HIGHER-ORDER AMPLITUDE SQUEEZING
OF PHOTONS PROPAGATING THROUGH A SEMICONDUCTOR

Nguyen Ba An*
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT
Photon amplitude $K^{th}$ power squeezing is studied when the coherent photon propagates through a semiconductor containing the exciton. If the exciton is prepared initially in a coherent state, the photon may become amplitude $K^{th}$ power squeezed. It is shown that, in the short-time limit, the photon squeezing in the $P$ direction does not appear at all while that in the $X$ direction is possible for all the amplitude powers $K$. In the latter case, the amount of squeezing is larger for higher power $K$. Dependences on all the system parameters as well as on the output light detection moment are investigated in detail.

MIRAMARE – TRIESTE
December 1996

*Permanent address: Institute of Physics, P.O. Box 429 Bo Ho, Hanoi 10000, Vietnam.
I. INTRODUCTION

Advanced optical measurement technologies, e.g., gravitational wave detection interferometers or active laser gyroscopes, have now been approaching the shot noise limit set by the quantum mechanics. The new strategy is circumventing the standard quantum limit to achieve an arbitrarily desired precision in experimental measurements. The circumvent appears possible in squeezed states. These are states in which fluctuations in one observable become smaller than fluctuations in a coherent state, at the cost of sacrificing fluctuations in the other conjugate partner observable to meet the Heisenberg uncertainty principle.

The conventional quadrature amplitude squeezing, or shortly amplitude squeezing, deals with the variance of linear combinations of field operators. Hillery introduced for the first time the concept of squeezing associated with the variables describing the real and imaginary parts of the square of the complex amplitude of the field. Recently, much research has been devoted to the so-called amplitude $K^{th}$ power squeezing which contains the conventional squeezing when $K = 1$ and the Hillery's when $K = 2$ as particular cases. In general, one speaks of the higher-order amplitude squeezing when $K = 2, 3, \ldots > 1$. It is worth noting that this type of higher-order squeezing should not be identified with that defined earlier by Hong and Mandel. Of particular interest is the situation under which the conventional squeezing is absent but the higher-order squeezing takes place. On the other hand, when squeezings of different powers $K$ coexist, it is essential to elucidate the $K$-dependence. Since techniques for measuring higher-order correlations in quantum optics are improving, the higher-order squeezing would be experimentally observed and find its various applications in the near future.

Many physical nonlinear systems have been shown to display squeezing with $K > 1$. A generic connection between the conventional and higher-order squeezings was found in the $M^{th}$ harmonic generation process. If the $M^{th}$ harmonic is squeezed in the conventional sense, then the fundamental should be $M^{th}$ power squeezed. It also seems that squeezing of all powers $K$ might occur at the same time, for instance, in Raman
scatterings. This however is true only for a short initial period of the time evolution. This paper studies photons passing through a finite size semiconductor. We show that, due simultaneously to both the exciton-photon and exciton-exciton interactions inside the semiconductor, the output light may in general become squeezed to any power $K$. However, there exist time intervals during which any $K^{th}$ power squeezing is possible. And, there also exist time intervals during which either only squeezing of certain powers $K$ appears or no squeezing occurs at all for any $K$. The exciton-induced mechanism of generation of squeezed photons in the conventional sense was proposed in Ref. 10. The present paper is a natural generalization of Ref. 10 to the higher-order squeezing case.

We organize our paper as follows. Section II formulates the problem starting with an effective bosonic Hamiltonian describing the interacting photon-exciton system. Section III solves the Hamiltonian for the time evolution in the secular approximation using the polaron picture. Section IV derives expressions for the photon amplitude $K^{th}$ power squeezing and analyzes the squeezing characteristic in the parameter space. Conclusion is the final section.

II. FORMULATION

Consider for simplicity a two-band semiconductor with direct band gap and allowed interband dipole transition. Under excitation the semiconductor is looked upon as a system of interacting electron-hole pairs. This fermionic system can be bosonized into an excitonic one as was done, e.g., in Ref. 10. The coupled light-semiconductor system can be described by an effective Hamiltonian in terms of exciton operators $a$, $a^+$ and photon operators $c$, $c^+$

$$H = E a^+ a + \omega c^+ c - g(a^+ c + c^+ a) + f a^+ a^+ a a$$

with $E$ and $\omega$ the exciton and photon energies ($\hbar = 1$), $g$ and $f$ the exciton-photon and exciton-exciton interactions whose analytic expressions are given in Ref. 10. Suppose that at $t = 0$ the light hits the semiconductor and leaves it at $t = T > 0$. The interaction
process takes place of course during the interval \([0,T]\). It is expected that input classical photons at \(t \leq 0\) may evolve into amplitude \(K^{th}\) power squeezed ones which are detected as the output light behind the semiconductor at \(t \geq T\). The output photon is said to be amplitude \(K^{th}\) power squeezed in \(P\) or in \(X\) directions if

\[
< (\Delta P(K, T))^2 > < \left| \frac{\langle F(K, T) \rangle}{4} \right|^2 \tag{2}
\]
or

\[
< (\Delta X(K, T))^2 > < \left| \frac{\langle F(K, T) \rangle}{4} \right|^2 \tag{3}
\]

respectively, where \(\Delta x = x - < x >\) with \(x\) any operator and \(< x >\) the quantum average of \(x\),

\[
P(K, T) = \frac{e^K(T) - e^{+K}(T)}{2t}, \quad X(K, T) = \frac{e^K(T) + e^{+K}(T)}{2},
\]
and

\[
F(K, T) = \left[ e^K(T), e^{+K}(T) \right] = e^K(T)e^{+K}(T) - e^{+K}(T)e^K(T).
\]

We shall solve (1) for the time evolution in the next section. In this section we (i) prove that the amplitude squeezed state of any power \(K\) is a nonclassical state and (ii) derive explicit expressions for \(F(K, T)\).

(i) Using the Glauber-Sudarshan representation of the density matrix\(^{11,12}\) we can write the variances as

\[
< (\Delta P(K, t))^2 > = \frac{\langle F(K, t) \rangle}{4} + \int \left\{ \Im z^K(t) - \Im < e^K(t) > \right\}^2 \mathcal{P}(z(t))dz(t), \tag{4}
\]

\[
< (\Delta X(K, t))^2 > = \frac{\langle F(K, t) \rangle}{4} + \int \left\{ \Re z^K(t) - \Re < e^K(t) > \right\}^2 \mathcal{P}(z(t))dz(t) \tag{5}
\]

where \(z(t)\) is a complex number characterizing the coherent state at time \(t\) and \(\mathcal{P}\) the weight function in the expansion of the density matrix in terms of coherent states. Clearly from (4) and (5) that, for any squeezed state, for which either (2) or (3) holds, the function \(\mathcal{P}\) cannot generally be interpreted as a probability distribution since it is not non-negative. As a result, the amplitude \(K^{th}\) power squeezed photons defined above are nonclassical for any \(K\).
Aiming at the calculation of the amplitude $K^{th}$ power squeezing, it is desirable to derive for the function $F(K,t)$ forms which are convenient in application to an arbitrary $K$. This task was in fact tried in Ref. 7 but the expression found there (see (2.7) in Ref. 7) looks too cumbersome and by no means convenient. It turns out that much simpler formulae can be obtained. Indeed, by virtue of the commutation relation $c(t)c^+(t) = c^+c(t) + K$ with $n_c(t) = c^+(t)c(t)$ the photon number operator, we are able to derive by mathematical induction two simple formulae for $F(K,t)$. The first formula is

$$F(K,t) = \prod_{j=1}^{K} (n_c(t) + j) - \prod_{j=0}^{K-1} (n_c(t) - j) \quad (6)$$

and the second formula is

$$F'(K,t) = \sum_{q=0}^{K} \frac{K!K^{(q)}}{(K-q)!q!} e^{+K-\gamma}(t)e^{K-\gamma}(t) \quad (7)$$

where $K^{(q)} = K(K-1)\ldots(K-q+1)$. Formula (6) is useful to get $F'(K,t)$ in terms of $n_c(t)$. For the first five amplitude powers, we get immediately from (6) (try the same by using (2.7) in Ref. 7)

$$F(1,t) = 1,$$

$$F'(2,t) = 2[n_c(t) + 1],$$

$$F(3,t) = 3[3n_c^2(t) + 3n_c(t) + 2],$$

$$F(4,t) = 8[2n_c(t) + 1][n_c^2(t) + n_c(t) + 3],$$

$$F'(5,t) = 5[5n_c^4(t) + 10n_c^3(t) + 55n_c^2(t) + 50n_c(t) + 21].$$

Formula (7), on the other hand, is helpful in evaluating expectation values because it is normally ordered.

III. PHOTON TIME EVOLUTION

To solve (1) for the photon time evolution we transform it into the polariton representation in which it reads$^{10}$
\[ H = \sum_{\nu=1}^{2} \Omega_{\nu} \alpha_{\nu}^{\dagger}(t) \alpha_{\nu}(t) + f \sum_{\nu=1}^{2} \sum_{\mu=1}^{2} \sum_{\xi=1}^{2} \sum_{\zeta=1}^{2} v_{\nu\mu} v_{\xi\zeta} \alpha_{\nu}^{\dagger}(t) \alpha_{\mu}^{\dagger}(t) \alpha_{\xi}(l) \alpha_{\zeta}(l). \]  

(8)

The two-branch polariton operators \( \alpha_{\nu}(t), \alpha_{\nu}^{\dagger}(t) \) are related to the photon operators as

\[ c(t) = \sum_{\nu=1}^{2} u_{\nu} \alpha_{\nu}(t), \quad c^{\dagger}(t) = \sum_{\nu=1}^{2} u_{\nu} \alpha_{\nu}^{\dagger}(t). \]  

(9)

The polariton energy \( \Omega_{\nu} \) and the transformation coefficients \( u_{\nu} \) and \( v_{\nu} \) are given by

\[ \Omega_{\nu} = \frac{1}{2} \left[ \omega + E + (-1)^{\nu} \sqrt{(\omega - E)^2 + 4g^2} \right], \]

\[ u_{\nu} = \left[ 1 + \frac{g^2}{(E - \Omega_{\nu})^2} \right]^{-1/2}, \]

\[ v_{\nu} = \frac{g u_{\nu}}{E - \Omega_{\nu}}. \]

The Heisenberg equation of motion of the polariton is solvable exactly in the secular approximation which retains only resonant terms in (8). Let us consider the near resonant situation when \( \delta = \omega - E \) is very small, i.e., \( |\delta/E| \ll 1 \). Then the secular approximation works well and the polariton time evolution can explicitly be derived in the form

\[ \alpha_{\nu}(t) = \exp \left\{ -i \left[ \Omega_{\nu} + \sum_{\mu=1}^{2} f_{\nu\mu} \alpha_{\mu}^{\dagger}(t) \right] t \right\} \alpha_{\nu}, \]

\[ \alpha_{\nu}^{\dagger}(t) = \alpha_{\nu}^{\dagger} \exp \left\{ i \left[ \Omega_{\nu} + \sum_{\mu=1}^{2} f_{\nu\mu} \alpha_{\mu}(t) \right] t \right\}. \]  

(10)

We have adopted the abbreviations \( b \equiv b(t = 0) \) with \( b \) any operator and \( f_{\nu\mu} = 2f_{\nu\mu}^{\dagger} \). Inserting (10) into (9) gives the time-dependence of the photon operator.

**IV. PHOTON AMPLITUDE KTH POWER SQUEEZING**

To examine the squeezing in \( X \) and \( P \) directions we introduce the following functions

\[ S(P, K, t) = \frac{4 \angle (\Delta P(K, t))^2}{| \langle F(K, t) \rangle |}. \]  

(11)

and

\[ \text{6} \]
\[
S(X, K, t) = \frac{4 < (\Delta X(K, t))^2 >}{< F(K, t) >}
\]

where \(\cdots\) denotes a normal ordering of the operators sandwiched between the colons. If \(-1 \leq S(X, K, t) < 0\) \((-1 \leq S(P, K, t) < 0\) the photon is said to be squeezed in the \(X\) (\(P\)) direction.

Making use of (4), (5), (9) and (10) we have obtained

\[
< (\Delta P(K, t))^2 > = \frac{1}{2} \left\{ (K!^2 \sum_{i=0}^{K} \sum_{j=0}^{K} \frac{u_1^{2K-i-j} u_2^{i+j}}{(K-i)! (K-j)! i! j!} \right.
\]
\[
\times \left[ \Re < \alpha_1^{+K-i}(t) \alpha_2^{+i}(t) \alpha_1^{K-i}(t) \alpha_2^{i}(t) >
- 2 \Re < \alpha_1^{K-i}(t) \alpha_2^{i}(t) > \right]
\]
\[
\left. - (2K)! \sum_{q=0}^{2K} \frac{u_1^{2K-q} u_2^q}{(2K-q)! q!} \Re < \alpha_1^{2K-q}(t) \alpha_2^q(t) > \right\}
\]

\[
< (\Delta X(K, t))^2 > = \frac{1}{2} \left\{ (K!^2 \sum_{i=0}^{K} \sum_{j=0}^{K} \frac{u_1^{2K-i-j} u_2^{i+j}}{(2K-q)! q!} \right.
\]
\[
\times \left[ \Re < \alpha_1^{+K-i}(t) \alpha_2^{+i}(t) \alpha_1^{K-i}(t) \alpha_2^{i}(t) >
- 2 \Re < \alpha_1^{K-i}(t) \alpha_2^{i}(t) > \right]
\]
\[
\left. - (2K)! \sum_{q=0}^{2K} \frac{u_1^{2K-q} u_2^q}{(2K-q)! q!} \Re < \alpha_1^{2K-q}(t) \alpha_2^q(t) > \right\}
\]

\[
< F(K, t) > = K! \sum_{i=1}^{K} \left\{ \frac{(K-q)! K^{(q)}}{q!} \right.
\]
\[
\times \left[ \sum_{i=0}^{K-q-1} \sum_{j=0}^{K-q-1} \frac{u_1^{2(K-q-i-j)} u_2^{i+j}}{(K-q-i)! (K-q-j)! i! j!} \right.
\]
\[
\times \left. < \alpha_1^{K-q-i}(t) \alpha_2^{K-q-j}(t) \alpha_1^{K-q-i}(t) \alpha_2^{i+j}(t) > \right\}.
\]

As seen from (13) to (15), we need to calculate the average of the product of polariton operators \(< \alpha_1^{+k}(t) \alpha_2^{+q}(t) \alpha_1^{p}(t) \alpha_2^{q}(t) >\) with \(k, q, p \) and \(r\) any integers including zeros.

Assume that the photon and the exciton are initially (at \(t = 0\)) in their coherent states characterized respectively by complex numbers \(z_c = \sqrt{N_c} \exp(i \theta_c)\) and \(z_a = \sqrt{N_a} \exp(i \theta_a)\) with \(N_c, \theta_c, N_a \) and \(\theta_a\) real. Using properties of the displacement operator \(D_b(z) = \exp(z b^+ - z^* b)\) and the orthogonality condition for the transformation coefficients \(u_1 e_1 +\)
\( u_2v_2 = 0 \), we are able to express the initial photon-exciton system in terms of the polaritons as

\[
|z_a, z_c| = D_a(z_a)D_c(z_c)|0> = D_{a1}(z_1)D_{a2}(z_2)|0> \equiv |z_1, z_2>
\]

where \( z_{1,2} = u_{1,2}z_c + v_{1,2}z_a \). The averaging over the photon and exciton state can then equivalently be carried out over the polariton state. Namely,

\[
<...> = <z_a, z_a|...|z_a, z_c> = <z_2, z_1|...|z_1, z_2>.
\]

Substituting (10) into (17) yields after lengthy and tedious operatoric manipulations

\[
<\alpha_1^+k(l)\alpha_2(l)\alpha_3(l)\alpha_4(l)> = \exp\{i[(k-p)\Omega_1 + (q-r)\Omega_2]t\}
\]

\[
\times Q_{k,q}(z_1; 0, f_{11}, q_{12}, -f_{11}, -r_{12}; f)
\]

\[
\times Q_{k,q}(z_2; k_{12}, f_{22}, -p_{12}, -f_{22}, 0; f)
\]

where

\[
Q_{k,q}(z; l, n, s, w; t) = z^{*k}z^q \exp\{i[k(l + m(k - 1)/2) + q(s(q - 1)/2) + w]t\}
\]

\[
- |z|^2 [1 - \exp[i(l + mk + n + sq + w)t]]
\]

with \( z \) complex, \( q, k \) integers and all the remaining arguments real.

Figures 1 and 2 draw the squeeze functions (11) and (12) versus \( E/t \) for the conventional amplitude squeezing \( K = 1 \) (short-dashed), amplitude-squared squeezing \( K = 2 \) (long-dashed) and amplitude-cubed squeezing \( K = 3 \) (solid). The parameters used for these figures are \( \omega/F = 1 \), \( g/E = 0.04 \), \( f/F = 0.004 \), \( N_a = N_c = 100 \) and \( \theta_a = \theta_c = 0 \). As followed from (19) and (11) to (18), the function \( S \) varies in time by means of trigonometrical functions. As a consequence of this, there exist three types of time domains. The first type is time domains within which squeezing does not occur at all. The second type is time domains within which only squeezing of certain powers \( K \) can arise. And, the third type is time domains within which squeezing of any amplitude power \( K \) is possible. The duration and position of such time domains are sensitively determined.
by the system parameters. For a fixed set of parameters one can adjust the semiconductor
optical size $L = v_g T$ ($v_g$ the group velocity of light inside the semiconductor), by which
the light beam propagates, to detect the squeezing of desired amplitude powers $K$. A
generic property can be obtained analytically in the short-time limit $ET \ll 1$ when both $z_1$ and $z_2$ are real. In this case, by virtue of (19) and (11) to (18), it is easy to derive

$$S(P, K, t) = p^2(K) t^2 + O(t^3)$$  \hspace{1cm} (20)

and

$$S(X, K, t) = -x^2(K) t^2 + O(t^3)$$  \hspace{1cm} (21)

where $p(K)$ and $x(K)$ are some cumbersome real functions of $K$ which increase for growing $K$. Equations (20) and (21) imply that, in the short-time limit, squeezing in the $P$
direction is absolutely impossible while squeezing in the $X$ direction always happens for
all amplitude powers as verified from Figs. 1 and 2. Figure 2 also confirms that, in the short-
time limit, the amount of squeezing in the $X$ direction increases with the amplitude
power $K$. We stress, however, that the above generic property only holds for real $z_1$ and
$z_2$. If either of $z_1$ and $z_2$ or both are complex, the situation changes qualitatively. To
see this we plot, in Fig. 3, $S(X, K, t)$ versus $Et$ with same parameters as in Fig. 2 but
$\theta_c = \pi/4$, i.e., both $z_1$ and $z_2$ are complex. It is manifest that, in the short-time limit, the
$K = 1, 2$ amplitude squeezing is impossible but the $K = 3, 4$ amplitude squeezing appears
and, the $K = 5$ amplitude squeezing is again impossible, ... in contrast to (21). More
information on the $K$-dependence of the amplitude squeezing is obtained from Fig. 4
where the output light is detected at $T = 0.05/E$ (see the figure caption). An alternative
way to study the dependence on both time and amplitude power is to plot the function
$S$ versus $K$ for various detection moments. This is done in Fig. 5 where the detection is
made at the moments $ET = 0.02, 0.05, 0.1, 0.15$ and 0.2. Up to $K = 10$ the following
conclusions can be drawn. For short times ($ET = 0.02, 0.05$), amplitude squeezing arises
for all powers $K$ in accordance with the generic property established above in the short-
time limit, Eq. (21). For a longer time ($ET = 0.1$), squeezing with $K \leq 10$ still occurs
but there appears a tendency of disappearance of squeezing for $K > 10$. For even longer times, there exist $T$-dependent critical powers $K_T$ such that squeezing takes place only for $K \leq K_T$ (here $K_T = 7$ for $ET = 0.15$ and $K_T = 5$ for $ET = 0.2$).

It is known that the photon-exciton interaction is strongest in the exact resonance $\omega = E$. The squeezing effect is thus also governed by the resonance condition. Figure 6 plots $S(X, K, t)$ as a function of detuning $\omega/E$ for different $K$ and $t$. The long-dashed, short-dashed and solid curves correspond to $\{K, Et\} = \{(1, 0.65), (2, 1.35), (3, 0.9)\}$, respectively. The values of $Et$ are chosen at minima in Fig. 2. Figure 6 transparently shows that, for all powers $K$ the squeezing effect is strongest at the resonance and decreases away from the resonance.

In general, disregarding the detection moment, the minima of the squeeze function $S$ are deeper for larger exciton (photon) average number $N_a$ ($N_e$). This was indeed justified\textsuperscript{10} for the conventional squeezing. Yet, for a given semiconductor optical length $L$, the detection moment $T$ is fixed. Then, it is more reasonable to investigate the dependence of the squeezing on the initial exciton average number at a given moment of detection $T$. The dependence of squeezing on the initial exciton average number $N_a$ is plotted in Fig. 7 using the parameters like in Fig. 1 but $\omega/E = 0.9$ and $ET = 0.6$. The $N_a$-dependence is quite nontrivial as recognized from Fig. 7. It is worth noticing that, for $N_a = 0$, amplitude-cubed squeezing is absent but amplitude-squared and conventional squeezings are present. The presence of squeezing for $N_a = 0$ and $\omega/E < 1$ could cause a surprise because no real excitons are available in this case. Here the role of the virtual exciton is played.

Finally, we discuss on the role of the exciton-exciton interaction. If there is no exciton-exciton interaction, i.e., $f = 0$, the photon-exciton system remains coupled but no squeezing can be generated from the initially non-squeezed states because of lack of nonlinearity. In this case, however, if the exciton is initially prepared in a squeezed state, then the squeezing of exciton can be transferred to the photon during the course of time evolution and vice versa\textsuperscript{14}. For initially non-squeezed excitons the condition $f \neq 0$ is necessary for the photon to be squeezed. But, for a given detection moment $T$, $f \neq 0$ is not a sufficient
condition. Also, it is not true that for a given $T$ the greater $f$ the better the squeezing. The nontrivial dependence on $f/E$ is shown in Fig. 8 for $K = 2$ and $ET = 1.35$.

V. CONCLUSION

In conclusion, we have shown that photon amplitude $K^{th}$ power squeezing can be generated from an initially non-squeezed exciton-photon system. The photon squeezing characteristic depends in a complicated manner on all the involved parameters $g/E$, $f/E$, $\omega/E$, $N_a$, $N_c$, $\theta_a$, $\theta_c$ and $ET$. If all the parameters but one are fixed, the squeezing seems to depend "periodically" on the varying parameter. In general, the choice of "best" parameters is difficult. However, in the short-time limit $LT \ll 1$, the photon squeezing in the $X$ direction is possible for all the amplitude powers $K$ and the higher $K$ the larger the amount of squeezing.

ACKNOWLEDGMENTS

This work was supported by SAREC, IAEA and UNESCO. The author would like to thank ICTP and its Condensed Matter Group for hospitality at Trieste.
REFERENCES


FIGURES

FIG. 1. Conventional amplitude squeezing (short-dashed curve), amplitude-squared squeezing (long-dashed curve) and amplitude-cubed squeezing (solid curve) in the $P$ direction as a function of $Et$. The parameters used are $\omega/E = 1$, $g/E = 0.04$, $f/E = 0.004$, $N_a = N_c = 100$ and $\theta_a = \theta_c = 0$. The horizontal straight line passing through 0 in this as well as in all other figures is traced just to guide the eye.

FIG. 2. Same as in Fig. 1 but for the squeezing in the $X$ direction.

FIG. 3. $S(X,K,t)$ as a function of $Et$ with the parameters $\omega/E = 1$, $g/E = 0.04$, $f/E = 0.004$, $N_a = N_c = 100$, $\theta_a = 0$ and $\theta_c = \pi/4$. The curves with decreasing length of dashing correspond to $K = 1, 2, 3$ and 4, respectively. The solid curve is for $K = 5$.

FIG. 4. $S(X,K,t)$ detected at $T = 0.05/E$ as a function of $K$ with the same parameters as in Fig. 3. The squeezing "selects" the amplitude power $K$: up to $K = 10$, only the $K = 3, 4, 7$ and 8 amplitude squeezing occurs while $K = 1, 2, 5, 6, 9$ and 10 squeezing does not appear.

FIG. 5. $S(X,K,t)$ as a function of $K$ for different detection moments. The curves with increasing length of dashing correspond to $Et = 0.02, 0.05, 0.1, 0.15$ and 0.2, respectively. The used parameters are as in Fig. 1.

FIG. 6. $S(X,K,T)$ versus detuning $\omega/E$ for $\{K, ET\} = \{1, 0.65\}$ (long-dashed curve), $\{2, 1.35\}$ (short-dashed curve) and $\{3, 0.9\}$ (solid curve). The other parameters are as in Fig. 1.

FIG. 7. $S(X,K,T)$ as a function of the initial exciton average number $N_a$ for $K = 1$ (long-dashed), $K = 2$ (short-dashed) and $K = 3$ (solid). The other parameters are as in Fig. 1 but $\omega/E = 0.9$ and $ET = 0.6$.

FIG. 8. $S(X,K,T)$ as a function of the scaled exciton-exciton interaction $f/E$ for $K = 2$ and $ET = 1.35$. The other parameters are as in Fig. 1 but $g = 10f$. 

13
Fig. 1

Fig. 2
Fig. 5

Fig. 6
Fig. 7

EXCITON AVERAGE NUMBER

Fig. 8

EXCITON-EXCITON INTERACTION