The conversion of photons into axions in a periodic external electromagnetic field with axion frequency is considered in detail by Feynman methods. The differential cross sections are given. It is shown that there is a resonant conversion for the considered process.
1. Introduction

As is known, an axion was postulated to explain why the strong interactions conserve the discrete symmetries P and CP in spite of the fact that the Standard Model of particle interactions as a whole violates those symmetries [1].

In the Peccei - Quinn scheme, the mass of the axion is essentially a free parameter that could plausibly be anywhere within the range of $10^{-12}$ eV to 1 MeV. The main axion window still allowed at present is $10^{-3}$ eV to $10^{-6}$ eV [2], where if the axion mass is $10^{-5}$ eV, it may constitute the dark matter in the Universe [3]. The axion properties and their phenomenological consequences have been studied in depth and some experiments trying to discover the axion are under way [3,4]. Almost all experiments so far designed to search for light axions make use of the coupling of the axion to photons. The conversion of axions into electromagnetic (EM) power in a resonant cavity was first suggested by Sikivie [5]. Various terrestrial experiments to detect invisible axions by making use of their coupling to photons have been proposed [6] and the first results of such experiments appeared recently [7,8]. Axion detections in experiments with axion frequency have been proposed in Ref. [9].

In our earlier works [10,11], we considered the conversion of photons into axions in the static and periodic EM fields by applying the Feynman diagram techniques. Now we also apply this method to consider the conversion of photons into axions in the $TE_{10}$ mode with a frequency equal to the axion mass (axion frequency).

2. Cross sections

The basis for photon - axion conversions in external (EM) field is the coupling of the axion to two photons [3,4]

$$L_{\gamma\gamma} = g_{\gamma} \frac{\alpha}{4\pi} \frac{\phi_0}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

Let us consider the conversion of a photon $\gamma$ with momentum $q$ into an axion $a$, with momentum $p$ in an external electromagnetic field in the $TE_{10}$ mode with axion frequency. The nontrivial solution of the $TE_{10}$ mode is given by [12]

$$H_z = H_0 \cos \left( \frac{\pi x}{a} \right) e^{ikz - i\omega t},$$
$$H_x = -\frac{ka}{\pi} H_0 \sin \left( \frac{\pi x}{a} \right) e^{ikz - i\omega t},$$
$$E_y = i\frac{\omega a_0}{\pi} H_0 \sin \left( \frac{\pi x}{a} \right) e^{ikz - i\omega t}.$$ 

Using the Feynman rules we get the following expression for the matrix element

$$\langle p | M | q \rangle = \frac{g_{\gamma\gamma}}{(2\pi)^2 \sqrt{p_0 q_0}} \left[ (\varepsilon_3(q' \tau) q_1 - \varepsilon_1(q' \tau) q_3) F_y + \varepsilon_1(q' \tau) q_0 F_x + \varepsilon_3(q' \tau) q_0 F_z \right],$$

where $g_{\gamma\gamma}$ is the coupling constant of the photon to the axion.
where \( p_0 \equiv q_0 + \omega_a \), \( \omega_a \) is the axion frequency, \( \varepsilon_\mu(q, \sigma) \) represents the polarization vector of the photon and \( g_{a\gamma} \equiv g_{\gamma} = \frac{\omega_a}{2 \pi^2} \times 10^{-7} \text{GeV}^{-1} \) [4]. The coupling constant in (2) is model dependent. In the Dine- Fischler- Srednicki- Zhitnitskii (DFSZ) model [6] \( g_{\gamma} \approx 0.36 \), and in the Kim- Shifman- Vainshtein- Zakharov (KSVZ) model [7] \( g_{\gamma} \approx -0.97 \) and [11]

\[
F_x = -\frac{8k aH_0(q_x - p_x) \cos[\frac{1}{2} a(q_x - p_x)] \sin[\frac{1}{2} b(q_y - p_y)] \sin[\frac{1}{2} c(q_z - p_z + k)]}{\pi |(q_x - p_x)^2 - \frac{\omega_a^2}{\omega_a^2}|(q_x - p_x)(q_y - p_y)(q_z - p_z + k)},
\]

\[
F_y = -\mu F_x, \quad F_z = k^{-1}(q_x - p_x)^{-1} \pi^2 a^{-2} F_x
\]

are form factors for the EM region.

In the following, we assume that the speed of light is unity and \( \omega_a \ll q \). From (2) and (3), it follows that when the momentum of the photon is parallel to the \( z \) axis then the differential cross section (DCS) vanishes. It implies that if the momentum of the photon is parallel to the \( z \) axis then there is no conversion. If the momentum of the photon is parallel to the \( x \) axis, i.e., \( q^\mu = (q, q, 0, 0) \) then DCS takes the final form

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{g_{a\gamma}^2 H_0^2 q^2 V^2 q^2 \left[ \omega_a^2 + \frac{\pi^2}{a^2} \right]^2}{2\pi^4 \left[ \frac{\omega_a^2}{\omega_a^2} - \frac{\pi^2}{a^2} \right]^2} \left( 1 + \frac{\omega_a}{q} \right) \cos^2 \left( \frac{\omega_a a}{2} \right)
\]

for \( \theta = 0 \), and

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{2g_{a\gamma}^2 H_0^2 a^2 q^2 \left( \omega_a q - \frac{\pi^2}{a^2} \right)^2}{2\pi^4 \left( 1 + \frac{\omega_a}{q} \right) \left( q^2 - \frac{\pi^2}{a^2} \right)} \cos^2 \left( \frac{a}{2} q \right) \sin^2 \left( \frac{b}{2} q + \omega_a \right)
\]

for \( \theta = \frac{\pi}{2}, \varphi' = 0 \), and

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{2g_{a\gamma}^2 H_0^2 a^2 b^2 \left( \omega_a q - \frac{\pi^2}{a^2} \right)^2 \left( 1 + \frac{\omega_a}{q} \right) \cos^2 \left( \frac{a}{2} q \right) \sin^2 \left( \frac{c}{2} q \right)}{2\pi^4 \left( q^2 - \frac{\pi^2}{a^2} \right)}
\]

for \( \theta = \frac{\pi}{2}, \varphi' = \frac{\pi}{2} \). Here we use the Taylor approximation: \( \sqrt{1 + 2 \frac{\omega_a}{q}} \approx (1 + \frac{\omega_a}{q}) \). From Eqs. (7), (8) it follows that, if \( q = \frac{\omega_a^2}{\omega_a a^2} \) and \( q = (2n + 1) \frac{\pi}{a} \) with \( n = 0, 1, 2, \ldots \) then DCS vanishes. In the limit \( \omega, q \rightarrow 0 \) then Eqs. (6), (7), and (8) become, respectively

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{g_{a\gamma}^2 V^2 H_0^2 q^2}{8\pi^2} \left( 1 + \frac{\pi}{qa} \right),
\]
It is easy to show that the cross section for the reverse process coincides exactly with the above results, so that for the conversion of photon - axion - photon, the cross section is the square of the previous evaluation.

3. Discussion

In order to estimate photon - axion conversions, from Eqs. (6), (7), and (8), if we choose \( \omega_a = 10^{-5} \text{eV} \), \( a = b = c = 100 \text{cm} \) then DCS’s depend on the momentum of photons which graph is shown in Fig. ?? . The solid curve corresponds to \( \theta = \frac{\pi}{2} \) and the dashed curve corresponds to \( \theta = 0 \). From Fig. ?? it implies that when the momentum of photons is parallel to the momentum of axions then photon - axion conversions are more probable at high energy. When the momentum of photons is perpendicular to the momentum of axions then it exists a resonant conversion at the value \( q = 0.03 \text{eV} \). This is the best case for photon - axion conversions, and in the limit \( 0 < q \), the DCS vanishes.

From Eq.(9) we see that in the limit \( \frac{q}{\omega_a} \ll 1 \) we have the result similar to that of a static field. However, it does not yield the same results, because in this case both the electric and magnetic components simultaneously give contributions. In C.G.S units, if \( \omega_a = 10^{-5} \text{eV} \) and \( g_{\gamma} = 0.36 \) ( the DFSZ model ) then Eq.(9) becomes

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{g_{\gamma}^2 H_0^2 a^3 c^2 (\omega_a - \frac{\pi}{a})^2}{8\pi^5 (\omega_a + \frac{\pi}{a})} \sin^2 \left( \frac{b}{2} (\omega_a + \frac{\pi}{a}) \right), \tag{10}
\]

and

\[
\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{g_{\gamma}^2 H_0^2 a^5 b^2 (\omega_a - \frac{\pi}{a})^2}{8\pi^5} \sin^2 \left( \frac{c\pi}{2a} \right). \tag{11}
\]

Finally, in this work we considered only the theoretical basis for experiments, other problems associated with the detection will be investigated in the future.

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References


