangle = 0 \) or \( \pi \), neither \( \langle \phi_s \rangle \) nor \( \langle \phi_f \rangle \) can come close to a CP-conserving point. In this sense, \( \langle \phi_k \rangle \) which has no correspondence in the MSSM limit, can be viewed as the source of finite CP violation in the NMSSM. In the conventions leading to (18), the MSSM limit is realized by the replacements

\[
\begin{align*}
    h_s S &\rightarrow \mu, & h_s A_s S &\rightarrow m^2_{12}, & k_s &\rightarrow 0,
\end{align*}
\]

where the MSSM parameters \( \mu \) and \( m^2_{12} \) are no longer dynamical fields like \( S \), instead they are background spurions that can appear only as mass insertions in the loop diagrams. It may be convenient to discuss the modifications in the topologies of the diagrams in Fig. 2 under the replacements (19). It is clear that diagrams (a), (c) and (e) vanish due to vanishing \( k_s \). However, as Fig. 3 shows explicitly, the three-loop diagrams (b) and (d) in Fig. 2 go over to two-loop diagrams \((\bar{b}), (\bar{d})\). On the other hand, the diagram (f) in Fig. 2 remains unaffected by the effective MSSM limit. A simple observation on Fig. 3 shows that \((\bar{b}), (\bar{d})\) and \((\bar{f})\) generate, respectively, the phases \( \text{Arg} [\mu A_j m^4_{12}^2] \equiv \phi_s - \phi_f \), \( \text{Arg} [m^4_{12}^2] \equiv \phi_s \) and \( \text{Arg} [m^4_{A^*}^2] \equiv \phi_f \) where each is expressed in terms of the physical phases in (12) for later use. It should be emphasized that these phases are precisely the ones appearing in the MSSM [19].

As is already seen from the modifications in the topologies of the diagrams in Fig. 2 and Fig. 3, the types of divergences of the diagrams change. Indeed, unlike (b) and (d) of Fig. 2, now \((\bar{b})\) and \((\bar{d})\) are only logarithmically divergent. This is, in fact, what is implied by the spurion character of the singlet field in the MSSM limit which generates the marginal operators \( \mu A_j m^4_{12}^2 \) and \( m^4_{A^*}^2 \) that are endowed with logarithmic divergences only. On the other hand, the MSSM limit does not alter the logarithmic structure of (13) apart from certain modifications in individual diagrams. Therefore, summing up the contributions of all diagrams, the radiative corrections to the vacuum energy takes the form

\[
14
\]
FIG. 3. The diagrams (b), (d) and (f) show, respectively, the form of the diagrams (b), (d) and (f) of Fig. 2 in the MSSM limit. Here the dot, triangle and square correspond to the insertions of the mass parameters, \(m_\lambda\), \(m^2\), and \(\mu\), respectively. The diagram (f) of Fig. 2 is not affected by the MSSM limit.

\[
(\Delta V)_{\text{MSSM}} = m^{4/3} \log \left( \frac{M^2_{\text{pl}}}{m^{2/3}} \right) \left[ c_s \cos(\phi_s + \delta_s) + c_{sf} \cos(\phi_s - \phi_f + \delta_{sf}) \right] \\
+ m^{2/3} M^2_{\text{pl}} c_f \cos(\phi_f + \delta_f)
\]

where in the conventions leading to (18) there is no Yukawa pollution in this limiting case. In this expression the weight factors and the phase shifts have the same meaning in (13) and (14), and the loop suppression factors are not factored out. Assuming again the relaxation of the phase shifts to the CP–conserving points in both short- and long-distance contributions, one finds that \(\langle \phi_f \rangle = 0 \) or \(\pi\) to an accuracy \(O(m^{2/3}/M^2_{\text{pl}})\). Then, necessarily \(\langle \phi_s \rangle = 0 \) or \(\pi\). Therefore, \(\phi_f \) (\(\phi_s\)) is determined solely by the short–distance (long–distance) dynamics. Moreover, both phases relax to CP–conserving points. As a result, in the MSSM limit (19) the phases in (16) and (18) go over the usual MSSM relaxation pattern leaving, however, \(\phi_k\) completely undetermined.

It is, in fact, the \(k_s \to 0\) limit that washes out any information about the fate of \(\phi_k\). Obviously, for \(k_s \equiv 0\), the NMSSM superpotential (1) possesses an additional \(U(1)_{\text{PQ}}\) symmetry, and thus, \(\langle \phi_k \rangle\) is nothing but the phase lifted by this global symmetry. Therefore, relaxation of the physical CP violating phases away from the CP–conserving points in (16) or (18) follows from the fact that this \(U(1)_{\text{PQ}}\) symmetry is broken by a dimensionless parameter \(k_s\) having no connection with the SUSY breaking mechanism.

V. CONCLUSIONS

The discussions and derivations in the text show that if one insists on a radiative induction of the \(\mu\) parameter, and postulates some spontaneously broken global symmetries in the SUSY breaking sector then

1. The soft terms possess dynamical phases so that the Lagrangian possesses a continuous, anomalous, global \(R\)–symmetry replacing the \(Z_3\) symmetry in the usual case
such that

- the strong anomaly of $U(1)_R$ symmetry relaxes the effective QCD vacuum angle to zero. This solves the strong CP problem.
- the corresponding pseudo-Goldstone boson acquires a mass $m_R \sim f_R m_\pi / M_{SUSY}$. This is the invisible axion whose mass can be well in the axion window in supergravity models.
- the QCD instanton effects can break $U(1)_R$ symmetry only to its $Z_3$ subgroup which is spontaneously broken together with the gauge symmetry. Thus, the domain wall problem persists in the form of axionic domain formation.

2. The nonvanishing CP violation effects in the tree level potential of the ordinary model relax to CP-conserving points when the pseudo-Goldstone bosons develop VEV’s. This relaxation property, however, implies by no means that the individual CP violating phases relax to CP conserving points. All CP-violating dynamical phases relax to CP-violating points that can be arbitrarily away (Mod $\pi$) from the CP-conserving points—a property not found in the MSSM.

3. Radiative corrections generate a strong sensitivity to short-distance physics due to the quadratic divergences of the non–marginal operators. There is no particular operator that gets a contribution only from the long distance physics. Therefore, the corresponding pseudo–Goldstone bosons have masses at the intermediate scale and their couplings to light fields are suppressed by $1/M_{SUSY}$.

4. Since the QCD vacuum angle has already relaxed, the CP-violating observables can receive contributions only from CKM phase and supersymmetric CP phases. There can be nonvanishing CP violation in the low energy Lagrangian even in the limit of vanishing Jarlskog invariant. The low energy constraints, that is, the upper bounds on the neutron and electron EDM’s, $\epsilon_K$, $\epsilon'_K/\epsilon_K$ and $\epsilon_B$ (that will be measured in near future) then imply heavy superpartner masses with appropriate flavor structures rather than vanishing of the SUSY CP phases.

5. In a non–GUT model like MSSM or NMSSM introduction of new flavor structures as in [31] may be an appealing way of avoiding the axionic domain walls. In coincidence with this requirement, even forgetting about EDM constraints, for pure supersymmetric CP violation to be able to saturate $\epsilon_K$ and $\epsilon'_K/\epsilon_K$ without violating $\text{BR}(B \to K^*\gamma)$, one surely needs new flavor structures beyond the minimal one [38]. These two seemingly unrelated problems, that is, the axionic domain walls and pure supersymmetric CP violation both require nonminimal flavor structures.

VI. ACKNOWLEDGEMENTS

The author is grateful to Goran Senjanović for numerous discussions and helpful comments about various aspects of this work. The author thanks to Charanjit Aulakh and Antonio Masiero for useful discussions and their careful reading of the manuscript.
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