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THE CHIRALLY SPLIT DIFFEOMORPHISM ANOMALY
AND THE $\mu$-HOLOMORPHIC PROJECTIVE CONNECTION

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Abstract

The relationship between the $\mu$-holomorphic projective connection and the action $\Gamma_{II}$ necessary to write down the chirally split diffeomorphism anomaly when it is shifted to the Weyl anomaly is given. Then, using the $\bar{\partial}$-Cauchy kernel on the complex plane to solve the $\mu$-holomorphic projective connection equation, we get the general expression for this type of projective connection. This enables us to compute the Green's functions contribution of the action $\Gamma_{II}$ to the shifting scheme.

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1 - Introduction

Since the mid-eighties, a large body of theoretical and mathematical literature has been devoted to the study of two-dimensional conformal field theories on Riemann surfaces without boundary [1,2]. These models are relevant in string theory and in the two-dimensional statistical system obeying certain boundary conditions at criticality. In particular, the dependence on the background geometry has been exploited to obtain effective actions for the two-dimensional quantum gravity. This has led to exciting developments in non-critical string theory [3,4]. Furthermore, most of the studies on the subject are concerned with lagrangian field theories on a two-dimensional Riemann manifold \((\Sigma, \mathbf{g})\) which are both Weyl and diffeomorphism invariant at the classical level [5-7].

At the quantum level, string theory exists in two entirely different versions: in the form of canonical quantization it appears as the representation theory of the algebras of Heisenberg, Virasoro, and Kac-Moody. In the second version, quantization is performed with the help of the Polyakov formalism which is geometric and can thus treat global objects [5]. In this formalism quantization involves functional integration over the matter field and on the zweibein. The first one is gaussian however; the second one is non-trivial and leads to two different settings depending on the gaugeslice of the zweibein we choose. In the conformal gauge obtained after transforming the zweibein by diffeomorphism and Weyl rescalings into a flat reference zweibein, the functional integration analysis leads to the Liouville theory [8]. This is a two-dimensional field theory of the scalar field of Weyl rescalings, called the Liouville mode. This represents the degree of freedom of the 2-dimensional gravity. The corresponding action yields, out off the critical dimension, a measure of the violation of the conformal symmetry at the quantum level.

One can, instead of using the conformal gauge, choose the light-cone or chiral gauge, which has a single non-vanishing metric mode, the Beltrami differential that represents the graviton, and recasts the theory in a local form by introducing the Wess-Zumino field defined by the Beltrami equation. In mathematical terms, this field is the projective coordinate that represents the isothermal (or projective) structure parametrized by the Beltrami differential. The resulting effective action is the Polyakov action that describes de Wess-Zumino field, and is invariant under reparametrizations, while its variation under Weyl rescalings produces the conformal anomaly. This gauge fixing scheme can be better understood if one makes use of conformal field theory on Riemann surfaces. A precious result that stems from the combination of algebraic geometry of Riemann surfaces and conformal field theory thereon, is that in critical dimensions, the quantum functional measure is basically the square modulus of a holomorphic function on moduli [9-11]. The holomorphic factorization consists in separating those correlation functions which depend on the projective coordinates from those which depend on their complex conjugates. But since these coordinates being solutions of the Beltrami equations [12-14], are holomorphic functionals of Beltrami differentials, the holomorphic factorization can be recast into separating functionals of the Beltrami differential and its complex conjugate \(\mu, \overline{\mu}\):

\[
\Gamma_{\mu}[\mu, \overline{\mu} ; R_0, \overline{R_0}]= \Gamma_{\mu}[\mu ; R_0] + \overline{\Gamma_{\mu}[\mu ; R_0]} \tag{1.1}
\]

where \(R_0\) is a background holomorphic projective connection in the conformal reference...
structure \( \{\bar{z}, z\} \) i.e. \( \partial R_0 = \frac{\partial R_0}{\partial \bar{z}} = 0 \); it is introduced so as to make the diffeomorphism anomaly well-defined; i.e. ensures the correct conformal covariance.

The chiral functional on the right-hand side of (1.1) depends holomorphically on the background conformal geometry which is parametrized by the pair \((\mu, R_0)\). This functional called the induced Polyakov action, serves as a "classical" action for the 2D quantum gravity in the light-cone gauge, i.e. \( ds^2 = (dz + \mu d\bar{z})dz \) and satisfies the chiral conformal "Ward identity:

\[
(\partial - \mu \partial - 2\partial \mu) \frac{\delta \Gamma}{\delta \mu} [\mu; R_0] = \frac{- k}{24\pi} (\partial^2 \mu + 2 R_0 \partial \mu + \partial R_0 \mu) \quad (1.2)
\]

and

\[\text{c.c.,}\]

expressing the anomalous breakdown of the diffeomorphism symmetry [12,15]. Here, \( k \) is the central charge of the chiral sector which measures the strength of the Virasoro algebra generated by the energy-momentum tensor of the original matter system. The anomaly; the right hand side of (1.2) represents the center-extension cocycle of the Virasoro algebra. The fact that this action depends only on the background conformal geometry suggests that the study of 2-dimensional conformal field theories on Riemann surfaces should rely on conformal geometry and thus a starting point for this study is to solve the conformal Ward identity (1.2). Accordingly, a unique solution on the complex plane was found by Polyakov in [16], the solution on the torus was constructed by Lazzarini and Stora in [12], and Zucchini has found the generalization of these solutions to a Riemann surface of higher genus [17]. In this case the solution is non-unique, since it is only up to addition of an arbitrary local holomorphic function due to the presence of zero modes.

Now we come to discussing the shifting of the Weyl anomaly to the chirally split diffeomorphism anomaly. Indeed, out of critical dimensions the Weyl anomaly can be changed to a diffeomorphism anomaly by extracting from the effective action a suitable local counterterm and this leads to exploit the holomorphic feature of the diffeomorphism anomaly which is expressed in (1.2). Local forms of this counterterm have been proposed in the literature [18,19]. However, the suitable form on an arbitrary compact Riemann surface without boundary was given by Knecht et al in [20]. They have found that three terms are involved and they have obtained, in the space of local functionals, the following equivalence equation for the s-cohomology:

\[
\Lambda (\Omega, g) + s [\Gamma_1 + \Gamma_2 + \Gamma_{\Pi} ] = \Lambda (C, \mu) + \overline{\Lambda (C, \mu)} ,
\]

where \( \Lambda (\Omega, g) \) is the Weyl anomaly which is a functional of the Weyl ghost \( \Omega \) and of the metric \( g \) on a Riemann surface \( \Sigma \), \( s \) is the BRST operator associated with the diffeomorphism group [21]. \( \Lambda (C, \mu) + \overline{\Lambda (C, \mu)} \) is the chirally split diffeomorphism anomaly which depends on the vector field \( C^x = c^x + \mu^x e^z \) (the combination of the diffeomorphism ghost \((c^x,c^z)\)) and on the Beltrami differential \( \mu \). \( \Gamma_1 \) is the Liouville action written in terms of a (1,1)-conformal field in a background which is designed to absorb the Weyl anomaly. \( \Gamma_{\Pi} \) is the
second term that requires the projective connection necessary to write down the chirally split form of the diffeomorphism anomaly. \( \Gamma_{III} \) is the third term that completes the elimination of the background to the benefit of the conformal class of metrics. The action \( \Gamma_{II} \) that we are interested in is given by:

\[
\Gamma_{II} = \frac{k}{12\pi} \left( \frac{d\tau \wedge dz}{2i} \right) \mu \zeta (R - R_0) + \text{c.c.},
\]

(1.4)

where \( R_0 \) is a holomorphic projective connection and \( R \) is a smooth (not holomorphic) projective connection. Indeed, we will give an explicit expression for the smooth projective connection that verifies the \( \mu \)-holomorphic condition [17]:

\[
\partial R = \partial \mu + 2R \partial \mu + \mu \partial R,
\]

(1.5)

Then, we get the iterative solution of (1.5) by using the techniques of the Cauchy kernel [12,13] on the complex plane.

The plan of this paper is as follows. In sect. II we shall review the basic results of the conformal and the projective geometry and we will clarify the origin of eq. (1.5). In sect. III we will give the general iterative solution of (1.5) on the complex plane and the basic ingredients to get such solution on the torus. Sect.IV is devoted to our conclusion and suggestions.

II - Conformal and projective structures on Riemann surface \( \Sigma \)

II-1 Riemann surface

A Riemann surface \( \Sigma \) of genus \( g \geq 0 \) is a connected topological manifold of real dimension 2 which is equipped with a complex structure: any two local systems of coordinates \((z, \bar{z})\) and \((z', \bar{z}')\) are related by a conformal (biholomorphic) transformation:

\[
z' = z'(z), \quad \bar{z}' = \bar{z}(\bar{z}). \tag{II.1}
\]

The natural tangent space basis is \( \partial = \partial / \partial z \), \( \bar{\partial} = \partial / \partial \bar{z} \).

A reference conformal structure, say \( P_0 \) on the surface \( \Sigma \) is a maximal atlas of local coordinates with holomorphic coordinates change (eq. (II.1)) whose generic coordinate is \( z \).

Let

\[
\Pi : \Sigma_{II} \rightarrow \Sigma \tag{II.2}
\]

be a universal covering of \( \Sigma \). As is well-known, \( \Sigma_{II} \) is a simply connected differentiable surface and \( \Pi \) is a local diffeomorphism.

Let \( K \) be the \( P_0 \)-holomorphic cotangent line bundle of \( \Sigma \) and \( \ell \) a \( P_0 \)-holomorphic line bundle on \( \Sigma \) such that \( \ell^{\otimes 2} = K \). Let \( O \) be a non-empty open subset of \( \Sigma \) and \( p, q \) half-integer numbers.

A conformal field \( \Psi \) of weights \( p, q \) on \( O \) is a smooth section of \( \ell^{\otimes 2p} \otimes \mathbb{C}^{\otimes 2q} \) on \( \Pi^{-1}(O) \). The
set of all such fields is an infinite dimensional complex vector space that we denote by $C_r^0(O)$.

II. 2 Beltrami differential

A Beltrami differential $\mu$ on $\Sigma$ is an element of $C^{-1,1}(\Sigma)$ satisfying the boundedness condition $\sup_{\Sigma} | \mu | < 1$. Then, they are differentials of the form $\mu = \mu_z \, dz \, \partial$ and they can be integrated versus quadratic differentials $\phi = \phi_z \, (dz) \partial$:

$$\langle \mu / \phi \rangle = \int_{\Sigma} \frac{d\tau \wedge dz}{z} \mu_z \phi_z.$$  \hspace{1cm} (II.3)

The set $Beltr(\Sigma)$ of Beltrami differentials on $\Sigma$ has a structure of topologically contractible complex Banach manifold [22]. $Beltr(\Sigma)$ parametrizes the set of all conformal structures on $\Sigma$. Indeed, to any Beltrami differential $\mu$ in $Beltr(\Sigma)$, there is associated a conformal structure $P_\mu$ on $\Sigma$ whose generic coordinate $Z$ is a local solution of the Beltrami equation

$$\overline{\partial} - \overline{\mu \partial} Z = 0,$$ \hspace{1cm} (II.4)

which expresses the criterion for conformality of the diffeomorphism

$$(z, \overline{z}) \rightarrow (\overline{Z}(z, \overline{z}), \overline{Z}(z, \overline{z})).$$ \hspace{1cm} (II.5)

However, the jacobian of this transformation being positive implies the following local invertibility condition

$$|\partial Z| \Delta - |\overline{\partial} Z| \phi 0,$$ \hspace{1cm} (II.6)

which is precisely the requirement that, locally $|\mu(z, \overline{z})| < 1$. Conversely $P_\mu$ determines $\mu$ through the local relation

$$\mu = \overline{\partial} Z \overline{\phi Z}$$ \hspace{1cm} (II.7)

where $Z$ is a generic coordinate of $P_\mu$. In such parametrization, we have the following identities

$$dz = \partial Z (dz + \mu \partial \overline{z})$$ \hspace{1cm} (II.8)

$$\partial \overline{z} = \frac{1}{\partial \overline{Z} (1 - \overline{\mu})} (\overline{\partial} - \mu \partial).$$ \hspace{1cm} (II.9)

In particular, one can verify that $P_0$; the reference structure corresponds to the vanishing Beltrami differential.
II. 3 Projective structure

Besides the complex structure on $\Sigma$, there is a projective structure on $\Sigma$ which is related to a projective connection $R$ that is an assignment to any coordinate $z$ of $P_0$ of a smooth function $R$ defined in the domain of $z$ with the following properties. On the overlapping domains of $z$ and $z'$, one has

$$R' = (\partial' z) \partial R - S(z', z), \quad (\text{II.} 10)$$

where

$$S(z'; z) = \partial \Delta_{z} \partial z' - \frac{1}{2} \partial \ln \partial z'$$

is the Schwarzian derivative. Further, there exists a Beltrami differential $\mu$ in $\text{Beltr} (\Sigma)$ such that $R$ is $\mu$-holomorphic, that is

$$(\partial - \mu \partial - 2 \partial \mu) R = \partial^3 \mu. \quad (\text{II.} 12)$$

To any element $R$ of the space of all projective connections satisfying eq.(II.12) there is canonically associated a projective structure $P(\mu, R)$ subordinated to $P_0$. By definition, this is a maximal coordinate covering contained in $P_0$ whose transition functions are restrictions of elements of the Möbius group $\text{PSL}(2, \mathbb{C})$ [23]. $P(\mu, R)$ determines $\mu$ and $R$ through (II.7) and the relation

$$R = S(z'; z), \quad (\text{II.} 13)$$

where $Z$ is a generic coordinate of $P(\mu, R)$.

Let us consider the operator

$$C = c^2 \partial + c^\xi \partial^\xi \quad (\text{II.} 14)$$

which is associated to the generating diffeomorphism operator $(\xi, \partial) = \xi^z \partial + \xi^\xi \partial^\xi$. One can verify that, under the mapping (II.5) the components $c^i$ and $c^z$ transform into $C^z$ and $C^{\xi}$ given by

$$C^z(z, \overline{z}) = \lambda [c^z(z, \overline{z}) + \mu_z^z(z, \overline{z}) c^\xi(z, \overline{z})] \quad (\text{II.} 15)$$

and

$$c.c. \quad (\text{II.} 15)'$$

where $\lambda \equiv \partial \mathcal{Z}$. We recognize that the field $c^i + \mu^z_z c^\xi$ appearing in (II.15) is the ghost field on which depends the diffeomorphism anomaly; the first term of the right hand side of (I.3). Moreover, we get from the action of $\partial_{\mathcal{Z}}$ on $C^z$ the relation

5
\[ \partial_x C^x = \frac{\lambda}{\bar{\lambda}} s \mu, \]  

(II.16)

where \( s \mu = (\bar{\partial} - \mu \partial + \partial \mu)(c^x + \mu c^x) \) is the BRST transformation of \( \mu \) [20], which is equivalent to \( s(\mu \lambda) = \bar{\partial} C^Z \) with \( s\lambda = \partial C^Z \). Furthermore, it is easy to get

\[ \partial_x C^x = \partial_x \hat{C}^x + \frac{\lambda}{\bar{\lambda}(1 - \mu \bar{\mu})}(\bar{\partial} - \mu \partial + \partial \mu)(\mu c^x) \]  

(II.17)

where \( \hat{C}^x \) is the holomorphic field introduced in [24]. However, the conformally covariant operator \( \partial_x^3 + 2 R_{zz} \partial_x + \partial_x R_{zz} \), with \( R_{zz} \) a holomorphic projective connection in the \((Z, \bar{Z})\) coordinates, acting on \( \hat{C}^x(Z, \bar{Z}) \) does not preserve its form under (II.5). To overcome this problem we assume that \( \hat{C}^x \) is locally holomorphic that is;

\[ \partial_x \hat{C} = 0 \]  

(II.18)

and then we get the following holomorphic property of the field \( c^x \):

\[ (\bar{\partial} - \mu \partial + \partial \mu)c^x = 0. \]  

(II.19)

Moreover, we have the covariant transformation law

\[ (\partial_x^3 + 2 R_{zz} \partial_x + \partial_x R_{zz}) \hat{C}^x = \lambda^2 (\partial^3 + 2 R \partial + \partial R)c^x, \]  

(II.20)

where \( R \) is a smooth projective connection defined by

\[ R_{zz}(Z) = \lambda^{-2}(R - S(Z; z)). \]  

(II.21)

The holomorphy condition \( \partial_x R_{zz}(Z) = 0 \) implies that

\[ (\bar{\partial} - \mu \partial - 2 \partial \mu) R = (\bar{\partial} - \mu \partial - 2 \partial \mu) S(Z, z). \]  

(II.22)

However, it is easy to verify from the Schwarzian expression given in (II.11) that

\[ (\bar{\partial} - \mu \partial - 2 \partial \mu) S(Z, z) = \partial^3 \mu \]  

(II.23)

and hence

\[ \bar{\partial} R = (\mu \partial + 2 \partial \mu) R + \partial^3 \mu \]  

(II.24)

which is the equation (I.5) introduced above. Thus, the smooth projective connection that appears in the term \( \Gamma_\mu \) takes its origin from the covariantization of the Bol's operator \( L_\mu = \partial^3 + 2 R \partial + \partial R \) by assuming the holomorphy of the conformal field \( \hat{C}^x \). On the other hand one can get form the expression of the action \( \Gamma_\mu \) the equations
\[ \frac{\delta \Gamma}{\delta \mu} = \frac{k}{12\pi} (R - R_0) \]  

(II.25)

and

\[ \left( \partial - \mu \partial - 2\partial \mu \right) \frac{\delta \Gamma}{\delta \mu} = \frac{k}{12\pi} \left[ (\partial - \mu \partial - 2\partial \mu)R + \mu \partial R + 2\partial \mu R_0 \right]. \]  

(II.26)

Then, by putting eq (II.24) in (II.26) we get

\[ \left( \partial - \mu \partial - 2\partial \mu \right) \frac{\delta \Gamma}{\delta \mu} = \frac{k}{12\pi} \left( \partial \mu + \mu \partial R + 2\partial \mu R_0 \right) \]  

(II.27)

which is the Ward identity (I.2)

III. The solution of (II.24) on the complex plane

Here, we develop the solution of eq.(II.24) as a perturbative solution in terms of the Beltrami differential by using the \( \tilde{\partial} \)-Cauchy kernel on the complex plane. This is given by [12]

\[ \tilde{\partial} \left( \frac{1}{z-w} \right) = -\pi \delta \Delta (\xi - w), \]  

(III.1)

where \( \delta \Delta \) is the delta function in two dimensions, such that for any complex function \( F \) we have the following relation

\[ \left( \tilde{\partial}^{-1} F \right) (z) = \int \frac{dw \wedge d\bar{w}}{2i\pi} \frac{F(w, \bar{w})}{z-w}. \]  

(III.2)

Now, we can rewrite (II.24) in the form

\[ R(z, \bar{z}) = \tilde{\partial}^{-1} (\partial^3 \mu + 2R \partial \mu + \mu \partial R) \]  

(III.3)

and then, with the help of (III.2) we get

\[ R_0(z, \bar{z}) = \int dm_i \frac{\partial^3 \mu_i + 2R \partial_i \mu_i + \mu_i \partial_i R_i}{z_{i01}}, \]  

(III.4)

where we have using the notation

\[ dm_i = \frac{dz_i \wedge d\bar{z}_i}{2i\pi}, \quad \mu_i = \mu(z_i, \bar{z}_i) \]

\[ \partial_i = \frac{\partial}{\partial z_i}, \quad R_i = R(z_i, \bar{z}_i) \]  

(III.5)

\[ z_{ij} = z_i - z \quad \text{with} \quad z_0 = z, \]
and the generic \( R_i \) is given by

\[
R_i = \int_{\mathcal{G}} \frac{\partial^3_{i+1} \mu_{i+1}}{Z_{i+1}} + \int_{\mathcal{G}} \frac{A_{i+1}}{Z_{i+1}} R_{i+1} \quad \text{(III.6)}
\]

with

\[
A_i = \left( \partial_i - \frac{1}{Z_{i-1}} \right) \mu_i, \quad A_0 = 1. \quad \text{(III.7)}
\]

Then, we get the projective connection that satisfies (11.24) as the sum of the formal series

\[
\sum_{N=1} W_N,
\]

where

\[
W_N = -3! \sum_{i=1}^{N} \left( \frac{\partial \mu_{i-1}}{Z_{i-1}} \right) \frac{\mu_N}{Z_{N-1}^{N-1}}. \quad \text{(III.8)}
\]

So, we have

\[
R(z, \zeta) = -3! \sum_{N=1}^{\infty} \prod_{i=1}^{N} \left( \frac{\partial \mu_{i-1}}{Z_{i-1}} \right) \frac{\mu_N}{Z_{N-1}^{N-1}}. \quad \text{(III.9)}
\]

However, the term \( W_N \) must be of the form

\[
W_N = -3! \int_{\mathcal{G}} \frac{1}{Z_{i+1}^2} \mu_1 \mu_2 ... \mu_N \quad \text{(III.10)}
\]

and then we were not be able to give explicitly the projective connection \( R(z, \zeta) \) as a functional of the Beltrami differential. However, we get the first three terms

\[
W_1 = -3! \int_{\mathcal{G}} \frac{1}{Z_{01}} \mu_1, \quad W_2 = -2.3! \int_{\mathcal{G}} \frac{1}{Z_{01}} \frac{2}{Z_{12}} \left( \frac{1}{Z_{12}} - \frac{1}{Z_{01}} \right) \mu_1 \mu_2, \quad W_3 = -4! \int_{\mathcal{G}} \frac{1}{Z_{01} Z_{12}^2} \left[ \frac{1}{Z_{12}} \left( \frac{1}{Z_{23}} \right) \left( \frac{1}{Z_{12}} - \frac{1}{Z_{23}} \right) + \frac{1}{Z_{01}} \left( \frac{1}{Z_{12}} - \frac{1}{Z_{23}} \right) \right] \mu_1 \mu_2 \mu_3 \quad \text{(III.11)}
\]

where

\[
\text{dm}_{123} = \prod_{k=0}^{i} \text{dm}_k.
\]

Moreover, we get explicitly the term contained in the sum \( \sum_{K=1}^{N} W_K \) that does not introduce the partial derivatives:
Now, let us give the contribution to the operator product expansion (OPE) of the model induced from the action \( \Gamma_u \). Indeed, by inserting eqs. (III.12) in (I.4) we get the action \( \Gamma_u \) at the third order of the perturbative series in \( \mu \).

\[
\Gamma_u = -k \left\{ \int \frac{dm_{01}}{z_{01}} \mu_0 |\mu_1 + 2 \int \frac{dm_{012}}{z_{01} z_{12}} \left( \frac{2}{z_{12}} - \frac{1}{z_{01}} \right) \mu_0 \mu_1 \mu_2 \right. \\
\left. -4 \int \frac{dm_{0123}}{z_{01} z_{12} z_{23}} \left[ \frac{1}{z_{01}} \left( \frac{2}{z_{23}} - \frac{1}{z_{12}} \right) + \frac{1}{z_{12}} \left( \frac{1}{z_{23}} - \frac{1}{z_{12}} \right) \right] \mu_0 \mu_1 \mu_2 \mu_3 + \ldots \right\} \quad (\text{III.14}).
\]

where

\[
\mu_0 = \mu(z), \quad dm_0 = \frac{dz \wedge dz}{2i\pi}.
\]

Then, the energy-momentum tensor is given by

\[
T_0 = \frac{\delta \Gamma_u}{\delta \mu_0} \bigg|_{\mu_0 = 0} = -\frac{k}{12} R_0. \quad (\text{III.15})
\]

The two-point functions read

\[
\langle T_0, T_0 \rangle = \frac{\delta \Delta \Gamma_u}{\delta \mu_0 \delta \mu_1} \bigg|_{\mu_0 = 0} = -\frac{k}{2} \frac{1}{f^4}. \quad (\text{III.16})
\]

Likewise, the three-point functions are

\[
\langle T_0, T_1, T_2 \rangle = \frac{\delta^3 \Gamma_u}{\delta \mu_0 \delta \mu_1 \delta \mu_2} \bigg|_{\mu_0 = 0} = -\frac{k}{2} \frac{2}{(f^3)^2}. \quad (\text{III.17})
\]

where we have considered \( z_{ij} = f \) as a constant for \( |i-j|=1 \). So, the contribution of the \( \Gamma_u \) Green’s functions can be ignored as we go to higher orders of the perturbative series and, then the important contribution for \( \Gamma_u \) is given by the holomorphic projective connection.
IV Conclusion and open problems

We have established that the action $\Gamma_n$, that is necessary to write down the chirally split diffeomorphism anomaly and then to shift this later to the Weyl anomaly, is expressed in terms of the $\mu$-holomorphic projective connection, i.e. that satisfies eq.(II.24). This type of projective connection is related to the covariantization of the $L_3$ Bol's operator. Then, by using the $\xi$-Cauchy kernel on the complex plane, we have given the general expression of this $\mu$-holomorphic projective connection. This enables us to compute the two and the three points functions from the action $\Gamma_n$ and to conclude that the important contribution of the action $\Gamma_n$ comes from the holomorphic projective connection.

The analogues of our results can be found on the torus by using the Weistrass quasielliptic function as the $\xi$-Cauchy kernel on the torus:

$$\xi(z, \bar{w}) = \frac{1}{\pi} \delta(z, w)$$

with

$$(\xi^{-1} F)(z) = \oint_{C} dz \xi(z, w) F(w, \bar{w})$$

for any complex function $F$ on the two-dimensional torus. Moreover, the generalization to any Riemann surface of genus $g \geq 2$ and to the supersymmetric case can be done by using the results introduced in [12,13,25,26].

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