CHARACTERISTICS OF THE CONDUCTANCE OF QUANTUM WAVEGUIDE CONTAINING AN ARRAY OF STUBS: NUMERICAL SIMULATION

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Abstract

We present the numerical investigations on the conductance of the quantum
waveguide containing an array of stubs with the use of the transfer matrix ap-
proach. The profiles of the conductance as functions of the stub length and of
the Fermi wave number of electrons depend on the number of the stubs as also
on the geometric configuration of the stubs. The conductance performance of this
system for disordered stub lengths and disordered stub intervals is examined in
detail. It is found that the localization length substantially depends on the type
of disorder, the extent of disorder and the Fermi wave number of electrons. The
influence of the stub interval disorder is less serious than that of the stub length
disorder. For the same extent of disorder, the localization length associated with
the stub length disorder is much shorter than that associated with the stub inter-
val disorder. Root-mean-square value of the conductance fluctuations depends on
the extent of disorder. We also present the statistical distribution of conductance
fluctuations in this disorder network structure. It is found that the statistical
distribution can be normal or log-normal, depending on the extent of disorder.
Finally, we find that the additivity of the inverse of the localization lengths cor-
responding to the individual disorders and their combined disorder seems to be
valid with good accuracy in the weak disorder regime.

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I. INTRODUCTION

Quantum coherence transport in low-dimensional mesoscopic systems with novel electronic and magnetic properties has attracted considerable attention recently. In mesoscopic systems typically of nm sizes, the mean free path of electrons at low temperature can be larger than the sample dimension. Therefore, the electron motion is dominated by the wave nature and is mainly governed by the quantum mechanics principle. Using high-mobility two-dimensional (2D) electron gas (2D) and split-gate technique, van Wees et al. and Wharam et al. have fabricated quantum point confinement (QPC) and observed that the two-terminal conductance of the QPC increases in quantum steps of 2e²/h upon varying the width of QPC’s with the gate voltage. Each step corresponds to the population of a new transverse subband.

Considerable interest has been focused to study the propagation of electrons along quantum wires of various geometries. A large number of theoretical calculations have been carried out for a T-shaped device consisting of a main wire of constant width and a stub of similar width perpendicular to the main wire. Many interesting conductance characteristics in this structure were revealed, such as sharp drop to zero transmission at certain values of the stub length, repeated periodically. These features may serve as a new type of transistor action.

Recently, Xia presented a one dimensional quantum waveguide theory for mesoscopic structures. With this theory the Aharonov-Bohm effect, the conductance behaviors of two stub quantum waveguide and various quantum-interference devices can be quantitatively discussed. Wu et al. have presented an efficient way of solving the electron-wave-propagation problem for serially connected quantum-wire devices. With this method, they have discussed the electron motion in the multiple stub waveguide, the multiple-stub T-shaped potential well and the one-dimensional mesoscopic crystal. The multiple-stub quantum wire structures have the special features that the sharp drops in conductance as a function of the stub length become extended to forbidden bands. Some properties of similar devices have been studied experimentally.

Experimental samples used in the measurements of the conductance of the microstructures usually have many leads (typically eight) with different lengths. The length of the stubs in the multiple stub quantum wire can be changed by controlling the gate voltages applied to the stubs. In practice, the interval between adjacent stubs may be different. A natural question that arises is what happens in the conductance characteristics of the multiple stub wire system when the number of the stubs increases to a large value, the stub lengths (which are regulated by the gate voltages) have a disordered modulation and the stub intervals also become disordered.

The purpose of this paper is to present the numerical results of quantum-mechanical calculations of the conductance for the quantum wire containing an array of stubs in terms of the transfer matrix method. We demonstrate the evolution of the conductance with the change of the stub length as the number of the stubs are increased. The variation of the conductance as a function of the Fermi wave number of the electrons is displayed for various modulation configurations of the stub lengths such as the double periodicity, the triple periodicity, and the Fibonacci sequence. In particular, we intend to show the effects of the disordered stub lengths and disordered stub intervals on the characteristics of the conductance in the quantum wire with an array of stubs. It is found that the localization length substantially depends on the disorder types, the degree of disorder, and the Fermi wave number of the electrons. We demonstrate the statistical distribution of the conductance fluctuations. It can be normal or log-normal, depending on the degree of disorder and the Fermi wave number of the electrons. The combined effect of both the stub length disorder and the stub interval disorder is also examined. We find that the additivity of the inverse of the localization lengths corresponding to the individual disorders and the combined disorder seems to be
valid with good accuracy in the weak disorder regime.

This paper is organized as follows. In Sec. II, we describe the particular structure of the model device and the calculation method. Section III presents calculational aspects and numerical results in detail. Section IV, finally, contains a brief summary of the main finding in this study.

II. MODEL, CALCULATION METHOD AND FORMULAS

A sketch of the quantum waveguide device containing an array of stubs is illustrated in Fig. 1. It consists of one main quantum wire and \( N \) stubs with different lengths \( \{ l_i \} \) corrected to the main wire. Therefore, \( N \) junctions are formed. The stub lengths are effectively defined by control signals applied to the ends of the stubs. This structure is assumed to be connected to two perfect semi-infinitely long leads on either sides of the \( N \) junction wire. When a potential difference is applied across the sample, the two perfect leads serve as the reservoirs of emitter and collector of electrons. We consider a one-dimensional quantum wire. This idealization to one dimension corresponds experimentally to a network of high-mobility quantum wires with narrow width such that only the lower subband is populated. The positions of the junctions are designated by \( x_i \) ( \( i = 1, 2, 3, \ldots, N \) ) and the distances between adjacent stubs, \( d_i = x_{i+1} - x_i \) ( \( i = 1, 2, 3, \ldots, N \) ), are allowed to be different. For simplicity, our calculations use single electron approximation neglecting e-e interaction. The basic building unit of the quantum wire network is a single "T-"shaped junction with a single stub. We first require its transfer matrix which relates the wave function coefficients of the electron at one side to those at the other side of the junction. The global transfer matrix that represents the electron wave propagation through the entire device can then be obtained by cascading all the individual transfer matrices in sequence. We have to use the Griffith boundary conditions at each junction owing to the requirement of the single valuedness of wave functions and the conservation of the current density

\[ \psi_1(x_i) = \psi_2(x_i) = \psi_3(x_i), \]  

(1a) and

\[ \sum_{i=1}^{N} \frac{1}{m^*_i} \frac{d\psi_i}{dx} |_{x_i} = 0, \]  

(1b)

where \( \psi_1, \psi_2, \) and \( \psi_3 \) represent the electron wave functions of the left side lead, the stub, and the right-side lead of the junction \( x_i \), respectively. All the derivatives are either outward or inward from the junction, and \( m^*_i \) denotes the electron effective mass in the region. In the approximation of single electron, consider an electron with the wave number \( k \) and the energy \( E(k) = \frac{\hbar^2 k^2}{2m^*} \) (for simplicity, assume all the electron effective masses in different leads to be same) propagating through the \( i \)-th "T-"shaped junction. The wave functions can be written as

\[ \psi_1 = a_1 e^{ikx} + a_2 e^{-ikx}, \]
\[ \psi_2 = c \sin[k(s - l_i)], \]
\[ \psi_3 = g_1 e^{ikx} + g_2 e^{-ikx}. \]

Here we have introduced the local coordinate system for each lead such that the direction is along the electron current direction and the origin is taken at the node of the network for the stub. We also assume that the electron wave function of the stub connected to the gate is a standing wave with zero point at the gate. For some cases (see next section), in order to meet the real physical condition, we may assume a wave peak at the stub gate, i.e., the wave function in the stub has the form of \( c \cos[k(s - l_i)]. \) Using the Griffith boundary conditions one can easily derive the transfer matrix at a given junction. For instance, at junction \( x_i \), we have

\[ M(kx_i) = U^{-1}(ikx_i) m(kl_i) U(ikx_i), \]
The global transfer matrix can be constructed by the product of the individual transfer matrices associated with each junction in order. We obtain

\[ M_{\text{tot}} = U^{-1}(i k z_N) m(k z_N) U^{-1}(i k x_i) m(k |j|) U(i k x_i). \]  

The transmission amplitude of the electrons is related to the element of \( M \) as \( t(k) = 1/M_{22} \). For a given configuration of an array of the stubs, we carry out the matrix multiplication. The two-terminal dimensionless conductance \( g \) of the device at small voltages can be written using Landauer’s formulas, \(^{15, 16}\) as

\[ g = G/(2 e^2/h) = |t|^2. \] 

### III. NUMERICAL RESULTS AND ANALYSES

#### A. Variation of the conductance profile with the stub length for multiple stub quantum wires

We first demonstrate the variation of the dimensionless conductance \( g \) of the multiple stub quantum wires with stub length and also by increasing the number of the stubs, as depicted in Figs. 2. For comparison with the theoretical results of the ideal two-dimensional electron waveguide model, \(^7\) we use the same parameter values as in Ref. 7. Of course, the finite width effect is neglected in our calculations. We take the electron effective mass \( m^* = 0.05 m_0 \), electron energy \( \epsilon \approx 0.08 eV \), the corresponding wave number \( k \approx 0.0341 \AA^{-1} \), and the distance between two stubs is \( d_0 = 95 \AA \) for Fig. 2(a) and \( d_0 = 45.7 \AA \) for Fig. 2(b). In order to match the physical condition that the conductance of the quantum wire should approach 1 in units of \((2 e^2/h)\) when the length of the stub goes to zero, i.e., in the absence of any junction, we need to assume that the electron wave function of the stub has a wave peak at the gate end of the stub. Consequently, the wave function in the stub takes the form of \( c \cos[k(s - l)] \). All the above transfer matrix elements should be simply changed with \( \sin(k l) \) replaced by \( -\cos(k l) \) and \( \cos(k l) \) replaced by \( -\sin(k l) \). This means that the \( k l \) is shifted by \( \pi/2 \) relative to the original one.

In Figs. 2(a) and 2(b), curve \( a \) corresponds to a single stub case, curve \( b \) to double stubs, curve \( c \) to triple stubs, and curve \( d \) to four stubs. Curves in Figs. 2(a) and 2(b) have been vertically offset for clarity. It is seen that the conductance curve oscillates periodically with the stub length for the single stub case. It is the standing wave in the stub that determines the periodical change of the conductance curve. This behavior is qualitatively in agreement with the two-dimensional theoretical model results. \(^7\) On increasing the number of the stubs, some interesting features in the conductance emerge. For instance, for two stub quantum wire with an interval \( d_0 = 95 \AA \) between the stubs, the conductance valley becomes broader and the conductance hump is narrower, compared with the single stub structure. The other important feature is that one strong resonant peak arises in the conductance valley region for the two stub case. The peak position nearly locates at \( kl \approx \pi/2 \) in the varying range \([0, 100] \AA \) of the stub length \( l \). The appearance of this sharp resonant peak can be understood as follows. We have assumed that the wave function of the electrons in the stub is \( c \cos[k(s - l)] \). When \( kl \approx \pi/2 \) the standing wave function has its minimum value close to zero at the node of the network ( \( s = 0 \) ). This is similar to placing a barrier at the node of the network and the junctions become analogous to the barrier structure. As the electrons bounce back-and-forth between two nodes of the network, a quasi-bound state may be formed at certain values of the stub length and the resonant tunneling process of electrons lead to the appearance of the sharp peak. On increasing the number of stubs, from two to four, the sharp peak width becomes broader; the second and third peaks emerge owing to the coupling of many junctions, similar to the case of the multiple barrier well structures. The inset in Fig. 2(a) shows...
the detailed profile of the resonant peaks of curve $d$ in a magnified scale. The coupling of the multiple junctions leads the split of the degenerate energy level. However, upon shortening the stub interval $d = d_0 - 47A$ (Fig. 2(b)), the nature of the conductance curve exhibits a different variation compared with the above-mentioned results of increasing the number of the stubs. This structural parameter seems to correspond to the shallower or narrower quantum well case in which the quasi bound states may not survive. The resonant peaks no longer appear and the narrow conductance valley rapidly extends to the conductance forbidden band. The conductance plateau is formed when the number of stubs is more than one. Small oscillatory structures are superimposed on the plateau when $N > 2$. For larger number of stubs, there are sharp peaks appearing at both edges of the conductance plateaus, as seen in curve $d$. On increasing the number of the stubs, the zero conductance band gets rapidly extended while the drop in conductance becomes very sharp. This broadening of the valleys and the formation of the conductance plateaus in the multiple-stub structures can be understood as the superposition of the higher order harmonic components in the transfer matrix elements. It leads the transition of the conductance trace from a sinusoid-like oscillation for a single stub case to the square wave-shaped variation for the multiple stub structures. From the consideration of a practical device, the formation of the flat conductance blocked band is most favorable and a much more useful situation because it will not require an extremely precise tuning of the stub length. On the contrary, the presence of the sharp resonant peaks in the conductance valley may be unfavorable for the switching device action. It follows that although the device structure is similar, the appropriate choice of the parameters is very important in the optimal design of the device.

B. Effect of different modulation fashions of stub lengths on the characteristics of conductance

We now show the feature of the dimensionless conductance $g$ as a function of $k$ for the regular stub array with the same stub length $l_0$ and the equal interval between adjacent stubs, $d = d_0 = l_0$. In order to meet the physical consideration that when the Fermi energy of electron approaches zero, the conductance should be zero, we choose the form of the standing wave of the stub to be $\sin[k(s - l_0)]$, i.e., a standing wave with the zero value at the gate end of the stub. The evolution of the conductance trace is demonstrated in Fig. 3 on increasing the number of the stubs. Curve $a$ corresponds to a single stub case, curve $b$ to the double stubs, curve $c$ to the triple stubs, curve $d$ to the five stubs, and curve $e$ to the twelve stubs. Each curve has been vertically shifted with respect to the preceding one for clarity. For a single stub quantum wire structure, it can be seen from curve $a$ that the conductance profile is in good agreement with the experimental single-mode results. On adding more stubs into the quantum wire, the original sinusoidal-like oscillatory conductance gradually develops the rectangular-wave shape. The narrow conductance valleys develop into the extended forbidden bands centered at the positions of $k l_0 = n\pi$ $(n = 0, 1, 2, ...)$.

The structural parameters are as follows: the standard length of the stubs is $l_0$, the distance between adja-
cent stubs is fixed at \( d = d_0 = l_0 \) and the number of the stubs is 34 corresponding to the Fibonacci number of the 8th generation of the Fibonacci sequence. Curves \( a - d \) in Fig. 4 correspond to the single periodic modulation with the same stub length of \( l_0 \), the double periodic modulation with the stub lengths taking values of \( \{l_0, 0.9l_0\} \) alternatively, the triple periodic modulation in which the stub lengths take the set of values \( \{l_0, 0.9l_0, 0.95l_0\} \), and the Fibonacci sequence modulation with dual values \( \{l_0, 0.9l_0\} \), respectively. For a comparison, we also show the result of the disordered stub length modulation with the disorder parameter \( w_t = 0.10 \). Curve \( e \) corresponds to the result of an ensemble averaged over 400 configurations for conductance as a function of the Fermi wave number. The consecutive curves are vertically offset for clarity. It is evident from these curves that the original rectangular wave shaped conductance curve is substantially distorted on introducing the modulation of the stub lengths. The complete conductance plateaus are split to several parts in the high Fermi wave number region. The narrow and deep conductance dips emerge in the conductance plateaus, strongly destroying the plateaus. The conductance blocked region do not remain clean as several sharp peaks appear in this region. As increasing complication of the stub length modulations, from curve \( a \) to \( d \), results in the conductance trace displaying more complicate pattern and developing a series of spike-like signals. However, when considering the disordered stub length modulation, after performing the ensemble averaging over configurations the conductance spectrum becomes smooth and the fine oscillatory structures have been significantly suppressed. The sharp edges of the rectangular wave shaped conductance are rounded. The conductance plateau becomes humped and its maximum value drops below the standard value \( (2e^2/h) \) in the high Fermi wave number region.

C. Features of conductance for a quantum wire containing an array of stubs with stub length disorder and stub interval disorder

We now present the calculated conductance as a function of the Fermi wave number \( k \) for the disordered stub length and the disordered stub interval modulations in the quantum wire containing an array of the stubs. To reduce the number of the parameters, we assume the average stub interval \( d_0 \) is equal to the average stub length \( l_0 \). We consider the quantum wire with \( N = 20 \) stubs. To produce disorder modulations, the stub lengths of the sample are determined by the form of \( l_i = (1 + w_t R_1) l_0 \) and the stub intervals are \( d_i = (1 + w_d R_2) d_0 \), where \( R_1 \) and \( R_2 \) represent the random function with a uniform distribution in the range \([–0.5, 0.5]\). \( w_t \) and \( w_d \) describe the extent of the disorder. Figure 5 shows the results of an ensemble averaging over 600 configurations for dimensionless conductance \( g \) as a function of the Fermi wave number \( k \), for two types of disorders, (a) stub length disorder \( w_t \), keeping the stub interval \( d_i = d_0 = 1 \) (constant) in units of \( l_0 \), (b) stub interval disorder \( w_d \), keeping the stub length \( l_i = l_0 \) constant. Each specific configuration in the ensemble averaging for a given curve corresponds to the same geometry \( L = 19 l_0 \) and the same extent of disorder for that curve. Curves \( a - e \) in Fig. 5(a) correspond to different values of \( w_t : 0.0, 0.05, 0.10, 0.15, \) and \( 0.20 \), respectively. However, curves \( a - e \) in Fig. 5(b) corresponds to different stub interval disorder of \( w_d : 0.0, 0.05, 0.10, 0.20, \) and \( 0.30 \), respectively. The consecutive curves in Figs. 5(a) and 5(b) are vertically offset for clarity. It is seen from these curves in Fig. 5(a) for the weak disorder of the stub lengths the ensemble averaged conductance traces still exhibit the periodic rectangular-wave shape but the dense fine oscillatory structures which were imposed upon the conductance plateaus in the earlier cases have been substantially smoothed and the sharp edges of the plateaus are rounded in the high Fermi wave number region. For the larger value of \( w_t \) the conductance curve is significantly disturbed. The conductance plateaus develop into lumps and the maximum values of the lumps decrease rapidly with the increase in the wave number \( k \), as seen in curve \( e \) of Fig. 5(a). These results can be understood as follows: Owing to the transfer matrix elements depending
only on the factor $k_l$, for the large wave number $k_l$, the small fluctuations of $l$ cause a large change of $k_l$; this leads the disorder effect to be considerably enhanced as compared with the small value of $k$. The destructive interference of the wave functions results in the decrease of the conductance. However, the positions of the humps remain unchanged, regardless of the disorder extent. It is worth pointing out that small peaks appear in the conductance blocked region owing to the presence of the disordered stub length modulation, corresponding to the formation of the quasi-bond state in the conductance gap.

For the stub interval disorder, the effect on the conductance is different from that of the stub length disorder, as shown in Fig. 5(b). For different disorder degrees, all the curves basically remain the modified periodic variation. The influence of the disorder stub interval on the conductance curve is three-fold: (1) the rectangular wave shape of the conductance evolve toward the hump-like shape; (2) the dense and fine oscillatory structures imposed upon the conductance curves are smeared out and the edges of the squared wave are rounded; (3) The width of the hump is narrowed down on increasing the extent of disorder. However, the maximum value of the conductance hump and its wave number position remain unchanged, located at $k_{l0} = (2n + 1)\pi/2$ ($n = 0, 1, 2, ...$). It is the property of the standing wave in the stubs that determines the hump-top positions. When $k_{l0} = (2n + 1)\pi/2$ the wave function amplitudes in the stubs take their maximum values at the nodes of the network, i.e., $|\sin((2n + 1)\pi/2)| = 1$. That means the electrons can pass through every junction without any reflection for these special Fermi wave numbers. The change of the stub interval only causes a phase shift of the propagating electron wave, of the form of $e^{i(k_{l0}z)}$. It does not lead to any observable variation in the conductance. So we can safely draw the conclusion that the stub interval disorder causes less serious effect on the conductance trace as compared with the stub length disorder. In particular, for special Fermi wave number $k$ as discussed above, the effect of the stub interval disorder can be entirely neglected.

D. Localization length, conductance fluctuations, and statistic distribution of conductance fluctuations

We now calculate the localization length for the multiple stub quantum wire structures with disorder stub length. The ensemble-averaged value of the logarithmic conductance ($-\langle \ln g \rangle$) is plotted in Fig. 6 as a function of the sample length $L$ in units of the average stub length $l_0$, for several values of the stub length disorder parameter, $w_l$, holding the Fermi wave number fixed, $k_{l0} = 1.7\pi$. The values of $w_l$ are 0.05, 0.10, 0.15, and 0.20 for curves a - d, respectively. 240 to 2400 configurations have been generated for each $L$ and $w_l$ combination. These numbers of configurations are good enough to statistically guarantee the averaged values to be convergent and stable. It is evident that all the curves exhibit a near-linear variation. This linear behavior indicates the exponential decay of the conductance. From the relation $\xi = -dL/d\langle \ln(g) \rangle = \xi_{UCF}$ we obtain the localization lengths for $w_l = 0.05, 0.10, 0.15$, and 0.20 to be 86.23, 21.25, 8.59, and 4.52, respectively. It follows that the localization length $\xi_{UCF}$ drastically decreases with increase in the value of $w_l$. With changing Fermi wave number of electrons also, $\xi_{UCF}$ varies.

The root-mean-square value of the dimensionless conductance fluctuation, $\Delta g = (\langle g^2 \rangle - \langle g \rangle^2)^{1/2}$, as a function of sample length $L$ is displayed in Fig. 7 for $k_{l0} = 1.7\pi$. Curves a - d correspond to different disorders, $w_l = 0.05, 0.10, 0.15$, and 0.20, respectively. For the weak disorder, $w_l = 0.05$, curve a shows a slowly rising function of $L$. The value of $\Delta g$ is less than 1 (in units of $(2e^2/h)$), differing from the prediction of the universal conductance fluctuation (UCF) theory. On increasing the disorder $w_l$, the localization length $\xi_{UCF}$ rapidly decreases as compared with the sample length. Hence here the electron transport process is far away from the ballistic regime and enters the diffusion regime.
conductance fluctuations are substantially suppressed for longer samples.

We now examine the statistical distribution of the conductance $g$. Although there is a general belief that the distribution of $g$ should be normal in the extended regime but log normal in the localized (diffusive) regime, it is still very illustrative in many cases to reveal numerically the statistical distribution of conductance. We display the histogram of the conductance distribution for 2400 configurations as a function of $g$ in Figs. 8(a) and 8(b) and as a function of $\ln(g)$ in Fig. 9, for some different Fermi wave numbers and different disorders $\omega_i$. The parameters are $N = 40, L = 39L_0$, (a) $k_0 = 1.7\pi, \omega_i = 0.01$, (b) $k_0 = 1.6\pi, \omega_i = 0.01$. While in Fig. 9 we have $k_0 = 1.6\pi, \omega_i = 0.15$. The dashed line indicates a Gaussian line shape to guide the eyes. The former two cases correspond to the weak disorder. The corresponding localization length $\xi$ are (a) $234.0L_0$ and (b) $2852.6L_0$, much larger than the sample length $39L_0$. In general, in the extended regime ($L < < \xi$), the distribution of $g$ should be of normal Gaussian line shape. Figure 8(a) exhibits the complete Gaussian distribution, but Figure 8(b) displays semi-Gaussian function with non-Gaussian tail. The latter distribution profile originates from the fact that the Fermi wave number is closer to the position of the conductance hump top, i.e. $k_0 = 1.5\pi$. For the single channel transport, the maximum dimensionless conductance is one and so the values of the conductance fluctuation can not exceed over one. Consequently, the conductance fluctuation closer to one is substantially suppressed and the conductance fluctuation exhibits a semi-Gaussian shape. As an opposite case the histogram of the conductance in the stronger disorder is plotted in Fig. 9 as a function of $\ln(g)$ for 2400 configurations. The localization length now is $30.3L_0$, comparable to sample length $39L_0$. The distribution is found to be semi-Gaussian shape with a non-Gaussian tail. Of course, the detailed profile of the distribution is varied with the change of the Fermi wave number of electrons.

To understand the combined effect of both the stub length disorder and the stub interval disorder, we investigate the variation of the localization length for the dual disordered quantum wire with different extents of the disorder. In contrast to the Anderson-localization problem with single parameter, the disorder in the dual disordered system is characterized by both the random stub length disorder ($\omega_l$) and the random stub interval disorder ($\omega_d$). From the numerical calculations we have found that the inverse of the localization lengths corresponding to different types of disorders fulfills a law of addition in some range of the parameter values. If one denotes the localization length associated with stub length disorder $\omega_l$ as $\xi_l$, that with the stub interval disorder $\omega_d$ as $\xi_d$, and that with the dual disordered sample with the disorder parameters $\omega_l$ and $\omega_d$ as $\xi_{ld}$, then they satisfy the relation $\xi_{ld}^{-1} = \xi_l^{-1} + \xi_d^{-1}$. We show in Table I the calculated localization lengths. It is seen from the table that this additive rule seems to be valid with good accuracy for the weak disorder, the relative error being less than 6%. However, for the strong dual disorder, this additive property is violated. It is worth pointing out the fact that the localization length associated with stub interval disorder $\omega_d$ is always much larger than that associated with the stub length disorder $\omega_l$ when $\omega_d = \omega_l$.

VI. SUMMARY AND REMARKS

We have presented the calculated conductance for the quantum wires containing an array of stubs as functions of the stub length and of the Fermi wave number of electrons. We found that the conductance plateau and the forbidden band are formed when the stub number is more than three. There are fine oscillatory structures superimposed upon the plateau when $N > 2$. For large number of stubs, the zero conductance band is rapidly extended and the simultaneous drop in conductance becoming very sharp. From the consideration of a practical device, the formation of the flat conductance blocked band is most favorable and a much more useful situation because it will not require an extremely precise tuning of the stub length. We investigate the effect of different modulation fashions of the stub lengths on the characteristics of conductance, such as the single periodic modulation...
tion, the double periodic one, the triple periodic one, and the Fibonacci sequence modulation. On increasing the complication of the stub length modulations, the conductance trace displays more complex pattern and develops a series of spike-like signals. We presented the calculated conductance as a function of the Fermi wave number $k$ for the disordered stub length and the disordered stub interval modulations in this system. We found that the effects of two types of disorders on the conductance trace are different. The effect of stub interval disorder is less serious than that of the stub length disorder. In some special value of the Fermi wave number of electrons the effect of the stub interval disorder can be neglected. The ensemble averaged value of logarithmic conductance varies almost linearly with the sample length. The slope of the variation determines the localization length $\xi$. The statistical distribution of conductance changes with the degree of disorder and the Fermi wave number. Finally, we have investigated the combined effect of dual disorder on conductance, i.e., both the stub length disorder and the stub interval disorder. For the weak disorder the inverse of the localization lengths corresponding to different types of disorders obeys a law of addition. These results should enrich our knowledge of electron transport behaviors and localization characteristics in quantum wires containing an array of stubs with two disorder parameters.

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References

Figure captions

FIG. 1 Schematic representation of a quantum wire containing an array of the stubs. \( \mathcal{N} \) junctions are formed. Their positions are designated by \( x_i \) \( (i = 1, 2, \ldots, \mathcal{N}) \). The stub lengths \( l_i \) and the stub intervals \( d_i = x_{i+1} - x_i \) can be different.

FIG. 2 (a) Calculated dimensionless conductance \( g \) as a function of the stub length for the quantum wire with stubs on increasing the number of the stubs, keeping the Fermi wave number of the electrons, \( k = 0.0347\AA^{-1} \) constant and the stub interval is \( d_0 = 0.05\AA \). Curves \( a - d \) correspond to different numbers of stubs: 1, 2, 3, and 4, respectively. The curves in figures have been offset vertically for clarity. (b) Same as (a), but having a shorter stub interval \( d_0 = 0.0475\AA \). The inset in Fig. 2(a) shows the detailed profile of the resonant peaks of curve \( d \) in a magnified scale.

FIG. 3 Calculated conductance \( g \) as a function of the Fermi wave number \( (k_{\text{F}}) \) for the quantum wire with multiple stubs on increasing the number of the stubs. The stub lengths take the same value as \( l_0 \) and the stub intervals are the same as \( d_0 = l_0 \). Curves \( a - e \) correspond to different numbers of stubs: 1, 2, 3, 5, and 12, respectively. The curves have been shifted vertically for clarity.

FIG. 4 Calculated conductance \( g \) as a function of the Fermi wave number \( (k_{\text{F}}) \) for the quantum wire with different modulations of stub lengths. The number of stubs is 34 and the stub interval is constant \( d_0 = 1 \) in units of the standard stub length \( l_0 \). Curves \( a - d \) correspond to the regular array (single periodic modulation), the double periodic modulation with the stub lengths taking values of \( \{l_0, 0.9l_0\} \) alternatively, the triple periodic modulation in which the stub lengths take the set of values \( \{l_0, 0.8l_0, 0.8l_0\} \), the Fibonacci sequence modulation with dual values \( \{l_0, 0.9l_0\} \), respectively. Curve \( e \) corresponds to the ensemble averaged over 400 configurations for conductance \( g \) in the quantum wire structure with disordered stub lengths and the disorder parameter is \( \omega_1 = 0.10 \). Two consecutive curves are vertically offset for clarity.
FIG. 5 Calculated conductance $g$ as a function of the Fermi wave number ($k_{0}$), ensemble-averaged over 600 configurations corresponding to the same number of stubs, $N = 20$, and the same extent of disorder for each curve. (a) for stub length disorder $w_{l}$; (b) for stub interval disorder $w_{d}$. Curves $a - e$ in (a) correspond to different values of $w_{l}$, 0.0, 0.05, 0.10, 0.15, and 0.20, respectively. However, curves $a - e$ in (b) correspond to different stub interval disorders $w_{d}$: 0.0, 0.05, 0.10, 0.20, and 0.30. The consecutive curves are vertically shifted for clarity.

FIG. 6 The averaged value of $-\langle \ln(g) \rangle$ plotted as a function of sample length $L$ for several values of $w_{l}$. The data points show a linear variation behavior. Curves $a - d$ correspond to $w_{l} = 0.05, 0.10, 0.15, and 0.20$, respectively. The value of Fermi wave number is $k_{0} = 1.7\pi$.

FIG. 7 The rms dimensionless conductance fluctuation $\Delta g$ plotted as a function of sample length $L$ for various values of $w_{l}$. Curves $a - d$ correspond to $w_{l} = 0.05, 0.10, 0.15, and 0.20$, respectively. The Fermi wave number is $k_{0} = 1.7\pi$.

FIG. 8 Histogram of the conductance distribution plotted as a function of $g$ for 2400 configurations. The geometric and physical parameter values are $N = 40$, $L = 39l_{0}$, and $w_{l} = 0.01$. The Fermi wave number is (a) $k_{0} = 1.6\pi$, (b) $k_{0} = 1.7\pi$. The dashed line is a Gaussian line shape to guide the eyes.

FIG. 9 Histogram of the conductance distribution plotted as a function of $\ln(g)$ for 2400 configurations. The parameter values are $N = 40$, $L = 39l_{0}$, $w_{l} = 0.15$, and $k_{0} = 1.6\pi$. It corresponds to the case of strong stub length disorder. The dashed line is a Gaussian line shape to guide the eyes.

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$\Delta(\xi^{-1}) = (\xi^{-1}_{\text{max}} - \xi^{-1}_{\text{min}})/(\xi^{-1}_{\text{max}})$, where $\xi_{\text{max}}$, $\xi_{\text{ave}}$, and $\xi_{\text{min}}$ are the localization length for the length disorder of the stubs, the distance disorder between the neighbor stubs, and the dual disorder, respectively.
Fig. 1

Fig. 2a