\( \Lambda_b \rightarrow \Lambda_c \) EXCLUSIVE WEAK DECAYS IN A LIGHT CONE MODEL

Faheem Hussain
Dongsheng Liu
and
Salam Tawfiq

MIRAMARE-TRIESTE
1. INTRODUCTION

Recently much attention has been focussed on the study of heavy flavour baryons, such as $\Lambda_c$ and $\Lambda_b$. Experimentally, besides the existing data for charmed baryons, events providing evidence for the semileptonic decays of the bottom baryon to the charm baryons have been observed in $Z$ decays by the ALEPH and OPAL collaborations at LEP [1]. We hope that more measurements of the bottom baryon will be available in the near future.

On the other hand, a lot of theoretical investigation for heavy baryon [2-6] decays have been carried out in the framework of the Heavy Quark Effective Theory (HQET) [7-10]. The HQET has a particularly strong predictive power when applied to the decays of heavy $\Lambda$-like baryons. This is because a heavy $\Lambda_Q$ is particularly simple in its spin structure due to the fact that the light degrees of freedom are in a scalar (spin zero) state. At the heavy quark limit, the heavy quark spin decouples from the gluon field and one can identify the heavy quark spin with the spin of the baryon. A well-known consequence of the HQET, to lowest order, for baryon weak decays is that the number of independent form factors governing the transition between the heavy $\Lambda$-like baryons decreases from six to one [2,5,6] and that this single form factor is normalized to unity at zero recoil. Even when the corrections arising from the finite quark mass are taken into account at $O(\frac{1}{m})$, there are still five relations among the six form factors, apart from a dimensional parameter [11]. Furthermore, in the case that the heavy quark mass limit is taken only for the parent baryon, while the daughter baryon is considered as light, just two independent form factors are necessary to describe the decays [2,5,6].

The $q^2$ dependance of the independent form factor is, however, not determined by the heavy quark symmetry. The pole dominance ansatz for the form factors has been used to evaluate the decay properties, such as the $q^2$ distribution of the decay rate, lepton energy spectrum, asymmetry parameters in the angular distribution, exclusive decay rates and so on [4,12,13]. One useful way to address this problem is to use model calculations based on assumptions for the hadronic wave function. The light-cone QCD [14] provides a convenient framework to describe hadronic states and has been successfully applied to evaluate form factors for the heavy meson weak decays [15], specially for large recoil or the low $q^2$ region, and for the electromagnetic form factors of the proton [16]. Recently, by incorporating the heavy quark symmetry, the heavy meson form factor has been reexamined in the light-cone formalism [17,18]. In the present work, we examine the heavy baryon decays in the light-cone formalism and calculate the form factors over all momentum transfer range in a given model.

In Section II of this paper, we first present the formalism needed to evaluate form factors in terms of the light cone wavefunction and then show that the heavy quark symmetry is explicit in this formalism in the limit of infinite quark mass.
In Section III we evaluate this single form factor within a valence dominance assumption. The study of the decay properties in terms of this form factor is reported in Section IV. Section V contains our conclusions.

2. LIGHT CONE FORMALISM AND HEAVY QUARK SYMMETRY

We begin with the general form for the matrix elements of the vector and axial vector currents, \( V^a = \bar{Q} \gamma^a \gamma^5 Q \) and \( A^a = \bar{Q} \gamma^a \gamma^5 \gamma^i Q \), inducing transitions between the \( \frac{1}{2}^+ \) heavy baryons \( B_Q \) and \( B_{Q'} \). Here, \( Q \) and \( Q' \) are the initial and final active quarks. Following the usual practice in heavy quark physics, we write the matrix elements as

\[
\langle B_Q; \lambda', v'|V|B_Q; \lambda, v \rangle = \bar{u}_{\lambda'}(v') [\gamma^\mu f_1 + \gamma^\mu f_2 + \gamma^\mu f_3] u_\lambda(v)
\]

\[
\langle B_Q; \lambda', v'|A^a|B_Q; \lambda, v \rangle = \bar{u}_{\lambda'}(v') [\gamma^\mu g_1 + \gamma^\mu g_2 + \gamma^\mu g_3] u_\lambda(v)
\]

(1)

where \( (\lambda, v) \) and \( (\lambda', v') \) are the helicities and the velocities of the initial and final baryons, respectively. The baryon spinor is normalized such that

\[
\langle u | \bar{u} \rangle = \sqrt{2} \delta_{\lambda \lambda'}
\]

In the light-cone formalism [14,19], a vector is written in terms of its plus, minus and two transverse components which are related to the usual components through

\[
p^+ = p^0 + p^3
\]

\[
p^{R,L} = p^1 \pm i p^2,
\]

with \( \vec{p}_L = (p^R, p^L) \)

For on-shell particles one has \( p^{-} = \frac{m^2 - p^2}{p^+} \).

In the following analysis, we are going to link the form factors to certain matrix elements which can be evaluated in the light-cone QCD.

To this end, we present a number of useful algebraic relations (here \( \Gamma \) is either \( I \) or \( \tau_3 \)):

\[
\bar{u}_I(v') \gamma^\mu \Gamma u_I(v) = \sqrt{v^+} \bar{u}_I(v') \Gamma u_I(v) = 0
\]

\[
\bar{u}_I(v') \gamma^\mu \Gamma u_I(v) = \frac{1}{2} \frac{v^+ L \pm u^+}{\sqrt{v^+ u^+}} \bar{u}_I(v') \Gamma u_I(v) = \pm \frac{1}{2} \frac{v^+ L - u^+ L}{\sqrt{v^+ u^+}}
\]

\[
\bar{u}_I(v') \gamma^\mu \Gamma u_I(v) = 0
\]

(3)

where the positive sign is associated with \( \Gamma = I \) and the negative sign with \( \Gamma = \tau_3 \). More useful relations are presented in the Appendix.

Substituting these relations in Eq.(1) leads to

\[
\langle \uparrow | J^+ | \downarrow \rangle = \frac{1}{2} \frac{v^+ u^+}{\sqrt{v^+ u^+}} (v^+ h_2 + u^+ h_3)
\]

\[
\langle \uparrow | J^L | \downarrow \rangle = \frac{1}{2} \frac{v^+ u^+}{\sqrt{v^+ u^+}} (v^+ h_2 + u^+ h_3)
\]

(4)

with \( J = V \) or \( A \), and correspondingly \( h_1 = f_3 \) or \( g_3 \). The plus (minus) sign is associated with \( V(A) \). When a particular reference frame is specified, further simpler relations can be set up. For instance, if we choose the initial baryon moving along the z-axis, we find that the third form factors are only connected with the matrix elements of the left components of the currents between opposite helicity states,

\[
h_3 = \frac{2}{v^+ u^+} (\langle \uparrow | J^L | \downarrow \rangle)
\]

(5)

Another example is that if we work in a Breit-like frame in which

\[
v^+ = v^+; \quad \vec{v}_L = -\vec{v}_L
\]

(6)

the first axial form factor is expressed by the matrix element of the plus component of the axial current,

\[
g_1 = (\langle \uparrow | A^+ | \downarrow \rangle)
\]

(7)

Here \( \vec{v}_L = (v^R, v^L) \).

From now on we use the Brodsky-Lepage approach of quantizing QCD on the light-cone in the light-cone gauge \( A^+ = 0 \) [14]. The advantages of this approach are well-known and will not be repeated here. The interested reader is referred to Ref. [14]. In the light-cone QCD, a hadronic state is described as a sum over all Fock states. The heavy \( B_Q \) baryon with momentum \( (P^+, \vec{P}_L) \) and helicity \( \lambda \) is described by the state

\[
\langle B_Q; P^+, \vec{P}_L, \lambda \rangle = \sum_{n, \lambda_i} \int \prod_i d^2 \vec{k}_L \delta^{16} \psi_{n, \lambda_i} (z_i, \vec{k}_L, \lambda_i) |n; z_i, P^+, \vec{P}_L, \vec{k}_L, \lambda_i \rangle
\]

(8)

where

\[
\prod_i d^2 \vec{k}_L = \prod_i d^2 \vec{k}_L \frac{16 \pi^2}{16 \pi}
\]

\[
\prod_i \delta^{16} \psi_{n, \lambda_i} (z_i, \vec{k}_L, \lambda_i)
\]

Here the Fock space state vector \( |n; z_i, P^+, \vec{P}_L, \vec{k}_L, \lambda_i \rangle \) represents a parton configuration with the quantum numbers of the \( B_Q \). The fraction of the longitudinal momentum the \( i \)th parton carries is denoted by \( x_i = P_i^+ / P^+ \), and \( \vec{k}_L \) is the transverse relative momentum of the \( i \)th parton with respect to the direction of motion of the baryon. \( \psi_{n, \lambda_i} (z_i, \vec{k}_L, \lambda_i) \) specifies the parton wavefunction representing the probability amplitude.
for a certain parton configuration. One of the advantages of this description is that the inner motion of the Fock state, represented by the wave function, is decoupled from the hadronic momentum $P$, except through the mass-shell condition $P^2 = M^2$. As we shall see later, the evaluation of form factors in the light cone QCD formalism is simpler than in other covariant descriptions of hadronic states.

As discussed in Ref. [14], unrenormalized QCD perturbation theory indicates that hadronic wavefunctions do not fall off sufficiently quickly as $k^2 \to \infty$. The usual way [14] to get around this difficulty is to introduce an ultra violet cut-off $\Lambda$ which simply truncates the Fock space, excluding all states with $k^2_{\perp} > \Lambda^2$. This is implied in Eq.(8) and in the rest of this section. However, in the next section, when we explicitly evaluate the form factors we will not use QCD perturbation theory to calculate the wavefunctions. Instead we will use model wavefunctions which parametrize the transverse distribution such that they fall off rapidly with increasing $k^2_{\perp}$.

The hadronic state, Eq.(8), is normalized as

$$
(B_Q^p; P', \lambda | B_Q; P, \lambda) = 2P^+(2\pi)^3 \delta^3(P - P') \delta_{\lambda, \lambda'}
$$

which leads to the normalization for the parton wave functions

$$
\sum_{n, \lambda} \int \prod_n \frac{dx_n d^2 k_{\perp}}{16\pi^3} |\psi_{n, \lambda}(x_n, \vec{k}_{\perp, n}, \lambda)|^2 = 1
$$

As far as the current operator responsible for weak decays is concerned, we refer to the plus component of the current operator as a good current, since it only contains the positive projections of the quark fields, namely that

$$
J^+ = \bar{\Psi} Q^- \Gamma V Q = \bar{\Psi}^{(\pm)} Q^- \Gamma \Psi^{(\pm)}
$$

where $\Psi^{(\pm)} = \Lambda_{\pm} \Psi$ with $\Lambda_{\pm} = \gamma^\pm/2$. The advantage of this property arises from the fact that, in the light cone quantization, the positive projected quark field can be expanded in terms of a set of creation and annihilation operators. The matrix elements of a good current turn out to have a simple form. This simplification does not occur in the transverse currents

$$
J_\perp = \bar{\Psi} \gamma_\perp \Gamma \Psi_Q = \bar{\Psi}^{(\pm)} (1 + \bar{P}_Q \gamma_\perp) \Gamma (1 + P_Q) \Psi^{(\pm)}
$$

(12)

where $P_Q = \frac{1}{E} [\vec{P}_Q, \vec{D}_\perp + \gamma_0 (\vec{b} + \vec{k}_{\perp})]$ which contain both the positive and negative projected fields ($\Psi_Q^{(\pm)} = P_Q \Psi^{(\pm)}$). In general, for the analysis of their matrix elements is more involved.

Substituting the heavy quark current (at the moment we restrict ourselves to the vector current) between the Fock state expansions, Eq. (8), for the baryonic states gives the weak decay matrix element which reads formally

$$
\langle \lambda' | V^- | \lambda \rangle = \sum_{n, \lambda, \lambda'} \int d\Gamma \bar{\psi}_{n, \lambda'}(x', \vec{k}_{\perp, n}, \lambda') \psi_{n, \lambda}(x, \vec{k}_{\perp, n}, \lambda) \delta_{\lambda, \lambda'} \Gamma^{\mu} u_{\lambda Q}
$$

(13)

dT is obtained from the measure in the Fock states, Eq. (8), after imposing energy momentum conservation, $Q = P - P'$, at the baryonic level and also detailed energy-momentum balance at the parton level. The actual expressions for $dT$ will depend on the choice of frame. This will be illustrated later. In Eq. (13), $u_{\lambda Q}$ and $\bar{u}_{\lambda Q}$ are the light-cone spinors for the active quarks involved in the weak decay.

For the good current, $\Gamma^\pm$ is just the gamma matrix $\gamma^\pm$ and the spinor algebra is trivial. In this case the r.h.s. of Eq.(13) is just a sum of overlap integrals that is quite analogous to the nonrelativistic result (see Fig.1a). However, for the transverse components the $\Gamma^\pm$ in general are more complicated expressions of the momenta. Also the vertex no longer conserves particle number since there will now be terms involving transitions $Q + W \to Q' + g + Q + g + W \to Q'$ as shown in Fig. 1b.

We now discuss the consequences of the heavy quark limit for the Fock state wavefunctions. The heavy quark limit can be used to constrain these wavefunctions. We shall consider baryons, like $A_+$ and $A_-$, which contain only one heavy quark. Since the creation of a heavy quark-antiquark pair is suppressed in the heavy quark limit, we shall assume that there is only one heavy quark parton in each Fock state.

In the rest of this paper we will be concerned with discussing the form factors in the transition $\Lambda_Q \to \Lambda_Q$, where $\Lambda_Q$ has the quantum numbers $Q [ud]$. For such a heavy baryon the helicity of the heavy quark can be identified with the helicity of the baryon at the heavy quark limit, where the heavy quark moves with the same velocity as the baryon and carries its spin. This is because in the $\Lambda_Q$ the light degrees of freedom are in a total spin zero state. Therefore

$$
\psi_{n, \lambda}(\lambda_Q \neq \lambda) = 0
$$

(14)

upto order $k_{\perp} \Lambda_0/M_0$ or $\Lambda_{QCD}/M$.

Let us now apply these properties to the form factors. Firstly, we consider the case that only the parent $\Lambda_Q$ is heavy while the daughter baryon may be either heavy or light. That is, at this stage, we impose constraints only on the parent baryon. In fact, this turns out to be sufficient to ignore some of the form factors.

Because of Eq.(14), the sum over the helicities of the initial heavy quark is reduced to one term and Eq. (5) becomes

$$
\Gamma_3 = \pm \sqrt{\sqrt{\alpha_s} \alpha_e} \sum_n \int d\Gamma u_{\lambda Q} \Gamma^\mu u_{\lambda'} \langle x', \vec{k}_{\perp} | \psi_{n, \lambda'}(x, \vec{k}_{\perp}, \lambda) |\lambda = \lambda'\rangle
$$

(15)

A detailed analysis in a reference frame in which the initial heavy baryon moves along the $z$-axis shows that the coupling of a left handed current with a helicity down quark, $\Gamma^\mu u_{\lambda'}$, does not depend on the longitudinal momentum of the quark and only contains small
components compared to the heavy quark mass $M$, such as $p_{TQ}/M$, $p/M$ and $q/M$ where $p_{TQ}$ is the transverse momentum of the heavy quark, $p$ is the momentum of a light quark parton and $q$ is the gluon parton momentum. All of them are of the order of $M$ and vanish as long as the decaying quark is infinitely heavy. Hence we get the immediate result

$$f_3 \text{ or } g_3 \sim \frac{\Lambda_{QCD}}{M} \to 0 \text{ when } M \to \infty.$$  

Recalling that these form factors are connected with the transverse current, we realize that form factors associated with good currents are sufficient to describe heavy baryon decays. Hence, the non-trivial form factors are obtained from the matrix elements

$$\langle J^+ | J^+ \rangle = \frac{1}{2} \frac{v^+ v^+ - v^+ v^+}{v^+ v^+} v^+ h_2$$

and

$$\langle J^+ | J^+ \rangle = \frac{1}{2} \frac{v^+ v^+ - v^+ v^+}{v^+ v^+} v^+ h_2$$

in the heavy quark limit for the initial $\Lambda_Q$.

Again, using the heavy quark symmetry, Eq. (14), in the velocity Breit frame, Eq. (6), we find that Eqs. (17) give

$$f_1 + f_2 = \frac{M'}{\sqrt{M_Q M}} \sum_{n_i} \left( \frac{M}{M'} \right)^{1/2} \int \frac{d \xi x_i d \xi' x_i'}{4 \pi \xi^2} \psi_n(x_i' \xi' \lambda_i') \psi_n(x_i \xi \lambda_i) \lambda_{Q} = 1$$

$$f_2 = g_2 = -\frac{1}{v^+} \frac{M'}{\sqrt{M_Q M}} \sum_{n_i} \left( \frac{M}{M'} \right)^{1/2} \int \frac{d \xi x_i d \xi' x_i'}{4 \pi \xi^2} \psi_n(x_i' \xi' \lambda_i') \psi_n(x_i \xi \lambda_i) \lambda_{Q} = 1$$

(18)

In deriving relations (18), we have used the algebraic relations, Eq. (3). We also have the momentum conservations relations in the Breit frame (Eq. 6):

$$M'(1 - x_Q) = M(1 - x_Q)$$

$$\tilde{E}_{\perp_{Q}} = \tilde{E}_{\perp_{A}} - 2(1 - x_Q)M_{\perp}$$

(19)

for the active quarks, $Q$ and $Q'$, and

$$M' x_i' = Mx_i$$

$$\tilde{E}_{\perp_{i}} = \tilde{E}_{\perp_{A}} + 2x_i M_{\perp}$$

(20)

for all other partons.

We have, thus, found two relations among these form factors and, therefore, there remain only two independent form factors. Also, in the light-cone QCD, they are just a sum of overlap integrals over Fock states, very similar to the non-relativistic form. The two independent form factors incorporate all the effects of the low-energy QCD interactions on the weak decay of heavy $\Lambda_Q$ baryons. One remark which should be stressed here is that we have set up four equations for the six form factors in the light-cone QCD at the limit where the decaying quark has a mass much larger than the QCD-scale, which should be a reasonable approximation scheme for $\Lambda_{c}$ and $\Lambda_{b}$ decays. These results are in agreement with the heavy quark symmetry analysis of Refs. [2,5,6].

The results above, Eqs. (16) and (18), are valid for heavy to light decays. It is easy now to proceed to the heavy to heavy transitions. Here we take the heavy quark limit in the final $\Lambda_Q$ baryon as well. In this limit, the velocity of the daughter heavy quark is the same as the daughter baryon, and also the wavefunction $\psi_n\{x_{i'}(\lambda_{Q'=1})\}$ is zero.

Thus we are left with just one form factor

$$f_1 = g_1 = \frac{M'}{\sqrt{M_Q M}} \sum_{n_i} \left( \frac{M}{M'} \right)^{1/2} \int \frac{d \xi x_i d \xi' x_i'}{4 \pi \xi^2} \psi_n(x_i' \xi' \lambda_i') \psi_n(x_i \xi \lambda_i) \lambda_{Q} = 1$$

$$f_2 = g_2 = f_3 = g_3 = 0.$$  

(21)

As one can see, this result, Eq. (21), for heavy to heavy $\Lambda$ transitions is the same as the heavy quark symmetry analysis of Refs. [2,5,6].

3. RELATIVISTIC CONSTITUENT QUARK MODEL

The symmetry results of the last section are as far as one can go without knowing the detailed Fock state wavefunctions. In this section, we compute the single form factor in heavy to heavy decays based on the dominance of valence quarks in the Fock-state expansion. This approach is similar in spirit to the relativistic constituent quark model used by Dziembowski and co-workers [19,20] to successfully calculate the electromagnetic form factors of nucleons.

In a valence quark dominant approximation, the $\Lambda_Q$ wavefunction used in our present work has the form

$$\psi_{\Lambda}(x_i \tilde{E}_{\perp}; \lambda_i) = \phi(x_i) \phi(\tilde{E}_{\perp}) \delta_{\lambda_i}([1 + \mu \gamma_5 C]u_{\perp} u_{\perp} \Lambda)$$

(22)

where the factor $([1 + \mu \gamma_5 C]u_{\perp} \Lambda)$ is the spin projector for the $\Lambda_Q$ baryon, which guarantees the antisymmetry of the helicity amplitude under the interchange of the two light quarks in a scalar state [2,5,21]. We take the 3rd quark as the heavy, active quark. Dziembowski and

* The equal velocity assumption for the final heavy quark, $x'_{Q'} = \frac{m_{Q'}}{M'}$, requires that $M' - m_{Q'} = M - m_{Q}$. This is the assumption usually made in the heavy quark effective theory.
The explicit forms of the helicity amplitude are listed in the Appendix. Weber [22] have also shown that the helicity structure in Eq. (22) follows from solving the Weinberg equation [23] using a separable scalar quark-quark interaction. We assume that there is no diquark clustering. The wavefunction, Eq. (22), also has explicit decoupling of the heavy quark spinor from the light spinors as expected from heavy quark symmetry. We also notice that because of Eq. (20), the peaks of the initial and final light quark longitudinal distributions exactly coincide in this frame. The rest of the wave functions are relatively flat near this peak. We thus take the $x_i$ distributions to be essentially proportional to delta functions at the equal velocity point. This allows us to do the integration in Eq. (26). We normalize the single form factor to unity at $q^2_{max}$ (or $v\cdot v = 1$) as required from heavy quark symmetry. Thus

$$\xi(v\cdot v') = [1 + c_1(v\cdot v' - 1) + c_2(v\cdot v' - 1)^2]\exp[-\rho^2(v\cdot v' - 1)]$$

with $\rho^2 = \frac{m^2_1}{2M}$, where $m$ is the light quark mass. The coefficients $c_1$ and $c_2$ are given by

$$c_1 = \frac{1}{4} \left(1 - \frac{1}{p^2} + \frac{3}{p^2} \right)$$
$$c_2 = \frac{1}{64} \left(1 - \frac{1}{p^2} + \frac{3}{p^2} \right)$$

The structure underlying this form factor is transparent. The exponential factor comes from the transverse Gaussian distribution and the polynomial is from the light quark helicity part. The exponential part of the form factor is the same as in the meson decay case since we use similar Gaussian distributions as in Eq. (23) [17,24] with the same parameter $\rho^2 = \frac{m^2_1}{2M}$. The slope of the form factor, $-\partial\xi/\partial(v\cdot v')$, at the zero recoil point, $v\cdot v' = 1$, is approximately $\rho^2$ if $\rho^2 \approx 1$. This exponential form for the meson form factor has been used by the ARGUS [25] collaboration to fit the decays $B^0 \rightarrow D^*(\rightarrow D^* \ell^+\ell^-)$. They obtain $\rho = 1.37 \pm 0.19 \pm 0.08$ with $|V_{cb}| = 0.50 \pm 0.06 \pm 0.02$. These are the values we will use in our numerical calculations in Sec. IV. We also investigate the sensitivity of the form factors, as well as exclusive decay rates, by using various values of the $\rho$-parameter.

With this single form factor available, we can also take the leading $1/\rho^2$ corrections into account, using the expansion of the heavy quark effective Lagrangian [11]. Thus, including $1/\rho^2$ corrections, we have

$$f_1 = \left[1 + \frac{2}{m_{Q'}} v\cdot v' + \gamma \right] \xi(v\cdot v')$$
$$f_2 = g_2 = \left[1 - \frac{2}{m_{Q'} v\cdot v'} - \gamma \right] \xi(v\cdot v')$$

where

$$f_1 = \frac{1}{2m_{Q'} v\cdot v' + 1} + \frac{4\alpha_s(m_{Q'})}{\pi} \gamma$$

with

$$\gamma = \frac{\log(v\cdot v' + \sqrt{(v\cdot v')^2 - 1})}{\sqrt{(v\cdot v')^2 - 1}}$$

and $\lambda = M_{Q'} - m_{Q'}$. Also $g_1 = f_1 + f_2 = \xi$. Now, for the longitudinal momentum distribution $\phi(x_i)$ and $\phi(x'_i)$. In the heavy quark limit, in deriving Eq. (21) we have already made the assumption that the Fock state wavefunctions have delta function support in $x_i$ and $x'_i$ with $x_i = \frac{2}{M}(x_i' = \frac{2}{M'}$). In the weak binding approximation one has $x_i = \frac{2}{M}(x_i' = \frac{2}{M'})$ also for the light quarks. This implies that the light quarks also have the same longitudinal velocity as the velocity of baryon. One therefore expects that also for the light quarks the distribution is peaked fairly sharply around the equal velocity point. We assume a factorized form for the longitudinal and transverse distributions. For the totally symmetric transverse momentum distribution we choose a Gaussian shape

$$\phi(k_{L}) = \exp\left[-\sum_{i=1}^{2} \frac{k_{Lx_i}^2}{6a^2}\right]$$

This ansatz of course, circumvents the ultraviolet problem mentioned in Section III.

It is well-known that such a wavefunction overlap calculation for the electromagnetic form factors of the nucleon is restricted to low values of $q^2$, i.e. for large recoil, since it fails to reproduce the correct power-law fall off at high $q^2$ as expected by perturbative QCD. However, in the case of heavy hadron decays, as Isgur has noted, [17], the momentum transfer of the light degrees of freedom is actually quite small. Most of the momentum transfer is due to the mass difference between the active heavy quarks. Hence, since the form factor in the heavy to heavy case, depends essentially on the overlap of the light quark distributions, we believe our calculations to be valid over the whole $q^2$ range, i.e. from high to low recoil.

The choice of the distribution, Eqs. (23) is also dictated by the fact that the final form of the form factor is well-behaved in the limit, $M, M' \rightarrow \infty$. On the other hand if one uses another distribution, for example, as in Ref. 20, which has the form

$$\exp\left[-\frac{\sum_{i=1}^{2} k_{Lx_i}^2}{6a^2}\right]$$

then it leads to a form factor which blows up in the limit.

Then, substituting Eq. (22), (23) in eq. (21) we find that the single form factor $(\xi = f_1 = g_1)$ has the form

$$\xi(q^2) = \frac{M'}{\sqrt{m_{Q'} q^2}} \sum_{\lambda_1 \lambda_2} \frac{\lambda_1 \lambda_2}{\lambda_1 M'} \int d\tau \phi(z'_i)\phi(z_i)\exp\left[-\frac{3}{6a^2} \sum_{i=1}^{2} (k_{Lx_i}^2 + k_{Lx'_i}^2)\right] \times a_1(\lambda'_2)a_1(\lambda_1)\bar{a}_2(\lambda_1)\bar{a}_2(\lambda'_2)[(1 + \lambda')\tau C][e_{X_i}]a_{\lambda'_2}[\pm \lambda C][e_{X'_i}][1 + \lambda \tau C]$$

where in the Breit-like frame we have

$$q^2 = (M - M')^2 - 4M'M'\beta_{L_{2}}^2 = q_{max}^2 - 4M'M'\beta_{L_{2}}^2$$

with

$$\beta_{L_{2}} = \left[1 + \frac{2}{m_{Q'} v\cdot v' + 1} + \frac{4\alpha_s(m_{Q'})}{\pi} \gamma \right] \xi(v\cdot v')$$

and $\lambda = M_{Q'} - m_{Q'}$. Also $g_1 = f_1 + f_2 = \xi$. Now, for the longitudinal momentum distribution $\phi(x_i)$ and $\phi(x'_i)$. In the heavy quark limit, in deriving Eq. (21) we have already made the assumption that the Fock state wavefunctions have delta function support in $x_i$ and $x'_i$ with $x_i = \frac{2}{M}(x_i' = \frac{2}{M'}$). In the weak binding approximation one has $x_i = \frac{2}{M}(x_i' = \frac{2}{M'})$ also for the light quarks. This implies that the light quarks also have the same longitudinal velocity as the velocity of baryon. One therefore expects that also for the light quarks the distribution is peaked fairly sharply around the equal velocity point. We assume a factorized form for the longitudinal and transverse distributions. For the totally symmetric transverse momentum distribution we choose a Gaussian shape

$$\phi(k_{L}) = \exp\left[-\sum_{i=1}^{2} \frac{k_{Lx_i}^2}{6a^2}\right]$$

This ansatz of course, circumvents the ultraviolet problem mentioned in Section III.

It is well-known that such a wavefunction overlap calculation for the electromagnetic form factors of the nucleon is restricted to low values of $q^2$, i.e. for large recoil, since it fails to reproduce the correct power-law fall off at high $q^2$ as expected by perturbative QCD. However, in the case of heavy hadron decays, as Isgur has noted, [17], the momentum transfer of the light degrees of freedom is actually quite small. Most of the momentum transfer is due to the mass difference between the active heavy quarks. Hence, since the form factor in the heavy to heavy case, depends essentially on the overlap of the light quark distributions, we believe our calculations to be valid over the whole $q^2$ range, i.e. from high to low recoil.

The choice of the distribution, Eqs. (23) is also dictated by the fact that the final form of the form factor is well-behaved in the limit, $M, M' \rightarrow \infty$. On the other hand if one uses another distribution, for example, as in Ref. 20, which has the form

$$\exp\left[-\frac{\sum_{i=1}^{2} k_{Lx_i}^2}{6a^2}\right]$$

then it leads to a form factor which blows up in the limit.

Then, substituting Eq. (22), (23) in eq. (21) we find that the single form factor $(\xi = f_1 = g_1)$ has the form

$$\xi(q^2) = \frac{M'}{\sqrt{m_{Q'} q^2}} \sum_{\lambda_1 \lambda_2} \frac{\lambda_1 \lambda_2}{\lambda_1 M'} \int d\tau \phi(z'_i)\phi(z_i)\exp\left[-\frac{3}{6a^2} \sum_{i=1}^{2} (k_{Lx_i}^2 + k_{Lx'_i}^2)\right] \times a_1(\lambda'_2)a_1(\lambda_1)\bar{a}_2(\lambda_1)\bar{a}_2(\lambda'_2)[(1 + \lambda')\tau C][e_{X_i}]a_{\lambda'_2}[\pm \lambda C][e_{X'_i}][1 + \lambda \tau C]$$

where in the Breit-like frame we have

$$q^2 = (M - M')^2 - 4M'M'\beta_{L_{2}}^2 = q_{max}^2 - 4M'M'\beta_{L_{2}}^2$$

(26)
4. \( \Lambda_b \rightarrow \Lambda_c \) DECAY PROPERTIES

With the form factors presented in Sec. III, we are able to calculate the decay properties for the process \( \Lambda_b \rightarrow \Lambda_c \ell^+\bar{\nu}_\ell \). The zeroth order form factor, \( \xi(q^2) \), and the \( \frac{1}{m_c} \) corrected form factors, Eqs. (29) and (30), are plotted in Fig. 2 with \( \rho = 1.37 \). As a comparison, the dipole form factor is also shown. We find that our zeroth order form factor \( \xi(q^2) \) is close to the dipole form. In Fig. 3, we show the form factors \( f_1 \) and \( f_2 \) varying with the parameter \( \rho \).

The differential \( q^2 \) distribution of the decay rate is plotted in Fig. 4, for the massless lepton limit, using the \( \frac{1}{m_c} \) corrected form factors, \( f_1 \), \( f_2 \), \( g_1 \) and \( g_2 \). We use the differential decay distribution expressed in terms of helicity form factors as in Ref. [4]. In Fig. 5, we show the differential lepton energy spectrum. The total exclusive decay rates for different values of \( \rho \) are listed in Table 1 along with the transverse and longitudinal rates. For comparison, we also list the rates using a dipole form factor and also the free quark decay (FQD) rate.

Considering the total rates, the \( \Lambda_b^0 \rightarrow \Lambda_c^+ \) rate can be seen to amount to \( \sim 74\% \) of the FQD rate. This implies that \( \sim 74\% \) of the semileptonic \( \Lambda_b \) decays are into the ground state \( \Lambda_c \). The remaining 26\% would then be filled by semi-leptonic transitions to excited charm baryon states.

As pointed out in Ref. [4], it is known that the total non-leptonic bottom baryon decay rate cannot be reliably estimated because of the presence of W exchange and Pauli interference contributions in addition to the non-leptonic free quark decay rate. One expects the total semi-leptonic branching ratio of bottom baryons to be roughly 5%. These estimates would then yield an exclusive \( \Lambda_b \rightarrow \Lambda_c \) (ground state) branching ratio of (3–4)%.

There is a considerable theoretical uncertainty in the determination of \( |V_{bc}| \) from fitting the \( B \rightarrow D^* \) decay on the basis of HQET [23]. The reason is that \( |V_{bc}| \) using HQET is essentially determined by the data points with the largest \( q^2 \) values, where the statistical precision is poor. Consequently there is a considerable theoretical uncertainty in our predictions for the decay rates arising from the uncertainties in \( |V_{bc}| \) and \( \rho^2 \).

5. CONCLUSION

In conclusion, we would like to stress a number of points. We have demonstrated that, as expected, the light-cone approach reproduces the Heavy Quark Symmetry results in the analysis of the form factors in heavy \( \Lambda \)-like baryon decays. Further we have shown that the good current is sufficient for the evaluation of the independent form factors. This makes the calculation of these form factors significantly easier, at least formally.
Appendix A

In this appendix we present matrix elements of some useful $\gamma$-matrices between light-cone spinors.

We use the standard representation of the $\gamma$-matrices.

$$
\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}, \quad \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
$$

(11)

with $\sigma^i$ being the usual Pauli matrices.

The light-cone spinors $u_\lambda(\bar{\ell})$ and the antispinors $v_\lambda(\bar{\ell})$, with velocity $\bar{\ell}$ and helicity $\lambda$, $\bar{(\ell, 1)}$, are defined by

$$
u_\lambda(\bar{\ell}) = \frac{1}{\sqrt{2v^+}}(v^+ + \gamma \cdot \vec{v})\chi, \quad (A2)$$

and

$$
u_\lambda(\bar{\ell}) = \frac{1}{\sqrt{2v^+}}(v^+ + \gamma \cdot \vec{v})\beta\chi, \quad (A3)$$

where $\beta = \gamma^0$ and $\alpha = \beta \gamma^i$ and

$$
\chi^+ = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \chi_\perp = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}
$$

(44)

It is straightforward to show that

$$
u_\lambda(\bar{\ell}) = \frac{1}{2\sqrt{v^+}} \begin{bmatrix} v^+ + 1 \\ v^+ - 1 \end{bmatrix}, \quad u_\lambda(\bar{\ell}) = \frac{1}{2\sqrt{v^+}} \begin{bmatrix} -v^L \\ v^+ + 1 \end{bmatrix}
$$

(45)

Similarly, the antispinors $v_\lambda(\bar{\ell})$ are given by

$$
u_\lambda(\bar{\ell}) = \frac{1}{2\sqrt{v^+}} \begin{bmatrix} v^+ - 1 \\ v^+ + 1 \end{bmatrix}, \quad v_\lambda(\bar{\ell}) = \frac{1}{2\sqrt{v^+}} \begin{bmatrix} -v^L \\ v^+ - 1 \end{bmatrix}
$$

(46)

where $v^L = v^1 \pm iv^2$.

The spinors are eigenstates of $\mathfrak{f}$, i.e. $\mathfrak{f}_\lambda u_\lambda = u_\lambda$ and they are normalized such that

$$u_\lambda u_{\lambda'} = -u_\lambda v_{\lambda'} = \delta_{\lambda\lambda'}.
$$

(47)

The matrix elements $\delta_{\lambda\lambda'} \frac{\bar{\lambda}(\bar{\ell}) \gamma_\lambda(\bar{\ell})}{\bar{\lambda}(\bar{\ell}) \gamma_\lambda(\bar{\ell})}$ for various choices of $\gamma$ are listed in Table 2. Table 3 represents some other important bilinears which have been used in Sec. III.

REFERENCES


Table Captions

Table 1 Partial helicity and total rates (in units of $10^{13} s^{-1}$) for $\Lambda_b \rightarrow \Lambda c \ell \nu$. Mass values used are: $M_{\Lambda_b} = 5.60$ GeV, $M_{\Lambda c} = 2.28$ GeV, $m_{\ell} = 4.73$ GeV, $m_c = 1.55$ GeV. We take $V_{c\ell} = 0.050$ and $\Delta m_{\ell\nu} = 0.11$.

Table 2 Matrix elements $\frac{\langle p_\ell | T_\mu | 0 \rangle}{\sqrt{\nu}}$.

Table 3 Light cone bilinears used in Sec. III. Here $s_i = x_i + \frac{m_i^2}{s}$ with $i = 1, 2$ and $3$ and $m_1 = m_2 = m$ is the light quark mass. $\vec{p}_{i\perp}$ represents the $i$th quark momenta transverse to the direction of the hadron.


S. Weinberg, Phys. Rev. 133B (1964) 1318; 139B (1965) 597;
F. Hussain, Ph.D. Thesis, Imperial College, London (1966);


### Table 1

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\Gamma_L$</th>
<th>$\Gamma_T^+$</th>
<th>$\Gamma_T^-$</th>
<th>$\Gamma_{total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.10</td>
<td>5.52</td>
<td>0.62</td>
<td>2.72</td>
<td>8.86</td>
</tr>
<tr>
<td>1.37</td>
<td>4.16</td>
<td>0.53</td>
<td>2.22</td>
<td>6.91</td>
</tr>
<tr>
<td>1.64</td>
<td>2.82</td>
<td>0.43</td>
<td>1.68</td>
<td>4.93</td>
</tr>
<tr>
<td>dipole</td>
<td>4.2</td>
<td>0.70</td>
<td>2.0</td>
<td>6.90</td>
</tr>
<tr>
<td>FQD</td>
<td>6.2</td>
<td>0.59</td>
<td>2.49</td>
<td>9.28</td>
</tr>
</tbody>
</table>

### Table 2

| $\lambda_{--\lambda}$ | $|\lambda_{--\lambda}|^{-1}$ | $|\lambda_{--\lambda}|^{-1}$ | $\lambda_{--\lambda}$ |
|------------------------|-------------------------------|-------------------------------|------------------------|
| $j_n$                  | $\frac{1}{2} s^+ \frac{1}{2} s^+ $ | $\frac{1}{2} s^+ \frac{1}{2} s^-$ | $\frac{1}{2} s^+ \frac{1}{2} s^+$ |
| $\gamma^+_{--\lambda}$ | $1$                            | $\pm 1$                       | $0$                     |
| $\gamma^-_{--\lambda}$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^-$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ |
| $\gamma^+_{--\lambda}$ | $\frac{1}{2} s^+ \frac{1}{2} s^-$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^-$ |
| $\gamma^+_{--\lambda}$ | $\frac{1}{2} s^+ \frac{1}{2} s^+$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^-$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ |
| $\gamma^+_{--\lambda}$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^-$ | $\pm \frac{1}{2} s^+ \frac{1}{2} s^+$ |

### Table 3

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\sqrt{\frac{1}{n^2} \left[ \gamma^+<em>{--\lambda} / \gamma^-</em>{--\lambda} \right]}$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\uparrow \uparrow$</td>
<td>$\frac{\lambda_1 \lambda_2}{\lambda_3 \lambda_4}$</td>
<td>$\frac{1}{2} \alpha_3$</td>
<td>$\uparrow \uparrow$</td>
<td>$\frac{1}{2} \alpha_3$</td>
</tr>
<tr>
<td>$\uparrow \downarrow$</td>
<td>$\lambda_1 \lambda_2$</td>
<td>$\frac{1}{2} \alpha_3$</td>
<td>$\uparrow \downarrow$</td>
<td>$\frac{1}{2} \alpha_3$</td>
</tr>
<tr>
<td>$\downarrow \uparrow$</td>
<td>$\lambda_1 \lambda_2 - \frac{1}{2} \lambda_3 \lambda_4$</td>
<td>$\downarrow \uparrow$</td>
<td>$\downarrow \uparrow$</td>
<td>$\frac{1}{2} \alpha_3$</td>
</tr>
<tr>
<td>$\downarrow \downarrow$</td>
<td>$\lambda_1 \lambda_2 - \frac{1}{2} \lambda_3 \lambda_4$</td>
<td>$\downarrow \downarrow$</td>
<td>$\downarrow \downarrow$</td>
<td>$\frac{1}{2} \alpha_3$</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1 Diagrams for matrix elements a) only for good currents $J^+$; b) additional terms contributing to $J^+$, $\mu \neq +$.

Fig. 2 The single zeroth order form factor $\xi$ and the form factors $f_1$ and $f_2$ with the leading corrections for $\Lambda_b \to \Lambda_c$ weak s.l. decays. The value of $\rho$ is taken as $1.37$. For comparison the dipole form factor is also plotted.

Fig. 3 Form factors $f_1$ and $f_2$ with different values of $\rho$.

Fig. 4 Differential $q^2$-rates for the process $\Lambda_b \to \Lambda_c \ell \nu_\ell$ with $\rho = 1.37$. Also shown are partial helicity rates.

Fig. 5 Differential $E_\ell$-spectra for $\Lambda_b \to \Lambda_c \ell \nu_\ell$. Also shown are partial helicity rates.
Fig. 4

Fig. 5