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BARYOGENESIS VIA NEUTRINO OSCILLATIONS

E.Kh. Akhmedov
National Research Centre Kurchatov Institute, Moscow 123182, Russian Federation
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

V.A. Rubakov
Institute for Nuclear Research of the Russian Academy of Sciences,
Moscow 117312, Russian Federation,
Institute for Cosmic Ray Research, University of Tokyo,
Tanashi, Tokyo 188, Japan
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

A.Yu. Smirnov
Institute for Nuclear Research of the Russian Academy of Sciences,
Moscow 117312, Russian Federation
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

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Abstract

We propose a new mechanism of leptogenesis in which the asymmetries in lepton numbers are produced through the CP-violating oscillations of “sterile” (electroweak singlet) neutrinos. The asymmetry is communicated from singlet neutrinos to ordinary leptons through their Yukawa couplings. The lepton asymmetry is then reprocessed into baryon asymmetry by electroweak sphalerons. We show that the observed value of baryon asymmetry can be generated in this way, and the masses of ordinary neutrinos induced by the seesaw mechanism are in the astrophysically and cosmologically interesting range. Except for singlet neutrinos, no physics beyond the Standard Model is required.
1. A number of mechanisms have been proposed to date to explain the observed baryon asymmetry of the Universe (for recent reviews see, e.g., [1]). One of the simplest possibilities, suggested by Fukugita and Yanagida [2], is that the baryon asymmetry has originated from physics in the leptonic sector. Namely, it was assumed that at temperatures well above the electroweak scale, lepton asymmetry was produced, which was then reprocessed into the baryon asymmetry by non-perturbative electroweak effects [3] – sphalerons [4]. According to ref. [2] the lepton asymmetry is generated in out-of-equilibrium, CP- and lepton number non-conserving decays of heavy Majorana neutrinos (for recent discussions see, e.g., ref. [5] and references therein).

In this Letter we propose a new realization of baryogenesis through leptogenesis which also makes use of the electroweak reprocessing of the lepton number into the baryon number. Like the Fukugita–Yanagida mechanism, our proposal requires only a mild extension of the Standard Model by introducing “sterile” (i.e., electroweak singlet) heavy neutrinos. However, our mechanism of leptogenesis is entirely different from that of ref. [2]: we suggest that asymmetries in lepton numbers were generated due to oscillations of these singlet neutrinos and their interactions with ordinary matter in the early Universe. Moreover, the novel feature of our scenario is that the total lepton number is not violated in these oscillations and/or interactions; an important ingredient is separation (rather than generation) of lepton number, i.e., its redistribution between different species of singlet neutrinos.

For this reason we do not necessarily require that singlet neutrinos be Majorana particles; Dirac “sterile” neutrinos are equally suitable (and even better in some respect) for our mechanism to work. Furthermore, in our case the values of the masses and couplings of singlet neutrinos are very different from those of ref. [2].

2. Let us consider the Standard Model extended by adding three types of Majorana neutrinos $N_a, a = A, B, C$ which interact with other particles only through their Yukawa couplings [6]. The corresponding Lagrangian can be written in the “Yukawa basis” (where the matrix of Yukawa coupling constants has been diagonalized) as follows,

$$\mathcal{L} = \bar{N}_{Ra} i \not{\partial} N_{Ra} + h_a \bar{l}_a N_{Ra} \Phi + \frac{M_{ab}}{2} N_{Ra}^T C N_{Rb} + h.c.$$  

Here $N_{Ra}$ are right-handed components of $N_a$, $h_a$ are (real) Yukawa coupling constants, $l_a$ are usual leptonic doublets, $\Phi$ is the Higgs doublet, and $M_{ab}$ is the mass matrix.

We suggest that the baryogenesis proceeds in the following way:
(i) In the course of the evolution of the Universe, singlet neutrinos are produced through their Yukawa couplings. The production mechanism of singlet neutrinos conserves CP, i.e. for each type equal numbers of particles and antiparticles (particles of opposite helicities in the Majorana case) are produced.

(ii) Once created, singlet neutrinos oscillate, and also interact with ordinary matter. None of these processes violates the total lepton number $L^{tot} = L + L_A + L_B + L_C$, where $L$ is the usual lepton number (we assume that Majorana masses are small enough, see below). However, CP is not conserved due to mixing in the singlet neutrino sector. Therefore, the initially created state with individual lepton numbers $L_A = L_B = L_C = 0$ evolves through the oscillations into a state in which $L_A \neq 0$, $L_B \neq 0$, $L_C \neq 0$ but still $L^{tot} = 0$. That is, the total lepton number gets unevenly distributed between different species.

(iii) Singlet neutrinos communicate their lepton asymmetry to ordinary neutrinos and charged leptons through their Yukawa couplings. We assume that the Yukawa couplings of $N_A$, $N_B$ and $N_C$ have certain hierarchy, so that neutrinos of at least one type, $N_A$, come into thermal equilibrium before the time $t_{EW}$ at which sphalerons become inoperative (the corresponding temperature is $T_{EW} \sim 100$ GeV) and those of at least one other type, $N_C$, do not equilibrate by $t_{EW}$. The neutrinos of the third type, $N_B$, may or may not come into thermal equilibrium by $t_{EW}$. To be specific, we discuss mostly the case when the Yukawa coupling of $N_B$ is relatively large so that $N_B$ equilibrate at temperatures exceeding $T_{EW}$ (the opposite case is treated in a similar way). In this case the lepton numbers $L_A$ and $L_B$ are communicated to the ordinary leptons before $t_{EW}$ whereas $L_C$ is not. Therefore (a fraction of) $L_A + L_B$ is reprocessed into baryon asymmetry by electroweak sphalerons; the lepton number $L_C$ is transferred to the active leptons only after the sphalerons have already switched off, so it has no effect on the baryogenesis.

Let us stress that if, for the sphaleron freezing effect, no baryon asymmetry would have been obtained: all three singlet neutrino species eventually transform into ordinary leptons, and since $L^{tot} = 0$, no net lepton charge would have been generated in the sector of ordinary leptons. Hence, the requirement that $N_C$ do not get into thermal equilibrium before $t_{EW}$ is crucial for our mechanism.

3. The evolution of the initially produced coherent neutrino state is described by the Schrödinger equation. In the Yukawa basis the effective Hamiltonian of the system at $T \gg M_s$ is
Here $U$ is the mixing matrix which relates the Yukawa states and the mass eigenstates $N_i$ ($i = 1, 2, 3$): $N_a = U_{ai}N_i$ (we assume that mixing is small, and choose the standard parametrization for $U$ [7]); $\hat{M}^2 = \text{diag} (M_1^2, M_2^2, M_3^2)$ is the matrix of mass eigenvalues; $k(t)$ is the neutrino momentum which depends on time due to the expansion of the Universe. The CP violation in the system is described by the CP-odd phase $\delta$ in the mixing matrix $U$.

The medium effects on the propagation of singlet neutrinos in plasma are described by complex potentials $\tilde{V}$ whose matrix is diagonal in the Yukawa basis,

$$
\tilde{V} = \text{diag} (\tilde{V}_A, \tilde{V}_B, \tilde{V}_C), \quad \tilde{V}_a = V_a - i\frac{\Gamma_a}{2}.
$$

The real and imaginary parts of $\tilde{V}$ are due to the coherent forward scattering and inelastic (and incoherent elastic) processes, respectively. The main contribution to the real potentials $V_a$ comes from the 1-loop self energy diagrams with ordinary lepton and Higgs doublets in the intermediate states. At temperatures above $T_{EW}$ the resulting potentials are [8]

$$
V_a = \frac{1}{8} h^2_\sigma T .
$$

The imaginary parts of the potentials $\Gamma_a$ describe the production and destruction of singlet neutrinos. We will be interested in the temperatures far exceeding the masses of singlet neutrinos. Therefore the rates of the $1 \leftrightarrow 2$ reactions which correspond to the absorptive parts of the self energy diagrams are suppressed by the factor $M_a/T$ and $2 \leftrightarrow 2$ reactions are more important. The main contributions to $\Gamma_a$ come from the Higgs exchange reactions $Q_L N_{Ra} \leftrightarrow t_R l_a$, $t_R N_{Ra} \leftrightarrow Q_L^c l_a$ and $l_a^c N_{Ra} \leftrightarrow t_R Q_L^c$ where $Q_L$ is the third-generation quark doublet and $l_a$ is the lepton doublet. The result [9] at $T \gg M_a$ (corrected to include the color factor for quarks) is

$$
\Gamma_a \simeq \frac{9 h^2_\tau}{64\pi^3} h^2_\sigma T .
$$

Here $h_t \simeq 1$ is the top quark Yukawa coupling. The rates of the elastic $2 \rightarrow 2$ scattering processes are proportional to $h^2_\sigma h^2_\tau$ instead of $h^2_\sigma h^2_\tau$. We will need very small Yukawa couplings of singlet neutrinos, so the elastic processes can be safely neglected. The imaginary parts $\Gamma_a$ of the neutrino potentials in medium can be formally regarded as widths describing the “decay” of the singlet neutrino states.
At late times the lepton asymmetry is stored in the longest living eigenstate of the Hamiltonian $\hat{H}$ which at $t \sim t_{EW}$ coincides with the mass eigenstate $N_3 \approx N_C$. The conditions that $N_1(\approx N_A)$ and $N_2(\approx N_B)$ come into thermal equilibrium before the time $t_{EW}$, while $N_3$ do not, are

$$\Gamma_{1,2}(T_{EW}) > H(T_{EW}), \quad \Gamma_3(T_{EW}) < H(T_{EW}),$$

where $H(T) = T^2/M_{Pl}^3$ is the Hubble parameter, $M_{Pl}^3 \equiv M_{Pl}/1.66\sqrt{g_*} \simeq 10^{18}$ GeV, and $\Gamma_3 \approx \Gamma_C + s_{13}^2 \Gamma_A + s_{23}^2 \Gamma_B$. Here $s_{13} \equiv \sin \theta_{13}$ and $s_{23} \equiv \sin \theta_{23}$ determine the admixtures of $N_A$ and $N_B$ in $N_3$.

The conditions (4) translate into bounds on the Yukawa couplings,

$$h_{A,B}^2 > 2 \cdot 10^{-14}, \quad h_C^2 < 2 \cdot 10^{-14},$$

and on mixing angles,

$$s_{13}^2 < 2 \cdot 10^{-14} h_A^{-2}, \quad s_{23}^2 < 2 \cdot 10^{-14} h_B^{-2}.$$

Eq. (5) implies a certain hierarchy between the Yukawa couplings, which, however, need not be very strong.

4. As described above, to find the baryon asymmetry we should calculate the asymmetry $\Delta_L(t_{EW})$ which was communicated to usual leptons by the time $t_{EW}$ at which sphalerons switch off. This asymmetry emerges because singlet neutrinos $N_3$ do not transfer their asymmetry to active neutrinos by the time $t_{EW}$ due to the smallness of $h_C$, $s_{13}$ and $s_{23}$. Since the total lepton number is conserved in all processes of interest, we have $\Delta_L(t_{EW}) = -\Delta_3(t_{EW})$ (up to a factor of order one that accounts for the distribution of the asymmetry between $B$, $L$ and $L_A + L_B$; we will not write this factor in formulas below), where $\Delta_3(t_{EW})$ is the asymmetry stored in $N_3$. The asymmetry $\Delta_3(t_{EW})$ can be found as follows. Let $S(t, t_0)$ be the evolution matrix corresponding to the Hamiltonian $\hat{H}$ (notice that $S(t, t_0)$ is not unitary since the Hamiltonian (1) is non-Hermitean). Then the probability to find $N_b$ at time $t$ if the state produced at $t_0$ was $N_a (a, b = A, B, C)$ equals $|S_{ba}(t, t_0)|^2$. Notice that singlet neutrinos are produced in the Yukawa eigenstates $N_a$ which then evolve independently. Since $N_a$ are created and destroyed in collisions of particles which are in thermal equilibrium, their production and destruction rates coincide. Taking this into account we can write the ratio of the number density of $N_3$ to the equilibrium density of one spin degree of freedom (in the approximation of Boltzmann statistics) at time $t_{EW}$ as
where \( t_i \) is the time at which the production of singlet neutrino begins. The asymmetry \( \Delta_3(t_{EW}) \) is the CP-odd part of the quantity (7).

In the case under consideration, where the mechanism of production of singlet neutrinos and the mechanism by which they communicate their lepton number to ordinary leptons is the same, the integration over the production time \( t_0 \) can be performed in a closed form. Indeed, the matrices \( S \) and \( S^\dagger \) obey \( \partial_{t_0} S = iS \dot{H}(t_0), \partial_{t_0} S^\dagger = -i\dot{H}^\dagger(t_0)S^\dagger \). From these equations and \( \Gamma = i(\dot{H} - \dot{H}^\dagger) \), one finds that \( \partial_{t_0} (SS^\dagger) = \Gamma(t_0)S^\dagger \). Using this relation one can readily perform the integration over the production time \( t_0 \) in Eq. (7),

\[
\frac{n_3(t_{EW})}{n_{eq}(t_{EW})} = 1 - \left[ S^M(t_{EW}, t_i)\bar{S}^M(t_{EW}, t_i) \right]_{33}.
\]

Here \( S^M \equiv U^\dagger SU \) is the evolution matrix in the mass eigenstate basis. The CP-odd part of this expression determines the asymmetry transferred to usual leptons by \( t_{EW} \) (and hence the generated baryon asymmetry)

\[
\Delta_L(t_{EW}) \equiv (n_L - n_L)/n_\gamma = -\Delta_3(t_{EW})
\]

\[
= \frac{1}{2} \sum_i |S^M_{3i}(t_{EW}, t_i)|^2_{CP-odd},
\]

where the factor 1/2 accounts for two helicity states of photon. The production of singlet neutrinos starts at very early times, so we set \( t_i = 0 \). Since \( T_{EW} \) is much smaller than all relevant energy parameters in the problem, one can formally let \( t_{EW} \to \infty \) in actual calculations.

5. The lepton asymmetry is produced mainly at the epoch \( t_L \) when the differences of the eigenvalues of the Hamiltonian \( \Omega_{ij} \equiv \Omega_i - \Omega_j \) become of order of the Hubble parameter \( H \): \( \Omega_{ij}(t_L) \sim 1/t_L \). Indeed, at \( t \ll t_L \) the wave functions essentially stay constant, whereas at \( t \gg t_L \) they undergo fast oscillations and due to averaging effects the asymmetry is strongly suppressed.

In what follows we will present the results for the most interesting range of the parameter space where the mass differences are relatively large, \( \Delta M^2 \gg (h_{ab}/8)^3 M_{Pl}^2 \) \( (h_{ab} \equiv h_a - h_b) \). \( \Delta M^2 \) being the typical value of \( \Delta M^2_{ij} \). In this case the mass terms dominate over potentials in \( \dot{H} \) at the epoch \( t_L \), so that \( \Omega_{ij} \sim \Delta M^2_{ij}/2T \) and the leptogenesis temperature is \( T_L \equiv T(t_L) \sim (M_{Pl}^2 \Delta M^2_{ij})^{1/3} \). At \( t = t_L \) we have
\[
\frac{|V_{ab}|}{H} = \frac{h_{ab}^2}{8} \left( \frac{M_P^2}{\Delta M^2} \right)^{1/3} \equiv \lambda \ll 1. \tag{9}
\]

This means that the potentials \( \tilde{V}_a \) can be treated perturbatively with \( \lambda \) being the expansion parameter. The lepton asymmetry (8) appears in the third order of perturbation theory, and therefore is suppressed by the cube of \( \lambda \). This can be seen in the mass eigenstate basis, where the potential has the form \( U^\dagger \tilde{V} U \). Indeed, \( \Delta L \) is a CP-violating observable, so it should be proportional to the invariant \( J = s_{12} c_{13} c_{23} c_{23} \sin \delta \); this invariant can be collected from \([U^\dagger V U]^3\) only. It is clear from eqs. (7) and (8) that \( \Delta L \) vanishes in the limit \( \Gamma \to 0 \), so one expects it to be proportional also to the phase of \( \tilde{V} \), i.e., to \( \sin \phi \equiv \Gamma /2V \simeq 2 \cdot 10^{-2} \). Therefore, up to a numerical constant we have an estimate

\[
\Delta L \sim J \lambda^3 \sin \phi. \tag{10}
\]

The calculations are simplified if \( |\Delta M_{13}^2| \ll |\Delta M_{12}^2|, |\Delta M_{23}^2| \). In this case, using Eq. (8), we obtain

\[
\Delta L = \frac{\Gamma(1/3)^3}{384} J \sin \phi \frac{h_{AC}^2 h_{AB}^2 h_{BC}^2 \cdot M_P^2}{(\Delta M_{13}^2)^{1/3}(\Delta M_{12}^2)^{2/3}}, \tag{11}
\]

where \( \Gamma(1/3) \simeq 2.68 \) is the value of the gamma function at 1/3. The estimate (10) as well as formula (11) are valid both in the case when \( N_2 \) equilibrate before \( t_{EW} \) and in the opposite case.

The asymmetry increases when the parameter \( \lambda \) approaches 1, i.e., the maximal effect for given \( h_a \) is expected when \( \Delta M^2 \sim (h_{ab}^2/8)^3 M_P^2 \). Notice, however, that these and smaller values of \( \Delta M^2 \) correspond to singlet neutrinos strongly degenerate in mass.

6. Let us present constraints on the parameters of singlet neutrinos and discuss the value of the asymmetry.

(i) To be consistent with the standard mechanism of nucleosynthesis, all singlet neutrinos, including the most weakly interacting one \( N_3 \), should decay before the nucleosynthesis epoch. The decay of \( N_3 \) at \( T \ll T_{EW} \) occurs due to its mixing with ordinary neutrino. Requiring that the decay rate of \( N_3 \) exceeds the inverse lifetime of the Universe at temperatures of order of a few MeV, and recalling Eq. (5), we obtain a lower bound on the mass, \( M_3 \geq 1 \text{ GeV} \). Alternatively, for \( M_3 \gg 1 \text{ GeV} \), we have a lower bound on the Yukawa constant, \( h_3^2 \simeq 10^{-16} (1 \text{ GeV}/M_3)^3 \) where \( h_3^2 \simeq h_C^2 + s_{13}^2 h_A^2 + s_{23}^2 h_B^2 \).
(ii) If the mass of $N_3$ is close to 1 GeV, and/or the Yukawa couplings of $N_A$ and $N_B$ are close to the bound (5), the decays of singlet neutrinos may lead to the reheating of the Universe after the electroweak epoch (but before nucleosynthesis), and hence to the dilution of the baryon asymmetry. This reheating is rather modest, however: given the constraints already imposed, the entropy density may increase at most by a factor of 10. In this case the baryon asymmetry produced before the reheating should be an order of magnitude larger than the observed one.

(iii) Baryon and lepton asymmetries should not be washed out before $T = T_{EW}$ by the Majorana mass itself. At $T \gg M_A$ the lepton number equilibration rate is suppressed with respect to the lepton charge conserving rate $\Gamma_A$, given by Eq. (3), by a factor $M_A^2/T^2$, so we have to require that $\Gamma_A(T_{EW})/(M_A^2/T_{EW}^2) \ll H(T_{EW})$, and similarly for $N_B$. The parameters of Majorana singlet neutrinos should therefore satisfy

$$1 \text{ GeV} \lesssim M_a \lesssim 100 \text{ GeV}, \quad h_A^2, h_B^2 \lesssim 10^{-10}. \quad (12)$$

The upper bounds here do not apply to Dirac singlet neutrinos.

The Lagrangian of the model leads, via the see-saw mechanism, to the generation of masses of the light (active) neutrinos: $m_a = h_a^2 v^2 / M_a$, where $v$ is the Higgs vacuum expectation value. The constraints (5) and (12) imply that the mass of the heaviest active neutrino is in the range $m_a = (10^{-2} - 10^{-3})$ eV. From the cosmological bound $m_a \lesssim 10$ eV we get a constraint which is somewhat stronger than Eq. (12), $h_A^2 \lesssim 10^{-11}$. For the lightest active neutrino, the constraints (5) and (12) lead to $m_C = (10^{-6} - 10^{-1})$ eV. (Notice that the mixing parameters of active neutrinos are unconstrained by our scenario as they are not directly related to the mixing matrix of singlet neutrinos.)

The above constraints imply that the condition (9) is indeed satisfied in a large part of the allowed parameter space. In particular, it holds in the two cases which we now turn to.

As follows from Eq. (11), in the case when two of the singlet neutrinos are relatively strongly interacting the desired lepton (and baryon) asymmetry $\Delta L \sim (a \text{ few}) \cdot 10^{-9}$ is obtained for generic values of the parameters subject to the above constraints. For example, for $h_A \sim h_B = 10^{-12}$, $\Delta M^2 \sim M_a^2$ and $M_a = 10$ GeV the correct asymmetry is generated provided that $J \gtrsim 10^{-3}$ which is certainly consistent with Eq. (6). The temperature of leptogenesis is $T_L \sim 10^7$ GeV. In this case two active neutrinos are relatively heavy, $m_A \sim m_B = (a \text{ few})$ eV, so that they can constitute the hot dark matter of the Universe. Moreover, oscillations between them can solve either the atmospheric or the solar neutrino problem, provided that their mass splitting is small.
A variant of our scenario makes use of weakly interacting $N_C$ and $N_B$. In this case two of the usual neutrino species have masses in the range $(10^{-6} - 10^{-1})$ eV, and the remaining one is relatively heavy. As an example, let us take $h_A^2 = 5 \cdot 10^{-14}$, $h_B^2 = 10^{-15}$, $h_C^2 < h_B^2$ and $M_a = 20$ GeV, which corresponds to the masses of usual neutrinos $m_A \sim 0.1$ eV, $m_B \sim 2 \cdot 10^{-3}$ eV and $m_C < m_B$. This variant fits particularly well into the mass pattern suggested by the solar and atmospheric neutrino data \[10\]. Given that $J \lesssim 10^{-2}$ due to the constraints analogous to Eq. (6), the correct value of asymmetry is obtained for $\Delta M^2 \lesssim 10^{-2}$ GeV$^2$. Thus, in this case the singlet neutrinos should be degenerate in mass. The temperature of leptogenesis is lower, $T_L \sim 3 \cdot 10^5$ GeV. Let us note in passing that the degeneracy of masses $M_a$ is helpful also for obtaining the desired baryon asymmetry for very small mixing angles $\theta_{ij}$.

The crucial feature of the suggested mechanism is that it works only if Yukawa couplings of all singlet neutrinos are small, $h_a \sim (10^{-8} - 10^{-6})$. This smallness can be explained, e.g., by the mixing of $N_a$ with very heavy right-handed neutrinos having Yukawa couplings of the same order of magnitude as those of quarks (and charged leptons) $h_q$. In this case $h_a \sim h_q \sqrt{M_a/M_R}$, and for our values of $h_a$ and $M_a$ the mass scale $M_R$ may be close to the Grand Unification scale, $M_R \sim 10^{16}$ GeV.

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[6] Our analysis can be applied with minor modifications to Dirac singlet neutrinos as well.


