NON-STANDARD MODEL COUPLINGS
IN WWV VERTEX

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1 Introduction

The standard model (SM) has proved to be extremely successful in describing recent experimental data from LEP-1 \cite{1} giving rise to attempts at extending the theory by proposing the existence of neutral heavy gauge bosons $Z'$ whose mixing with the standard model $Z$ reflects the Higgs structure of the theory \cite{2}. Direct experimental evidence of SM is of course restricted to vector–boson–fermion interactions only and the LEP-1 precision data \cite{3} have provided a bound on the vacuum polarisation due to boson loops. This upper bound indirectly confirms the fundamental $SU(2)_L \times U(1)_Y$ symmetry of the interaction of the vector bosons with one another.

But the direct investigation of the tri-linear and quadri-linear vector-vector interaction is indispensable for the verification of the $SU(2)_L \times U(1)_Y$ theory \cite{4}. The experiment at the CERN $e^+e^-$ collider LEP-2 on $W^+W^-$ pair production has the aim of examining electroweak theory quantitatively. The well known and theoretically much studied process

\begin{equation}

e^+ + e^- \rightarrow W^+ + W^-

\end{equation}

allows us not only to determine various properties of the $W$–boson but also to measure the tri-linear couplings $W^+W^-V$ where $V$ stands for $\gamma$ and $Z$. This is expected to provide a direct test of the underlying $SU(2)_L \times U(1)_Y$ gauge symmetry.

Recently, Babich, Pankov and Paver \cite{5} have investigated the effects of neutrino and electron mixing with exotic heavy leptons on the reaction (1) in the framework of electroweak models based on $E_6$ gauge symmetry. Kuo, Park and Zralek \cite{6} have also examined the above process in the context of the $SP(6)_L \times U(1)_Y$ model. Babich, Pankov and Paver have discussed the possibility of obtaining constraints on $Z - Z'$ mixing from studies of the above process at the LEP-1 collider. Kuo, Park and Zralek have found that there are significant deviations in the total cross-sections from the SM results due to the presence of additional gauge bosons $Z'$ and $W'$ in the model.

In their work, Babich, Pankov and Paver have studied the possible indirect effects of $Z'$ bosons at future higher energy $e^+e^-$ colliders starting from the SM tri-linear couplings but including the $Z'$ couplings to the left–handed and right–handed fermions. In the presence of extra $U(1)_Y$ charges in the mass eigenstate basis are given by

\begin{equation}

- \mathcal{L} = J_{\text{int}}^a A^a + J_\gamma^\gamma Z_{1\gamma} + J_\gamma^Z Z_{2\gamma}.

\end{equation}

where the relevant coupling constants are given in their paper \cite{5}. But they have used the standard model tri-linear vector–vector coupling to obtain the differential and total cross-sections for the process (1).

However, the most general effective Lagrangian for the tri-linear coupling $W^+W^-V$ incorporating constraints from Lorentz invariance and electromagnetic gauge invariance has nine coupling constants \cite{7}. This number is reduced to six if CP invariance is imposed and to five if additionally C invariance is also included. Of these five couplings, three appear in the SM with definite values.

The purpose of the present work is to consider the process (1) in terms of the coupling parameters in the most general WWV vertex, thus extending the expressions for

\begin{equation}

e^+ + e^- \rightarrow W^+ + W^-.

\end{equation}
the cross-sections used Babich et al. in their study of the $Z - Z'$ mixing at future $e^+e^-$ colliders. Of particular interest here is the classical set of WWV coupling strengths $K_V$ and $A_V$ which are related to the $W$-magnetic dipole and the electric quadrupole moments respectively.

$$\mu_W = \frac{e}{2m_W} (1 + \kappa_V + \lambda_V)$$

$$Q_W = -\frac{e}{m_W} (\kappa_V - \lambda_V)$$

(3)

with $\kappa_V = 1$ and $\lambda_V = 0$ in the SM. The possibility of probing these couplings at $e^+e^-$ and hadron colliders has been extensively investigated in the literature. As it is well known, the process (1) is described in the SM by three Feynman diagrams for photon, neutrino and $Z$-exchanges denoted by $A_\gamma$, $A_\nu$ and $A_Z$. Babich, Pankov and Paver have assumed that new physics induces a small change of these amplitudes from the SM prediction so that in the $K_V$ model, there are now 5 Feynman diagrams corresponding to photon, neutrino, heavy neutrino, $Z_1$ and $Z_2$ exchanges. As they have explained in detail, the $s$-channel $Z_2$ and the $t$-channel $N$ exchange amplitudes arise only in the case of non-vanishing mixing angles, and the SM result is obtained by taking all mixing angles equal to zero. Similarly, if we take $K_V = 1$ and $\lambda_V = 0$ in our expressions for the cross-sections, we get the SM results given by Bilchak and Stronghair among many other authors.

However, in the process (1), the WW$_{V}$ and WWZ vertices both enter as we will see in Section 3 and the anomalous couplings are horribly intermingled so that it is difficult to obtain bounds on their values from this process. Therefore as emphasized by Choi and Schrempp, it is desirable to have separate tests of the photon and $Z$-meson anomalous couplings. For this purpose, it is important to study the process

$$e^- + \gamma \rightarrow W^- + \nu_e$$

(4)

and this has also been done by many authors. Choi and Schrempp have given very compact expressions obtained from algebraic manipulation packages and Yehudai has given the amplitudes for the process (4) in the helicity basis. Very recently, Philipsen has made a detailed study of this process but he has given no analytic expressions for the helicity amplitudes. One of our objectives is to calculate the helicity amplitudes for the processes (1) and (4) following the method of Yehudai. We may also mention that Renard has calculated the helicity amplitudes for the process (1) considering the general invariants.

2 Triple Gauge Boson Vertex

The most general WWV vertex is derived from the effective Lagrangian,

$$L_{WWW} = g_{WWW}(i\gamma^\mu (W^\mu)^*_V W^\nu V^\rho - W^\mu V^\nu W^\rho))$$

(5)

(6)

where $W_{\mu\nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} = W_{\mu\nu} - \partial_{\mu} V_{\nu}$. Therefore for the vertex shown in Fig.1, we have

$$\Gamma_{WWW}^{\alpha\beta} = g_{WWW} \left[ Q^\alpha Q^\beta \left( \frac{f_2}{m_W} P^\alpha P^\beta Q^\gamma + f_3 \left(P^\alpha g^{\gamma\delta} - P^\gamma g^{\alpha\delta}\right) \right) \right]$$

(6)

where $Q_{\alpha\beta} = \epsilon_{\alpha\beta\gamma\delta} P_{\gamma\delta}$, $P = q + \bar{q}$, $Q = q - \bar{q}$ and

$$f_1 = g_1 + \frac{s}{2m_W} \lambda_V$$

(7)

$$f_3 = g_1 + \kappa_V + \lambda_V q^2 - \bar{q}^2$$

$$f_5 = g_1 + \lambda_V q^2 - \bar{q}^2$$

$$f_7 = -\frac{1}{2} \lambda_V$$

We will in the following keep only the C and P conserving couplings $g_1$, $\kappa$ and $\lambda$ and will choose

$$g_{WWW} = e$$

$$g_{WWW} = e \cot \theta_W$$

(8)

where $\theta_W$ is the weak mixing angle of the standard model.

Now following Bilenky, Kneur, Renard and Schildknecht, we can write the C and P conserving part of the Lagrangian in the form,
and introduce parameters representing deviation from the SM values of the various couplings. Defining, 

$$y_{\pm} = \lambda_{\pm} A,$$ 

we rewrite $L_1$ in the form 

$$L_1 = -i\epsilon A (W^+ W^- + W^- W^+) + F_{\mu\nu} W^{\mu\nu}$$ 

$$-i\epsilon Z_{\mu\nu} W^{\mu\nu} - i\epsilon Z_{\mu\nu} W^{\mu\nu}$$ 

$$-i\epsilon \cot \theta_{W} \lambda_{W} W^{\mu\nu}$$ 

$$+ i\epsilon \cot \theta_{W} \lambda_{W} W^{\mu\nu}$$ 

which is well-suited for an analysis of experimental data.

From this expression it is clear that if we require the validity of global $SU(2)_L$ symmetry, we must have 

$$\delta z = x_{s} = x_{Z} = 0$$ 

but $y_{s} = \cot \theta_{W} y_{s} \neq 0$. The number of free parameters is then only 1 i.e. $\lambda_{s}$. On the other hand, if we re-express $L_1$ in terms of the unmixed $W^{\mp}, B_{s}$ fields via the standard orthogonal transformation with angle $\theta_{W}$ and demand the exclusion of only $SU(2)_L$ violating term $W^{\mu\nu} W^{\mu\nu}$, we find that $\delta z, x_{s}, y_{s}$ are not now zero and $x_{Z} = -\tan \theta_{W} x_{s}$. Excluding dimension-6 terms $y_{s}$ and $y_{s}$, we are left with two free parameters $\delta z$ and $x_{s}$ and imposing further the condition that the term $B_{s} W^{\mu\nu} W^{\mu\nu}$ be absent since it gives tree-unitarity-violating contribution to vector-meson scattering, we are left with only one free parameter $x_{s}$. These considerations were motivated by the desire to reduce arbitrary deviation of the tri-linear couplings from their standard model values.

3 The Differential and Total Cross-Section for the Process $e^{+}e^{-} \rightarrow W^{+}W^{-}$

The differential cross-section for the process $e^{+}e^{-} \rightarrow W^{+}W^{-}$ is given by

$$\frac{d\sigma}{d \cos \theta} (e^{+}e^{-} \rightarrow W^{+}W^{-}) = \frac{\sqrt{1-4m_{W}^{2}/s}}{32\pi m_{W}^{2}} [d\sigma_{AA} + d\sigma_{ZZ} + d\sigma_{\nu\nu} + d\sigma_{AZ} + d\sigma_{AU} + d\sigma_{Z\nu}]$$

where

$$d\sigma_{AA} = 2e^{4} A(k, \lambda, s, t, u)$$

$$d\sigma_{ZZ} = 2 \left[ \frac{e^{2} \left( g_{s}^{2} + g_{A}^{2} \right) \cot^{2} \theta_{W}}{(s - \sec^{2} \theta_{W})^{2}} \right] A(k', \lambda', s, t, u)$$

$$d\sigma_{\nu\nu} = g^{2} E(s, t, u)$$

$$d\sigma_{AZ} = -4 \left[ \frac{e^{2} g_{s} g_{A} \cot \theta_{W}}{(s - \sec^{2} \theta_{W})^{2}} \right] A'(k, \lambda', s, t, u)$$

$$d\sigma_{AU} = -e^{2} g^{2} I(k, \lambda, s, t, u)$$

$$d\sigma_{Z\nu} = g \left[ \frac{4 \left( g_{s}^{2} + g_{A}^{2} \right) \cot \theta_{W}}{(s - \sec^{2} \theta_{W})^{2}} \right] I(k', \lambda', s, t, u)$$

i.e., $d\sigma_{AA}$ is the differential cross-section from photon exchange, $d\sigma_{ZZ}$ that from $Z$-exchange, $d\sigma_{\nu\nu}$ that from neutrino exchange and $d\sigma_{AZ}, d\sigma_{AU}$ and $d\sigma_{Z\nu}$ arise from $\gamma Z, \gamma \nu$ and $ZW$ interference respectively. Here we have used

$$A(k, \lambda, s, t, u) = \left( \frac{ut}{m_{W}^{2} - 1} \right) \left[ \frac{1}{4} + \frac{s}{4} \right] \left( 1 + \kappa' + \lambda' + \frac{s}{m_{W}^{2}} \right)$$

$$A'(k', \lambda', s, t, u) = \left( \frac{ut}{m_{W}^{2} - 1} \right) \left[ \frac{1}{4} - \frac{s}{4} \right] \left( 1 + \kappa + \lambda' + \frac{s}{m_{W}^{2}} \right)$$

$$E(s, t, u) = \left( \frac{ut}{m_{W}^{2} - 1} \right) \left[ \frac{1}{4} + \frac{s}{4} \right] \left( 1 + \kappa' + \lambda + \frac{s}{m_{W}^{2}} \right)$$

$$I(k, \lambda, s, t, u) = \left( \frac{ut}{m_{W}^{2} - 1} \right) \left[ \frac{1}{4} - \frac{s}{4} \right] \left( 1 + \kappa' + \lambda + \frac{s}{m_{W}^{2}} \right)$$

We have also used the notation $\kappa = \kappa_{s}, \lambda = \lambda_{s}, \kappa' = \kappa_{Z}, \lambda' = \lambda_{Z}$.

Our expression for the differential cross-section agrees with that of Bilchak and Strouehair if we put $\lambda = 0$ and $\lambda' = 0$. Moreover, the predictions of the SM are also obtained by setting $\kappa = \kappa' = 1$ and $\lambda = \lambda' = 0$. The total cross-section is given by ($m_{W} = 1$),

$$\sigma = \frac{(0.624)^{2}}{32\pi m_{W}^{2}} \left( 1 - \frac{s}{4} \right) \left[ \sigma_{AA} + \sigma_{ZZ} + \sigma_{\nu\nu} + \sigma_{AZ} + \sigma_{AU} + \sigma_{Z\nu} \right]$$
where

\[
\sigma_{AA} = \frac{4e^4}{s^3} G_1(\kappa, \lambda, s) \frac{g^2}{\beta} \left[ \frac{e^2(g_1^2 + g_2^2) \cot^2 \theta_W}{(s - \sec^2 \theta_W)^2} \right] G_1(\kappa', \lambda', s)
\]

\[
\sigma_{Zz} = \frac{4e^4}{s^3} G_2(\kappa, \kappa', \lambda, \lambda') \frac{g^2}{\beta} \left[ \frac{e^2(g_1^2 + g_2^2) \cot^2 \theta_W}{(s - \sec^2 \theta_W)^2} \right] G_2(\kappa', \lambda', \lambda', s)
\]

\[
\sigma_{Zv} = \frac{4e^4}{s^3} G_3(\kappa, \lambda, s) \frac{g^2}{\beta} \left[ \frac{e^2(g_1^2 + g_2^2) \cot^2 \theta_W}{(s - \sec^2 \theta_W)^2} \right] G_3(\kappa', \lambda', \lambda', s)
\]

\[
\sigma_{Zr} = \frac{4e^4}{s^3} G_4(\kappa, \lambda, s) \frac{g^2}{\beta} \left[ \frac{e^2(g_1^2 + g_2^2) \cot^2 \theta_W}{(s - \sec^2 \theta_W)^2} \right] G_4(\kappa', \lambda', \lambda', s)
\]

with the definitions

\[
G_1(\kappa, \lambda, s) = \frac{2\pi^2}{3} \beta \left[ \frac{e^2}{2s} \left( \frac{1 + \kappa^2 + \lambda^2 - 2\lambda + 2\lambda \kappa}{s^2} \right) \right]
\]

\[
G_2(\kappa, \kappa', \lambda, \lambda') = \frac{2\pi^2}{3} \beta \left[ \frac{e^2}{2s} \left( \frac{1 + \kappa + \lambda}{s} \right) \right]
\]

\[
G_3(\kappa, \lambda, \lambda') = \frac{2\pi^2}{3} \beta \left[ \frac{e^2}{2s} \left( \frac{1 + \kappa + \lambda}{s} \right) \right]
\]

\[
G_4(\kappa, \lambda, s) = \frac{2\pi^2}{3} \beta \left[ \frac{e^2}{2s} \left( \frac{1 + \kappa + \lambda}{s} \right) \right]
\]

Putting \( \kappa = \kappa' = 1 \) and \( \lambda = \lambda' = 0 \) we get the results of Babich, Pankov and Paver. We note that in the total cross-section we have integrated from \( \tau = -\frac{1}{2}(1 + \beta) \) to \( \tau = -\frac{1}{2}(1 - \beta)^2 \) unlike Babich et al. who have integrated from \( Z_1 \) to \( Z_2 \). Thus if we wish to generalize the model of Babich et al. including the anomalous couplings, we will have to use our expressions (14).

### 4 Helicity Amplitudes and Longitudinally Polarised Cross-Section

In calculating the helicity amplitudes, we follow the method of Yehudai and write the matrix element in the form

\[
M_{\sigma}^{\lambda^{-}\lambda'}(\theta) = \sqrt{s} \frac{1}{2} \left[ \frac{\sigma_{AA}}{\beta} \frac{d\sigma_{AA}}{dz} + \frac{d\sigma_{Zz}}{dz} + \sum_{i, j} \left( \frac{d\sigma_{Zz}}{dz} \right) \right] + \sum_{i, j} \frac{d\sigma_{Zr}}{dz}
\]

where \( z = \cos \theta \),

\[
\frac{d\sigma_{AA}}{dz} = 4F_1 \\
\frac{d\sigma_{Zz}}{dz} = 2g_2 \cdot F_2 \\
\frac{d\sigma_{Zr}}{dz} = -4g_1 \cdot g_2 \cdot \text{Re} \chi, F_3 \\
\frac{d\sigma_{Zr}}{dz} = 4g_1 \cdot g_2 \cdot \text{Re} \chi, F_3 \\
\frac{d\sigma_{Zr}}{dz} = 2 \left( \frac{g_1}{g_2} \right)^2 |\chi|^2 F_2 \\
\frac{d\sigma_{Zr}}{dz} = 2 \left( \frac{g_1}{g_2} \right)^2 \text{Re} \chi, F_3 \\
\frac{d\sigma_{Zr}}{dz} = 2 \left( \frac{g_1}{g_2} \right)^2 \text{Re} \chi, F_3
\]

the \( \chi \)'s are the \( z_1 \) and \( z_2 \) propagators.

\[
\chi(s) = \frac{s}{s - M^2 + i\Gamma M}
\]

and \( g_{1, 2} = e_{1, 2} \omega_w, g_{1, 2} = g_{1, 2} \omega_{2, 2}, g_{1, 2} \omega_{1, 2} \omega_{2, 2} \).

\[
F_1 = 2 \frac{dE_0}{du} (\sigma_{AA}, \kappa, \lambda) \\
F_2 = \frac{1}{16} A(stu, \kappa, \lambda) \\
F_3 = b(stu, \kappa, \lambda)
\]
\( J_0 = \max(1, |\Delta \lambda|) \) and the \( d \)-functions are the following

\[
\begin{align*}
& \quad d_{2,1}^{2,1} = \pm \frac{\sqrt{u} - 1}{s \beta^2} (t - u + \beta s) \\
& \quad d_{2,1}^{2,1} = \pm \frac{\sqrt{u} - 1}{s \beta^2} (t - u + \beta s) \\
& \quad d_{2,1}^{2,1} = \frac{1}{2s \beta} (t - u + s \beta) \\
& \quad d_{2,1}^{2,1} = \mp \frac{1}{2s \beta} (t - u - s \beta) \\
& \quad d_{1,1}^{2,1} = \frac{1}{s \beta} \sqrt{u - 1}.
\end{align*}
\]

The reduced matrix elements then become

\[
\begin{align*}
\langle -1 \rangle & = \frac{1}{s \beta} \sqrt{u - 1} \\
\langle -1 \rangle & = -\beta H_{2,1}^{1,1} \\
\langle -1 \rangle & = -\beta \text{Re} \left[ \frac{v}{2s \beta} B_{2,1}^{1,1} \right]
\end{align*}
\]

where the coefficients are given in the table below \((V = \gamma \text{ and } Z)\),

<table>
<thead>
<tr>
<th>( \lambda^- \lambda^+ )</th>
<th>( A^{1,1} )</th>
<th>( B_{2,1}^{1,1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (+,+) )</td>
<td>( \sqrt{2} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( (-,+) )</td>
<td>( 2\beta - 2 \cos \theta + r )</td>
<td>( \frac{1}{\sqrt{2}} \left[ g_1^{-1} \lambda^+ + \lambda^+ \right] )</td>
</tr>
<tr>
<td>( (0,+) )</td>
<td>( \sqrt{2} \beta \cos \theta )</td>
<td>( g_1^{-1} \lambda^+ + 2 \lambda^+ )</td>
</tr>
<tr>
<td>( (0,0) )</td>
<td>( 2\beta - 2 \cos \theta + r )</td>
<td>( g_1^{-1} \lambda^+ + 2 \lambda^+ )</td>
</tr>
</tbody>
</table>

Several useful conclusions on the contributions of the various couplings to the different helicity amplitudes may be drawn from Table 1:

(i) Since the neutrino exchange term contains the factor \( \delta_L \), the right-handed amplitudes are entirely determined by the photon and \( Z \) exchange. Furthermore, since \( B_{2,1}^{1,1} = 0 \) for \((+,+)\) and \((-,+)\) amplitudes, the left-handed amplitudes for production of vector bosons of opposite transverse helicities is entirely determined by neutrino exchange. Thus polarised electrons will be useful in discriminating photon-induced and \( Z \)-induced couplings, \( \kappa_V \) and \( \lambda_V \).

(ii) The amplitudes for the production of transverse vector mesons of the same helicity \((++,--\) or \(--,++)\) depend on the vector meson coupling \( g_1^T \) and quadrupole interactions \( \kappa_V \) and \( \lambda_V \), as well as on the CP violating couplings \( \kappa_V \) and \( \lambda_V \).

(iii) The amplitudes for the production of longitudinal vector mesons \((0,0)\) depend on \( g_1^L \) and on the dipole interactions \( \kappa_\gamma \) and \( \kappa_\gamma \).

(iv) The amplitudes for the longitudinal-transverse components \((0,\pm)\) or \((\pm,0)\) depend on all couplings. We note that these results agree with those of Bohm et al. when we take \( \kappa_V = 1 \) and \( \lambda_V = 0 \).

From Table 1, it is straightforward to obtain the differential cross-section for longitudinal polarisation. Setting \( m_{W^0} = 1 \), we have the amplitudes,

\[
\begin{align*}
M_{L}^{0} & = \frac{1}{2s \beta} \sqrt{u - 1} \left( \frac{1}{s} - \frac{1}{s - m_W^2} \right) \\
M_{R}^{0} & = \frac{1}{2s \beta} \sqrt{u - 1} \left( \frac{1}{s} - \frac{1}{s - m_W^2} \right)
\end{align*}
\]

Therefore the differential cross-sections can be written as

\[
\begin{align*}
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_3(s, t) \\
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_4(s, t) \\
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_5(s, t) \\
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_6(s, t) \\
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_7(s, t) \\
\frac{d\sigma^{00}}{dt} & = \frac{\alpha^2}{16s \beta} \frac{1}{s^2} F_8(s, t)
\end{align*}
\]

and

\[
\frac{d\sigma^{00}}{dt} = \frac{\alpha^2}{16s \beta} \frac{1}{s} \left( \frac{1}{s - m_W^2} \right) [\alpha^2 + \alpha^2 + \alpha^2 + \alpha^2 + \alpha^2 + \alpha^2 + \alpha^2 + \alpha^2]
\]

where \( s_W = \sin \theta_W \), \( \cos \theta_W = \cos \theta_W \), \( v = (4s_W - 1)/4s_W \), \( a = -1/4s_W \) and

\[
\begin{align*}
F_3(s, t) & = -4 \frac{1}{s^2} \Delta(s, t) \left( t^2 - \frac{4}{s^2} \right) \\
F_4(s, t) & = -4 \Delta(s, t) (s + 2)^2 \\
F_5(s, t) & = \frac{4}{s^2} \Delta(s, t) \left( t + \frac{4}{s} \right) \left( 2 + \kappa_s \right)
\end{align*}
\]

with

\[
\Delta(s, t) = t^2 - 2t + 1 + st.
\]
Thus if we put $\kappa = 1$, we get back the results of Lemoine and Veltman and it is interesting to see that in these amplitudes the term $s+2$ of Lemoine and Veltman has been replaced by $s\kappa + 2$ but the amplitudes do not depend on $\lambda$ as we have already stated.

5 Helicity Amplitudes and Differential Cross-Section for the Process $\gamma e \rightarrow W\nu_e$

The electroweak interaction can also be studied through the production of $W^\pm$ bosons on photon-electron scattering and as the energy becomes greater than $M_W$ the cross-section is expected to be sizeable. Indeed the process $\gamma + e \rightarrow W^\pm + \nu_e$ offers the possibility of studying the $\gamma W W$ vertex and its Yang-Mills form unencumbered by the Z-boson.

Once again we use the most general C and P conserving $\gamma W W$ interaction Lagrangian in the form,

$$L = \kappa \left[ W_{\mu} W^{\nu} A^\nu - W_{\mu} A^\nu W^\nu + \kappa W_{\mu} W^\nu W^\nu + \frac{\lambda}{m_W} W_{\mu} W^\nu W^\rho W^\sigma \right]$$

and then for the vertex $\gamma W W$ with momenta as shown below.

![Diagram](image)

we get

$$\Gamma^{\gamma W W} = \kappa \left[ g^{\mu \nu} \left( p^\mu + p^\nu - \frac{\lambda}{m_W} \frac{q^\mu q^\nu}{m_W^2} \right) + g^{\mu \nu} \left( \frac{\lambda}{m_W} \frac{q^\mu q^\nu}{m_W^2} \right) + g^{\mu \nu} \left( \frac{\lambda}{m_W} \frac{q^\mu q^\nu}{m_W^2} \right) \right]$$

Hence for the process $\gamma(k) + e^-(p_1) \rightarrow W^+(p) + \nu_e(p_2)$

we obtain for the two Feynman diagrams

the matrix element in the form,

$$iM^{\kappa} = \beta e^2 u_e(p_2) \left[ \gamma^\mu A^\nu \right. + \frac{\lambda}{m_W^2} \frac{q^\mu q^\nu}{m_W^2} \left. + \frac{\lambda}{m_W^2} \frac{q^\mu q^\nu}{m_W^2} + \frac{\lambda}{m_W^2} \frac{q^\mu q^\nu}{m_W^2} \right]$$

where $s = (k + p)^2$, $t = (k - p)^2$ and the matrix element has to be contracted with the polarisation vectors $e^\mu_+ (k)$ and $e^\nu_+ (p)$ of the photon and $W-$meson respectively. As before we write the matrix element after factorizing out the $d$-function $d_{\lambda + X + \lambda}(\theta)$ where $J_0 = \max(\alpha + \frac{1}{2}, \alpha + \frac{1}{2})$, i.e. we write

$$M^{\lambda \kappa}(e) = A^{\lambda \kappa}$$

$$M^{\lambda \kappa}(W) = \frac{\kappa}{2} \frac{B^{\lambda \kappa}}{t - m_W^2}$$

The coefficients $A$ and $B$ are given in Table 2:

<table>
<thead>
<tr>
<th>$\lambda \kappa$</th>
<th>$A^{\lambda \kappa}$</th>
<th>$B^{\lambda \kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(++)$</td>
<td>0</td>
<td>$2 + \Delta \kappa$</td>
</tr>
<tr>
<td>$(-)$</td>
<td>$\sqrt{2}$</td>
<td>$2(3 - \cos \theta) - 2r(1 - \cos \theta) + 2\Delta \kappa - \lambda (1 - \cos \theta)$</td>
</tr>
<tr>
<td>$(--)$</td>
<td>0</td>
<td>$\frac{\lambda}{r}$</td>
</tr>
<tr>
<td>$(++)$</td>
<td>0</td>
<td>$-\frac{1}{r} [2r^2 + \Delta \kappa r + \lambda (1 - r)]$</td>
</tr>
<tr>
<td>$(+$0)</td>
<td>0</td>
<td>$-\frac{1}{\sqrt{2}r} [4r + \Delta \kappa (1 + r) + \lambda (1 - r)]$</td>
</tr>
<tr>
<td>$(0-)$</td>
<td>$-\frac{1}{\sqrt{2}r}$</td>
<td>$\frac{1}{\sqrt{2}r} [1 - \cos \theta + r(1 + \cos \theta) + \Delta \kappa + \lambda \cos \theta]$</td>
</tr>
</tbody>
</table>

where $\Delta \kappa = \kappa - 1$. 

\[11\]
The differential cross-section can be obtained by summing over the squares of the helicity amplitudes. Alternatively we can obtain
\[ \Sigma |M|^2 \]
using for a heavy boson the spin sum,
\[ \Sigma \varepsilon^{\nu} \varepsilon^{\mu} = -g^{\nu \mu} + q^\mu q^\nu/m_H^2 \]
but for a photon of momentum \( k^\mu \) we can use the axial gauge sum,
\[ \Sigma \varepsilon^{\nu} \varepsilon^{\mu} = -g^{\nu \mu} + k^\nu p^\mu + k^\mu p^\nu / p \cdot k \]
where \( p^\mu \) is a four-vector different from \( k^\mu \). We have taken \( p = p_t \). This then gives the result
\[ \frac{d\sigma}{d\Omega} (\gamma e \to W \nu) = \frac{G_F^2}{64\pi^3 s} \left( 1 - \frac{m_W^2}{s} \right) T(s,t,u) \]
where
\[ T(s,t,u) = -16 \left( \frac{u}{s} + \frac{t}{2m_W^2} \right) \]
\[ + \frac{2}{s} [u(t - m_W^2)] - 8ut \left( 1 + \frac{2u}{m_W^2} \right) \]
\[ - 8st \left( \frac{u}{m_W^2} - 1 \right) (2 + \kappa - \lambda - 1) \]
\[ + 16su \left( \frac{t}{2m_W^2} - 1 \right) (2 + \kappa - \lambda - 1 - \frac{\lambda}{m_W^2} (t - m_W^2)) \]
\[ + \frac{\lambda}{m_W^2} u^2(t + m_W^2) \]
\[ + 16(us + 2m_W^2 t) (1 - \frac{m_W^2}{s})^2 \]
\[ + \frac{1}{(s - m_W^2)^2} \left[ 8(2 + \kappa - \lambda - 1) ut \left( \frac{1 + m_W^2}{m_W^2} \right) \right. \]
\[ - 16 \left( 2 + \kappa - \lambda - 1 + \frac{m_W^2 - t}{m_W^2} \right) \left. \frac{u}{m_W^2} \right) \]
\[ + \frac{\lambda}{m_W^2} u^2(t + m_W^2) \]
\[ + 2(2 + \kappa - \lambda - 1)^2 \left( \frac{m_W^2 - t}{m_W^2} \right)^2 \]
\[ + 4(2 + \kappa - \lambda - 1) \left( 2 + \kappa - \lambda - 1 + \frac{m_W^2 - t}{m_W^2} \right) \left. \frac{u}{m_W^2} \right) \]
\[ - 8 \left( 2 + \kappa - \lambda - 1 + \frac{m_W^2 - t}{m_W^2} \right) \left. \frac{u}{m_W^2} \right) \]
\[ + \frac{\lambda}{m_W^2} u^2(t - m_W^2) \right) . \]

A detailed comparison of this expression with the calculations of Robinson and Rizzo \(^9\) has not been attempted in the limit \( \kappa = 1 \) and \( \lambda = 0 \) but we may note that the first term of \( T(s,t,u) \) is the same. We have written \( T(s,t,u) \) in such a way that it is easy to see the origin of each term from the square of the matrix element. In this form it will be easier to pinpoint the deviations from the SM predictions, especially because this process is known to be very sensitive to the change in \( \kappa \) from unity \(^10\).

6 Results and Discussion

The numerical evaluation for the process \( e^+e^- \to W^+W^- \) has been done using \( m_W = 80.33 \) GeV, \( m_Z = 91.54 \) GeV, \( \sin^2 \theta_W = 0.23 \) and \( a = e^2/4\pi = 1/137.06 \). The differential cross-sections for the standard and non-standard model parameters are shown in Figs.1-6. The differential cross-section in the SM case is strongly peaked near \( \theta = 180^\circ \) and this effect is more pronounced at higher energies. For non-standard parameters, increasing the values of the parameters \( \kappa, \kappa', \lambda \) and \( \lambda' \) increases the differential cross-section.

The separate contributions of the different Feynman diagrams for the process \( e^+e^- \to W^+W^- \) are shown in Fig.5 from which it will be seen that the contribution from neutrino exchange is the most dominant. For \( \theta < 165^\circ \), the \( \nu \nu \) and \( Z\nu \) interference terms are negative but above this angle, all terms are positive.

The differential cross-section for longitudinally polarised vector boson for the GWS model and non-standard models are shown in Figs.6 and 7. It has been found that the peak decreases at higher energies and shifts towards larger angles. Non-standard values of the parameters \( \kappa \) increases the cross-section. For \( 200 \) GeV, the longitudinally polarised cross-section is about a factor 10 smaller than the cross-section including the transverse vector boson.

The total cross-sections as a function of energy for both standard and non-standard parameters are shown in Figs.8 and 9. The total cross-section for SM at tree level has a peak of about 17pb at about 200GeV. At higher energies the cross-section decreases. This is due to the strong cancellations between different contributions to the process \( e^+e^- \to W^+W^- \) which is a characteristic of gauge theories (Fig.10).

The inclusion of non-standard values of the parameters \( \kappa, \kappa', \lambda \) and \( \lambda' \) has significant effect on total cross-section which becomes higher for higher values of \( \kappa \) and \( \lambda \) for example, the total cross-section is about 7pb for \( \lambda = 0 \) but 62pb for \( \lambda = 1 \) and 230pb for \( \lambda = 2 \) at \( \sqrt{s} = 500 \) GeV. The contributions for different diagrams to the total cross-section for non-standard values of the parameters are shown in Figs.11 and 12.

Very recently Bilenky, Kneur, Renard and Schildknecht \(^{11}\) have considered in great detail how to extract the \( W^\pm \) spin-density matrix and the \( W^\star, W^\star^\prime \) spin correlations from future data. They have obtained 95% confidence-level bounds on the parameters and have concluded that LEP-2 operating around \( E_{e+} \sim 190 \) GeV will allow us to deduce bounds approximately given by
\[ |\delta_2, |\delta_3, |\delta_4| \leq 0.04 \text{ to } 0.15 \]
where \( \delta_2 = g_{WWZ} - \cot \theta_W, \delta_3 = \kappa - 1 \) and \( u_3 = \lambda \). Our input parameters are within this range but we have also taken different values to highlight the effects of these parameters.
Like Bilenky et al. \cite{10}, we have also noticed strong correlations between variations of the non-standard couplings.

The numerical work on the process $\gamma e \rightarrow W \nu_e$ will be reported separately.

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Figure 1: The angular differential cross-section for \( e^+e^- \rightarrow W^+W^- \) for centre of mass energy 175, 200, 250, 300 GeV; \( \theta \) is the angle between \( \vec{e}^+ \) and \( \vec{W}^+ \).

Figure 2: The angular differential cross-section for \( e^+e^- \rightarrow W^+W^- \) for centre of mass energy 260, 400, 500 GeV; \( \theta \) is the angle between \( \vec{e}^+ \) and \( \vec{W}^+ \).
Figure 3: The angular differential cross-section for $e^+e^- \rightarrow W^+W^-$ for GWS model, $x=2$, $x'=3$, $t'=1$, $t=2$, when one of the parameter varies, the others remain fixed at GWS value.

Figure 4: The angular differential cross-section for $e^+e^- \rightarrow W^+W^-$ for $x=2$, $x'=3$, $t'=1$, $t=2$, when one of the parameter varies, the others remain fixed at GWS value.
Figure 5: Contributions to the differential cross-section for $e^+e^- \rightarrow W^+W^-$ from various diagrams as function of $\theta$ for $\sqrt{s} = 250$ GeV.

Figure 8: The differential cross-section for longitudinally polarized vector boson for QWS model at c.m. energy 200, 250 and 300 GeV.
The differential cross-section for longitudinally polarized vector boson for $x=1, 2, 3$ ($\sqrt{s} = 200$ GeV).

Figure 7: The differential cross-section for longitudinally polarized vector boson for $x=1, 2, 3$ ($\sqrt{s} = 200$ GeV).

Figure 8: Variation of total cross-section $\sigma$ as function of centre of mass energy ($\sqrt{s}$) for CMS model, $x=1, 2, 3$ and $x'=3$. When one of the parameter varies, the others remain fixed at CMS value.
Figure 9: Variation of total cross-section $\sigma$ as function of c.m. energy ($\sqrt{s}$) for $\nu=5, x=3, y=3, z=3$ and $\beta=2$. When one of the parameters varies, the others remain fixed at CMS value. $\nu$ is the relative ratio of the TMD and DLA contributions.

Figure 10: Contributions to the total cross-section for $e^+e^- \rightarrow e^+e^-$ from various diagrams as function of c.m. energy ($\sqrt{s}$) in CMS model.
Figure 11: Contributions to the total cross-section for $e^+e^- ightarrow W^+W^-$ from various diagrams as function of c.m. energy ($\sqrt{s}$) for $\lambda = 1$.

Figure 12: Contributions to the total cross-section for $e^+e^- ightarrow W^+W^-$ from various diagrams as function of c.m. energy ($\sqrt{s}$) for $\lambda' = 1$. 