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SIMPLE DERIVATION OF THE PICARD-FUCHS EQUATIONS FOR THE SEIBERG-WITTEN MODELS

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ABSTRACT

A closed form of the Picard-Fuchs equations for $N = 2$ supersymmetric Yang-Mills theories with massless hypermultiplet are obtained for classical Lie gauge groups. We consider any number of massless matter in fundamental representation so as to keep the theory asymptotically free.

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1 Introduction

Recently duality has become a very important tool both in supersymmetric Yang-Mills theories and string theory. Seiberg and Witten[?] have used duality and holomorphy to obtain the exact prepotential of $N = 2$ SYM theory with gauge group $SU(2)$. (For a review see: e.g.[?], [?] and [?].)

The key point in $N = 2$ SYM models was the discovery of a hyperelliptic curve with r complex dimensional moduli space (r is the rank of the gauge group) with certain singularities, which gives information about the low energy Willsonian effective action. The Seiberg-Witten data is a hyperelliptic curve with a certain meromorphic one form (E_{u_i}, λ_{SW}).

Indeed, the prepotential of $N = 2$ SYM theory in the Coulomb phase can be described with the aid of a family of complex curves with the identification of the vacuum expectation value (v.e.v) a_i and its dual a_i^D with the periods of the curve

$$a_i = \oint_{\alpha_i} \lambda_{SW} \quad \text{and} \quad a_i^D = \oint_{\beta_i} \lambda_{SW}, \tag{1}$$

where α_i and β_i are the homology cycles of the corresponding Riemann surface.

There are two well known methods for finding the periods and thereby the prepotential. The first method is to calculate the periods directly from the above integrals. This method has been developed in[?] and [?]. They explicitly calculated the full expansion of the renormalized order parameters using the method of residues. By this method, they worked out explicitly the perturbative corrections as well as the one and two instanton contributions to the effective prepotential.

On the other hand, one may use the fact that the periods $\Pi = (a_i, a_i^D)$ satisfy the Picard-Fuchs equations. Probably from the Picard-Fuchs equations one can obtain the prepotential in an analytic way, which is for example, important in the instanton calculus. Recently some of these equations have been obtained in [?] and[?]. Also in [?] we obtained a simple closed form of the Picard-Fuchs equations for Pure $N = 2$ SYM theories for classical Lie gauge groups. The Picard-Fuchs equations can also be obtained from the mirror symmetry in Calabi-Yau manifold[?].

In this article we extend the results of [?] to obtain a closed form for $N = 2$ SYM theories with classical Lie gauge groups which have massless matter in fundamental representation.

The hyperelliptic curves for classical gauge groups with any number of matter in the fundamental representation are known [?],[?] and [?]. Although in some cases different hyperelliptic curves have been proposed for the same gauge group and the same hypermultiplet contents, but it was shown in [?] by explicit calculations up to two instanton processes, that the corresponding effective prepotentials are the same for all these different curves. This equivalence results from the fact that the effective prepotential is unchanged under analytic reparametrizations of the classical order parameters[?].

The Seiberg-Witten data (E_{u_i}, λ_{SW}) for classical gauge groups with n_f massless matter in fundamental representation have been proposed as follows (see: [?],[?] and [?])

$$\begin{aligned} y^2 &= p^2(x) - G(x), \\ \lambda_{SW} &= (\frac{G'}{2G}p - p')\frac{xdx}{y}, \end{aligned} \tag{2}$$

where

$$p = x^{m+\epsilon} - \sum_{i=2}^{m} u_i x^{m+\epsilon-i} \tag{3}$$

with $m = r+1$, $i = 2, 3, ...$, $\epsilon = 0$ for A_r series and $m = 2r$, $i = 2, 4, ...$, $\epsilon = 0$ for B_r and D_r series, and $m = 2r$, $i = 2, 4, ...$, $\epsilon = 2$ for C_r series, and u_i's, the Casimirs of the gauge groups. Also

$$\begin{aligned} &G = \Lambda^{2m-n_f}(x - \delta_{2m-1,n_f})^{n_f} && for\ A_r \\ &G = \Lambda^{2m-2-2n_f}x^{2+2n_f} && for\ B_r \\ &G = \Lambda^{2m+4-2n_f}x^{2n_f} && for\ C_r \\ &G = \Lambda^{2m-4-2n_f}x^{4+2n_f} && for\ D_r. \end{aligned} \tag{4}$$

Note that the D_r series has an exceptional Casimir, t, of degree r, but in our notation we set $u_{2r} = t^2$.

From the explicit form of λ_{SW} and the fact that the λ_{SW} is linearly dependent on the Casimirs, setting $\frac{\partial}{\partial u_i} = \partial_i$ one can see

$$\begin{aligned} \partial_i \lambda_{SW} &= -\frac{x^{m+\epsilon-i}}{y}dx + d(*), \\ \partial_i \partial_j \lambda_{SW} &= -\frac{x^{2m+2\epsilon-i-j}}{y^3}p(x)dx + d(*). \end{aligned} \tag{5}$$

The procedure of derivation of the Picard-Fuchs equations is to find proper linear combinations of $\frac{x^{m-i}}{y}dx$ and $\frac{x^{2m-i-j}}{y^3}p(x)dx$ which give total derivative, then by integration from two sides and using (??) and (??), one can find second order differential equations for the periods.

For example, from the second equation of (??) one can find the following identity for the periods $\mathcal{L}_{i,j;p,q}\Pi = 0$ where

$$\mathcal{L}_{i,j;p,q} = \partial_i \partial_j - \partial_p \partial_q, \quad i + j = p + q \tag{6}$$

2 B_r and D_r Cases

From (??) the proposed hyperelliptic curve for these gauge groups with $n_f \leq m-k-1$ massless matter in the fundamental representation are

$$y^2 = p^2 - \Lambda^{2m-2k-2n_f} x^{2k+2n_f}, \tag{7}$$

where $k=1$ for B_r and $k=2$ for D_r.

By direct calculation one can see that

$$\frac{d}{dx}\left(\frac{x^n}{y}\right) = (n-k-n_f)\frac{x^{n-1}}{y} - (m-n_f-k)\frac{x^{m+n-1}}{y^3}p + \sum_{i=2}^{m}(m-k-n_f-i)u_i\frac{x^{m+n-1-i}}{y^3}p \tag{8}$$

Now from equation (??), we can find the second order diffrential equations for the periods ($\mathcal{L}_n\Pi = 0$) as follows

$$\mathcal{L}_n = (k+n_f-n)\partial_{m-n+1} + (m-n_f-k)\partial_2\partial_{m-n-1} - \sum_{i=2}^{m}(m-k-n_f-i)u_i\partial_i\partial_{m-n+1}. \tag{9}$$

Here $n = 2s-1$ and $s = 1, ..., r-1$. Note that for $s = r$ equation (??) does not give the second order differential equation with respect to u_i. So by this method we can only obtain $r-1$ differential equations. Another equation can be obtained by following linear combination

$$D_0 = (k+n_f-m)d\left(\frac{x^{m+1}}{y}\right) + \sum_{i=2}^{m}(m-k-n_f+i)u_i d\left(\frac{x^{m+1-i}}{y}\right) \tag{10}$$

or

$$\begin{aligned} D_0 &= \lambda_{SW} - \Big(\sum_{i=2}^{m} i(i-2)u_i\frac{x^{m-i}}{y} + \sum_{j,i=2}^{m} iju_iu_j\frac{x^{2m-i-j}}{y^3}p \\ &- (m-n_f-k)^2\Lambda^{2m-2n_f-2k}\frac{x^{2n_f+2k}}{y^3}p\Big)dx. \end{aligned} \tag{11}$$

Note that, for the case of $n_f = m-k-1$, one cannot write the last term in the above equation as the form of $\frac{x^{2m-i-j}}{y^3}p(x)dx$, so it only gives the second order differention equation for the periods in the case of $1 \leq n_f \leq m-k-2$, which is

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2)u_i\partial_i + \sum_{j,i=2}^{m} iju_iu_j\partial_i\partial_j - (m-n_f-k)^2\Lambda^{2m-2k-2n_f}\partial^2. \tag{12}$$

where for $SO(m+1)$

$$\begin{aligned} \partial^2 &= \partial_{m-n_f-k}\partial_{m-n_f-k} \qquad n_f \ odd \\ \partial^2 &= \partial_{m-n_f-2k}\partial_{m-n_f} \qquad n_f \ even \end{aligned} \tag{13}$$

and for $SO(m)$

$$\begin{aligned} \partial^2 &= \partial_{m-n_f-k}\partial_{m-n_f-k} \qquad n_f \ even \\ \partial^2 &= \partial_{m-n_f-k-1}\partial_{m-n_f-k+1} \qquad n_f \ odd \end{aligned} \tag{14}$$

For the case of $n_f = m-k-1$ we should add an extra term to D_0 to cancel the last term, which is

$$D = D_0 + \Lambda^2 d(\frac{x^{m-1}}{y}). \tag{15}$$

so the last term of (??) changes to

$$\Lambda^2 \sum_{i=2}^{m} (i-1) u_i \partial_2 \partial_i. \tag{16}$$

Equations (??) and (??) give a complete set of the Picard-Fuchs equations for the periods for gauge groups B_r and D_r with any number of massless matter so as to keep the theory asymptoticlly free. ($n_f \leq m-k-1$).

3 C_r Case

First let us write the Picard-Fuchs equations for the pure gauge theory.[1] The proposed curve for pure $N=2$ SYM with gauge group $SP(m)$ is [?]

$$x^2 y^2 = p^2 - \Lambda^{2m+4} \tag{17}$$

where

$$p = x^{m+2} - \sum_{i=2}^{m} u_i x^{m+2-i} + \Lambda^{m+2} \quad i = 2, 4, ..., m \tag{18}$$

By direct calculation one can see that

$$\frac{d}{dx}(\frac{x^n}{z}) = n\frac{x^{n-1}}{z} - \frac{x^n p'}{z^3} p, \tag{19}$$

where $z = xy$. So from (??) we have the following second order differential for the periods

$$\mathcal{L}_n = -n\partial_{m-n+3} + (m+2)\partial_2\partial_{m-n+1} - \sum_{i=2}^{m} (m+2-i) u_i \partial_i \partial_{m-n+3}. \tag{20}$$

here $n = 2s+1$ and $s = 1, ..., r-1$. Just as for the B_r and D_r cases, there is a difficulty for $s = r$, again, we have only $r-1$ differential equations. One can see that, the last equation can be obtained by the following linear combination

$$D_0 = -(m+2)d(\frac{x^{m+3}}{z}) + \sum_{i=2}^{m} (m+2+i) u_i d(\frac{x^{m+3-i}}{z}) - (m+2)^2 \Lambda^{m+2} d(\frac{x}{z}) \tag{21}$$

which gives a second order differential equation for the periods

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2) u_i \partial_i + \sum_{j,i=2}^{m} ij u_i u_j \partial_i \partial_j + (m+2)^2 \Lambda^{m+2} \sum_{i=0}^{m} (m-i) u_i \partial_m \partial_{i+2}. \tag{22}$$

[1]Here our notation for C_r gauge group is different from the one in[?].

here $u_0 = -1$.

Let us return to our task of obtaining the Picard-Fuchs equations for C_r gauge group with $n_f \leq m+1$ massless matter in the fundamental representation. From (??) the proposed curve for this theory is

$$y^2 = p^2 - \Lambda^{2(m+2-n_f)} x^{2n_f} \tag{23}$$

One can see that

$$\frac{d}{dx}(\frac{x^n}{y}) = (n-n_f)\frac{x^{n-1}}{y} - (m+2-n_f)\frac{x^{m+n-1}}{y^3}p + \sum_{i=2}^{m}(m+2-n_f-i)u_i\frac{x^{m+n-1-i}}{y^3}p \tag{24}$$

which gives the following differential equations

$$\mathcal{L}_n = (n_f-n)\partial_{m-n+3} + (m+2-n_f)\partial_2\partial_{m-n+1} - \sum_{i=2}^{m}(m+2-n_f-i)u_i\partial_i\partial_{m-n+3}. \tag{25}$$

As in the previous cases from this method we obtain only $r-1$ differential equations. The last equation can be obtained from the following linear combination

$$D_0 = (n_f-m-2)d(\frac{x^{m+3}}{y}) + \sum_{i=2}^{m}(m+2-n_f+i)u_i d(\frac{x^{m+3-i}}{y}) \tag{26}$$

or

$$\begin{aligned} D_0 &= \lambda_{SW} - (\sum_{i=2}^{m} i(i-2)u_i\frac{x^{m+2-i}}{y} + \sum_{j,i=2}^{m} iju_iu_j\frac{x^{2m+4-i-j}}{y^3}p \\ &- (m+2-n_f)^2\Lambda^{2m+4-2n_f}\frac{x^{2n_f}}{y^3}p)dx. \end{aligned} \tag{27}$$

Similar to B_r and D_r cases, for $n_f = 1$ and $n_f = m+1$, the last term in the above expression cannot be rewriten in the form of $\frac{x^{2m+4-i-j}}{y^3}p(x)dx$, so we should add an extra term to D_0. For the case of $2 \leq n_f \leq m$, the above equation gives a second order differential equation

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2)u_i\partial_i + \sum_{j,i=2}^{m} iju_iu_j\partial_i\partial_j - (m+2-n_f)^2\Lambda^{2m+4-2n_f}\partial^2. \tag{28}$$

where $\partial^2 = \partial_{m+2-n_f}\partial_{m+2-n_f}$ for even n_f and $\partial^2 = \partial_{m+1-n_f}\partial_{m+3-n_f}$ for odd n_f. For $n_f = m+1$ D_0 should change to

$$D = D_0 + \Lambda^2 d(\frac{x^{m+1}}{y}), \tag{29}$$

so the last term of equation (??) changes to $\Lambda^2 \sum (i-1)u_i\partial_2\partial_i$, and for $n_f = 1$

$$D = D_0 - \frac{(m+1)^2}{u_m}\Lambda^{2(m+1)}d(\frac{x}{y}), \tag{30}$$

and the last term of equation (??) changes to

$$(m+1)^2\frac{\Lambda^{2m+2}}{u_m}\sum_{i=0}^{m-2}(m+1-i)u_i\partial_{i+2}\partial_m \tag{31}$$

where $u_0 = -1$.

4 A_r Case

Consider gauge group $SU(m)$ with $n_f \leq 2m-2$ massless hypermultiplets[2] in the defining representation of the gauge group. From (??) the hyperelliptic curve for this model is

$$y^2 \;=\; p^2(x) - \Lambda^{2m-n_f} x^{n_f} \tag{32}$$

where

$$p(x) = x^m - \sum_{i=2}^{m} u_i x^{m-i} \tag{33}$$

By direct calculation one can see that

$$\frac{d}{dx}(\frac{x^n}{y}) = (n - \frac{n_f}{2})\frac{x^{n-1}}{y} - (m - \frac{n_f}{2})\frac{x^{m+n-1}}{y^3}p + \sum_{i=2}^{m}(m - \frac{n_f}{2} - i)u_i \frac{x^{m+n-1-i}}{y^3}p \tag{34}$$

From (??) we can find the second order differential equation for the periods ($\mathcal{L}_n \Pi = 0$) as follow

$$\mathcal{L}_n = (\frac{n_f}{2} - n)\partial_{m-n+1} + (m - \frac{n_f}{2})\partial_2 \partial_{m-n-1} - \sum_{i=2}^{m}(m - \frac{n_f}{2} - i)u_i \partial_i \partial_{m-n+1}. \tag{35}$$

where $n = s-1$ and $s = 2, ..., r-1$. As before, for $s = r$ (??) does not give the second order differential equation. Moreover, here for $s = 1$ the same difficulty arises as well. So by this method we can only find $r-2$ equations. Two other equations can be obtaind by considering a particular linear combination of $d(\frac{x^j}{y})$.

Consider the following linear combination

$$D_0 = (\frac{n_f}{2} - m)d(\frac{x^{m+1}}{y}) + \sum_{i=2}^{m}(m - \frac{n_f}{2} + i)u_i d(\frac{x^{m+1-i}}{y}) \tag{36}$$

or

$$D_0 = \lambda_{SW} - (\sum_{i=2}^{m} i(i-2)u_i \frac{x^{m-i}}{y} + \sum_{j,i=2}^{m} iju_i u_j \frac{x^{2m-i-j}}{y^3}p - (m - \frac{n_f}{2})^2 \Lambda^{2m-n_f}\frac{x^{n_f}}{y^3}p)dx. \tag{37}$$

which gives the second order differential equation for the periods in the case of $0 \leq n_f \leq 2m-4$ and take the following form

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2)u_i \partial_i + \sum_{j,i=2}^{m} iju_i u_j \partial_i \partial_j - (m - \frac{n_f}{2})^2 \Lambda^{2m-n_f}\partial^2. \tag{38}$$

where $\partial^2 = \partial_m \partial_{m-n_f}$ for $n_f \leq m-2$ and $\partial^2 = \partial_2 \partial_{m-l-2}$ and $l = -1, 0, ..., m-4$ for $m-1 \leq n_f (= m+l) \leq 2m-4$. For the cases of $2m-3 \leq n_f \leq 2m-2$, one should add an extra term to D_0 as follows

[2]For $n_f = 2m-1$, because of Λ dependent term $((x - a_0\Lambda)^n_f$ where the coefficient a_0 comes from instanton calculations), there is a difficulty, which also arises in the massive case. So we postpone it to future study.

For $n_f = 2m-3$

$$D = D_0 + \frac{3}{2}\Lambda^3 d(\frac{x^{m-2}}{y}), \tag{39}$$

so

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2)u_i\partial_i + \sum_{j,i=2}^{m} iju_iu_j\partial_i\partial_j + \frac{3}{2}\Lambda^3\sum_{i=2}^{m}(i-\frac{3}{2})u_i\partial_3\partial_i + \frac{3}{4}\Lambda^3\partial_3. \tag{40}$$

For $n_f = 2m-2$

$$D = D_0 + \Lambda^2 d(\frac{x^{m-1}}{y}), \tag{41}$$

so

$$\mathcal{L}_r = 1 + \sum_{i=2}^{m} i(i-2)u_i\partial_i + \sum_{j,i=2}^{m} iju_iu_j\partial_i\partial_j + \Lambda^2\sum_{i=2}^{m}(i-1)u_i\partial_2\partial_i. \tag{42}$$

Finally, the last differential equation for the periods can be obtained from the following linear combination (which is analogous to $d(\frac{1}{y})$ in the pure case [?])

$$E_0 = (m-\frac{n_f}{2})d(\frac{x^m}{y}) + \frac{n_f}{4}(n_f-2m-4)u_2 d(\frac{x^{m-2}}{y}) + (m-\frac{n_f}{2})^2\sum_{i=3}^{m} u_i d(\frac{x^{m-i}}{y}). \tag{43}$$

which gives the second order differential equation for periods in the cases of $1 \leq n_f \leq 2m-3$.

$$\begin{aligned}\mathcal{L}_0 \;=\; & c_2(e-2)u_2\partial_3 + \sum_{i=2}^{m}(i-m)e^2u_i\partial_{i+1} + \sum_{i=3}^{m} c_i e u_i\partial_{i-1}\partial_2 + \sum_{i=2}^{m} c_2(e-i)u_2u_i\partial_3\partial_i \\ & + \sum_{i,j=2}^{m}(i-m)e^2u_iu_j\partial_{i+1}\partial_j + e^2\frac{n_f}{2}\Lambda^{2m-n_f}\partial^2. \end{aligned} \tag{44}$$

where $\partial^2 = \partial_m\partial_{m-n_f+1}$ for $n_f \leq m-1$ and $\partial^2 = \partial_2\partial_{m-l-1}$, $l = 0, ..., m-3$ for $m \leq n_f (= m+l) \leq 2m-3$ and also $e = (m-\frac{n_f}{2})$, $c_i = me + \frac{in_f}{2}$. For the case of $n_f = 2m-2$ one should add an extra term as follows

$$E = E_0 - (m-1)\Lambda^2\frac{d}{dx}(\frac{x^{m-2}}{y}) \tag{45}$$

and then the last term in equation (??) changes to

$$(1-m)\Lambda^2(\partial_3 - \sum_{i=2}^{m}(1-i)u_i\partial_3\partial_i). \tag{46}$$

At the end, note that equation (??) is not valid for $SU(2)$ with $n_f = 1$ as it should be. This inconsistency comes from the $d(\frac{1}{y})$ term in (??). One can see that the Picard-Fuchs equation for this case is obtained from the following combination

$$D = D_0 + \frac{9}{32}\frac{\Lambda^3}{u}d(\frac{x^2-3u}{y}), \tag{47}$$

and gives the well-known result

$$\mathcal{L}_1 = 1 + (4u^2 + \frac{27}{64}\frac{\Lambda^6}{u})\partial_u^2. \tag{48}$$

5 Conclusion

To compare our results for the groups of rank $r \leq 3$ with the results of [?] and [?], let us, for example, consider $SU(4)$ with one massless matter. From the equations (??), (??) and (??) we have

$$
\begin{aligned}
\mathcal{L}_0 &= 8u_2\partial_3 + 49u_3\partial_4 - 217u_3\partial_{22} + (8u_2^2 - 224u_4)\partial_{23} + 117u_2u_3\partial_{24} \\
&+ (49u_3^2 + 128u_2u_3)\partial_{34} + (49u_3u_4 - \frac{49}{2}\Lambda^7)\partial_{44}, \\
\mathcal{L}_1 &= \partial_4 - 7\partial_{22} + 3u_2\partial_{24} + u_3\partial_{34} - u_4\partial_{44}, \\
\mathcal{L}_3 &= 1 + 3u_3\partial_3 + 8u_4\partial_4 + 4u_2^2\partial_{22} + (9u_3^2 + 16u_2u_4)\partial_{33} + 16u_4^2\partial_{44} + 12u_2u_3\partial_{23} \\
&+ (24u_3u_4 - \frac{49}{4}\Lambda^7)\partial_{34},
\end{aligned} \tag{49}
$$

where $\partial_{ij} = \partial_i\partial_j$. One can check that these equations are linear combinations of those of[?].

To summarize, we have obtained a closed form for the Picard-Fuchs equations for $N = 2$ SYM theories with classical Lie gauge groups which have massless matter in the fundamental representation.

Note: After the completion of this work, I received the paper[?] which has considerable overlap with our work.

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