Abstract

In this work we consider the theory of sound amplification under the hopping perpendicular transport in a semiconductor superlattice. It is shown that phonons propagating close to the axis of superlattice demonstrate high values of the increment, exceeding the rates of the phonon scattering on natural isotopes. Effects, restricting the value of increment, including the current induced heating of electrons, are discussed. Influence of the phonon spectrum modification in superlattice on the value of increment is examined.
1 Introduction.

Semiconductor quantum superlattices have inspired considerable interest for about twenty years. A number of interesting phenomena related with electron and phonon transport have been studied. Among them there are Bloch oscillations due to electron minibands formation [1], hopping transport [2, 3], domain origination and related electric oscillations [4], phonon miniband formation [5], etc.

Among the variety of nonequilibrium effects in superlattices there is a phenomenon of sound amplification and generation. Sound amplification in a superlattice under the parallel electron transport have been considered in many works (see for example, [6]). In this case the main difference with respect to the classical problem of the sound generation by the electron drift [7] comes from the modification of the electron-phonon interaction formfactor due to the change of the phonon modes structure. Here we concentrate on the phonon generation under the perpendicular electron transport. If an external bias is strong enough, then electron minibands become broken and a superlattice transforms to a system of weakly coupled multiple quantum wells (QWs). In this case transport in the system is due to the hopping between the adjacent QWs [3], unless the resonant tunneling between the different subbands is realized. Hopping processes between the weakly coupled QWs are relatively slow due to strong localization of the electron wave function within the individual QW. Since that, quasiequilibrium state is reached rapidly in each of the QWs. However, the whole electron system is nonequilibrium due to applied electric field. This fact can, in principle, lead to the instability of an external wave, interacting with electrons. In this work we consider such an instability for acoustic phonons, coupled to electrons via the deformation potential. Note, that electrons in each of the QWs are quasiequilibrium, and interaction of phonons with electrons, involving intrawell electron transitions can result only in the decay of phonons. Phonon amplification can arise only due to the processes, involving interwell electron transitions. As we mentioned above, in general, the probability of the latter transitions is much less than that of the former ones. However, some phonons can participate only in interwell processes due to the energy and momentum conservation requirement. It is phonons, that can become unstable and demonstrate positive increment.

The paper has the following structure. In section 2 we describe the system under consideration and derive main formulae for the phonon increment. In section 3 the phenomenon of electron gas heating, which restricts the maximal value of the increment, is discussed. In section 4 the influence of the phonon modes modification in a superlattice due to the mismatch of the elastic properties of the materials of the QW and the barrier layers is analyzed. Then we make a conclusion.
2 Description of the system and main formulae for the phonon increment.

We consider a superlattice in which at zero electric field only one subband is populated (Fig. 1a). We assume that the system is monopolar, which corresponds, for example, to the doped superlattice. However, analogous results are valid for the bipolar system, where carriers can be due to the electrons and holes generation under illumination. If a uniform electric field is applied, then a system of equidistant energy levels (Stark ladder) is formed (Fig.1b). The energy distance between the Stark ladder levels is \( \Delta = \frac{eF}{d} \), where \( F \) is the value of electric field and \( d \) is the period of superlattice. If \( \Delta \) exceeds considerably the width of the superlattice miniband, then the wave function of an electron becomes predominantly localized within an individual QW. If scattering induced broadening of the Stark ladder levels is less than \( \Delta \), then perpendicular transport in the system is due to the so called Wannier-Stark hopping between the adjacent QWs [8]. Such hopping can be due to interaction of electrons with phonons, impurities, etc.

Energy spectrum of the system is

\[
E_{n,k} = E_0 - n\Delta + \frac{\hbar^2 k^2}{2m^*},
\]

where \( E_0 \) is a constant, which depends on the choice of the energy reference level, \( k \) is the absolute value of the electron 2D wave vector \( \vec{k} \), corresponding to the electron motion along the layers, and \( n \) is an integer. We use two-well approach for the electron wave function [2, 9]:

\[
\Psi_{n,\vec{k}} = \frac{1}{\sqrt{S}} \exp \left( i\vec{k}\vec{\rho} \right) \psi_n(z),
\]

\[
\psi_n = \chi(z - nd) - \frac{t}{\Delta} \left( \chi(z - (n + 1)d) - \chi(z - (n - 1)d) \right).
\]

Here \( \vec{\rho} = (x, y) \) is 2D coordinate of an electron in the plane of the layers, \( z \) axis is parallel to the superlattice axis, \( S \) is the normalizing area, \( \chi \) is normalized wave function of an electron in the individual QW. The value of \( t \) has the order of the width of the superlattice miniband at \( F = 0 \) and is defined by the equation

\[
t = V \int_{(1)} \chi(z - nd)\chi(z - (n + 1)d)dz,
\]

where \( V \) is the height of the barriers, the integration goes over the region of the \( n \)-th QW, and we assumed that \( V \gg \Delta \). The wave function (2) is an approximate form of the Stark ladder states under the condition \( t \ll \Delta \). Note, that wave function (2) is not normalized correctly. However, under the calculation of the matrix elements of the interwell and intrawell electron transitions under the condition \( t \ll \Delta \) this introduces a small error and that is why we do not introduce in (2) an additional factor, taking into account the correct normalization.

Under the condition \( t/\Delta \ll 1 \) hopping interwell transitions are much slower than intrawell ones, and this allows us to assume that a quasiequilibrium is reached in each of the QWs.
According to this, the distribution function for electrons, associated with the \(n\)-th QW, is
\[
f_n = \left(1 + \exp \left( E_{\mu,k} - E_{e}^{(n)} \right) / T_e \right)^{-1},
\]
where \(E_{e}^{(n)}\) is the quasi Fermi energy of the \(n\)-th QW and \(T_e\) is the electron temperature in energy units. If the electric field is uniform, then electron concentrations \(n_e\) in each QW are equal, and
\[
f_n = \left(1 + \exp \left( \frac{h^2 \kappa^2}{2m^*} - E_{\mu} \right) / T_e \right)^{-1},
\]
where \(E_{\mu}\) is the Fermi energy, referred to the energy \(E_{\alpha,\beta}\) and is related with \(n_e\) as
\[
n_e = \frac{m^* T_e}{\pi \hbar^2} \ln \left(1 + \exp \left( \frac{E_{\mu}}{T_e} \right) \right).
\]
The approaches introduced determine the electron subsystem.

For an acoustic phonon with wave vector \(q\), interacting with electrons, one can write down the kinetic equation:
\[
\frac{dN_q}{dt} = P^{(em)}_q (N_q + 1) - P^{(ab)}_q N_q,
\]
where \(N_q\) is the number of phonons in the system, and \(P^{(em,ab)}_q\) are the probabilities of phonon assisted electron transitions with the phonon emission and absorption, respectively. We define the phonon increment \(\alpha_q\) as
\[
\alpha_q = P^{(em)}_q - P^{(ab)}_q.
\]
It is obvious that such an increment determines the dynamics of the phonon number when it is high and stimulated emission exceeds spontaneous emission. If \(\alpha_q < 0\), then we have phonon decay (such a situation is common in the case of the equilibrium electron subsystem), and if \(\alpha_q > 0\), then one has generation of phonons. Note, that we do not put into equation (7) terms responsible for the phonon-phonon interaction due to the lattice anharmonicity, as well as for the phonon scattering on isotopes and other defects. This means that positive values of \(\alpha_q\) should be compared to the characteristic rates of the mentioned processes.

Before we proceed to the direct calculation of \(\alpha_q\), we can define a class of phonons for which the increment can be positive. It is convenient to split \(\alpha_q\) into two contributions:
\[
\alpha_q = \alpha^{(inter)}_q + \alpha^{(intra)}_q,
\]
where \(\alpha^{(inter)}_q\) is due to the interwell electron transitions and \(\alpha^{(intra)}_q\) is due to the intrawell electron transitions. Since electrons in each of the QWs are in quasiequilibrium, there should be \(\alpha^{(intra)}_q < 0\). Then, under the condition \(t / \Delta < 1\), in general \(|\alpha^{(intra)}_q| \gg |\alpha^{(inter)}_q|\). However, for particular phonons, propagating almost parallel to the superlattice axis, \(\alpha^{(intra)}_q \approx 0\). This is because for such phonons it is impossible to fulfill simultaneously energy and momentum conservation laws. Indeed, for intrawell transitions we have
\[
\bar{k} = \bar{k} + q_{||}, \quad \frac{h^2 k^2}{2m^*} = \frac{h^2 k^2}{2m^*} \pm \hbar \omega_q,
\]
where \( \tilde{k}, \tilde{k}' \) are the initial and final wave vectors of an electron, \( \bar{q}_|| \) is a projection of \( \bar{q} \) on the \((x,y)\) plane, \( +(-) \) sign is for the phonon emission (absorption), and \( \hbar\omega_q \) is the phonon energy. If we fix \( \hbar\omega_q \), then it is possible to fulfill both of the equations in (10) only if \( \hbar q_|| > m^*\omega_q/k \).

If electrons are degenerate \((k_F^3 > 7e)\), then important \( k \) are close to the Fermi wave vector \( k_F \), and we can write down a condition for the phonons, which can be involved in the intrawell transitions:

\[
q_|| > q_{\text{crit}} = \frac{v_s}{v_F} q_z.
\]  

where \( v_s \) is the sound velocity, \( v_F \) is the Fermi velocity, and \( q_|| \) and \( q \) are the absolute values of \( \bar{q}_|| \) and \( \bar{q} \). If \( q_|| < q_{\text{crit}} \), then intrawell transitions are switched off, and one can expect to obtain positive \( \alpha_q \) due to the interwell transitions. For typical electron concentrations and material parameters the ratio \( v_s/v_F \) is about \( 10^{-2} \).

Let us calculate the value of \( \alpha_q \). We will deal with the relatively high frequency phonons \((\hbar\omega_q \text{ of the order of several units of } meV)\). For such phonons the main mechanism of the electron-phonon interaction is deformation potential. If one took into account the anisotropy of the phonon spectrum, then the formulae for \( \alpha_q \) would be rather complex. However, as we mentioned above, we actually deal only with the phonons, propagating close to the superlattice axis. Most often, superlattices are composed from the cubic crystals, and superlattice axis coincides with the \((100)\) direction. In this case the phonons under consideration are pure longitudinal phonons, characterized by the sound velocity \( v_s = \sqrt{c_{11}/\rho} \), where \( c_{11} \) is the elastic constant and \( \rho \) is the material density. In this section we disregard the modification of phonons due to mismatch of the elastic properties of the QW and the barrier layers (the latter effect is analyzed in section 4).

Under the mentioned conditions for \( \alpha_q \) we have

\[
\alpha_q^{(\text{inter})} = N_{QW} \frac{2\pi E_1^2}{S L_z \hbar \omega_q} q^2 f_{\text{inter}}^{(12)}
\]

\[
\left[ \sum_{\tilde{k}, \tilde{k}'} f_n(k)(1 - f_{n+1}(k')) \delta_{\tilde{q}_||, \tilde{q}_||'} \delta(E_{n,k} - E_{n+1,k'} - \hbar\omega_q) - \right.
\]

\[
\left. \sum_{\tilde{k}, \tilde{k}'} f_n(k)(1 - f_{n+1}(k')) \delta_{\tilde{q}_||, \tilde{q}_||'} \delta(E_{n,k} - E_{n+1,k'} + \hbar\omega_q) + \ldots \right],
\]

where \( N_{QW} \) is the number of QWs in the superlattice, \( L_z \) is normalizing length of the system in \( z \)-direction, \( E_1 \) is the deformation potential constant. First two terms in the brackets are due to the transitions from the \( n \)-th QW to the \((n + 1)\)-th QW with emission and absorption of the phonon, respectively, \( \ldots \) denotes the analogous terms for the electron transitions from the \( n \)-th QW to the \((n - 1)\)-th QW, \( q_z \) is \( z \)-projection of \( \bar{q} \), and \( f_{\text{inter}}^{(12)} \) is a formfactor of the electron-phonon interaction for the interwell transitions:

\[
f_{\text{inter}}^{(12)} = \left| \int_{-\infty}^{+\infty} \psi_n \psi_{n+1} \exp(i q_z z) \, dz \right|^2.
\]
For the intrawell transitions one has a similar expression

\[
\alpha_{i}^{(\text{intra})} = N_{QW} \frac{2\pi \hbar^{2}}{S_{z}} q^{2} J_{q}^{(\text{intra})} \left[ \sum_{k,k'} f_{s}(k) (1 - f_{s}(k')) \delta_{\varepsilon - \hbar \omega} \delta(E_{n,k} - E_{n,k'} - \hbar \omega_{i}) + \right. \\
\left. \sum_{k,k'} f_{s}(k) (1 - f_{s}(k')) \delta_{\varepsilon + \hbar \omega} \delta(E_{n,k} - E_{n,k'} + \hbar \omega_{i}) \right],
\]

where \( J_{q}^{(\text{intra})} \) is a formfactor of the electron-phonon interaction for the intrawell transitions:

\[
J_{q}^{(\text{intra})} = \int_{-\infty}^{+\infty} \psi_{c}^{2} \exp(iqz) \, dz. \tag{15}
\]

Performing summation in equations (12) and (14), one gets

\[
\alpha_{q} = J_{q}^{(\text{inter})} \left[ P(\Delta - \hbar \omega_{q}) - P(\Delta + \hbar \omega_{q}) + P(-\Delta - \hbar \omega_{q}) - P(-\hbar \omega_{q}) + J_{q}^{(\text{intra})} [ P(-\hbar \omega_{q}) - P(\hbar \omega_{q})] \right], \tag{16}
\]

where we introduce function \( P \) according to

\[
P(c) = \left( \frac{m^{*}}{\hbar^{2}} \right)^{3/2} \int_{E^{*}}^{\infty} \frac{dE}{\sqrt{(1 + \exp(\frac{E - K_{\omega}}{T_{c}})) (1 + \exp(\frac{E + K_{\omega}}{T_{c}})) \sqrt{E - E^{*}}}} \left( \frac{m^{*}}{2\hbar^{2} q^{2}} \right)^{2} \tag{17}
\]

It is easy to analyze (17) in the zero temperature limit. If \( T_{c} = 0 \), then \( P(c) \) can be written as

\[
P(c) = \left( \frac{m^{*}}{\hbar^{2}} \right)^{3/2} \int_{E_{c} - c}^{E_{c}} \frac{dE}{\sqrt{E - E^{*}}} \tag{18}
\]

In this case \( P(c) \neq 0 \) only if \( c > 0 \) and \( E^{*} < E_{c} \). For small \( q \) the value of \( E^{*} \) can be less than \( E_{c} \) only if \( c \) is small. This means that the phonon generation is possible for small \( q \) and \( \hbar \omega_{q} \sim \Delta \). In this case \( \alpha_{q}^{(\text{intra})} \approx 0 \), in accordance with the energy and momentum consideration, discussed above, and approximately \( \alpha_{q} = J_{q}^{(\text{inter})} [ P(\Delta - \hbar \omega_{q}) - P(\hbar \omega_{q} - \Delta)] \). The first term is due to the phonon emission upon the electron hopping to the adjacent lower QW, and the second term is due to the phonon absorption upon the electron hopping to the adjacent higher QW. One can see, that if he disregards the dependence of \( J_{q}^{(\text{inter})} \) on the phonon energy, then \( \alpha_{q} \) is odd function of \( \Delta - \hbar \omega_{q} \). At \( \hbar \omega_{q} < \Delta \) \( \alpha_{q} > 0 \) and at \( \hbar \omega_{q} > \Delta \) \( \alpha_{q} < 0 \). It is easy to estimate the maximal value of \( \alpha_{q} \) as a function of the phonon energy at constant \( q \), \( \alpha_{\text{max}} \):

\[
\alpha_{\text{max}} = \omega_{q} J_{q}^{(\text{inter})} \sqrt{\frac{2m^{*} \hbar^{2}}{\pi \rho \hbar^{2} v_{QW}} \sqrt{\frac{E_{c}}{q|}}}. \tag{19}
\]
Here under the determination of maximum we disregard the dependence of $J_{qz}^{\text{(inter)}}$ on $\hbar \omega_q$ since $J_{qz}^{\text{(inter)}}$ is a relatively smooth function of the phonon energy. The maximum is realized for the phonon energy

$$(\hbar \omega_q)_{\text{max}} = \Delta - 2 \sqrt{\frac{\hbar^2 q^2_{\parallel}}{2m^*}}. \tag{20}$$

We see, that $\alpha_{\text{max}}$ diverges as $q_{\parallel}$ goes to zero, and the energy range of the phonon generation shrinks simultaneously. However, there are several factors, which cancel such a divergence of $\alpha_{\text{max}}$. The first factor is the finite value of the electron temperature. From equations (17) and (20) it is obvious that the zero temperature limit is valid if $T_c \ll \sqrt{E_{\text{e}} \hbar^2 q^2_{\parallel}/(2m^*)}$. This roughly establishes a limit where equation (19) is valid. The second factor is that $q_{\parallel}$ cannot be less than the reciprocal dimension of the sample along the QW layers.

Before we present the numerical results for $\alpha_{\text{max}}$, we discuss some points concerning the calculation of $J_{qz}^{\text{(inter)}}$. To do this, one needs to know the wave functions $\chi$. Usually, the quantization energy of electrons in the individual QW, $E_{\text{w}}$, is of the order of tens of meV, while $\Delta$ in our case is of the order of units of $meV$. This means that this is a good approach to use $\chi$, corresponding to the rectangular QW at $F = 0$. A similar approximation is also good for the calculation of $l$, provided that $\Delta \ll \sqrt{(V - E_{\text{w}}) \hbar^2/(2m^* b^2)}$, where $b$ is the thickness of the barrier. Let us introduce functions $\chi_{1,2}$ as $\chi_1 = \chi(z - nd)$, $\chi_2 = \chi(z - (n + 1)d)$. Then for $J_{qz}^{\text{(inter)}}$ we have

$$J_{qz}^{\text{(inter)}} = \left[ \int_{-\infty}^{\infty} \exp(\imath q_{\parallel} z) \left( \chi_1 \chi_2 + \frac{l}{\Delta} (\chi_2^2 - \chi_1^2) \right) dz \right]^2, \tag{21}$$

where in the integral there remains only terms proportional to the first degree of the small factor $\exp(-\sqrt{2m^* (V - E_{\text{w}}) \hbar^2/h^2})$. It is easy to obtain that if $V \gg E_{\text{w}}$, then approximately the contribution from the term $\chi_1 \chi_2$ is less than the contribution from the term $l/\Delta (\chi_2^2 - \chi_1^2)$ with the factor $\Delta \sqrt{2m^* b^2/(h^2 V)} \ll 1$. If we disregard the term $\chi_1 \chi_2$ in (21), then we get

$$J_{qz}^{\text{(inter)}} = 4 \left( \frac{l}{\Delta} \right)^2 \sin^2 \frac{q_{\parallel} d}{2} J_{qz}^{\text{(intr)}}. \tag{22}$$

This gives a guide how to select a structure to obtain a high value of the increment at a given phonon energy. First, it is necessary to select the period of the superlattice according to the condition $q_{\parallel} d = \pi + 2\pi l$, where $l$ is an integer. Since $J_{qz}^{\text{(intr)}}$ decays rapidly as $q_{\parallel} d$ increases, it is better to deal with small $l$. Since it should be $\Delta \sim \hbar \omega_q$, it is necessary then to choose the height of the barrier $V$ to have the ratio $l/\Delta$ not too low to have high increment, but not too high to allow the fast intrawell relaxation.

In Fig.2 we depict the dependence of the phonon increment on the phonon energy for $q_{\parallel} = 10^6 \text{ m}^{-1}$ at several values of the electron temperature. The calculations were done for the following parameters: $a = 4.5 \text{ nm}$, $b = 6 \text{ nm}$, $V = 0.6 \text{ eV}$, $\rho = 5360 \text{ kg/m}^3$, $v_s = 4708 \text{ m/s}$, $m^* = 0.067 \text{ m}_0$, $l_1 = 7 \text{ eV}$, $n_s = 2 \cdot 10^{15} \text{ m}^{-2}$, $\Delta = 3 \text{ meV}$. For such a structure $t \approx 0.3 \text{ meV}$. In general, we try to adhere to the parameters, characteristic for the GaAs/AlGaAs superlattice,
though the model of the constant effective mass is crude for the calculations of $t$ and more sophisticated calculations are necessary for the real heterostructures. In the figure we show only the energy region where the increment is positive. For $\hbar \omega_q > \Delta$ the increment is negative and behaves almost antisymmetrically with respect to $\hbar \omega_q = \Delta$. As we see, the value of $\alpha_q$ decreases rapidly with the increase of the electron temperature. This is because of the spread of the Fermi distribution function near the Fermi energy, which at $\hbar \omega_q < \Delta$ decreases the probability of the phonon emission (equal to $J_z^{(\text{inter})} P(\Delta - \hbar \omega_q)$) and increases the probability of the phonon absorption (equal to $J_z^{(\text{inter})} P(\hbar \omega_q - \Delta)$). In Fig.3 the dependences of $\alpha_q(\hbar \omega_q)$ are shown for a set of $q_\parallel$ at $T_e = 4K$. As it is expected, $\alpha_q$ is large for phonons, whose direction of propagation is close to the superlattice axis. The value of the increment can be as high as several units of $10^8 \text{ s}^{-1}$. The main mechanism of the phonon losses is elastic scattering on the isotopes. The rate of this process for the phonons with the energy $3\text{ meV}$ in GaAs is equal $2.5 \cdot 10^6 \text{ s}^{-1}$ [10], which is substantially less than the obtained increment even at relatively high temperature. This proofs the possibility to observe the phonon generation.

3 Electron gas heating under the hopping transport.

As we mentioned in the previous section, the main reason, restricting the maximal value of the increment, is the finite value of the electron temperature. Great increments can be achieved at low $T_e$. This means that it is necessary to cool the sample to low temperature. However, the current flow in the superlattice leads to the heating of electrons, and this effect restricts the minimal value of $T_e$ and, correspondingly, the maximal value of increment.

To calculate the effect we use the energy balance equation, which follows from the kinetic equation for electrons:

$$\frac{\partial f_k}{\partial t} = \sum_{\tilde{k}} \left[ W_{\tilde{k},k} f_{\tilde{k}} (1 - f_k) - W_{k,\tilde{k}} f_k (1 - f_{\tilde{k}}) + W_{\tilde{k},k} f_{\tilde{k}} (1 - f_{\tilde{k}}) - W_{k,\tilde{k}} f_k (1 - f_k) + W_{\tilde{k},k} f_{\tilde{k}} (1 - f_k) - W_{k,\tilde{k}} f_k (1 - f_{\tilde{k}}) \right],$$

(23)

where $f_k$ is the electron distribution function in a QW, $f_{\tilde{k}}^{\uparrow, \downarrow}$ are electron distribution functions in the adjacent higher and lower QWs, $W_{\tilde{k},k}^{\uparrow, \downarrow}$ are the probabilities of the electron intrawell transitions to the higher and lower QW states with wave vector $\tilde{k}$, $W_{k,\tilde{k}}$ is the probability of the intrawell transitions. The energy balance equation follows from equation (215) after the multiplications on the electron energy and summation over $\tilde{k}$:

$$\frac{d \langle E \rangle}{dt} = Q^{(\text{inter})}(T_e, T_{\text{lat}}) - Q^{(\text{int})}(T_e, T_{\text{lat}}),$$

(24)

where $\langle E \rangle$ is the average electron energy, $T_{\text{lat}}$ is the lattice temperature, $Q^{(\text{int})}$ is the energy flux to the QW subsystem due to the interwell electron transitions (first four terms in equation (23)),
and \( Q^{\text{intra}} \) is the energy flux from the electrons to the lattice due to the intrawall transitions (the last two terms in equation (23) with the minus sign). Obviously, \( Q^{\text{intra}} = 0 \) if \( T_e = T_{\text{lat}} \). At small temperature difference it can be written as \( Q^{\text{intra}} \approx (T_e - T_{\text{lat}})/\tau_e \), where \( \tau_e \) is the energy relaxation time of the 2D electrons. However, \( Q^{\text{intra}} \neq 0 \) even if \( T_e = T_{\text{lat}} \) due to the energy transfer upon interwell transitions between the shifted by electric field levels. This is the reason of electron heating. In the stationary state from equation (24) one has an equation for the determination of \( T_e \) at given \( T_{\text{lat}} \).

We assume that all transitions are due to the deformation potential interaction with acoustic phonons. In this case for the structure described in section 2, for the dependence of electron temperature on the lattice temperature we obtained the dependence depicted in Fig.4. One can see that there is a minimal electron temperature about \( 4K \). This value brings about the restriction of the phonon increment for the chosen structure.

Note, that at some parameters electron transitions due to the interaction with impurities can be faster than those ones due to the phonons. Since the former mechanism is elastic, at each electron interwell transition to the lower QW an electron gains the energy \( \Delta \) with respect to the bottom of the QW subband. This should lead to the increase of \( Q^{\text{inter}} \). At the same time \( Q^{\text{intra}} \) remains, being due to the interaction with phonons, since the intrawell elastic scattering on impurities does not lead to the energy transfer. As a result, the electron heating should become stronger.

4 Effect of the “phonon superlattice” formation on the value of increment.

Above we assumed that phonons in the system are the same as in the bulk material. However, the layered character of the structure influences not only electron, but also phonon spectrum. Actually, in a superlattice phonon miniband formation takes place. If the layers of the superlattice are composed from the cubic materials, grown along (100) direction, then for the longitudinal phonons, propagating along the superlattice axis, the dispersion equation is [11]

\[
\cos q_x d = \cos(q_1 a + q_2 b) - \frac{\xi^2}{2} \sin q_1 a \sin q_2 b,
\]

where \( q_x \) is the superlattice phonon wave vector, \(-\pi/d < q_x < \pi/d\), \( q_{1,2} = \omega/v_{s}^{(1,2)} \), where \( v_{s}^{(1,2)} \) are the longitudinal sound velocities along the (100) direction in the QW and the barrier layers, respectively, and \( \xi \) is determined by the equation

\[
\xi = \frac{v_{s}^{(2)} \rho_2 - v_{s}^{(1)} \rho_1}{\sqrt{v_{s}^{(1)} v_{s}^{(2)}} \rho_1 \rho_2}.
\]

Here \( \rho_{1,2} \) are the densities in the QW and the barrier layers, respectively. The value of \( \xi \) reflects the degree of the elastic properties mismatch in the different layers of the superlattice. Often
in actual superlattice this mismatch is relatively small. In this case the main changes of the phonon structure takes place near the boundaries of the phonon Brillouin band, \( q_s = \pm \pi /d \), where the phonon folding and stop band formation are realized [11].

To calculate the increment of the superlattice phonons one should perform the procedure of their quantization. Having done that, we obtain for the increment expressions, similar to (12) and (16), but with different formfactors for the interwell and intrawell transitions, \( j^{(\text{inter})} \), \( j^{(\text{intra})} \):

\[
J^{(\text{inter})}_{q_s,l} = \frac{1}{q_s^2} \left| \int_{-\infty}^{+\infty} dz \frac{du_z}{d\lambda_2} \left[ \lambda_1 \lambda_2 + \frac{1}{\Delta} (\lambda_2^2 - \lambda_1^2) \right] \right|^2, \tag{27}
\]

\[
J^{(\text{intra})}_{q_s,l} = \frac{1}{q_s^2} \left| \int_{-\infty}^{+\infty} dz \frac{du_z}{d\lambda_1} \right|^2. \tag{28}
\]

Here \( u_z \) is the displacement in the eigen phonon mode with wave vector \( q_s \) and the phonon subband number \( l \). \( u_z \) satisfies the condition

\[
u_z(z + d) = u_z(z) \exp(iq_sz) \tag{29}
\]

and the normalization rule

\[
\int \frac{u_z(z)^2}{d\lambda} dz = 1, \tag{30}
\]

where the integration goes over the period of the superlattice. Under such definitions one should substitute the parameters of the QW material into equation for \( P \), (17). Note, that from (29) follows the relation between the interwell and intrawell formfactors, similar to (22).

Since we deal with the phonons with small \( q_s \), we suppose that the phonon energy and spectrum do not depend on it. Then, we assume that the deformation potential constant is the same in the QW and the barrier layers.

In Fig.5 we present the dependence of \( J^{(\text{inter})} \) and \( J^{(\text{intra})} \) on the phonon energy for the structure described in section 2 in the case of the phonon propagation along the superlattice axis. For the calculations we used the parameters \( v_0 = 5214 \text{ m/s}, \rho_2 = 4400 \text{ kg/m}^3 \) and for the QW layers parameters were the same as in the calculations for the bulk phonons. This corresponds to the GaAs/Al\(_{0.6}\)Ga\(_{0.4}\)As superlattice [12]. As we see, the modification of the formfactor occurs mainly in the vicinity of the phonon stopbands, where for the bulk phonons maximums of the formfactor are realized. This modifies the value of the increment. In Fig.6 the obtained dependences of the increment on the energy at different \( q_s \) at \( T_s = 4 \text{K} \) are shown. The structure was chosen to satisfy the condition that phonons with \( \hbar \omega_q = \Delta \) are close to the second phonon stop band. One can see that the appearance of additional maxima and minima in the formfactor modifies the value of increment. In particular, for some phonons the value of the increment can be enhanced substantially.
5 Conclusion.

In conclusion, in this work we have considered the problem of the acoustic phonon generation in a superlattice under the perpendicular hopping electron transport. We have shown that phonons, propagating close to the superlattice axis, can demonstrate large increments of the order of several units of $10^8 \text{ s}^{-1}$, which is substantially greater than the rate of elastic scattering of such phonons by isotopes. The value of the increment is restricted mainly by the finite electron temperature, which can not be made arbitrary small due to the electron heating upon the current flow. We also analyzed the influence of the modification of the phonon modes structure due to the mismatch of elastic properties of the materials, composing the QW and the barrier layers.

It is worth to mention the difference between our results and the classical one for the sound generation by the electron drift [7]. In the latter case the phonon generation occurs due to the population inversion for phonons, created by the asymmetry of the electron distribution function in the momentum space. In our case the phonon generation is realized also due to some kind of the population inversion, created by the electric field. However, in the classical case to achieve positive increment one has to satisfy the condition $v_{dr} > v_n$, where $v_{dr}$ is the drift velocity of electrons, proportional to the current. In the case of the hopping electron transport in the superlattice there is no such condition. Though both electric current and phonon increments are proportional to the same value $(I/\Delta)^2$, the appearance of the population inversion is not directly related with the value of electric current.

The important point is that during our calculations we assumed that our structure is ideal, i.e. the spectrum of the Stark ladder is equidistant. However, in realistic structures it is not the case due to fluctuations of the layers parameters or due to the domains formation [4]. This means, that to obtain actual value of the increment, it is necessary to perform averaging over the set of present values of $\Delta$, $n_e$, etc. This topic will be the subject of the subsequent investigations.

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References


Figure captions

Fig.1. Electron superlattice in the zero electric field, (a), and Stark ladder formation in the strong enough electric field, (b). In (b) the notations of the parameters of the structure are marked.

Fig.2. The dependence of the phonon increment on the phonon energy at several temperatures for $q_{||} = 10^6 \text{ m}^{-1}$. The parameters of the structure are given in the text.

Fig.3. The dependences of the phonon increment at $T_c = 4K$ for several values of $q_{||}$.

Fig.4. Electron heating upon the hopping electron transport. Calculated dependence of the electron temperature on the lattice temperature. One can see that the current flow establishes a lower limit on the electron temperature.

Fig.5. The dependence of the formfactor of the electron–phonon interaction on energy for the bulk phonons and superlattice phonons, propagating along the superlattice axis.

Fig.6. The dependence of the phonon increment on the phonon energy at $T_c = 4K$ for several values of $q_{||}$, obtained for the superlattice phonons.
Fig. 2
Figure 3

Increment, $10^8 \text{s}^{-1}$

Phonon energy, meV

$\omega_{10}^i \mathbf{b}$
Electron temperature, K

Lattice temperature, K

Fig. 4
Fig. 5

Electron-phonon formfactor, $10^{-2}$

Phonon energy, meV

bulk media
superlattice
Increment, $10^8 \text{ s}^{-1}$

FIG. 6

photon energy, meV