ON NEUTRINO-MIXING-GENERATED LEPTON ASYMMETRY
AND THE PRIMORDIAL HELIUM-4 ABUNDANCE

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Abstract

In this article we discuss lepton asymmetry effect on BBN with neutrino oscillations. We argue that asymmetry much smaller than 0.01, although not big enough to influence directly the nucleosynthesis kinetics, can effect considerably BBN indirectly via neutrino oscillations. Namely, it distorts neutrino spectrum and changes neutrino density evolution and the pattern of oscillations (either suppressing or enhancing them), which in turn effect the primordial synthesis of elements. We show that the results of the paper X. Shi et al., Phys. Rev. D 60, 063002 (1999), based on the assumption that only $L > 0.01$ will influence helium-4 production, are not valid. Instead, the precise constraints on neutrino mixing parameters from BBN are presented.

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There exists an interesting interplay between lepton asymmetry and neutrino oscillations in the early Universe. As it was noticed in [?, ?, ?, ?] neutrino oscillations can generate lepton asymmetry, besides their well known ability to erase it [?, ?, ?]. On the other hand, lepton asymmetry (no matter if neutrino-mixing generated or pre-existing one) can suppress neutrino oscillations [?, ?] and has also the remarkable ability to enhance them [?]. Consequently, in the presence of neutrino oscillations, lepton asymmetry exerts much complex influence on Big Bang Nucleosynthesis (BBN) via oscillations, than in the simple case without oscillations.

In this work we will discuss the indirect effect of lepton asymmetry on primordial nucleosynthesis via neutrino oscillations. This paper is provoked by the publication “Neutrino-Mixing-Generated Lepton Asymmetry and the Primordial He-4 Abundance” by X. Shi, G. Fuller, and K. Abazajian, published in Phys. Rev. D 60, 063002 (1999) ref. [?] (hereafter SFA). As we understood from their paper and some other recent publications [?, ?, ?] there exists some shallow understanding of the role of the lepton asymmetry in BBN with oscillations. And we would like on the first place to clarify this subject.

In SFA the study of the lepton asymmetry effect on BBN is based on the assumption that only asymmetry bigger than 0.01 at the freeze-out of the $n - p$ transitions may have an appreciable impact on the primordial abundance of helium-4 $Y_p$. Hence, the authors estimate the effect of the asymmetry on BBN after it has been enhanced up to 0.01. Such a consideration is certainly valid for the simple case of nucleosynthesis without oscillations! There are exhaustive studies on that subject [?], which results, concerning the neutrino degeneracy effect on nucleosynthesis, the authors of SFA reproduce in general.

However, in the case of nucleosynthesis with oscillations the assumption that only asymmetry bigger than 0.01 effects nucleosynthesis, is no longer valid. It was first noticed in the original works [?, ?], that in the case of BBN with neutrino oscillations even very small lepton asymmetries $L << 0.01$ (either initially present [?], or dynamically ‘neutrino-mixing’ generated [?]), although not big enough to influence nucleosynthesis directly, may considerably effect BBN indirectly through oscillations. In these works a very precise account of the evolution of the neutrino and antineutrino distribution functions and their spectral distortions, and the evolution of the asymmetry was provided in the BBN calculations.\(^2\)

In the present work we calculate the net effect of small lepton asymmetries $L << 0.01$ on BBN and obtain precise cosmological constraints on neutrino mixing parameters.

In the presence of oscillations, lepton asymmetry affects BBN indirectly through its feedback effect on:

1. the evolution of the neutrino and antineutrino number densities [?, ?], which play an essential role in the kinetics of nucleons at $n/p$-freeze-out;

2. the neutrino and antineutrino spectrum distortion [?, ?], which is important for the correct

\(^2\)We are really sorry that the authors of SFA had to rediscover the importance of this account, but we cannot agree neither that they were the first to provide the account, nor that they provided this account accurately.
calculation of the neutrino number densities and weak interaction rates in \( n - p \) transitions (see the following eq. (2));

(3) the neutrino oscillation pattern. Namely, \( L \) may suppress or enhance oscillations, leading, correspondingly, to underproduction or overproduction of primordial helium-4 in comparison with the case without asymmetry account. The suppression may be strong enough to allow a substantial alleviation of the nucleosynthesis bounds on the neutrino mixing parameters. The effect on BBN of a suppression due to a \textit{relic} neutrino asymmetry was first discussed in [?] and calculated in detail taking into account (1) and (2) in [?]. While the suppression due to \textit{neutrino-mixing generated} asymmetry and its effect on BBN was first calculated in [?]. It was recently shown [?] also that lepton asymmetry is capable of enhancing the oscillations and thus strengthening the BBN bounds on the neutrino oscillation parameters.

These three effects are typical for the case of BBN \textit{with oscillations}.

(4) In case when \( L \) is [?] or grows [?] big enough > 0.01, it can also influence directly the kinetics of the \( n - p \) transitions, depending on the sign of \( L \).

It is essential that in the presence of oscillations lepton asymmetry has a more complex influence on BBN (1)-(4) than in the simple case without oscillations. The correct study should follow selfconsistently the evolution of the neutrino ensembles, the evolution of the lepton asymmetry as well as the evolution of the neutron and proton number densities. So that the complete effect of the asymmetry throughout its evolution (growth or damping) during the nucleosynthesis epoch could be registered. Such an exact study was provided for small neutrino mass differences \( \delta m^2 \leq 10^{-7} \text{ eV}^2 \) for the resonant case in [?] and in the nonresonant case in [?, ?].

In what follows we present the results of a precise investigation of the asymmetry effect on BBN via neutrino oscillations and provide a comparison with an artificial case without the account of asymmetry in order to extract the net effect of the asymmetry on BBN. Finally, we obtain accurate cosmological constraints on the oscillation parameters.

We discuss the case of active-sterile neutrino oscillations assuming mixing present just in the electron sector \( \nu_i = U_{i\ell} \nu_\ell \ (\ell = e, s) \), following the line of work in ref. [?]. The set of kinetic equations describing simultaneously the evolution of the neutrino and antineutrino density matrix \( \rho \) and \( \bar{\rho} \) and the evolution of the neutron number density \( n_n \) in momentum space reads:

\[
\frac{\partial \rho(t)}{\partial t} = H_{\rho \nu} \frac{\partial \rho(t)}{\partial p_{\rho \nu}} + i [H_{\rho \nu}, \rho(t)] + i \sqrt{2} G_F \left( \pm \mathcal{L} - Q/M_W^2 \right) N_{\alpha} \left( \alpha, \rho(t) \right) + O(G_F^2),
\]

\[
(\partial n_n/\partial t) = H_{\rho \nu} \left( \partial n_n/\partial p_n \right) +
\int d\Omega(\epsilon^-, p, \nu) \left| \mathcal{A}(\epsilon^-p \to \nu n) \right|^2 \left[ n_{e^-} - n_p (1 - \rho_{LL}) - n_n \rho_{LL} (1 - n_{e^-}) \right]
- \int d\Omega(\epsilon^+, p, \bar{\nu}) \left| \mathcal{A}(\epsilon^+ n \to p \bar{\nu}) \right|^2 \left[ n_{e^+} - n_n (1 - \bar{\rho}_{LL}) - n_p \bar{\rho}_{LL} (1 - n_{e^+}) \right].
\]
\[ \alpha_{ij} = U^*_{ie} U_{je}, \]  
where \( \alpha_{ij} \) is the momentum of electron neutrino, \( n \) stands for the number density of the interacting particles, \( d\Omega_{ij} \) is a phase space factor and \( A \) is the amplitude of the corresponding process. The plus sign in front of \( L \) corresponds to neutrino ensemble, while minus sign corresponds to antineutrino ensemble. Actually, we solve nine equations selfconsistently: four equations for the components of the neutrino density matrix, another four for the antineutrino density matrix following from eq. (1), and one for the neutron number density eq. (2).

The first term on the right hand side of equations (1) and (2) describes the effect of Universe expansion. The second term in (1) is responsible for neutrino oscillations, the third accounts for forward neutrino scattering off the medium and the last one accounts for second order interaction effects of neutrinos with the medium. \( H_0 \) is the free neutrino Hamiltonian. \( L \) is proportional to the fermion asymmetry of the plasma and is essentially expressed through the neutrino asymmetries \( L \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}, \) where \( L_{\mu, \tau} \sim (N_{\mu, \tau} - N_{\mu, \tau})/N_\gamma \) and \( L_{\nu_e} \sim \int d^3p (\rho_{LL} - \bar{\rho}_{LL})/N_\gamma. \) The ‘nonlocal’ term \( Q \) arises as a \( W/Z \) propagator effect, \( Q \sim E_\nu T. \) It is important for the nonequilibrium active-sterile neutrino oscillations to provide a simultaneous account of the different competing processes, namely: neutrino oscillations, Hubble expansion and weak interaction processes.

Neutrino and antineutrino ensembles evolve differently as far as the background is not \( CP \) symmetric. Besides, the evolution of neutrino and antineutrino ensembles may become strongly coupled due to the growing electron asymmetry term and hence, the evolution of \( \rho \) and \( \bar{\rho} \) must be considered simultaneously.

Moreover, it is extremely important for the correct account of the role of the asymmetry on BBN to study the asymmetry evolution and the neutron number density evolution in \( p \)-space selfconsistently with the evolution of neutrino and antineutrino ensembles involved in oscillations! This looks obvious as far as there exists asymmetry-oscillations interplay – oscillations change neutrino-antineutrino asymmetry and it in turn affects oscillations, and, besides, neutrino \( \rho_{LL} \) and antineutrino \( \rho_{LL} \) number densities enter the kinetic equations for nucleons. However, usually in many papers the growth of asymmetry is calculated, and then, when it has reached values around 0.01, its influence on BBN kinetics is estimated. Thus, the asymmetry influence (1)-(3) on BBN during its growth till 0.01 cannot be caught. We will demonstrate in this work that this very influence may give up to 10% relative change in primordial helium-4. Therefore, the indirect influence of lepton asymmetry on BBN should be carefully accounted for during asymmetry’s full evolution.

It is essential also, that the equations should follow neutrino evolution in momentum space, i.e. enabling to account precisely for the distortion of the neutrino spectrum due to oscillations and asymmetry. This approach was demonstrated \[ ?, ? \] in detail for the case of small mass differences and it helped to precise the constraints on the neutrino squared mass differences \( \delta m^2 \) by almost an order of magnitude in comparison with the previous studies (see fig. 8 from \[ ? \]). Working with mean energies and equilibrium spectrum is tempting of course due to the simplicity
of the analysis, however, is not correct. We have stressed in our previous works the importance of the proper account of the spectrum distortion and asymmetry for the BBN with active-sterile oscillations and we have provided this account in \[?, ?, ?\]. Besides, many papers have discussed separately the questions of the dynamical evolution of the asymmetry (see for example \[?, ?, ?, ??\]) or of the correct account of the spectrum distortion for nonequilibrium neutrinos \[?, ?\].

It is really not an easy task to solve exactly the system of eqs. (1)-(2). Especially, in the case of a rapid asymmetry growth more than 1000 bins may be required for the accurate description of the neutrino spectrum, however, this is the correct way to study this topic. We have described the spectrum using in general 1000 bins, and sometimes for the resonant case up to 5000 bins. The equations were integrated for the characteristic period from the electron neutrino decoupling till the $n/p$ freeze-out at 0.3 MeV. We have calculated the value of the primordially produced helium-4 with neutrino oscillations for the full range of the model’s parameters values, namely for $\sin^2(2\theta)$ ranging from $10^{-3}$ to maximal mixing and $\delta m^2 \leq 10^{-7}$ eV$^2$. For smaller mixing parameters the effect on helium-4 is negligible \[?\]. The exact feedback effect of the asymmetry on the neutrino ensembles evolution, neutrino spectrum distortion and neutrino oscillations, was numerically followed. Hence, the total effect of the asymmetry on BBN, indirect via its interplay with oscillations and direct on the kinetics of $n - p$ transitions, was obtained numerically.

In fig. 1 the impact in helium-4 due to oscillations and asymmetry is presented as a function of the neutrino square mass differences. For comparison the curve corresponding to the artificial case without the account of the asymmetry is presented also. The difference between the two curves measures the net asymmetry effect on BBN with oscillations. It is obvious, that for the range of oscillation parameters discussed, the total effect of the asymmetry is a reduction in $Y_p$ in comparison with the case without asymmetry account. This reduction can be as big as 10%, which is considerable on the background of our recent knowledge from primordial helium measurements \[?\]. As it is obvious from the figure, small $\delta m^2$ are also constrained from BBN considerations. The obtained constraints on $\delta m^2$ are by several orders of magnitude more severe than the constraints obtained in SFA (see fig.4 there\[4\]) on the basis only of the kinetic effect of the asymmetry.

In fig. 2 we present a comparison of the iso-helium-4 contours, $Y_p = 0.245$, for the resonant case, obtained without the account of the asymmetry, with the contours obtained with the account of the asymmetry. The area to the left of the curves is the allowed region of the oscillation parameters.

The numerical analysis showed that in the case of small mass differences we discuss and

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\[3\]Therefore, it is quite amazing now in 1999 to see published statement in (SFA) about the existence in literature only of “BBN calculations based on a constant asymmetry and a thermal neutrino spectrum” “overly simplistic” and with “inaccurate results”. It is easy to judge that this same paper (SFA) may be considered overly simplistic comparing the calculated in it “semianalytically” distorted spectrum distributions with the precisely calculated spectra presented in \[?\].

\[4\]or the same figure reproduced in another publication of the same authors, namely fig.3 in the first reference in \[?\].
naturally small initial asymmetry, the growth of the asymmetry is less than 4 orders of magnitude. Hence, beginning with asymmetries of the order of the baryon one, the asymmetry does not grow enough to influence directly $n - p$ transitions. Consequently, the apparently great asymmetry effect (as seen from the curves) is totally due to the indirect effects (1-3) of the asymmetry on BBN. The maximal asymmetry effect is around 10% 'underproduction' of $Y_p$ in comparison with the case of BBN with oscillations but without the asymmetry account. The total effect of oscillations, with the complete account of the asymmetry effects, is still an overproduction of helium-4, although considerably smaller than in the calculations neglecting asymmetry. Therefore, nucleosynthesis constraints on the mixing parameters of neutrino are alleviated considerably due to the asymmetry effect.

The case of nonresonant active-sterile oscillations was already discussed [?] and investigated in detail in [?]. It was shown that the effect of the asymmetry on BBN with oscillations, in case it was initially of the order of the baryon one, is negligible. However, in case it was initially bigger than $10^{-7}$, it may also have a crucial effect on BBN through its effect on oscillations [?]. In the last work a complete exact numerical study of the asymmetry effect on BBN with oscillations was provided for a wide range of initial asymmetry values ($10^{-10} - 10^{-2}$). In fig. 4 of the original paper [?] the iso-helium contours for the case with pre-existing asymmetry ($L = 10^{-6}$) and the case without asymmetry effect (dashed curves) were presented. It is obvious that in the discussed nonresonant case the strong asymmetry effect again is due to its indirect influence on nucleosynthesis. However, it is not so straightforward. For the nonresonant case the asymmetry account reflects into alleviating BBN constraints on mixing parameters for big $\theta$ due to the suppression of oscillations but strengthening the constraints for small $\theta$ due to the enhancement of oscillations. For more details see the original paper [?].

We would like also to stress, that in the nonresonant case of small mass differences oscillations, due to the complex interplay between oscillations and asymmetry, antineutrinos and neutrinos undergo resonance almost simultaneously. This is easy to grasp as far as the asymmetry has a fast oscillating sign-changing behavior due to which both the neutrino and antineutrino ensembles are able to experience resonance. Consequently, the effect on helium-4 does not depend on the initial sign of $L$, in case the asymmetry is small enough not to have a direct kinetic effect on $n - p$ transitions, i.e. $L << 10^{-2}$ (contrary to the case of direct $L$ influence when the sign of $L$ is important, as far as in one case it leads to overproduction and in the other to underproduction of helium-4 [?].)

And last but not least, we are really amused, that the authors of SFA after frankly declaring that they do not know if the evolution of the lepton asymmetry represents a true chaos or not, do continue working with this not clear understanding, and, moreover, they continue exploiting it fabricating models and constraints [?], before clarifying the situation with the “chaotic” behavior of $L$.

In this work we have proved that lepton asymmetry, by orders of magnitude less than 0.01,
although not big enough to influence nucleosynthesis directly, can considerably effect nucleosynthesis indirectly via oscillations, changing the pattern of neutrino oscillations, neutrino densities evolution and neutrino spectrum. In the resonant case we have obtained precise cosmological constraint on neutrino oscillation parameters $\delta m^2$ and $\theta$ accounting for the dynamical evolution of the neutrino asymmetry, its interplay with oscillations and its effect on primordial production of helium-4.\(^5\)

The constraints and conclusions of previous works \([?, ?, ?, ?, ?, ?, ?]\) concerning asymmetry effect on BBN with oscillations will change considerably when a proper selfconsistent account for (a) the complete effect of the asymmetry (1)-(4) during its whole evolution in nucleosynthesis epoch; (b) the neutrino spectrum distortion; (c) and the exact kinetics of nucleons is provided using the kinetic equations in momentum space. The role of the mixing-generated neutrino asymmetry in BBN is considerable and should be accounted for precisely.

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\(^5\)The constraints for the nonresonant case were obtained in our previous work \([?]\).
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0.04

-9.0

\Delta Y_p

log( \delta m^2 [\text{eV}^2])

0.00

0.01

0.02

0.03

0.04

-8.0

-7.5

-7.0