NEUTRINOLESS DOUBLE BETA DECAY
AND THE SOLAR NEUTRINO PROBLEM

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1 Introduction

The results of the solar neutrino experiments obtained so far can be considered as an indication of existence of nonzero neutrino masses and mixing. The data are well described in terms of neutrino resonant transitions $\nu_e \rightarrow \nu_{\nu}(\nu) [1]$, or vacuum oscillations $[2] \nu_e \leftrightarrow \nu_{\nu}(\nu)$, with parameters (see refs. [3] for latest analyses):

$$\Delta m^2 \cong (0.6 - 1.2) \times 10^{-5} \text{eV}^2, \quad \sin^2 2\theta \cong (0.6 - 1.4) \times 10^{-2} \text{ or } (0.65 - 0.85),$$

(1)

and

$$\Delta m^2 \cong (0.5 - 1.1) \times 10^{-10} \text{eV}^2, \quad \sin^2 2\theta \cong 0.75,$$

(2)

respectively, where $\Delta m^2 \equiv m_2^2 - m_1^2$, $m_{1,2}$ being the masses of two mass eigenstate neutrinos $\nu_{\nu,\bar{\nu}}$, and $\theta$ is the lepton mixing angle in vacuum. The values of $\Delta m^2$ in (1) determine a neutrino mass scale associated with the solar neutrino problem:

$$m_0 = \sqrt{\Delta m^2} \cong 3 \times 10^{-3} \text{eV}. \quad (3)$$

What are the implications of the results (1) and (2) for the present and future searches for neutrinoless double beta ($\beta\beta_{0
\nu}$) decay: $(A,Z) \rightarrow (A,Z + 2) + e^{-} + e^{-}$ [5]? As is well known, these searches are sensitive to the existence of massive Majorana neutrinos $\nu_j$, coupled to the electron in the weak charged lepton current. For relatively small masses of $\nu_j$, $m_j < 30 \text{ MeV}$, the $\beta\beta_{0\nu}$ decay amplitude is proportional to the $(\nu_j \nu_i)$ element, $m_{ij}$, of the neutrino Majorana mass matrix. It can be written as [6]:

$$m_{\text{rec}} = \sum_{j} \eta_j |m_{ij}|^2,$$

(4)

where $\eta_j = \pm 1$, $\eta_j^CP$ is the CP-parity of the neutrino $\nu_j$, and $U_{ij}$ determines the admixture of the $\nu_i$ state in the $\nu_j$ state $^3$. The results of the $\beta\beta_{0\nu}$-decay searches imply the following upper bound on the value of $m_{\text{rec}}$ [7]: $|m_{\text{rec}}| < (1 - 2) \text{ eV}$. Future experiments

\[1\] Let us note that solution (2) is disfavored by the data on the neutrino burst from the supernova SN1987A [4].

\[2\] We assume for simplicity in the present study that CP-parity is conserved in the leptonic sector and that weak interactions with right-handed currents do not play an essential role in the $\beta\beta_{0\nu}$ decay.

\[3\] $U_{ij}$ is an element of the unitary lepton mixing matrix defined by $\nu_L = \sum_j U_{ij} \nu_{\nu,L,i} = e, \mu, \tau$. 

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with enriched isotopes $^{76}$Ge and $^{100}$Mo [8] will be sensitive to values of $|m_{ee}|$ as small as $|m_{ee}| \approx (0.1 - 0.3) \, \text{eV}$. We will refer to the effective mass having a value between the existing upper bound and the indicated limit of sensitivity,

$$m_{ee}^{\text{obs}} \cong (0.1 - 1.0) \, \text{eV},$$

as "observable" effective Majorana mass.

Simplest schemes of neutrino mass generation based on the see-saw mechanism [9-11] do not allow to reconcile solutions (1) and (2) of the $\nu_{\mu}$-problem with a value of $m_{ee}$ in the interval (5). Indeed, the most straightforward interpretation of an observation of the $(\beta\beta)_{0n}$ decay would be that of $\nu_3$ practically coinciding with the lightest Majorana neutrino $\nu_1$ possessing a mass $m_1 \cong m_{ee}^{\text{obs}}$. It follows then from the see-saw scenario that the masses of the other two light neutrinos should lie in the intervals $m_2 \cong (1 - 100) \, \text{keV}$, and $m_3 \cong (0.1 - 10) \, \text{MeV}$. Obviously, in this case solutions (1) and (2) are impossible. On the other hand, e.g., solution (1) implies $m_1 < m_2 \equiv m_{ee}$, and the see-saw mechanism with quadratic hierarchy gives $m_3 \sim (1 - 10) \, \text{eV}$ in the range of interest. However, the admixture $U_{33}$ of $\nu_3$ in $\nu_e$ is typically predicted to be of the order of the parameter describing the mixing between the first and the third generations in the quark sector: $|U_{3e}| \lesssim 5 \times 10^{-5}$ (see e.g., refs. [11]). This gives $|m_{ee}| \lesssim 2.5 \times 10^{-4} \, \text{eV} << m_{ee}^{\text{obs}}$.

In the present letter we analyze the phenomenological implications of a neutrino physics solution of the $\nu_{\mu}$-problem ((1) or (2)) and the existence of $(\beta\beta)_{0n}$ decay having a rate in the range of sensitivity of the present and future $(\beta\beta)$-decay experiments [7,8]. Schemes with three light massive neutrinos are discussed in detail. Consequences for the mechanisms of neutrino mass generation are considered. We comment also on the possibility to accommodate two other elements in the indicated schemes: i) a value for one of the light neutrinos in th. region of $(5 - 7) \, \text{eV}$ - such a neutrino can play the role of a "hot" dark matter component [2], ii) oscillations $\nu_3 \leftrightarrow \nu_e$ (or $\nu_3 \leftrightarrow \nu_\mu$) with $\Delta m^2 \cong 10^{-2} \, \text{eV}^2$ and $\sin^2 2\theta \cong (0.4 - 0.6)$, which can explain the suggested deficit of muon neutrinos in the atmospheric neutrino flux [13].

### 2 The Case of Strong Neutrino Mass Hierarchy

Suppose that the masses of the three light Majorana neutrinos $\nu_{1,2,3}$ obey the hierarchy relation $m_3 \ll m_2$, and $m_1 \ll m_2$, or $m_1 < m_2$. The $\nu_{\mu}$-problem can be solved then by $\nu_\mu \rightarrow \nu_e$ conversion if $m_3 \cong m_{ee}$ and the $e - \mu$ flavour mixing corresponds to one of the intervals in (1). A value of $|m_{ee}|$ in the interval (5) can only be due to a sufficiently large admixture of the $\nu_3$ state with a mass $m_3 > 0.1 \, \text{eV}$ in the $\nu_e$ state:

$$|m_{ee}| \cong m_3 |U_{3e}|^2.$$  \hspace{1cm} (6)

The values of $|U_{3e}|$ and $m_3$ are constrained by the null results of the oscillation experiments performed at reactors [14,15] and accelerators [16,17]. Since $m_1 < m_2 \cong m_{ee}$, the oscillation length associated with $\Delta m^2_{32}$ is much longer than the relevant source-detector distances and the indicated oscillation experiments are sensitive only to $\Delta m^2_{32} = m_3^2 - m_2^2 \cong m_3^2 (\cong \Delta m^2_{32})$. As a consequence, the $\nu_\mu \leftrightarrow \nu_e$ and $\nu_\tau \leftrightarrow \nu_e$ oscillation probabilities depend only on $|U_{3e}|^2$, taking the simple form of the probability of two-neutrino oscillations with effective mixing parameters $\sin^2 2\theta_{\nu_e} = |U_{3e}|^2 |U_{3e}|^2$. As follows from (6) the $\nu_e$-oscillation probability can be rewritten in terms of the oscillation parameters $\Delta m^2_{32} \cong m_3^2$ and $\sin^2 2\theta_{\nu_e} \equiv \sin^2 2\theta$. For the probability $P(\nu_\mu \rightarrow \nu_e)$ (disappearance experiments) one has $\sin^2 2\theta = 4|U_{3e}|^2(1 - |U_{3e}|^2)$, and it follows from (6) that

$$\Delta m^2_{32} \cong m_3^2 = \frac{4m_{ee}^2}{(1 - \sqrt{1 - \sin^2 2\theta})^2}.$$  \hspace{1cm} (7)

For small values of $\theta$ eq. (7) reduces to $\Delta m^2_{32} \cong 16m_{ee}^2/\sin^4 2\theta$. The parameter $\sin^2 2\theta$ determined above enters also into the expression for the $\nu_e \leftrightarrow \nu_\mu$ oscillation probability. Indeed, the searches for $\nu_\mu \leftrightarrow \nu_e$ oscillations imply $|U_{3e}|^2 < 2 \times 10^{-2}$, while the negative results of the searches for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations [19] lead to the constraint $|U_{3e}|^2 < 10^{-2}/|U_{3e}|^2$. Consequently, $|U_{3e}|^2 << 1$ and one has $\sin^2 2\theta \cong 4|U_{3e}|^2(1 - |U_{3e}|^2)^2 - |U_{3e}|^2 \cong \sin^2 2\theta$.

The dependence (7) of $\Delta m^2_{32}$ on $\sin^2 2\theta$ for different values of $|m_{ee}|$ is shown in Fig. 1 together with the existing bounds on these two parameters and the planned sensitivities of the future experiments [20]. As follows from Fig. 1, for $|m_{ee}| > 0.1 \, \text{eV}$ we have $m_3 > 2 \, \text{eV}$.
and \( \sin^2 2\theta < 0.2 \). If \( |m_{\nu_3}| > 0.3 \) eV one gets \( \sin^2 2\theta < 0.08 \) and \( m_3 \approx 4m_{\nu_3}/\sin^2 2\theta > 15 \) eV. For the cosmologically interesting values of \( m_3 = (3 - 7) \) eV the allowed mixing equals \( \sin^2 2\theta = (0.03 - 0.20) \). Future oscillation experiments at accelerators (NOMAD, CHORUS [20]) will be able to cover large part of the region of interest. In the case of negative results the allowed domains will be around \( \Delta m^2_3 = (3 - 5) \) eV\(^2\) and \( \sin^2 2\theta = (0.1 - 0.2) \), or at \( m_3 > 30 \) eV. For \( m_3 > 30 \) eV the cosmological bound implies that \( \nu_3 \) should be unstable.

A generic feature of the possibility being considered is the strong mass hierarchy, \( m_{12}/m_3 \ll 10^{-3} \), and the relatively large mixing between the first and the third families of leptons. This feature is a consequence of the inequality \( |m_{\nu_3}| \gg m_1 \) which implies a fine tuning of the elements of the \( \nu_3 \) Majorana mass matrix, \( m^{\nu_3} \), at the level of

\[
\xi \equiv \frac{m_1}{|m_{\nu_3}|} \leq \frac{m_0}{m_{\nu_3}} \leq 10^{-2}.
\]

Indeed, as can be shown, the ratio \( \xi \) is related to the determinant \( D_m \equiv (m_{12}m_{23} - m_{13}^2) \) of the \( \nu_{12} \rightarrow \nu_{13} \) submatrix of \( m^{\nu_3} \): \( D_m \leq \xi \). Consequently, \( D_m \ll 10^{-2} \). Thus \( \rho \ll 10^{-2} \) and \( \nu_{13} \) will be the dominant lepton mixing.

Consider the implications of the above results for the see-saw mechanism of neutrino mass generation. According to this mechanism, the Majorana mass matrix \( m^{\nu_3} \) is given by

\[
m^{\nu_3} = -m_D M_R \overline{m_D}^T,
\]

where \( m_D \) is the neutrino Dirac mass matrix, and \( M_R \) is the the Majorana mass matrix of the right-handed (RH) neutrino components \( \nu_R \). We will assume that the Dirac mass matrices of the neutrinos and the charged leptons are similar in structure to the up and down quark mass matrices. Correspondingly, the \( \nu_e \rightarrow \tau \) flavour mixing resulting only from the Dirac neutrino and charged lepton mass matrices is exceedingly small. An enhancement of the \( \nu_e \rightarrow \tau \) mixing can take place then due to a special structure of the Majorana mass matrix \( M_R \) [21].

It is convenient to consider the problem in the neutrino Dirac basis \( \nu'_1, \nu'_2, \nu'_3 \), in which the neutrino Dirac mass matrix, \( m_D \), is diagonal: \( m_D = \text{diag}(m_{D1}, m_{D2}, m_{D3}) \). Suppose that in this basis the matrix \( M_R \) has the following form:

\[
M_R = \begin{pmatrix}
M_1 & 0 & M \\
0 & M_2 & 0 \\
M & 0 & M_3
\end{pmatrix},
\]

where for simplicity \( M \) and \( M_j \) \((j = 1, 2, 3) \) are considered to be real. The components \( \nu'_{1L} \) and \( \nu'_{1R} \) decouple from the other neutrino components, and \( \nu'_{2L} \cong \nu'_{2R} \equiv \nu_{2L} \) acquires a Majorana mass \( m_3 \cong m_{12}/M_1 \). The see-saw mechanism produces an \( \nu_e \rightarrow \nu_\tau \) mixing

\[
U_{\nu_\ell} \cong \frac{m_{1\ell}m_{2\ell}M}{M_1m_{12} - M_3m_{1D}} \cong m_{1\ell} \frac{M}{m_{12}M_1},
\]

and masses of \( \nu_1 \) and \( \nu_3 \)

\[
m_1 = \frac{m_{1\ell}}{M_1}, \quad m_3 = \left( \frac{m_{1\ell}}{m_{12}} \right)^2 \frac{D_m}{M^2},
\]

where \( D_m \equiv M_1m_2 - M^2 \) is the determinant of the 1 \(-\) 3 submatrix of \( M_R \). In eq. (12) we took into account the fact that the rotation to the flavour basis gives a small correction to the \( \nu_e \rightarrow \nu_\tau \) flavour mixing, and have neglected \( M_3m_{1D} \) in comparison with \( M_1m_{12} \). Since \( m_{1\ell}/m_{12} \sim m_\nu/m_\mu \sim (3 - 5) \cdot 10^{-3} \), we and \( m_\nu \) being the \( u \)- and t-quark masses.

According to (12), the needed enhancement of the \( \nu_e \rightarrow \nu_\tau \) mixing implies a hierarchy between the elements of \( M_R \): \( M/M_1 \gg 10^3 \). As follows from (13), the determinant \( D_m \) should be small to ensure the necessary enhancement of the mixing for small values of the ratio \( m_1/m_3 \). Moreover, as can be shown using eqs. (10) and (11),

\[
\frac{D_m}{M^2} \cong \frac{D_m}{m^2_3} \cong \xi \ll 3 \cdot 10^{-2},
\]

i.e., the fine tuning of the elements of (11) is precisely the same as in the light neutrino Majorana mass matrix \( m^{\nu_3} \). The see-saw mechanism transforms the fine tuning problem

*The Dirac basis is related to the flavour one by a Cabibbo-Kobayashi-Maskawa type mixing matrix. This matrix generates small \( \nu_e \rightarrow \mu \) and \( \nu_\mu \rightarrow \nu_\tau \) family mixing.
from the light neutrino sector to the heavy one. This in turn implies a strong mass hierarchy of the heavy Majorana neutrino masses:

\[
\frac{m_2^2}{M^2} = \frac{2}{M^2}.
\]  

To estimate the elements of the matrix \( M_R \) we use as an input the values \( m_{1D} = (0.5 - 7) \) MeV, and \( m_{2D}/m_{1D} = (0.4 - 3.0) \cdot 10^4 \) - typical for charged leptons and quarks. Then from (13) - (15) and for the maximal value \( m_3 \approx m_2 \approx 3 \cdot 10^{-3} \) eV we get for the elements of \( M_R \) (in GeV): \( M_f \sim (0.08 - 16) \cdot 10^6, M \sim (0.03 - 30) \cdot 10^6 (U_{31}/0.1), M_2 \sim (0.11 - 60) \cdot 10^{12} (U_{32}/0.1)^2 \).

With diminishing \( m_4 \) these masses increase as \( \sim (0.1 - 2.0) \) GeV and \( m_3 \sim 3 \cdot 10^{-3} \) eV one has \( M_f = (0.25 - 200) \cdot 10^9 \) GeV which can be of the order of or much smaller than \( M_3 \). Thus, the explanation of the inequality \( m_3 << m_3 \) requires a strong hierarchy between the elements of the matrix \( M_R, M_3 \) : \( M \sim 1 \cdot 10^{-3} : 10^{-6} \). The fine tuning of the elements of \( M_R \) is minimal when \( m_3 \) is comparable with \( M_2 \); the spectrum in this case has a rather peculiar form: \( m_3 >> m_3 \sim m_3 \). Let us note that the higher order corrections to the ratio \( M_3/M_3 \) affecting condition (14) are estimated to be \( \sim 10^{-3} \).

We shall outline next several approaches permitting to explain the required properties of the matrix \( M_R \). Its texture, hierarchy of elements as well as the smallness of the determinant of the \( 1 \cdot 3 \) submatrix can be a consequence of a certain family symmetry \( G \). The elements of the matrix \( M_R \) can appear as bare mass terms conserving \( G \), or/and can be generated by couplings to new scalar fields, \( \sigma \), singlets of \( SU(2)_L \times U(1) \). These scalar fields have, in general, nonzero \( G \)-charges, \( G_\sigma \), and acquire nonzero vacuum expectation values, \( \sigma_0 \), thus spontaneously breaking the symmetry \( G \). Let us consider several simplest possibilities.

1. The texture (11) can be easily generated by bare mass terms and by interactions with a field \( \sigma \) if, e.g., \( G = U(1) \), the \( G \)-charges of the neutrinos are \( G(\nu^\prime_{1L,R}, \nu^\prime_{2L,R}, \nu^\prime_{3L,R}) = (1, 0, -1), G_2 = 2, \) and \( G(\phi) = 0 \), where \( \phi \) is the standard Higgs doublet. However, condition (14) implies a fine tuning between the bare mass \( M \) and the contributions from the interaction with \( \sigma \).

Instead of bare mass terms for \( \nu^\prime_{3R} \) one can introduce couplings with additional scalar fields. Suppose that: \( G = SU(2) \), the scalar fields \( \sigma_{11}, \sigma_{13}, \sigma_{33} \) form a triplet \( \sigma \) of \( G \), \( \nu^\prime_{1R} \) and \( \nu^\prime_{3R} \) are in a doublet of \( G \) and interact with \( \sigma \), whereas \( \nu^\prime_{1R} \) is a \( G \)-singlet. The Higgs potential can be easily arranged in such a way that only one component of the triplet \( \sigma \) acquires a nonzero vacuum expectation value. If, e.g., this component coincides with \( \sigma_{33} \), only the \( M_{23} \) element of the \( 1 \cdot 3 \) submatrix of \( M_R \) will be nonzero. In general, the basis in which only one component of the triplet develops a nonzero vacuum expectation value differs from the neutrino Dirac basis by a rotation on a certain angle \( \alpha \). Thus, the vacuum expectation values of the components of the triplet in the neutrino Dirac basis will be related as \( \sigma_{33} : \sigma_{11} = \sin^2 \alpha : \cos \alpha \sin \alpha : \sin^2 \alpha \). Correspondingly, the same relation will take place for the components of the \( 1 \cdot 3 \) submatrix of \( M_R \). For \( \sin \alpha \approx 10^{-3} \), one reproduces the necessary hierarchy of the elements and, moreover, the determinant \( D_M \) is identically zero. A sufficiently small value of \( D_M \) can then arise as a result of weak violation of the symmetry \( G \).

2. Suppose that the symmetry \( G = U(1) \) is broken by the vacuum expectation value of \textit{only one} singlet scalar field \( \sigma \) which carries a charge \( G_\sigma \). The value of the \( M_3 \) element of the matrix \( M_R \) (11) can be related to the \( G \)-charge \( G_\sigma \) of the corresponding mass term operator as follows

\[
M_3 = \frac{h_3 \sigma_0}{M_3} \frac{G_\sigma}{G_0},
\]

where \( M_0 \) is a (bare) mass parameter and \( h \) is the constant of the \( \nu^\prime_{1R} \) and \( \sigma \) Yukawa coupling. Prescribing \( G(\nu^\prime_{1L,R}, \nu^\prime_{2L,R}, \nu^\prime_{3L,R}) = (1, 1/2, 0) \) and \( G_\sigma = 1 \), we get: \( M_3 = M_0, M = M_3 = M_0 \left( \frac{2}{M_3} \right), M_1 = M_0 \left( \frac{2}{M_3} \right)^3 \). All other elements of \( M_R \) are zero since they have no \( G \)-invariant interactions with \( \sigma \). The ansatz (16) ensures the needed relation between the elements of \( M_R \) (\( D_M = 0 \)). For \( \sigma_0/M_0 \approx 10^{-3} \) we get the requisite hierarchy of the elements of \( M_R \). Weak violation of the \( G \) symmetry at the scale of \( (10^{-3} - 10^4) \) GeV will generate the mass of the lightest RH component. The models reproducing the ansatz (16) should contain, e.g., an additional scalar field \( \phi' \) with \( G_\phi' = 2 \), having specific mass and couplings (e.g., \( M_{13} = M_0 \) trilinear coupling \( h_{M_0} \phi' \sigma \phi' \), and Yukawa coupling to \( \nu^\prime_{1R} \) with a constant \( h' = h \)).

3. One can make use of the fact that the \( 1 \cdot 3 \) submatrix of \( M_R \) is diagonalized by a rotation on angle \( \theta \approx 10^{-3} \) and its eigenvalues obey the hierarchy \( M^2_1/M^2_3 \approx 10^{-9} \). Suppose that due to a certain symmetry \( G \) the matrix \( M_R \) is diagonal and \( M^2_1 \leq M^2_3 \), whereas \( M^2_3 = 0 \). (The RH component \( \nu^\prime_{3R} \) acquires a mass due to weak violation of \( G \), so that...
3 Three Mass-Degenerate Neutrinos

Consider a highly degenerate neutrino mass spectrum: \( m_1 \approx m_2 \approx m_3 = m_0 \), with \( m_0 \geq 0.1 \) eV. The effective neutrino mass parameter can be written then as

\[
m_{\text{eff}} = m_0 \sum_{i=1,2,3} |U_{ei}|^2 \eta_i,
\]

and, in general, all three components give appreciable contributions in the sum. The solar neutrino deficit is explained by \( \nu_e \) conversion to, e.g., \( \nu_\mu \) if \( |m_2 - m_3| = \Delta m^2_{31}/2m_0 \approx m_0^2/2m_0 \leq (0.5 - 5) \times 10^{-5} \) eV, or by vacuum oscillations provided \( |m_3 - m_2| \approx (1 - 5) \times 10^{-10} \) eV. It is possible also to describe the atmospheric neutrino deficit in this case as caused by the oscillations \( \nu_e \leftrightarrow \nu_x \), which would require \( |m_3 - m_1| \approx 10^{-2} \) eV/2m0 = (0.5 - 5) \times 10^{-2} eV. With such a highly degenerate neutrino mass spectrum the reactor and accelerator data [14-17] do not impose any limits on the mixing.

There are several phenomenological possibilities depending on the relative signs of \( \eta_j \), \( j = 1, 2, 3 \), and on the values of the mixing parameters. If \( \eta_1, \eta_2, \) and \( \eta_3 \) have the same CP-parity, one obtains taking into account the unitarity of the lepton mixing matrix: \( m_{\text{ee}} = m_0 \). Consequently, the existing data on the (\( \beta \beta \))m decay implies: \( m_0 \leq (0.4 - 0.5) m_0 \) and a maximal allowed value of \( m_0 \approx (4 - 5) \) eV. For the vacuum oscillation solution values of \( \sin^2 2\theta \) up to the maximal one are not excluded, making possible even stronger cancellation and values of \( m_0 \) up to the upper bound on the electron neutrino mass obtained in the tritium \( \beta \)-decay experiments: \( m_0 \leq 7 \) eV [23]. In the case of the small mixing solution (1) one has \( m_0 \approx |m_\alpha| \) independently of the relative CP parities of the neutrinos \( \nu_j \).

Let us consider now the possibility of the third neutrino \( \nu_3 \) having an appreciable admixture in \( \nu_e \). The resonant conversion of the solar neutrinos will be effective if the lightest component \( \nu_1 \) dominates in \( \nu_e \) i.e., if \( |U_{e1}| > |U_{e2}|, |U_{e3}| \). Therefore the maximal compensation effect takes place when \( \eta_2 = \eta_3 = -\eta_1 \). The expression for \( m_{\text{ee}} \) reduces to (18). For an appreciable admixture of the \( \nu_3 \) state in \( \nu_e \), the region of the large mixing angle solution extends to \( \sin^2 2\theta \approx 0.95 \) [24,25]. The allowed region of parameters enlarges (see Fig. 2) and values of \( m_0 \) up to the experimental upper limit of 7 eV are possible.

We shall analyze next the relevant implications for the mechanisms of neutrino mass generation. The CP-parities of all Majorana neutrinos are the same (\( \eta_i = \eta_j = \eta_k \)), the Majorana mass matrix \( m^{\text{maj}} \) in the flavour basis can be represented as

\[
m^{\text{maj}} = m_0 \cdot I + \delta \eta \delta \eta^\dagger,
\]

where \( I \) is the unit matrix and \( \delta \eta \) is a "small" correction matrix generating the neutrino mass splitting and the mixing. The neutrino conversion (oscillation) parameters are determined \( \delta \eta \) by the matrix

\[
H \equiv \frac{m_0}{E} \cdot \delta \eta \delta \eta^\dagger.
\]

In order to have \( \Delta m^2_{21} \), or/and \( \Delta m^2_{31} \leq 10^{-5} \) eV², the elements of \( \delta \eta \) (at least some of them) should be of the order of or smaller than \( m_0^2/m_0 \approx (10^{-4} - 10^{-5}) \) eV. It is natural to
suggest that $m_0$, the main contribution in $m^{\mu\nu}$, is generated by interactions which preserve a family symmetry $G$, whereas $6\Phi$ results from $G$-breaking interactions. Suppose that the lepton doublets $\psi_i$ form a triplet of a horizontal group $G$ (e.g., $G = SO(3)$). The $m_0$ term in (19) can be generated by $G$ invariant interaction of the Higgs doublets with a superheavy $SU(2)_L$ triplet Higgs field $\Delta_3$ ($\Delta^\mu_3, \Delta^\tau_3, \Delta^\nu_3$) carrying a zero $G$-charge,

$$h_0 \sum_{i>\mu,\tau} \psi^*_i C \psi_i \Delta^3 + h.c.,$$  

(21)

where $h_0$ is a Yukawa constant. A sufficiently small vacuum expectation value $\Delta_0$ of $\Delta$ can be induced by a quartic coupling of $\Delta$ with the standard Higgs doublet $\Phi$ and an additional singlet field $\sigma$: $\Delta \Phi \Phi \sigma$. The singlet $\sigma$ can naturally acquire a vacuum expectation value $\sigma_0$ by the Yukawa couplings of leptons with $\Phi$. One finds that, e.g., $6\Phi_{\nu} \sim m_\nu^2 \Phi_\nu^2 \ln M^2 / m^2 \lesssim 3 \cdot 10^{-5} m_0$. The matrix $\delta m^2$ may be generated, e.g., by Planck-scale interactions [27]:

$$\delta m^2 = \frac{a_{\nu}}{M_P^2} \psi^*_i C \tau^i \psi_i \Phi^T \tau^i \Phi + h.c., \quad i, i' = e, \mu, \tau,$$

(22)

where $M_P$ is the Planck mass. For $a_{\nu} \sim 1$, one has $\delta m^2_\nu \sim 10^{-5}$ eV, which gives $\Delta m^2_3 \sim m^2_3$. The constants $a_{\nu}$ can be flavour universal [27]. One obtains in this case the large mixing angle MSW solution (1). If $a_{\nu}$ have values spread over an order of magnitude, $a_{\nu} \sim (10^{-1} - 1)$, the small mixing angle solution is reproduced.

The matrix $\delta m^2$ can appear also as a result of the seesaw mechanism. The relevant parameters can take values necessary to explain both the solar neutrino problem and the atmospheric neutrino deficit. For $M_R = M_0 \bar{I}$, with $M_0 = 10^{11}$ GeV one gets $\delta m^2_\nu \sim 10^{-5}$ eV, and $\delta m_{ee} \sim 10^{-2}$ eV, so that $\Delta m^2_3 \sim 2 m_0 \delta m_{ee}$ and $\Delta m^2_1 \sim 2 m_0 \delta m_{ee}$, are precisely in the requisite intervals.

In the schemes considered above neutrinos acquire masses due to couplings to superheavy particles at large energy scales, respecting a family symmetry (interactions with $\Delta$), Majorana mass terms of the RH neutrino components, etc.), whereas the masses of the charged particles as well as the lepton mixing are generated by the interactions with the standard Higgs doublet, which break this symmetry. An alternative possibility is that the Higgs doublet couplings also respect the family symmetry and neutrinos acquire equal masses via the seesaw mechanism. The hierarchical structure of the mass spectrum of the charged particles (quarks, charged leptons) may result from a certain pattern of the vacuum expectation values of the scalar horizontal multiplets. Such a possibility can be realized in terms of the universal seesaw mechanism [28].

### 4 Two Mass-Degenerate Neutrinos

Let us discuss the modification of the above scenario with only two almost degenerate neutrinos, say $\nu_1$ and $\nu_2$, having masses in the region of interest, $m_1 \equiv m_2 \equiv m_0 \lesssim 0.1$ eV. The third neutrino $\nu_3$ has a mass which can differ considerably from $m_0$. As before, the $\nu_3$-deficit is explained by $\nu_3 - \nu_\mu$ conversion (or oscillations) induced by a small mass difference of the degenerate neutrinos: $(m_2 - m_1) / m_0 \equiv m_2^2 / (2 m_0^2) \lesssim 10^{-3}$. The effective Majorana neutrino mass parameter can be written as

$$m_{ee} = m_{12} + \eta m_0 |U^2_{e3}|^2,$$

(23)

where $m_{12} \equiv |U_{e3}| (1 + (\eta - 1)|U_{e3}|^2)$ is the contribution due to the degenerate neutrinos. If the CP-parities of $\nu_1$ and $\nu_2$ are opposite, i.e., if $\nu_1$ and $\nu_2$ form a pseudo-Dirac neutrino [29,30], then $|m_{12}| = m_0 |1 - 2|U_{e3}|^2|$. Depending on the value of $m_3$ we get different phenomenological implications.

For $m_3 << m_0$ one has $\Delta m^2_3 \sim |\Delta m^2_3| \sim m_3^2 \lesssim 10^{-2}$ eV and the predicted $\nu_3 \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillation effects can be in the range of sensitivity of the future experiments [20]. The existing limits from the oscillation searches imply an upper bound on $|U_{e3}|$ which makes the contribution due to $\nu_3$ in $m_{ee}$ negligibly small. Thus, in what regards the $(\beta\beta)_{0m}$ decay, the present case reduces to the two neutrino case (18) analyzed in the previous section.

If $m_3 \gtrsim m_0$, there are several possibilities. The case $|m_{12}| < 0.1$ eV is equivalent, as far as the $(\beta\beta)_{0m}$ decay is concerned, to the one considered in section 2. New features appear when the contributions in $m_{ee}$ due to $\nu_3$ and the $m_{12}$ term are comparable. For values of
m_3 below the cosmological bound, m_3 \leq 30 \text{ eV}, one has, taking into account the results of the oscillation experiments [14-17], m_3|U_{e3}|^2 \leq 0.5 \text{ eV}. The allowed region shown in Fig. 2 is shifted to smaller (larger) values of m_0 when m_{12} and m_{33} have the same sign (opposite signs). In particular, |m_{12}| can be close to the upper bound on the electron antineutrino mass: |m_{12}| \approx (5 - 7) \text{ eV}. In the latter case the term m_{12} in m_{ee} should be cancelled by the contribution from v_s, which implies m_{33}|U_{e3}|^2 \approx (4 - 8) \text{ eV}. Using the bound on |U_{e3}|^2 from the oscillation experiments one finds m_3 \geq (200 - 400) \text{ eV}. Obviously, the neutrino v_3 should be unstable.

Finally, the case when m_3 \geq |m_{ee}|, m_0 << |m_{ee}| is equivalent from a phenomenological point of view to the one discussed in section 2.

Two highly degenerated neutrinos appear in a theory with a horizontal symmetry G, with two families having nonzero G-charges and the third family being a singlet of G. This possibility can be realized if neutrinos acquire masses due to an interaction with the triplet \Delta as in eq. (21) (now two families form a doublet of G), or in terms of the see-saw mechanism. Suppose G = U(1) and the neutrinos have G-charges \( \left( \frac{1}{2}, -\frac{1}{2}, 0 \right) \), then G-conserving bare mass terms and/or interactions with scalar singlets having zero G charges will generate the matrix M_0 with only nonzero elements M_{13} = M_{31} and M_{33}. Such a matrix leads via the see-saw mechanism to a light Majorana neutrino with mass m_3 and a light pseudo-Dirac neutrino with mass m_0, given by

\[ m_3 = \frac{m_{13}^2}{M_{13}}, \quad m_0 = \frac{m_{13}m_{33}}{M_{13}}. \tag{24} \]

For M_{13} = 10 M_{13} we obtain from (24) m_3 \approx 10 \text{ eV} and m_0 \sim (0.01 - 0.1) \text{ eV} (and even a stronger hierarchy is possible). Small mass splitting between the two Majorana components of the pseudo-Dirac neutrino is generated by weak violation of the G-symmetry by M_{13}. If the basis in which the neutrino states have definite G-charges differs from the flavour basis by a rotation on an angle \( \theta_3 \) of the v_1 and v_2 states, one finds that the mixing angle appearing in the expressions for v_3 and v_3 states is given by tan \( 2 \theta_3 \approx 1/tan 2 \theta_3 \). For the value \( \theta_3 \sim \theta_3 \approx 13^\circ \) we get \( \sin^2 2 \theta_3 \approx 0.8 \), i.e., such a scheme can accommodate the large mixing solution of the \( \nu_\odot \) problem.

Two almost mass-degenerate neutrinos may appear in the models with radiative mechanism of neutrino mass generation, like the Zee model [31]. Simplest models predict typically near to maximal mixing between the mass-degenerate states and maximal mixing of two flavour neutrinos. This is disfavored by the MSW solution of the \( \nu_{\odot} \) problem. Moreover, the two almost mass-degenerate Majorana neutrinos usually have opposite CP parities, their contribution to m_{ee} has the form m_{12} \sim \Delta m^2/2m_0 and turns out to be small. One can avoid the maximal mixing and the suppression of m_{ee} by modifying the Zee model, e.g., by introducing Higgs doublets with couplings to the lepton fields which are antisymmetric in the flavour indices.

5 Conclusions

In the present paper we have limited our discussion to the case of only three light massive Majorana neutrinos. One could consider also schemes with larger number of light neutrino states, for instance, with light sterile neutrinos \( \nu_{\mu L} \). This, obviously, opens up new possibilities for reconciliation of an observable \( (\beta/\beta)_{\odot} \) decay rate with the solar and atmospheric neutrino data.

A \( (\beta/\beta)_{\odot} \) decay with half-life in the range of the sensitivity of the future experiments can be caused by a mechanism not directly related to the exchange of light Majorana neutrinos, for example, by weak interactions with right-handed currents, exchange of heavy neutrinos \( (m > 30 \text{ MeV}) \) or other heavy neutral Majorana particle(s), etc. [5]. In these cases the usual schemes of neutrino mass generation need not be modified in order to obtain the MSW or the vacuum oscillation solution of the \( \nu_{\odot} \) problem.

In conclusion, we have shown that from a phenomenological point of view it is not difficult to reconcile the particle physics solution of the \( \nu_{\odot} \) problem and an observable \( (\beta/\beta)_{\odot} \) decay induced by exchange of Majorana neutrinos with an effective neutrino mass \( (0.1 - 1.0) \text{ eV} \). Moreover, some proposed schemes can accommodate also the solution of the atmospheric neutrino problem and contain at least one neutrino state with mass in the cosmologically interesting range. Neutrino oscillations which can be probed in future laboratory experiments are predicted in schemes with strong hierarchy between the neutrino masses. However, as we show, such a reconciliation will have serious implications for the
mechanisms of neutrino mass generation. The implications depend on the relation between
the mass of the lightest neutrino $m_1$, the solar $\nu_\odot$-problem mass scale $m_0$, and the effective
Majorana mass $m_{\text{ee}}$. There are two extreme cases.

If $m_1 < m_0 << m_{\text{ee}}$ (strong mass hierarchy), the elements of the LH flavour neutrino
Majorana mass matrix should be "tuned" to obey a certain relation with a relative precision
of $m_1/m_{\text{ee}} \leq 10^{-2}$. In the see-saw mechanism, such a relation may result from a similar one
between the elements of the mass matrix of the RH neutrinos. The later could be explained
in turn by the presence of a family symmetry and strong hierarchy of masses of the RH
neutrinos.

The case $m_1 \gtrsim m_{\text{ee}}$ implies a strongly degenerate light neutrino mass spectrum. Such
a spectrum can be produced by new interactions at high mass scale which respect a certain
family symmetry. The latter can be weakly broken by low mass scale interactions of the
usual Higgs doublet, or by gravitational effects. In the intermediate case $m_1 < m_0 \approx m_{\text{ee}}$
both the tuning condition should be satisfied and two light neutrinos should be almost
mass-degenerate. The stronger the degeneracy the weaker the "tuning", and vice versa.

Part of the work of S.T.P. for the present study was done during a visit to the L.P.T.H.E.
and L.A.L. of the Centre d’Orsay in the spring of 1993. He wishes to thank L. Oliver and
S. Julians for stimulating discussions, and O. Fene and the other members of L.P.T.H.E.
for the kind hospitality extended to him during his visit. A. Yu. S. would like to thank Z.
Berezhiani, G. Dvali and G. Senjanovic for illuminating discussions. The work of S.T.P.
was supported in part by the Bulgarian National Science Foundation via grant PH-16.

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Figure Caption

Fig. 1. The dependence of $\Delta m^2 = m_3^2$ on $\sin^2 2\theta = 4|U_{e3}|^2 (1 - |U_{e3}|^2)$ (solid lines) for different values of $|m_{ee}|$ (see eq. (7)). The regions of parameters excluded by the Goesgen [14], BEBC [16] and IPR [15] oscillation experiments are depicted. Shown are also the regions of sensitivity of the future oscillation experiments CHORUS, NOMAD, P-803 and P-860 [20].

Fig. 2. Values of the mass $m_0$ of the degenerate neutrinos and the mixing parameter $|U_{e1}|^2$ for which the MSW and vacuum oscillation solutions of the $\nu_0 - \nu_1$ problem can be reconciled with observable Majorana mass $m_{ee} = (0.1 - 1.4)$ eV. Solid lines correspond to two neutrino contributions in $m_{ee}$ and to two-neutrino oscillations/conversions. The regions of the large mixing MSW solution are hatched; the small mixing solution is shown as a vertical line at $|U_{e1}|^2 \cdot 1.7 \cdot \eta_0 \cong 0.1$. For an appreciable contribution of the third neutrino state in $m_{ee}$ the regions are larger: the dashed lines correspond to the case of three degenerate neutrinos, the dotted lines correspond to the case of large $m_3$, so that $m_0 \cdot |U_{e3}|^2 = 0.5$ eV. The upper bound on the electron antineutrino mass from the tritium experiments is also shown.
Figure 2.