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ON THE BOUND STATES OF $p-$ AND $(p+2)$-BRANES

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ABSTRACT

We study bound states of D-p-branes and D-$(p+2)$-branes. By switching on a large magnetic field F on the $(p+2)$ brane, the problem is shown to admit a perturbative analysis in an expansion in inverse powers of F. It is found that, to the leading order in $1/F$, the quartic potential of the tachyonic state from the open string stretched between the p- and $(p+2)$-brane gives a vacuum energy which agrees with the prediction of the BPS mass formula for the bound state. We generalize the discussion to the case of m p-branes plus 1 $(p+2)$-brane with magnetic field. The T dual picture of this, namely several $(p+2)$-branes carrying some p-brane charges through magnetic flux is also discussed, where the perturbative treatment is available in the small F limit. We show that once again, in the same approximation, the tachyon condensates give rise to the correct BPS mass formula. The role of 't Hooft's toron configurations in the extension of the above results beyond the quartic approximation as well as the issue of the unbroken gauge symmetries are discussed. We comment on the connection between the present bound state problem and Kondo-like problems in the context of relevant boundary perturbations of boundary conformal field theories.

1. Introduction

The question of bound states of different p-branes have played an important role in the understanding of various dualities. For example, the $SL(2,Z)$ duality symmetry of type IIB, predicts bound states of n number of 1-branes carrying R-R and m number of the ones carrying NS-NS charge with n and m being relatively prime [?, ?]. Indeed a 1-brane carrying the above charges is expected to have a tension given by the BPS expression:

$$\mathrm{T}(n,m) = T\sqrt{\frac{n^2}{\lambda^2} + m^2} \tag{1.1}$$

where λ is the string coupling constant and $T = \frac{1}{2\pi\alpha'}$. Throughout we will set $\alpha' = 1/2$. Since $\mathrm{T}(n,m)$ is less than the sum of the tensions of individual 1-branes, i.e. $T(n/\lambda + m)$, this system is expected to form a bound state. This was shown by Witten [?] by considering the world-sheet theory of n D-branes [?] which is a supersymmetric $N = 8$, $U(n)$ gauge theory. The NS-NS charge is introduced by giving vacuum expectation value to field strengths that correspond to a source transforming as m-th rank anti-symmetric tensor representation of $U(n)$. As a result, the $SU(n)$ part of $U(n)$ develops a mass gap showing the formation of a bound state, while the $U(1)$ part which corresponds to the center of mass motion gives rise to the same degrees of freedom as that of the fundamental string. The above expression for the tension follows from the Born-Infeld action for the center of mass $U(1)$ taking into account the above expectation value of its field strength. This is in agreement with the fact that the fundamental string is in the same $SL(2,Z)$ orbit as (n,m) string.

Starting from this result and applying various T and S-dualities one can arrive at statements about the existence of bound states of other p-brane systems. For example let us compactify two of the transverse directions of the (n,m)- string on a torus. Performing two T duality transformations one gets a system of n D-3-branes carrying m units of NS-NS 1-brane charge through the non-zero expectation value of the electric field. Further application of S-duality turns this system to that of n D-3-branes with m units of D-1-

brane charge. As a result of S-duality transformation the original non-zero expectation value of electric field is transformed to a non-zero magnetic field on the torus. This is in agreement with the existence of the Chern-Simons term $\int F \wedge A_2$ in the 3-brane world volume action where A_2 is the R-R 2-form. As noted in [?], the presence of this term implies that, when F has non trivial expectation value, the 3-brane carries D-1-charge. In this picture the quantization of D-1-charge is just a consequence of the quantization of the magnetic flux on the torus. One can easily see that under these duality transformations the tension given in eq. (??) is transformed to the following expression:

$$\mathrm{T}(n,m) = \frac{TV}{2\pi^2\lambda}\sqrt{n^2 + \frac{4\pi^4 m^2}{V^2}} \tag{1.2}$$

where V is volume of the torus. Note that although in the above discussion we restricted ourselves to 3- and 1-brane system, by T-duality it also applies to a system of $(p+2)$- and p-branes. In the above approach the p-brane charge appeared due to the expectation value of the magnetic field on the $(p+2)$-brane. However one can ask the question what happens when there are separate $(p+2)$- and p-branes. It is known that this system is not supersymmetric due to the number of mixed Neumann-Dirichlet (ND)-directions being 2 [?]. Moreover the lowest mode of the string stretched between the p- and $(p+2)$- branes has the mass given by:

$$m^2 = -\frac{1}{2} + \frac{b^2}{\pi^2} \tag{1.3}$$

where b is the distance between the two branes. As a result when $b^2 \leq \frac{\pi^2}{2}$ the above mode becomes tachyonic. One expects [?] that the dynamics of this tachyon will give a ground state in accordance with what we saw above from the application of dualities on (m,n)-string [?]. In particular, since the ground state energy of a system of separated p- and $(p+2)$- branes is strictly greater than the BPS tension for the bound state, the tachyon should provide a negative contribution to the vacuum energy of the $p-(p+2)$ system by the amount given by the difference between the bound state tension and the sum of the individual tensions.

In this paper we will study various configurations of p and $(p+2)$-branes, where, considering the tachyon potential to the quartic order, we show that to the leading approximation in small (or large) volume limit the above expectation turns out to be correct. In section 2, we study the tachyon potential when a $(1, N)$ system (i.e. a single $(p+2)$-brane carrying N units of p-brane charge) is brought near a single p-brane (i.e. a $(0, 1)$ system). In the Appendix we give the string derivation of the quartic potential of the tachyon. We also generalize this result when $(1, N)$ system is brought near $(0, m)$ system consisting of m p-branes. In section 3, we discuss the T-dual situation of the previous section. Thus we consider N $(p+2)$-branes where one of the branes carries a p-brane charge via a unit magnetic flux and arrive at the bound state formation by minimizing the tachyon potential. In this section we also discuss what happens when an $(N, 1)$ system is brought near a (1,0) system, which is the exact T-dual of the situation considered in section 2. In section 4, we discuss (N, m) bound states obtained by taking N $(p+2)$-branes with m units of magnetic fluxes. In this case again we find bound states. In section 5, we show that the above results can be extended beyond the quartic order and the resulting solution turns out to be related to 't Hooft's torons [?]. We also discuss the issue of the unbroken gauge symmetries in the presence of these torons and verify that the massive Kaluza-Klein spectrum agrees with T-duality. In section 6, we make some concluding remarks and in particular show that string duality implies the existence of non-trivial fixed points in a class of boundary conformal field theories that are perturbed by some relevant boundary operators. In fact the cases considered in the present paper are just the situations when the relevant operators are almost marginal. However the string duality predicts that this phenomenon must continue to hold even when the relevant operators are far from being marginal and there is a precise prediction on the value of the g-function (i.e. disc partition function) at the new fixed point.

2. Bound State of $(1, N)$ and $(0, 1)$ Systems for Large N/V

Let us consider an $(1, N)$ system where N units of p-brane charge has been introduced by turning on N units of the magnetic flux on the torus. Now imagine bringing a single p-brane close to the $(1, N)$ system. The sum of the two tensions is

$$\mathrm{T}(1,N)+\mathrm{T}(0,1)=\frac{TV}{2\pi^2\lambda}\Big(\sqrt{1+\frac{4\pi^4N^2}{V^2}}+\frac{2\pi^2}{V}\Big). \tag{2.1}$$

On the other hand, the combined system which carries the charge $(N+1, 1)$ should have a tension

$$\mathrm{T}(1,N+1)=\frac{TV}{2\pi^2\lambda}\sqrt{1+\frac{4\pi^4(N+1)^2}{V^2}}. \tag{2.2}$$

The first point to notice is the fact that the two tensions differ at order $1/\lambda$ and therefore one should be able to understand the formation of the bound state by minimizing the tree level potential for the tachyons. The potential at the minimum should account for the difference between the two tensions. For large N/V, the difference, $\delta\mathrm{T} = \mathrm{T}(1, N + 1) - \mathrm{T}(1, N) - \mathrm{T}(0, 1)$, between these two values of tension goes as:

$$\delta\mathrm{T}=\frac{-TV^2}{8\pi^4\lambda N(N+1)}+\mathcal{O}\Big(\frac{V^3}{N^3}\Big), \tag{2.3}$$

and therefore one may hope to find a solution in perturbation theory. This limit can be achieved either by taking small volume limit for a fixed N, or by taking large N limit for a fixed volume. As we shall see below, the minimization of the tachyon potential to the quartic order reproduces the difference (??).

As mentioned above, the tachyon field is the lowest mode of the open string stretched between the p- and $(p + 2)$-branes. On the p-brane side the string satisfies the Dirichlet boundary condition, while on the $(p + 2)$-brane side the Neumann condition. In refs. [?, ?, ?], the quantization of open strings in the presence of constant magnetic field has been discussed. The effect of the magnetic field on the $(p+2)$-brane can be incorporated by modifying the boundary condition of the open string from Neumann to a mixed boundary condition given by

$$\partial_\sigma X = -i\pi F \partial_\tau X \tag{2.4}$$

where X is the complex coordinate on the torus and $F = \frac{2\pi N}{V}$ is the quantized magnetic field. Taking into account the Dirichlet boundary condition on the p-brane side one has the following mode expansion for X:

$$\begin{aligned} X &= i\sum_{n=1}^{\infty} a_n \phi_n(\sigma,\tau) - i\sum_{n=0}^{\infty} \tilde{a}_n^\dagger \phi_{-n}(\sigma,\tau) \\ \phi_n(\sigma,\tau) &= (n-\frac{1}{2}-\epsilon)^{-\frac{1}{2}} e^{-i(n-\frac{1}{2}-\epsilon)\tau} cos[(n-\frac{1}{2}-\epsilon)\sigma + \pi\epsilon)] \end{aligned} \tag{2.5}$$

where

$$\epsilon = \frac{1}{\pi} arctan(\pi F) \tag{2.6}$$

There is also a similar mode expansion for fermions. Note that in the above mode expansion we do not have a zero mode for X due to the DN boundary condition. This is to be contrasted with the NN case studied in refs. [?, ?, ?] where X has a zero mode x which satisfies a modified commutation relation $[x, \bar{x}] = 1/F$ due to the presence of magnetic field. This resulted in a degeneracy of states $FV/2\pi = N$ due to the phase space factor. In the present case since there is no zero mode of X, we do not have this degeneracy. This can also be understood by starting from NN string with magnetic field on both sides with fluxes being M and N units respectively. The multiplicity of states in this case will be $M + N$. Now taking $M \to \infty$ one arrives at the DN string that we are considering, but with a multiplicity factor which goes to infinity. This multiplicity factor is just the choice of the position of the D-end of the string, which in our problem is fixed to be the position of the p-brane.

Taking into account the zero point energy of the bosons and fermions, one finds that the lowest modes of the states in the Neveu-Schwarz sector, when the p-brane is inside the $(p+2)$-brane (i.e. $b = 0$) have masses:

$$m^2 = (2n-1)(\frac{1}{2} - \epsilon) \tag{2.7}$$

Here n labels the Landau levels obtained by applying $\tilde{a}_0^\dagger$. The tachyon of course appears only at the lowest Landau level i.e. $n = 0$. Now let us take the large N/V limit. From eq. (??) it follows that $\epsilon \to \frac{1}{2} - \frac{V}{2\pi^3 N} + \mathcal{O}(\frac{V^2}{N^2})$. The mass square of the tachyon to the leading order is therefore $-V/2\pi^3 N$.

Let us now discuss the quartic interaction term in the tachyon potential. Since in the large N/V limit, the Neumann boundary condition on the $(p+2)$-brane end becomes effectively Dirichlet, the string stretched between the two branes becomes DD. This means that in this limit the tachyon vertex operator which involves twist fields for bosons and fermions (twisting by $\frac{V}{2\pi^3 N}$) goes over to that of the DD- scalar $\partial_\sigma X$. We expect therefore that the quartic term in the potential starts from order $(\frac{V}{N})^0$. If there is a non-zero contribution to the quartic term at this leading order, then minimizing the potential with respect to the tachyon field shows that its vacuum expectation value (vev) is of order $\sqrt{\frac{V}{N}}$ and, correspondingly, the vacuum energy is of order $(\frac{V}{N})^2$: we will see that in fact it agrees with the BPS prediction (??). The quartic term can be calculated directly at the string level by inserting four twist fields corresponding to the tachyon fields on a disc and removing the reducible diagrams arising from the exchange of the two $U(1)$ gauge fields. One can then take the limit $V/N \to 0$ in which case the on-shell momenta of the tachyons satisfy the massless condition and the potential can be calculated by taking the zero momentum limit. We have given the details of this computation in the Appendix. Denoting the tachyon field by χ, the resulting potential upto the quartic term (and upto the relevant order in V/N) is given by:

$$\mathcal{V} = -\frac{V}{2\pi^3 N}\chi\bar{\chi} + \frac{1}{2\pi^2}(1 + \frac{1}{N})(\chi\bar{\chi})^2 \tag{2.8}$$

which has a minimum at $\chi\bar{\chi} = \frac{V}{2\pi(N+1)}$. The resulting value of the potential at this minimum is $-V^2/8\pi^4 N(N+1)$. This is exactly the desired value to this order as we saw above when we considered the difference δT between the tension of the bound state and the sum of the individual ones given in eq.(??).

In fact the quartic term in eq.(??) is what one would have obtained directly from a D-term in a supersymmetric effective field theory on the p-brane world volume. In the large N/V limit, the $U(1)$ gauge field coming from the $(p+2)$-brane has a kinetic term which comes with the tension $\frac{V}{2\pi^2}\sqrt{1+\frac{4\pi^4 N^2}{V^2}} \to N$. This means that the tachyon field carries $g_1 = 1/\sqrt{N}$ units of charge with respect to this $U(1)$ field, while the charge $g_2 = 1$ with respect to the $U(1)$ gauge field coming from the p-brane. The D-term therefore gives rise to the quartic term in eq.(??) where the factor $(1+\frac{1}{N})$ is just the usual factor $(g_1^2+g_2^2)$.

We postpone the discussion of higher order terms in the tachyon potential to section 5, where we find it easier to discuss it in the T-dual version of the present case, which will be analyzed in sections 3 and 4.

Now let us discuss what happens when we have m parallel p-branes near a $(1,N)$ system. In this case the difference between the BPS bound for the tension and the sum of the tensions of the individual systems is easily seen to be:

$$\delta T = -\frac{TV^2}{8\pi^4\lambda}\frac{m}{N(N+m)} \tag{2.9}$$

We would now like to see if the minimization of the tachyon potential would account for this difference.

When the transverse distance between the m p-branes vanishes, there is a $U(1)\times U(m)$ gauge group, where the off-diagonal generators of $U(m)$ arise from the open strings stretched between pairs of p-branes. The tachyon fields appear, as before, from the strings stretched between the $(p+2)$-brane and any one of the p-branes. There are m of these tachyon fields that transform as a fundamental representation of $U(m)$ and carry $1/\sqrt{N}$ charge with respect to the first $U(1)$ factor. Let us label these tachyons by χ^i for $i = 1, ..., m$. The scalars of the off-diagonal gauge fields will be labelled by $A_\pm^{(ij)}$ and the ones along the Cartan directions by $A_\pm^{(ii)}$ and $A_\pm^{(00)}$, where the subscripts $\pm$ indicate the eigenvalues of the rotation operator in the torus directions. The quartic term in the potential to the leading order in the V/N is just obtained from the supersymmetric $U(1)\times U(m)$ gauge theory. Recalling the $U(1)$ charge of the tachyon, the potential involving the tachyons and the scalar partners of the gauge fields, upto the quartic terms is easily seen to be:

$$\mathcal{V} = -\frac{V}{2\pi^3 N}\sum_i |\chi^i|^2 + \frac{1}{2\pi^2}\frac{1}{N}(\sum_i |\chi^i|^2)^2 + \frac{1}{2\pi^2}\sum_i \left(\sum_{j\neq i}(|A_+^{(ij)}|^2 - |A_+^{(ji)}|^2) - |\chi^i|^2\right)^2$$
$$+\frac{1}{\pi^2}\sum_i\sum_{j>i}|\sum_k (A_+^{(ik)}A_-^{(kj)} - A_+^{(kj)}A_-^{(ik)}) - \bar{\chi}^i\chi^j|^2 + \frac{1}{\pi^2}\sum_i|\sum_k \chi^k A_-^{(ki)}|^2 \qquad (2.10)$$

If we set all the scalar partners of the gauge fields to be zero then the tachyon potential is just the same as in the $m=1$ case considered above, except that $|\chi|^2$ in eq.(??) is replaced by the $U(m)$ invariant combination $(\sum_i |\chi^i|^2)$. In any case, by using the $U(m)$ invariance, we can set all $\chi^i = 0$ for $i = 2, ..., m$ and χ^1 to be real. Then solving for χ^1 we would find the same value of the potential as in the previous case, namely eq.(??), instead of the desired value eq.(??). What is wrong with this ansatz of setting all the scalar partners of the gauge fields to zero is that, the resulting solution is unstable. Indeed by plugging in the vev of χ in eq.(??) for the potential, one finds that all the scalars $A_+^{(1j)}$ for $j = 2, ..., m$ become tachyonic, therefore they should also acquire vev's. Actually by using the residual $U(m-1)$ symmetry we can set all $A_+^{(1j)} = 0$ for $j = 3, ..., m$ except for $j = 2$. By giving vev to $A_+^{(12)}$, one finds that the scalars $A_+^{(2j)}$ become tachyonic for $j = 3, ..., m$, necessitating a vev to one of these scalars. Clearly this will continue. Thus many of the scalar partners of the gauge fields also acquire vev's.

To solve the equations, it is convenient to introduce the following notation:

$$B^0 = \sum_i |\chi^i|^2 - \frac{1}{2\pi}\frac{m}{N+m}V$$
$$B^k = \sum_{j\neq k}(|A_+^{(kj)}|^2 - |A_+^{(jk)}|^2) - |\chi^k|^2 + \frac{1}{2\pi}\frac{1}{N+m}V$$
$$X^{(ij)} = \sum_k (A_+^{ik}A_-^{kj)} - A_-^{ik}A_+^{kj)}) - \bar{\chi}^i\chi^j$$
$$X^i = \sum_k \chi^k A_-^{(ki)} \qquad (2.11)$$

Note that not all the B's are independent; they satisfy the relation $B^0 + \sum_k B^k = 0$. The potential can then be rewritten as:

$$\mathcal{V} = -\frac{TV^2}{8\pi^4\lambda}\frac{m}{N(N+m)} + \frac{1}{2\pi^2}(\frac{1}{N}(B^0)^2 + \sum_k (B^k)^2) + \frac{1}{\pi^2}\sum_i\sum_{j>i}|X^{(ij)}|^2) + \frac{1}{\pi^2}\sum_i |X^i|^2 \quad (2.12)$$

The first term on the right-hand side of the above equation is just the right value for the BPS saturation. All the other terms above are positive definite and hence must be zero individually. Thus we have the equations:

$$B^0 = B^k = X^{(ij)} = X^i = 0 \quad (2.13)$$

The number of equations can be counted easily. B's are all real, and noting the fact that they are not all linearly independent, these give m real equations, while $X^{(ij)}$'s and X^i's are complex and therefore they provide $m(m+1)$ real equations. Thus the total number of real equations is $m(m+2)$. The total number of variables (apart from the one corresponding to overall $U(1)$) is $2m(m+1)$ real, of which m^2 can be set to zero, by using the $U(1) \times U(m)$ gauge symmetry. To see this explicitly, by using the gauge symmetry, we can set $\chi^i = 0$ for $i > 1$, as well as $A_+^{(ij)} = 0$ for $j > i+1$. This still leaves the Cartan subgroup, by use of which we can also set χ_1 and $A_+^{(i,i+1)}$ to be real. Plugging this gauge slice in the equations (??), one can show with some effort, that there is a unique solution (modulo the overall $U(1)$ namely translating all the $A_+^{(ii)}$ by the same amount), namely, $\chi^1 = \sqrt{\frac{V}{2\pi}\frac{m}{N+m}}$ and $A_+^{(k,k+1)} = \sqrt{\frac{V}{2\pi}\frac{m-k}{N+m}}$ and all the other fields equal to zero.

This proves, that there is a unique solution to the equations of motion, with non-trivial vev's for the tachyon and some other massless fields, such that the potential at this solution exactly compensates for the difference in the tension predicted by the BPS formula for the (m, N) system.

3. $(N, 1)$ Bound State for Large V

Let us now consider the situation where we have N $(p+2)$-branes and one of them is carrying a p-brane charge due to a unit flux of magnetic field on the torus. This is exactly the T and S-dual of the problem considered by Witten [?]. In our case we will be able to analyze the problem explicitly because the instability and the consequent mass gap arise from perturbative string states. BPS bound for the tension for this system is:

$$T(N,1) = \frac{TV}{2\pi^2\lambda}\sqrt{N^2 + \frac{4\pi^4}{V^2}} \tag{3.1}$$

while for the individual systems $(1,1)$ together with $(N-1)$ of the $(1,0)$ systems the tension is

$$\frac{TV}{2\pi^2\lambda}\sqrt{1 + \frac{4\pi^4}{V^2}} + (N-1)\frac{TV}{2\pi^2\lambda} \tag{3.2}$$

The difference between these two values is small in the large volume limit and in fact to the leading order in $1/V$ is

$$-\frac{T\pi^2}{\lambda}\frac{(N-1)}{NV} \tag{3.3}$$

This difference, as before, should be provided by the minimization of the potential. Upon dimensional reduction, the resulting $(p+1)$-dimensional world volume theory will contain the usual $U(1)^N$ gauge fields $A^{(ii)}$, $i = 1, ..., N$ that appear from the string going from i-th brane to itself. Besides this there will be also the string states stretched between two different branes. If the transverse distance between these branes is zero then in the absence of magnetic flux on the first brane (say), one would obtain this way $U(N)$ gauge fields on the $(p+2)$-brane world volume with $A^{(ij)}$ for $i \neq j$ being the off diagonal gauge fields. Upon dimensional reduction on the torus, the Wilson lines provide two scalars $A^{(ij)}_{\pm}$ with the reality condition $(A^{(ij)}_{+})^* = A^{(ji)}_{-}$.

What happens when we turn on the unit magnetic flux on the first brane? The magnetic field in this case is $1/V$ and once again applying the results of [?, ?, ?] we find that there are tachyons in the sectors of strings stretched between the first and some other branes. More explicitly there is a mass shift for the fields $A^{(1i)}_{\pm}$ and their masses are

$$M^2_{\pm} = (1 \mp 2)\frac{2\pi}{V} \tag{3.4}$$

Thus $A^{(1i)}_{+}$ is tachyonic and signals an instability of the vacuum. The corresponding vertex operator involves twist fields that twist by an amount $1/V$. In the large volume limit these operators go over to the standard untwisted vertex operators for the Wilson lines.

We will again consider in this section only upto quartic term in the potential and restrict to the leading order in $1/V$, which is expected to be of order $(1/V)^0$. This is obtained by a dimensional reduction of $U(N)$ gauge theory. Note that, unlike in the previous section, the tachyon carries a unit charge under the $U(1)$ of the first brane that has the unit magnetic flux. This can be seen by looking at the corresponding gauge kinetic term which in the large volume limit scales to the factor V just as for the $U(1)$ gauge fields coming from the other branes. As a result in the large volume limit, the quartic term can be computed from an effective gauge group $U(N)$ instead of $U(1) \times U(N-1)$. The relevant terms in the potential can be shown to be:

$$\begin{aligned} \mathcal{V} &= \frac{V}{2\pi^2}\left(\frac{2\pi}{V}\sum_{i=2}^{N}(-|A_+^{(1i)}|^2 + 3|A_+^{(i1)}|^2) + \frac{1}{2\pi^2}\sum_i\left(\sum_{j\neq i}(|A_+^{(ij)}|^2 - |A_+^{(ji)}|^2)\right)^2\right. \\ &\quad \left. + \frac{1}{\pi^2}\sum_i\sum_{j>i}|\sum_{k\neq i,j}(A_+^{(ik)}A_-^{(kj)} - A_+^{(kj)}A_-^{(ik)})|^2\right) \end{aligned} \tag{3.5}$$

Exactly as in the previous section, we can recast the above potential in a convenient form by defining:

$$\begin{aligned} B^1 &= \sum_{j\neq k}(|A_+^{(1j)}|^2 - |A_+^{(j1)}|^2) - \frac{2\pi^3}{V}\frac{N-1}{N} \\ B^k &= \sum_{j\neq k}(|A_+^{(kj)}|^2 - |A_+^{(jk)}|^2) + \frac{2\pi^3}{V}\frac{1}{N}, \qquad for \quad k>1 \\ X^{(ij)} &= \sum_k(A_+^{ik}A_-^{kj)} - A_-^{ik}A_+^{kj)}) \end{aligned} \tag{3.6}$$

Note that not all the B's are independent; they satisfy the relation $\sum_k B^k = 0$. The potential can then be rewritten as:

$$\mathcal{V} = -\frac{\pi^2}{V}\frac{N-1}{N} + \frac{V}{2\pi^2}\left(\frac{4\pi}{V}\sum_{i=2}^{N}|A_+^{(i1)}|^2 + \frac{1}{2\pi^2}\sum_k(B^k)^2 + \frac{1}{\pi^2}\sum_i\sum_{j>i}|X^{(ij)}|^2\right) \tag{3.7}$$

The first term on the right-hand side of the above equation is just the right value for the BPS saturation. All the other terms above are positive definite and hence must be zero individually. Thus we have the equations:

$$A_+^{(i1)} = B^k = X^{(ij)} = 0 \tag{3.8}$$

The number of equations can be counted easily. B's are all real, and noting the fact that they are not all linearly independent, these give $(N-1)$ real equations, while $X^{(ij)}$'s and $A_+^{(i1)}$ are complex and therefore they provide $(N+2)(N-1)$ real equations. Thus the total number of real equations is N^2+2N-3. The total number of variables (apart from the one corresponding to overall $U(1)$) is $2(N^2-1)$ real, of which $(N-1)^2$ can be set to zero, by using the $U(1)\times U(N-1)$ gauge symmetry. Thus the number of variables modulo gauge transformation is exactly equal to the number of equations. To see this explicitly, by using the gauge symmetry, we can set $A_+^{(ij)} = 0$ for $j > i+1$. This still leaves the Cartan subgroup, by use of which we can also set $A_+^{(i,i+1)}$ to be real. Plugging this gauge slice in the equations (**??**), one can show with some effort, that there is a unique solution (modulo the overall $U(1)$ namely translating all the $A_+^{(ii)}$ by the same amount), namely, $A_+^{(k,k+1)} = \sqrt{2\pi^3\frac{N-k}{VN}}$ and all the other fields equal to zero.

This proves, that there is a unique solution to the equations of motion such that the potential at this solution exactly compensates for the difference in the tension predicted by the BPS formula for the $(N,1)$.

We can now ask the question what happens when we bring m more $(p+2)$-branes near the above bound state. This system is exactly the T dual to the situation we have considered in section 2. The resulting system will again form a $(N+m,1)$ bound state carrying $(N+m)$ units of $(p+2)$-brane charge and 1 unit of p-brane charge exactly as discussed in the previous section, but now the expectation values are obtained from eqs.(**??**) and (**??**), with N replaced by $(N+m)$. This means that the original vev's of $A_+^{j,j+1)}$ for $j=1,...,N-1$, change from $\sqrt{2\pi^3\frac{N-j}{NV}}$ to $\sqrt{2\pi^3\frac{N+m-j}{(N+m)V}}$. On the other hand, these fields are massive fields in the $(N,1)$ bound state. Indeed, while the imaginary parts of these fields are the Goldstone bosons, that are eaten by the gauge fields, the real parts of these fields, as can be seen from eq.(**??**) for the scalar potential, acquire a mass term through their vev's of the form

$$\frac{V}{4\pi^4}\chi^i\mathcal{C}_{ij}\chi^j, \quad \chi^i \equiv 2\sqrt{2\pi^3\frac{N-i}{NV}}Re(A^{(i,i+1)}) \tag{3.9}$$

where $i, j = 1, ..., N-1$ and $\mathcal{C}_{ij}$ is the Cartan matrix of $SU(N)$. Thus in the new ground state of $(N+m, 1)$ bound state, some of the massive fields intrinsic to $(N, 1)$ bound state acquire vev's. We would now like to ask the question whether one can obtain the new ground state without discussing the details of the intrinsic massive fields of the $(N, 1)$ bound state. In other words we would like to integrate all the massive fields and focus on the tachyonic and massless fields. The only massless field in the $(N, 1)$ bound state is the center of mass $U(1)$ multiplet, while in the $(m, 0)$ system we have massless $U(m)$ vector multiplet. What are the tachyonic states. At first sight it may appear that the fields $A_+^{(i,N+a)}$ for $i = 1, ..., N$ and $a = 1, ..., m$ are tachyonic. But one can see by using eq.(**??**) for the potential, that in the presence of the non-trivial vev's that go into the formation of $(N, 1)$ bound state, only the fields $A_+^{(N,N+a)}$ are tachyonic with mass square given by $-\frac{2\pi}{VN}$. Furthermore, these tachyon fields have cubic interactions of the form $-\frac{V}{2\pi^4}\chi^{N-1}\sum_{a=1}^m(|A_+^{(N,N+a)}|^2 - |A_+^{(N+a,N)}|^2$.

The fact that the tachyon mass square is $-2\pi/NV$ rather than $-2\pi/V$ can be directly understood without going into the details of the intrinsic structure of $(N, 1)$ bound state. The reason is that in the process of formation of the bound state the $U(N)$ group is broken down to $U(1)/Z_N$. As a result, the magnetic field is quantized in units of $1/NV$. The presence of the cubic interaction has the following consequence. In the large volume limit one would have naively expected that the order V^0 quartic couplings that we are interested in will have a $U(m+1)$ symmetry. However if one integrates out the massive fields χ^i then one gets an extra contribution to the quartic term

$$-\frac{V}{4\pi^2}\mathcal{C}^{N-1,N-1}\sum_{a=1}^m(|A_+^{(N,N+a)}|^2 - |A_+^{(N+a,N)}|^2)^2 = -\frac{V}{4\pi^4}\frac{N-1}{N}\sum_{a=1}^m(|A_+^{(N,N+a)}|^2 - |A_+^{(N+a,N)}|^2)^2 \tag{3.10}$$

where $\mathcal{C}$ with superscripts is the inverse of the Cartan matrix and is just the propagator of the χ fields as follows from the mass term eq.(**??**). Note that this term has only the

$U(1) \times U(m)$ symmetry and not the accidental $U(m+1)$ symmetry. This is because the reducible diagrams involving the massive intermediate states involve the cubic term and the mass term which have only the $U(1) \times U(m)$ symmetry. The net effect of the details of the $(N,1)$ bound state is summarized in this extra term in the quartic potential.

Now we can proceed to minimize the potential involving only the tachyon and massless fields. As before using the $U(m)$ symmetry we can set all the fields $A_+^{(N+a-1,N+b-1)} = 0$ for $a = 1,...,m-1$ and $b > a+1$ and set $A_+^{(N+a-1,N+a)} \equiv \sqrt{v^a}$ for $a = 1,...,m$ to be real. Minimizing the potential one finds, just as before, that $A_+^{(N+a-1,N+b-1)} = 0$ for $a = 1,...,m-1$ and $b < a$ and $A_+^{(N+a-1,N+a-1)} = A_+^{(N,N)}$ for $a = 1,...,m-1$. Furthermore, v^a satisfies the equation

$$\mathcal{C}_{ab}v^b - \frac{N-1}{N}\delta_{a1}v^1 = \frac{2\pi^3}{NV}w_a \tag{3.11}$$

where $a,b = 1,...,m$, $\mathcal{C}$ is the Cartan matrix of $SU(m+1)$ and $w_a = \delta_{a1}$ are just the Dynkin coefficients of the fundamental representation of $SU(m+1)$. Note that the first term on the left-hand side of the above equation comes from the usual quartic terms that have $U(m+1)$ symmetry but the second term is due to the additional quartic term eq.(??) that appears after integrating out the massive modes. Solving this equation one finds that $v^a = 2\pi^3 \frac{m-a+1}{(N+m)V}$ for $a = 1,...,m$. The minimum value of the potential is

$$-\frac{\pi^2}{VN}\frac{m}{N+m} \tag{3.12}$$

which is exactly the difference, to order $1/V$, between the BPS bound for the $(N+m,1)$ system and the sum of the individual BPS bounds for $(N,1)$ and $(m,0)$ systems.

We can now compare the above result with the one obtained in section 2 where we considered the bound states of $(1,N)$ system with $(0,m)$ system. Indeed this system is T-dual to the ones involving the bound state of $(N,1)$ and $(m,0)$ systems. The extra term in the quartic potential eq.(??) that arose by integrating out the massive modes has an exact parallel in section 2, where it came from the fact that the charge of the tachyon with respect to the $U(1)$ of the $(1,N)$ bound state system was $\sqrt{1/N}$.

4. (N,m) Bound States for Large V

Let us now consider the bound states of N $(p+2)$- branes that carry a net m units of p-brane charge. Let $m = kN + s$, where k is a non-negative integer and s is between 0 and $N-1$. If $s = 0$ then we can realize this system by giving k units of flux in each $(p+2)$-brane. There are no tachyons and the system is just that of N copies of $(1,k)$ systems whose relative separations are flat directions. Let us then take s to be between 1 and $N-1$. First consider the case when s and N are relatively prime. We can realize this system by giving $(k+1)$ units of magnetic flux on each of the first s $(p+2)$-branes and k units each on the remaining $N-s$ branes. The sum of the tensions of this system is

$$\frac{T}{2\pi^2\lambda}\Big(s\sqrt{V^2+4\pi^4(k+1)^2}+(N-s)\sqrt{V^2+4\pi^4k^2}\Big) \tag{4.1}$$

On the other hand the BPS bound for a (N,m) system is

$$\frac{T}{2\pi^2\lambda}\sqrt{N^2V^2+4\pi^4(Nk+s)^2} \tag{4.2}$$

We expect that the difference between these two values will be provided by the minimization of the potential. This difference in the large volume limit is

$$\delta \mathrm{T} = -\frac{T\pi^2}{\lambda}\frac{s(N-s)}{NV}. \tag{4.3}$$

The world volume theory of the system just described has a $U(s)\times U(N-s)$ gauge symmetry. This is because strings stretched between two of the first s or the remaining $N-s$ branes have identical boundary conditions at the two ends and therefore they become massless when the relative separations vanish, thereby giving rise to the off-diagonal gauge fields. The tachyon fields $A_+^{(i,a)}$ for $i = 1,...,s$ and $a = s+1,...,N$ are the ground states of the open string stretched between one of the first s branes and one of the remaining $N-s$ branes. Obviously the tachyon fields transform as a fundamental representation of $U(s)$ and $U(N-s)$. Since the difference between the magnetic fluxes at the two ends of the string are 1 unit each, it follows from [?, ?, ?], that the multiplicity of these tachyons is one and their mass squares are $-2\pi/V$ each.

The quartic term in the potential involving massless and tachyonic fields to order $(\frac{1}{V})^0$, is obtained just by dimensional reduction from $(p+2)$ to p-brane world-volume. The relevant terms in the potential are then:

$$
\begin{aligned}
\mathcal{V} \;&=\; \frac{V}{2\pi^2}\left(\frac{2\pi}{V}\sum_{i=1}^{s}\sum_{a=s+1}^{N}(-|A_+^{(ia)}|^2+3|A_+^{(ai)}|^2)+\frac{1}{2\pi^2}\sum_i\left(\sum_{j\neq i}(|A_+^{(ij)}|^2-|A_+^{(ji)}|^2)\right)^2\right.\\
&\left.+\frac{1}{\pi^2}\sum_i\sum_{j>i}|\sum_{k\neq i,j}(A_+^{(ik)}A_-^{(kj)}-A_+^{(kj)}A_-^{(ik)})|^2\right). \qquad (4.4)
\end{aligned}
$$

Here the indices i,j,k run from 1 to N unless stated otherwise. We can again recast this potential into a convenient form by defining:

$$
\begin{aligned}
B^k \;&=\; \sum_{j\neq k}(|A_+^{(kj)}|^2-|A_+^{(jk)}|^2)-\frac{2\pi^3}{V}\frac{N-s}{N}, \qquad \text{for } 1\le k\le s\\
B^k \;&=\; \sum_{j\neq k}(|A_+^{(kj)}|^2-|A_+^{(jk)}|^2)+\frac{2\pi^3}{V}\frac{s}{N}, \qquad \text{for } s<k\le N\\
X^{(ij)} \;&=\; \sum_k(A_+^{(ik)}A_-^{(kj)}-A_-^{(ik)}A_+^{(kj)}) \qquad (4.5)
\end{aligned}
$$

As before, not all the B's are independent; they satisfy the relation $\sum_k B^k=0$. The potential can then be rewritten as:

$$
\mathcal{V}=-\frac{\pi^2}{V}\frac{s(N-s)}{N}+\frac{V}{2\pi^2}\left(\frac{4\pi}{V}\sum_{i=1}^{s}\sum_{j=s+1}^{N}|A_+^{(ji)}|^2+\frac{1}{2\pi^2}\sum_k(B^k)^2+\frac{1}{\pi^2}\sum_i\sum_{j>i}|X^{(ij)}|^2\right). \quad (4.6)
$$

The first term on the right-hand side of the above equation is just the right value for the BPS saturation. All the other terms above are positive definite and hence must be zero individually. The number of equations can be counted easily. B's are all real, and noting the fact that they are not all linearly independent, these give $(N-1)$ real equations. $X^{(ij)}$'s are complex and therefore they provide $N(N-1)$ real equations. Finally $A_+^{(ji)}=0$ for $i=1,...,s$ and $j=s+1,...,N$ give $2s(N-s)$ real equations. Thus the total number of real equations is $(N^2+2Ns-2s^2-1)$. The total number of variables (apart from the one corresponding to overall $U(1)$) is $2(N^2-1)$ real, of which $(s^2+(N-s)^2-1)$ can be set to zero, by using the $U(s)\times U(N-s)$ gauge symmetry. Thus the number of variables modulo gauge transformation is exactly equal to the number of equations. However, in this case, since the gauge group is smaller, there could be several gauge inequivalent solutions. We have not analyzed this problem in detail, but in the following we show that there always exists at least one solution, which is compatible with the BPS bound.

Let us take the ansatz $A_+^{(ij)} = 0$ for $j \neq i, i+1$, and $A_+^{(ii)} = A_+^{(11)}$ and $A_+^{(i,i+1)} \equiv \sqrt{v^i}$ are real. $X^{(ij)}$'s are then identically zero. From the expression for B^i's it follows, that v^i's for $i = 1, ..., N-1$ must satisfy:

$$\sum_{j=1}^{N-1} \mathcal{C}_{ij} v^j = \frac{2\pi^3}{V} \delta_{is} \equiv \frac{2\pi^3}{V} w_i \tag{4.7}$$

where $\mathcal{C}$ is the Cartan matrix of $SU(N)$. It is clear that the w_i's are just the Dynkin coefficients of the s-th rank anti-symmetric tensor representation of $SU(N)$. Thus if α_i are the simple roots of $SU(N)$ labelling the i-th point in the Dynkin diagram, then $\sum_i \alpha_i v^i$ is $2\pi^3/V$ times the highest weight of the s-th rank anti-symmetric tensor representation. Explicitly the solution is $v^r = 2\pi^3 r(N-s)/NV$ for $r \leq s$ and for $r > s$, $v^r = 2\pi^3 s(N-r)/NV$. The minimum value of the potential is

$$\mathcal{V}_{min} = -\frac{1}{\pi} v^s + \frac{V}{4\pi^4} \sum_{i,j=1}^{N-1} v^i v^j \mathcal{C}_{ij} = -\pi^2 \frac{s(N-s)}{NV} \tag{4.8}$$

This is exactly the desired difference in eq.(??). We do not know whether the above solution is unique, but we believe that for m and N coprime it is so, modulo the gauge transformations.

In the above discussion, actually we have not really used the fact that s and N are relatively prime. In fact the solution described above also seems to hold for the case when s and N are not relatively prime. The minimum of the potential of course gives the correct BPS bound for this situation also.

Let us consider now the case where s and N have a greatest common divisor c, that is $s = r.c$ and $N = n.c$, for some positive integers r and n that are relatively prime, with $r < n$, since $s < N$. Then this system is equivalent to c identical copies of $(n, kn+r)$ bound states which are each BPS states. Therefore, there will be no mutual force between these

c copies. We will now show, that there exist other solutions giving the correct minimum. To exhibit this, consider the ansatz

$$\begin{aligned} A_+^{(ij)} &= 0, \qquad\qquad j \neq i, i+c, \\ A_+^{(ii)} &= A_+^{(11)}, \\ A_+^{(i,i+c)} &= real. \end{aligned} \tag{4.9}$$

Denoting $v^i = (A_+^{(i,i+c)})^2$, we can see that the equations of motion arising from the potential (??) are consistent with this ansatz provided v's satisfy the equation:

$$\sum_{b=1}^{n-1} \mathcal{C}_{ab} v^{i+(b-1)c} = \frac{2\pi^3}{V}\delta_{ar} \equiv \frac{2\pi^3}{V} w_a, \qquad i = 1, ..., c \tag{4.10}$$

where $\mathcal{C}$ is the Cartan matrix of $SU(n)$. Thus we have c sets of uncoupled equations, each involving $SU(n)$ Cartan matrices. w's are just the Dynkin coefficients of the r-th rank anti-symmetric tensor representation of $SU(n)$. It is clear from the previous analysis that the potential at the minimum is

$$-c\pi^2 \frac{r(n-r)}{nV} \tag{4.11}$$

This is exactly the difference expected from eq.(??).

In the next section we will discuss the surviving gauge group beyond the quartic approximation we used so far, finding that, indeed, in the coprime case it is just the center of mass $U(1)$, whereas in the non-coprime case it is of rank c.

5. Higher Order Terms and Torons

In the last sections, we restricted the analysis to the quartic potential for the tachyonic fields. In the Appendix we show how this potential arises from a tree level string computation involving four external tachyon vertex operators, after integrating out the massive Kaluza-Klein modes $\phi(n_1, n_2)$ corresponding to the Cartan directions. These are the only massive modes which are relevant at the quartic level.

In this section we address the question of higher order contributions to the tachyon potential. We will discuss this in the context of the examples presented in sections 3 and 4 in which the relevant perturbation parameter is $1/V$. We will use the effective field theory on the world-volume of the $(p+2)$-brane and will carry out a Kaluza-Klein analysis in the presence of a magnetic flux on the two-torus part of the world-volume.

The higher order contributions to the effective potential can come either from higher derivative terms in $(p+2)$ dimensions (like F^4) or from the reducible diagrams of higher point functions. It is easy to see that the former contributions are suppressed in the large volume limit. However, the reducible diagrams involving vertices due to F^2 term give rise to contributions which are of the same order in volume as the quartic term considered previously. In fact it is easy to see this by noting that the n-point vertices scale as $V^{\frac{n-4}{2}}$ and the massive propagators scale as V. It then follows that the term $|\chi|^{2n}$ scales as V^{n-2}. The above massive modes include, in addition to the ϕ's, also the higher Landau levels. The fact that these massive modes contribute to the higher order terms in the tachyon potential implies that they also acquire expectation values. A more convenient way of including these infinite sets of modes, when minimizing the potential, is to work directly with the $(p+2)$ dimensional field strength.

As a simple example, let us discuss the $(2,1)$ system considered as a bound state of a $(p+2)$-brane which has one unit of magnetic flux on it with another $(p+2)$-brane. The world-volume theory of the $(p+2)$ -branes involves a $U(2)$ theory, where the $U(1)$ corresponding to the first brane has one unit of magnetic flux. If (x_1, x_2) are the compact world-brane coordinates of the $(p+2)$-brane, corresponding to a torus T^2 with volume $V = R_1 R_2$, the gauge potential can be taken to be $A_1^{(1,1)} = 0$ and $A_2^{(1,1)} = 2\pi x_1/V$, with field strength $2\pi/V$. Writing $U(2) = SU(2) \times U(1)/Z_2$, where $U(1)$ corresponds to the center of mass degree of freedom, we can decompose the background gauge potential as $A_z = -\frac{i\pi x_1}{2V}(I + \sigma_3)$, where $z = x_1 + ix_2$ and I is the identity matrix, one can see that the field strength of the center of mass $U(1)$ has a half unit of flux. The contribution to the energy of the center of mass $U(1)$ due to the half unit flux is $\pi^2/2V^2$ and saturates the BPS bound. This implies that the contribution of the $SU(2)$ part to the ground state energy should vanish. In order to see this, let us expand the $SU(2)$ gauge potential around the background as follows:

$$A_z = A_z^0 + \phi\sigma_3 + \xi\sigma_+ + \bar{\eta}\sigma_-, \qquad A_{\bar{z}} = (A_z)^\dagger. \tag{5.1}$$

where $\sigma_\pm = \sigma_1 \pm i\sigma_2$ and $A_z^0 = -i\pi x_1\sigma_3/2V$. Note that A_z^0 obeys the following boundary conditions:

$$\begin{aligned} A^0(x_1, x_2 + R_2) &= \Omega_2 A^0(x_1, x_2), \\ A^0(x_1 + R_1, x_2) &= \Omega_1 A^0(x_1, x_2), \end{aligned} \tag{5.2}$$

where $\Omega_2 = 1$, $\Omega_1 = \exp(i\pi x_2\sigma_3/R_2)$ and ΩA means the gauge transformation of A by Ω. These are in fact the boundary conditions of an $SU(2)$ toron:

$$\Omega_2(R_1)\Omega_1(0) = -\Omega_1(R_2)\Omega_2(0). \tag{5.3}$$

The boundary conditions for the fluctuations ϕ, ξ and η are dictated by the requirement that A satisfies the same boundary conditions as A^0. This implies that the fluctuations are periodic under $x_2 \to x_2 + R_2$ while under $x_1 \to x_1 + R_1$ they transform as:

$$\phi \to \phi, \qquad \xi \to e^{2\pi i x_2/R_2}\xi, \qquad \eta \to e^{2\pi i x_2/R_2}\eta. \tag{5.4}$$

The F^2 term corresponding to $U(2)$ is the sum of the ones corresponding to the center of mass $U(1)$ and the relative $SU(2)$. The $SU(2)$ contribution is of the form $\frac{1}{2}\int_{T^2}(F_3^2 + |F_+|^2)$, where:

$$\begin{aligned} F_3 &= \frac{\pi}{V} - i(\partial_{\bar{z}}\phi - \partial_z\bar{\phi}) + |\eta|^2 - |\xi|^2 \\ F_+ &= D_z\xi + D_z^\dagger\eta + 2i\bar{\phi}\xi - 2i\phi\eta. \end{aligned} \tag{5.5}$$

Here $D_z = \partial_z - i\pi x_1/V$ and $D_z^\dagger = -\partial_{\bar{z}} + i\pi x_1/V$. Since the $SU(2)$ contribution is the sum of two integrals of semi-positive definite functions on T^2, the minimum energy configuration is obtained when $F_3 = 0$ and $F_+ = 0$. We can recover, in this framework, the result on the quartic tachyon potential discussed in the appendix if in the term $\int_{T^2} F_3^2$ we restrict ourselves to the lowest Landau level (i.e., $D_z\xi = 0$) and integrate out the massive Kaluza-Klein modes of ϕ. The lowest Landau level is explicitly given by

$$\Psi_0 = (-2\pi i\tau)^{1/4} e^{-\pi x_1^2/V} \theta_3(\tau|\bar{z}); \qquad \tau \equiv i\frac{R_1}{R_2}, \tag{5.6}$$

where θ_3 is a Jacobi theta function. Note that since ϕ is neutral its Kaluza-Klein expansion is just the usual Fourier mode expansion whereas ξ and η are expanded in the Landau modes given by $(D_z^\dagger)^n\Psi_0$. However, as an inspection of equation (??) reveals, due to the fact that the massive modes of ϕ acquire non-trivial expectation value, the higher Landau modes cannot be ignored.

The most convenient way of including these higher modes is to look for the solutions to the equations $F_3 = F_+ = 0$ on T^2. As shown in [?, ?], there exist such zero field strength configurations with non-zero Z_2 flux arising from the twisted boundary conditions (??). (For a discussion of torons in the context of D-branes see [?].) For example, by making a suitable gauge transformation we can map the boundary conditions (??) to the the constant ones $\Omega_1 = i\sigma_1$ and $\Omega_2 = i\sigma_3$. It is clear that the zero gauge potential satisfies the latter boundary conditions, and therefore a gauge potential obeying (??) is a pure gauge.

In the above gauge in which $\Omega_1 = i\sigma_1$, $\Omega_2 = i\sigma_3$ it is easy to see that the relative $SU(2)$ is broken. The kinetic term for the gauge potential A_μ, where μ refers to the $p+1$ noncompact directions, comes from $|F_{\mu z}|^2$. Then the massless gauge fields are independent of the T^2 coordinates $z, \bar{z}$. Since A_μ satisfies the boundary condition (??) this implies that it should commute with σ_1 and σ_3. It follows that it can only be the identity matrix, that is, only the center of mass $U(1)$ is unbroken.

This argument can be extended to the case of N number of $(p+2)$-branes with m units of magnetic flux distributed among them, as discussed in section 4. Following the same steps as above, one is lead to consider the $SU(N)$ toron configuration corresponding to a

Z_N-twist given by

$$\Omega_2(R_1)\Omega_1(0) = \Omega_1(R_2)\Omega_2(0)e^{2i\pi m/N}. \tag{5.7}$$

Following 't Hooft, we can write constant Ω's satisfying this condition in terms of the matrices P and Q introduced in [?] as:

$$\Omega_1 = P^m, \qquad \Omega_2 = Q. \tag{5.8}$$

Then the question of the unbroken gauge symmetry reduces to finding constant (i.e., independent of z and $\bar{z}$) $SU(N)$ matrices A which commute with P^m and Q. Since Q is diagonal then A must be diagonal. To analyse the conditions coming from $[A, P^m] = 0$, one has to distinguish two cases: (i) N and m being coprime and (ii) not coprime, i.e., $N = cn$ and $m = cs$ for coprime integers n and s. In case (i) the $SU(N)$ is completely broken whereas in (ii) there is an unbroken rank $c-1$ gauge group, in addition to the center of mass $U(1)$.

The last point we would like to make concerns the spectrum of the massive K-K modes. We would like to show that the massive spectrum in the toron background is indeed the correct one. For the $(1, N)$ system (the example considered in section 2), let us consider the spectrum of the massive K-K modes of the open string from the $(p+2)$-brane to itself. Due to the presence of N units of magnetic flux, the momenta p_1 and p_2 are rescaled [?] by a factor $1/\sqrt{1+(\frac{2\pi^2 N}{V})^2}$ which in the small volume limit becomes $V/2\pi^2 N$. This implies that the momenta are given by $p_i = \frac{R_i}{\pi}(n_i + \frac{k_i}{N})$ for $k_i = 0, 1, \ldots, N-1$ and arbitrary integer n_i. On the other hand, in the dual $(N, 1)$ system due to Z_N twistings along the two one-cycles of the torus the K-K spectrum in the background of the $SU(N)$ toron is given by momenta that lie in a shifted lattice. Since the boundary conditions $\Omega_1 = P$ and $\Omega_2 = Q$ commute upto the center of the group, it is clear that they can be simultaneously diagonalized in the adjoint representation. Thus we can find a basis of $U(N)$ generators such that $\Omega_i^{-1} J_{(k_1,k_2)} \Omega_i = e^{i2\pi\frac{k_i}{N}} J_{(k_1,k_2)}$. With a little bit of effort one can show that there is a one-to-one correspondence between the N^2 generators of $U(N)$ and the twists (k_1, k_2) with $k_i = 0, 1, \ldots, N-1$. Thus the K-K momenta in the toron background are $\frac{2\pi}{R_i}(n_i + \frac{k_i}{N})$, where n_i and k_i are as before. Thus the two spectra agree, if one takes into account the T-duality $R_i \to 2\pi^2/R_i$.

6. Conclusions

In this paper we have studied the formation of bound states between p and $(p+2)$-branes. One of the simplifying features of this analysis was that the dynamical mechanism responsible for the bound state can already be studied at the string tree level. In fact the binding energy should appear entirely at the tree level, as it scales like inverse power of string coupling. We have argued here that the tachyons that appear in the open strings stretched between p and $(p+2)$-branes stabilize the potential. In general, however, determination of the minimum of the potential requires the exact tree level potential. In the present work we have considered limiting cases of large or small volume. In sections 2,3 and 4 we determined the binding energy from the potential containing up to quartic order terms in the tachyon fields and showed that it agrees with the prediction of the BPS formula. As we discussed in section 5, to the leading order in $1/V$ also terms of higher order in the tachyon potential do contribute, after integrating out the massive modes. This is because these massive modes are becoming massless in the large V limit. Or in other words, in the renormalisation group language, irrelevant operators are becoming marginal in that limit. Of course, as a result of this the expectation value of the tachyon fields are expected to be modified by higher order terms. However, as we saw in section 5, the value of the potential at the minimum does not change. Moreover, in section 5 we discussed the surviving gauge group at the minimum and showed that for the (N, m) system it is $U(1)$ in the coprime case and it is $(U(1))^c$ in the non-coprime case, c being the greatest common divisor of N and m.

One can ask the question, what happens in the finite volume case. In this case, one needs the exact tree level potential involving tachyonic and massless fields. However, the string dualities (in particular the $SL(2,Z)$ of type IIB) predicts the existence of the bound states.

This means that in the sigma-model framework, if one turns on the relevant boundary operators corresponding to the tachyons, there should exist a non-trivial fixed point, and the so-called g-function (namely the tree level partition function) should be given exactly as predicted by the BPS bound. Thus type IIB $SL(2,Z)$ symmetry predicts non-trivial fixed points in a whole variety of boundary conformal field theories that are perturbed by relevant boundary operators. Let us consider the case of the bound state between $(1,N)$ and $(0,1)$ systems discussed in section 2. Since the operators in question are twist operators that represent string stretched between two different branes (say between a $(p+2)$ and a p-brane, the anti-twist operator being the one that reverses the orientation) it is clear that in an arbitrary correlation function involving n twist and n anti-twist operators, the operators are ordered, i.e. two adjacent operators on the boundary of the disc must be twist and anti-twist respectively; they cannot be of the same type. This is exactly the kind of situation one encounters in the spin-1/2 Kondo problem [?], however the precise operators appearing in the Kondo problem involves Sine-Gordon type fields whereas in our case we have complicated twist fields. What is common between the two problems is the appearance of cocycles that are in the spin-1/2 representation, and in this sense we can refer to them as some generalization of the Kondo problem.

In the more general example discussed at the end of section 2 (namely the bound state of $(1,N)$ and $(0,m)$ systems) as well as in sections 3 and 4, we have several branes and minimization of the potential involves giving vev's simultaneously to strings stretched between different pairs of branes. In all of these cases there is an underlying A_n type Dynkin diagram. The operator $A_+^{(i,i+1)}$ takes one from the i-th brane to $(i+1)$-th one and its complex conjugate takes one the opposite way. The number of these operators is equal to the number of simple roots. The cocycles $S^i_\pm$, coming with these operators, for i running over the simple roots, satisfy the condition

$$\begin{aligned} S_+^i . S_+^j &= 0, \qquad for j \neq i+1 \\ S_-^i . S_-^j &= 0, \qquad for j \neq i-1 \end{aligned}$$

$$S^i_+ . S^j_- \;=\; 0, \qquad for j \neq i \tag{6.1}$$

Thus $S^i_\pm$ are just the generators corresponding to the simple roots (and their negatives respectively) in the fundamental representation of A_n algebra. In any given correlation function, one has to take the trace over the products of these cocycles, appearing in the order the operators are inserted on the boundary of the disc. The operators that multiply these cocycles involve either the twisted or untwisted operators depending on whether they are relevant or marginal (not truly marginal) operators. The cases we have considered in this paper are the ones where the relevant operators in question are nearly marginal. It is an interesting open question whether such boundary perturbations, when the relevant operators are far from being marginal, admit non-trivial fixed points and if so whether the disc partition functions (i.e. the g-functions) at the new fixed points are in accordance with the values predicted by the string duality. The fact that there is a tachyon instability and that the BPS bound implies that the potential is bounded below, suggests strongly that such non-trivial fixed points should exist.

There is another direction in which one can apply the results of this paper, namely the systems involving p, $(p+2)$ and $(p+4)$-branes [?]. Again the existence of $(p+2)$ branes, break the supersymmetry and give rise to tachyons, whose vev's should restore the supersymmetry and saturate the BPS bounds.

Finally, one can ask what our analysis would imply on the bound states between a D-3-brane and a fundamental string. Indeed this is just S-dual of the present discussion. Under S-duality, a D-3-brane with magnetic flux on a torus, becomes a D-3-brane with electric field in the non-compact 1+1 world volume. On the other hand, D-string becomes an infinitely long fundamental string. The tachyon which was an elementary open string stretched between the D-3-brane and D-string, and carried electric charge with respect to the $U(1)$ of the D-3-brane, now becomes an open D-string stretched between the D-3-brane and the infinitely long NS-string, carrying a magnetic charge with respect to the $U(1)$ of the D-3-brane. Thus, in the world volume $U(1)$ theory of the D-3-brane, there should appear tachyonic magnetic monopoles (solitonic states), as one brings the fundamental string close to it. This situation is similar to the case studied by Polchinski and Strominger [?] in connection with breaking the $N=2$ 4-d space-time supersymmetry by turning on field strengths in type IIA compactification on a Calabi-Yau space. In their discussion, the theory becomes tachyonic near the conifold singularity (with the would-be massless soliton becoming tachyon), and stabilizing the potential with respect to this tachyon field, restores the supersymmetry. Their discussion can also be extended to the $N=4$ case (by considering type IIA on $K_3 \times T^2$ with non-trivial field strengths) and a similar phenomenon happens when one shrinks appropriate 2-cycles of K_3. The would-be massless solitonic state, that enhance some $U(1)$ to $SU(2)$, becomes tachyonic whose vev again restores the supersymmetry.

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Appendix

In this Appendix, we will compute the 4-pt. function of tachyons, that was required for the determination of the quartic potential in section 2. Tachyon is the ground state of the open string stretched between the $(p+2)$-brane (with large magnetic field $2\pi N/V$) and the p-brane. As described in section 2, the string modes in this sector are not integer moded. In fact they are shifted by $\epsilon = \frac{1}{\pi}\arctan(V/2\pi^2 N) = V/2\pi^3 N$ to the leading order in V/N If X^2 and X^3 are the coordinates on the torus, then the complex combination $X = X^2 + iX^3$ is twisted by ϵ. Thus the corresponding vertex operators involve the twist fields (together with the twist fields for the fermionic partners). Let us denote by σ_+ the bosonic twist field that generates this boundary condition. Then σ takes one from the p-brane to the $(p+2)$-brane and σ_- (the anti-twist field) the opposite way. We are then interested in the correlation function of 2 σ_+'s and 2 σ_-'s, on the boundary of a disc (or equivalently the upper half plane, which is what we shall be using here) with an ordering such that 2 twist fields (or the 2 anti-twist fields) are never adjacent to each other. Of course we can fix three of the positions using the $SL(2,R)$ invariance of the upper-half plane. Let the positions of the two twist fields be $x_1 = 0$ and $x_3 = 1$ and the two anti-twist fields be $x_2 = x$ and $x_4 = \infty$, where x is real and lies between 0 and 1. This means that the boundary conditions on the interval $(-\infty, 0)$ and $(x, 1)$ are Dirichlet, and on the other two intervals, the twist (or anti-twist fields) ensure the right boundary condition of Neumann shifted by the magnetic field. In the large magnetic field limit, the latter go over to Dirichlet condition. On the Dirichlet intervals $(-\infty, 0)$ and $(x, 1)$, the value of x must be the position of the p-brane (say X_0). However, going from the first to the second Dirichlet interval, the string can wind around various cycles of the torus, which would give the world-sheet instanton contributions. First let us consider the quantum part of the correlation function, namely with winding number zero sector.

Following the work of Dixon et al [?], we can compute the normalized correlator

$$\tilde{G}(z, w, x_i) = \frac{\langle \partial_z X(z,\bar{z}) \partial_w \bar{X}(w,\bar{w}) \sigma_+(x_1)\sigma_-(x_2)\sigma_+(x_3)\sigma_-(x_4)\rangle}{\langle \sigma_+(x_1)\sigma_-(x_2)\sigma_+(x_3)\sigma_-(x_4)\rangle} \tag{A.1}$$

Then $\tilde{G}$ has the following form:

$$\begin{aligned}\tilde{G} = -\omega(z)\tilde{\omega}(w) \quad & \Big[\quad \frac{\epsilon(z-x_1)(z-x_3)(w-x_2)(w-x_4)}{(z-w)^2} \\ & + \quad \frac{(1-\epsilon)(z-x_2)(z-x_4)(w-x_1)(w-x_3)}{(z-w)^2} + A(x_i)\Big]\end{aligned} \tag{A.2}$$

where

$$\omega(z) = (z-x_1)^{\epsilon}(z-x_2)^{(1-\epsilon)}(z-x_3)^{\epsilon}(z-x_4)^{(1-\epsilon)} \tag{A.3}$$

and $\tilde{\omega}(z)$ is the same as ω but ϵ replaced by $(1-\epsilon)$. $A(x_i)$ is an arbitrary function of x_i's subject to the boundary condition that the winding number is zero, namely $\int_a^b dX = 0$, where a and b are respectively points on the two Dirichlet intervals. This can be made more explicit by considering another normalized correlator:

$$\tilde{H}(\bar{z},w,x_i)=\frac{\langle\partial_{\bar{z}}X(z,\bar{z})\partial_w\bar{X}(w,\bar{w})\sigma_+(x_1)\sigma_-(x_2)\sigma_+(x_3)\sigma_-(x_4)\rangle}{\langle\sigma_+(x_1)\sigma_-(x_2)\sigma_+(x_3)\sigma_-(x_4)\rangle} \tag{A.4}$$

The Dirichlet boundary condition on the two intervals, implies that $\tilde{H}(\bar{z},w,x_i)=-\tilde{G}(\bar{z},w,x_i)$, as one approaches the cuts along the two Dirichlet intervals. Note that with this condition, the boundary conditions on the remaining two intervals corresponding to $(p+2)$-branes with magnetic flux, namely $e^{i\pi\epsilon}\tilde{G}=e^{-i\pi\epsilon}\tilde{H}$, are automatically ensured due to the cuts in ω. The zero instanton sector, namely $\int_a^b(dz\tilde{G}+d\bar{z}\tilde{H})=0$, implies (taking $w\to\infty$ and setting the values of x_i using $SL(2,R)$ mentioned above) that

$$A(x)=\frac{\int_0^x(z-x)\omega(z)}{F(x)}=x(1-x)\partial_x\log F(x) \tag{A.5}$$

where $F(x)$ is the hypergeometric function

$$F(x)=\int_0^x\omega(z)=\int_0^1 dx z^{-\epsilon}(1-z)^{-1+\epsilon}(1-zx)^{-\epsilon} \tag{A.6}$$

Recalling the OPE of the stress energy tensor $T(z)=-:\partial_zX\partial_z\bar{X}:$ with the primary field $\sigma_-(x)$, we can deduce a differential equation with respect to x for the correlation function $G(x)=\langle\sigma_+(0)\sigma_-(x)\sigma_+(1)\sigma_-(\infty)\rangle$ (after fixing the x_i's as mentioned above), and using eq.(??), we find, exactly as in ref.[?]:

$$G(x)=[x(1-x)]^{-2\Delta}\frac{1}{F(x)},\qquad \Delta=\frac{1}{2}\epsilon(1-\epsilon) \tag{A.7}$$

where Δ is the dimension of the twist field.

Now we can also include the instanton sectors. For a winding sector labelled by the lattice vector $L=n_1R_1+in_2R_2$ for X between the two Dirichlet intervals, we have $X(z,\bar{z})=\frac{L}{F(x)}(\int_a^P dz\omega+\int_a^P d\bar{z}\bar{\omega})$ and similarly $\bar{X}(z,\bar{z})=\frac{\bar{L}}{F(x)}(\int_a^P dz\omega'+\int_a^P d\bar{z}\bar{\omega}')$. One can then show, that the instanton action contributes to the correlation function multiplicatively, by the exponential of the classical action, which is $e^{-\tau(n_1^2R_1^2+n_2^2R_2^2)}$, where

$$\tau=\frac{F(1-x)}{2\pi^2\sin(\epsilon)F(x)} \tag{A.8}$$

is positive and real for real x between 0 and 1.

The complete tachyon vertex operator in (-1)-ghost picture is $e^{-\phi}e^{ip_\mu X^\mu}\sigma_\pm e^{\pm i(1-\epsilon)\phi_1}$, where ϕ is the bosonization of the superghost and ϕ_1 is the bosonization of the fermionic partners of X and $\bar{X}$. Finally, p_μ is the $(p+1)$ dimensional world-volume momenta, satisfying the onshell condition $\sum_\mu p_\mu^2 = m^2 = \epsilon$. Since the total ghost charge of ϕ on the disc must be -2, we must insert two operators in (-1) ghost picture and two in 0-ghost picture. The latter are obtained by applying the picture changing operators on the (-1)-picture vertex operator. Taking both the twist fields in (-1)-ghost picture, we see that only the world-volume part of the picture changing operator contributes, with the final result for the 4-pt. function $I(x)$ involving the 2-tachyons and their complex conjugates:

$$I(x) = p_1.p_3 x^{s-1}(1-x)^{t-1}\frac{1}{F(x)}\sum_{n_1,n_2} e^{-\tau(n_1^2R_1^2+n_2^2R_2^2)} \tag{A.9}$$

where we have also included all the instanton sectors. In the above expression, s and t are the Mandelstam variables $(p_1+p_2)^2$ and $(p_2+p_3)^2$ respectively, where p_i are the on-shell world-volume momentum carried by the operators at x_i.

Few comments are in order. As $x \to 0$, $F(x) \to 1$ while $F(1-x) \to sin(\epsilon)(-\log(x) + \log(\delta))$ where $\log(\delta) = 1/\epsilon$ in the limit $\epsilon \to 0$, which is the case of interest to us here. Thus in $x \to 0$ limit, $I(x) \to p_1.p_3 x^{s-1}(\frac{x}{\delta})^{(n_1^2R_1^2+n_2^2R_2^2)/2\pi^2}$, which just means that the fields σ_+ and σ_- go to an intermediate open string state from the p-brane to itself and carrying the winding numbers n_1 and n_2. Indeed in the sector of the open string from the p-brane to itself, there are precisely such winding states. $C(n_1,n_2) \equiv \delta^{-\frac{1}{4\pi^2}(n_1^2R_1^2+n_2^2R_2^2)}$ (modulo some phases) are just the structure constants for this 3-pt. function. Integrating x near 0 we find a contribution of the following form:

$$\sum_{n_1,n_2}\frac{p_1.p_3}{s+m^2}|C(n_1,n_2)|^2 \qquad m^2 = (n_1^2R_1^2+n_2^2R_2^2)/2\pi^2 \tag{A.10}$$

This result can be understood in terms of factorized diagrams arising from the following three point functions:

$$\sum_{n_1,n_2}(C(n_1,n_2)\bar{\chi}\partial_\mu\chi A_\mu(n_1,n_2)+c.c.)+\sum_{(n_1,n_2)\neq(0,0)}(mC(n_1,n_2)\chi\bar{\chi}\phi(n_1,n_2)+c.c.) \quad \text{(A.11)}$$

where $A_\mu(n_1,n_2)$ for $n_1=n_2=0$ is the $U(1)$ gauge field of the p-brane and for non-zero n_1 or n_2 is the massive vector bosons carrying the winding numbers (n_1,n_2) with scalar partners $\phi(n_1,n_2)$. Note that for zero winding sector the coupling $C(0,0)$ to the $U(1)$ gauge field is 1 and it measures the charge of the tachyon with respect to this $U(1)$.

Now let us consider the other limit $x\to 1$. The intermediate state now is the open string stretched between the $(p+2)$-brane carrying the magnetic field to itself. Now $\tau^{-1}\to -2\pi^2\sin^2(\epsilon)\log[(1-x)/\delta]$ goes to zero. Thus, in order to get an interpretation of intermediate states, we must do a Poisson resummation over the integers n_1 and n_2. The result is

$$I(x)=p_1.p_3x^{s-1}(1-x)^{t-1}\frac{2\pi^3\sin(\epsilon)}{VF(1-x)}\sum_{n_1,n_2}e^{-\frac{\pi^2}{\tau}(\frac{n_1^2}{R_1^2}+\frac{n_2^2}{R_2^2})} \quad \text{(A.12)}$$

From the asymptotic behavior given above, we conclude that $I(x)\to\frac{1}{N}p_1.p_3(1-x)^{t-1}$ $(\frac{(1-x)}{\delta})^{\frac{1}{2\pi^2N^2}(n_1^2R_2^2+n_2^2R_1^2)}$. The $1/N^2$ in the exponent can be understood by considering the $(p+2)$-$(p+2)$ open string with the modified boundary conditions due to the magnetic field. Indeed one can easily work out the zero mode contribution to the annulus partition function for this sector with the result that effectively radii R_1 and R_2 are scaled by $1/N$. For zero windings n_1 and n_2, if one integrates the above behaviour of $I(x)$ near $x=1$, one finds a pole $p_1.p_3/t$, with coefficient $1/N$. This pole is due to the exchange of the $U(1)$ gauge field on the $(p+2)$-brane, and the coefficient $1/N$ signifies that the corresponding charge of the tachyon is $1/\sqrt{N}$. Just as in the $x\to 0$ case, we can identify the massive poles in the t-channel as arising from the following cubic terms in the effective action:

$$\sum_{n_1,n_2}(\tilde{C}(n_1,n_2)\bar{\chi}\partial_\mu\chi\tilde{A}_\mu(n_1,n_2)+c.c.)+\sum_{(n_1,n_2)\neq(0,0)}(\tilde{m}\tilde{C}(n_1,n_2)\chi\bar{\chi}\tilde{\phi}(n_1,n_2)+c.c.) \quad \text{(A.13)}$$

where $\tilde{C}(n_1,n_2)=\frac{1}{N}\delta^{-\frac{1}{4\pi^2N^2}(n_1^2R_1^2+n_2^2R_2^2)}$, the mass $\tilde{m}=m/N$ and the fields $\tilde{A}$ and $\tilde{\phi}$ are the $(p+2)$-brane analogues of the fields A and ϕ.

To obtain the quartic term in the potential, we must remove the reducible diagrams arising from the cubic terms in eqs.(??) and (??). Including the exchanges of both the vector fields A_μ's and the scalar fields ϕ's in the s-channel and using the identity $(p_1-p_2).(p_3-p_4) = 4p_1.p_3 + s$ and similarly for the t-channel we find that the reducible diagrams contribute:

$$\sum_{n_1,n_2}\Big((\frac{4p_1.p_3}{s+m^2}+1)|C(n_1,n_2)|^2+(\frac{4p_1.p_3}{t+\tilde{m}^2}+1)|\tilde{C}(n_1,n_2)|^2\Big) \tag{A.14}$$

After removing the reducible diagrams from the complete 4-pt. amplitude $\int_0^1 dx I(x)$, we can take the $V \to 0$ limit, when the tachyons become massless and the on-shell momenta can be set to zero. The resulting quartic coupling is:

$$\sum_{n_1,n_2} e^{-\pi N(n_1^2\frac{R_1}{R_2}+n_2^2\frac{R_2}{R_1})}+\sum_{n_1,n_2}\frac{1}{N}e^{-\frac{\pi}{N}(n_1^2\frac{R_1}{R_2}+n_2^2\frac{R_2}{R_1})} \tag{A.15}$$

This expression gives the quartic coupling $|\chi|^4$ in the effective action involving the infinite set of scalar fields $\phi(n_1,n_2)$ and $\tilde{\phi}(n_1,n_2)$ whose mass squares are of order V. The minimization of this potential (including the cubic terms discussed above) results in an infinite set of equations for the scalar fields $\phi(n_1,n_2)$, $\tilde{\phi}(n_1,n_2)$ and χ. Solving the equations for $\phi(n_1,n_2)$ and $\tilde{\phi}(n_1,n_2)$ one finds that, to the leading order, $\phi(n_1,n_2) = -\frac{C(n_1,n_2)}{m}|\chi|^2$ and similarly for $\tilde{\phi}$'s. Substituting this solution results in an effective quartic potential for χ with the quartic coupling being just the $n_1=n_2=0$ term of eq. (??). This is the result we have used in section 2.

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