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The acoustomagnetoelectric effect (AME) in a semiconductor superlattice (SL) has been studied for an acoustic wave in the region $q l \gg 1$. The expression for the acoustomagnetoelectric current $j^{AME}$ was calculated in the $\tau = \text{constant}$ approximation. The result indicates that the existence of $j^{AME}$ in SL may be due to the finite gap band and the periodicity of the electron spectrum. It is shown, in particular, that when $\omega_q > \Delta$ and $\Delta \gg T$ ($\omega_q$ is the frequency of the acoustic phonons; $2\Delta$ is the miniband width of SL and $T$ is the temperature measured in energy unit) the SL behaves as a monopolar semiconductor bulk material and in that case $j^{AME} \rightarrow 0$. 
Introduction

It is well known that when an acoustic wave propagates through a conductor it is accompanied by a transfer of energy and momentum to the conducting electrons. This gives rise to what is called the acoustoelectric effect (AC) [1, 2]. Recently, Mensah et al. have studied this effect in superlattice (SL) [3].

However, in the presence of a magnetic field the acoustic wave propagating in the conductor can induce another effect called the acoustomagnetoelectric effect (AME). The acoustomagnetoelectric effect is actually the generation of AME current (if the sample is short circuited in the Hall direction), or AME field (if the sample is open) when a sample placed in a magnetic field $H$ carried an acoustic wave propagating in a direction perpendicular to $H$.

The AME effect was first predicted theoretically by Grinberg and Kramer [4] for bipolar semiconductors and was observed experimentally in bismuth by Yamada [5]. The explanation given to the effect in [4] was that when an acoustic wave is propagating through a conductor having a bipolar conducting immersed in a magnetic field with the direction of propagation perpendicular to the field, a potential difference (or a short-circuit current) will appear in the third direction. This is due to the fact that the absorption of the acoustic wave causes equal fluxes of electrons and holes in the direction of propagation of the acoustic wave. The magnetic field then deflects these fluxes in the opposite direction, thus giving rise to the onset of the acoustomagnetoelectric field.

Epshtein and Gulyaev [6] later studied this effect in a monopolar semiconductor. In this specimen they observed that the AME effect occurs mainly because of the dependence of the electron relaxation time on the energy, i.e. $\tau(\varepsilon_p)$ and that when $\tau = \text{constant}$ the effect vanishes. The physics behind the existence of this effect is that the perturbation of the electron distribution function under the influence of the sound flux differs significantly from the perturbation that is caused by the electric field, so that, depending on their
energy the effect of the sound flux, will prevail for some electrons while the
effect of the compensating electric field will prevail for others. As a result when
the total acoustoelectric (longitudinal) current is equal to zero, the specimen
will manifest mutually compensating "partial" currents generated by different
energy groups of electrons. When this happens the energy dependence of the
electron momentum relaxation time will cause the average mobilities of the
electrons in these partial currents, in general, to differ. If an external magnetic
field is applied perpendicularly to the direction of the sound flux, the
Hall currents generated by these groups will not, in general, compensate one
another, and a nonzero acoustomagnetoelectric effect will result.

Kaganov et al. [7] upon studying this effect in a metal with an arbitrary
conduction-electron dispersion law, found out that the effect is very sensitive
to the structure of electron spectrum. As a result it can even exist at $\tau =$
constant.

It is necessary to note that like the classical magnetic field, this effect also
were the first to note this and later it was observed by Salaneck et al. [9]
in bismuth. Recently, Margulis and Margulis [10] have studied the quantum
acoustomagnetoelectric effect due to Rayleigh sound waves. They suggested
in their study that the ratio of AME current ($j^{AME}$) to that of the AC current
($j^{AC}$) will be of the order of the ratio of conductivities $\sigma_{xy}/\sigma_{yy}$, which is large
for degenerate electron gas by the parameter $\Omega/\nu$ ($\Omega$ - cyclotron frequency;
$\nu$ - the frequency of electron collisions).

We present in this paper the AME effect in a semiconductor Superlattice
(SL) on which, in our opinion, no work has been done. Furthermore, we think
the study of this effect may present itself as an interesting work so far as
applications in radioelectronic systems are concerned [10]. It may also help in
understanding the properties of SL material. It will be seen that due to the
anisotropic nature of the dispersion law, the AME effect is observed at $\tau =$
constant. It is also dependent nonlinearly on the SL parameters \((\Delta, d)\).

\((2\Delta - \text{miniband width}; d - \text{period of SL}); \text{temperature } T; \text{ and the frequency } \omega_q (\omega_q \text{ is the frequency of acoustic phonons and } q \text{ is its wavenumber}).

Finally, it depends substantially on the field \(H\), the quantity \(\Omega_r\) serving as a measure of the magnetic field strength and that the ratio of \(j^{AME}/j^{AC} = \Omega_r\).

It will also be noted that as \(\omega_q > \Delta; \Delta \gg KT\) the SL behaves as a bulk monopolar semiconductor and \(j^{AME} \to 0\) as expected \([6]\).

The paper is organized as follows. In Section 2 we outline the theory and conditions necessary to solve the problem and in Section 3 we discuss the results and draw some conclusions.

**Theory**

Following the method developed in \([3]\) we calculate the AME current in SL. The acoustic wave will be considered in the region \(ql \gg 1\) (\(l\) is the electron mean free path) and then treated as monochromatic phonons (frequency \(\omega_q\)).

The problem will be solved in the quasi-classical case, i.e. \(2\Delta \gg \tau^{-1}\) (\(\hbar = 1\)).

The magnetic field will also be considered classically, i.e. \(\Omega < \nu; \Omega \ll T\) (\(T\) is the temperature in energy unit).

The density of the acoustoelectric current in the presence of magnetic field can be written in the form

\[
j^{AC} = \frac{2e}{(2\pi)^3} \int U^{AC} \psi_i(\vec{P}, -\vec{H}) \, d^3p
\]

where

\[
U^{AC} = \frac{2\pi \phi}{\omega_q s} |G_{p-q,p}|^2 \left[ f(\varepsilon_{p-q}) - f(\varepsilon_p) \right] \delta(\varepsilon_{p-q} - \varepsilon_p + \omega_q) +
+ |G_{p+q,p}|^2 \left[ f(\varepsilon_{p+q}) - f(\varepsilon_p) \right] \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q)
\]

Here \(\phi\) is the sound flux, \(s\) is the velocity of sound, \(f(\varepsilon_p)\) is the distribution function, \(G_{p+q,p}\) is the matrix element of the electron-phonon interaction and \(\psi_i(\vec{P}, -\vec{H})\) is the solution of the kinetic equation given by

\[
\frac{e}{c} (\vec{V} \times \vec{H}) \frac{\partial \psi_i}{\partial p} + \vec{W}_p \{ \psi_i \} = V_i
\]
$V_i$ is the electron velocity and $\widehat{W}_p\{\ldots\} = \left(\frac{\partial f}{\partial \varepsilon}\right)^{-1} \widehat{W}\left(\frac{\partial f}{\partial \varepsilon}\right)$. The operator $\widehat{W}$ is assumed to be hermitian [7]. In the $\tau$ approximation $\widehat{W}_p = \frac{1}{\tau}$. We shall seek the solution of Eq.(3) as

$$\psi_i = \psi_i^{(0)} + \psi_i^{(1)} + \ldots$$

(4)

Substituting Eq.(4) into Eq.(3) and solving by the method of iteration, we get for the zero approximation, i.e. in the absence of magnetic field ($H = 0$)

$$\psi_i^{(0)} = V_i \tau$$

(5)

and for the first approximation

$$\psi_i^{(1)} = -\frac{\tau^2 e}{mc} (\vec{V} \times \vec{H})_i$$

(6)

where $i = x, y, z$.

Inserting (5) and (6) into (1) and taking into account of the fact that

$$|2\rho_{r}\rangle = |2\rho_{p}\rangle$$

we obtain for the acoustoelectric current the expression

$$j^{AC} = -\frac{e\phi}{2\pi^2s \omega_q} \int |G_{p+q,p}\rangle^2 [f(\varepsilon_{p+q}) - f(\varepsilon_p)] \times$$

$$\times \{V_i(p + q) \tau - V_i(p) \tau\} \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q) \ d^3p -$$

$$-\frac{e^2 \phi \tau^2}{2\pi^2 mc \omega_q \varepsilon} \int |G_{p+q,p}|^2 [f(\varepsilon_{p+q}) - f(\varepsilon_p)] \times$$

$$\times \{(\vec{V}(p + q) \times \vec{H})_i - (\vec{V}(p) \times \vec{H})_i\} \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q) \cdot d^3p$$

(7)

The matrix element of the electron–phonon interaction for $qd \ll 1$ is given as

$$|G_{p,q}|^2 = \frac{|A|^2 q^2}{2\sigma \omega_q}$$

(8)

where $A$ is the deformation potential constant, $\sigma$ is the density of the SL.

In solving Eq.(7) we shall consider a situation whereby the sound is propagating along the SL axis (ox) and the magnetic field $H$ is parallel to the $oz$
axis. Under such orientation the first term in Eq.(7) is responsible for the acoustoelectric current and the solution is found in [3].

The second term is the acoustomagnetoelectric current and is expressed as

\[
j_y^{AME} = -\frac{e|\Lambda|^2q^{2}\tau^2\Omega}{4\pi s\omega_y^2\rho} \int [f(\varepsilon_{p+q}) - f(\varepsilon_p)] \times \\
\times \{V_x(p + q) - V_x(p)\} \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_y) \, d^3p
\]

(9)

where \( \Omega = \frac{eH}{mc} \).

The distribution function \( f(\varepsilon_p) \) in the usual form is given by

\[
f(\varepsilon_p) = \frac{\pi dn}{mTl_0(\frac{\Delta}{T})} \exp(-\varepsilon_p/T)
\]

(10)

where \( n \) is the electron density, \( m \) is the mass of electron and \( l_0(X) \) is the modified Bessel function of the zero order.

The energy \( \varepsilon(p) \) of the SL in the lowest miniband is given using the usual notation by

\[
\varepsilon(p) = \frac{P_x^2}{2m} + \Delta(1 - \cos P_x d)
\]

(11)

Hence

\[
\frac{\partial \varepsilon}{\partial p} = V_x(p) = \Delta \sin P_x d
\]

(12)

Substituting (10), (11) and (12) into (9) and after cumbersome calculation we obtain for non-degenerate electron gas the following expression

\[
j_y^{AME} = \frac{e|N|^2nq^{2}\phi\tau^2\Omega d}{s\omega_y^2\sigma} \cdot \Theta(1 - b^2) \cdot \\
\cdot \sinh \frac{\omega_y}{2T} \sinh \left[ \frac{\delta}{T} \cos \left( \frac{qd}{2} \right) \sqrt{1 - b^2} \right]
\]

(13)

where

\[
b = \omega_y/2\Delta \sin(qd/2)
\]
Discussion and Conclusion

The result in Eq.(13) can be written in terms of the acoustoelectric current as

\[ j_y^{AME} = j_x^{AC} \Omega \tau \]  \hspace{1cm} (14)

The AME current depends on the magnetic field \( H \), the quantity \( \Omega \tau \), serving as a measure of the magnetic strength. The ratio of \( j_y^{AME} / j_x^{AC} \) is equal to \( \Omega \tau \).

This result is quite interesting as a similar ratio calculated for the case of QAME due to Rayleigh sound wave was of that order [10]. In their case \( \Omega \tau \gg 1 \) (quantized magnetic field) and the sample is a bulk material.

In our opinion, the mechanism responsible for the existence of the AME effect in SL may be due to the finite band gap and the periodicity of the electron spectrum (dispersion law) along the \( x \) axis and not the dependence of \( \tau \) on \( \varepsilon_p \).

The calculation was done on the basis of \( \tau = \text{constant} \) and according to [6] the AME effect should be zero. However, for \( \omega_q > \delta \) and \( \Delta \gg T \) when the SL behaves as a bulk monopolar semiconductor with parabolic law of dispersion \( j_y^{AME} \to 0 \) as expected for \( \tau = \text{constant} \) [6]. This is readily deduced from the conservation laws. The nonlinear dependence of \( j_y^{AME} \) on the SL parameter \((\Delta, d)\) and the frequency \( \omega_q \) and particularly the strong spatial dispersion of \( j_y^{AME} \) once again can only be attributed to finite band gap and periodicity of the spectrum along the \( x \) axis.

In conclusion, we have studied the acoustomagnetoelectric effect in a semiconductor superlattice and noted that it exists at \( \tau = \text{constant} \) and has a strong spatial dispersion.
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