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HYDRODYNAMIC FLUCTUATIONS AND LIGHT SCATTERING IN HOT ELECTRON GAS OF SEMICONDUCTORS

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Hydrodynamical fluctuations of the electron gas are the low-frequency and long-range stochastic excitations over steady state of the system. These fluctuations are responsible for the set of the physical phenomena which occur for both equilibrium and nonequilibrium conditions (for example, the current noises, the light scattering, etc.). We investigate the hot electron plasma that can be characterized by two time parameters – the electric charge decay time $\tau_W$ and the electron energy relaxation time $\tau_U$. The two spatial lengths – screening length $l_D$ and the energy relaxation length $l_T$ – correspond to these times and are important for the space-nonuniform effects. The hydrodynamical fluctuations of such a plasma were the subject of the numerous studies. However, the most advanced results have been formulated in two following cases: $\tau_u < \tau_w$, $\omega \tau < 1$, $q \tau < 1$, $\tau \omega < 1$, $q \tau < 1$, where $\omega$ and $q$ are frequency and wave vector of the hydrodynamical fluctuations.

In this work we develop the theory of the hydrodynamical fluctuations of the hot electron plasma for more general case when it is possible to drop the above mentioned limitations. Our consideration is based on the Boltzmann-Langevin kinetic equation for the fluctuations of the electron distribution function $\delta F$. The solution of this equation is found under typical criteria for the hot electron plasma: $\tau < \tau_w$, where $\tau_w$ is the time of relaxation of the momentum and energy of the electrons, $\tau_u$ is the electron-electron scattering time. It is shown that the fluctuation $\delta F$ can be expressed via the two fluctuating parameters: $\delta n(\omega, q)$, $\delta T(\omega, q)$ and via the initial steady state distribution function. The fluctuating parameters $\delta n$ and $\delta T$ mean the fluctuations of the magnitudes of the electron density and temperature. For them the hydrodynamic equations are deduced and as a result, the following correlation functions are calculated: $< \delta n \delta n >_{\omega, q}$, $< \delta T \delta T >_{\omega, q}$, $< \delta n \delta T >_{\omega, q}$. The analysis of these correlators shows the next features of the general results which are not restricted by above criteria: i. There are cross-over correlations of $\delta n$ and $\delta T$, that mean the mutual influence of the electron density fluctuations and their temperature. ii. The time and space dependences of the fluctuations strongly differ from that under the above mentioned limits. In particular, the frequency dependences of the correlation functions no longer are the Lorentz-like forms. iii. It is important that these features are characteristic, as well, for the equilibrium electron plasma under arbitrary relation between $\tau_w$ and $\tau_U$.

The above results are applied to the calculation of the light scattering by the electron plasma fluctuations. It is shown that the cross-correlation effect gives the essential contribution into the cross-section of the light scattering.
1. INTRODUCTION

Hydrodynamical fluctuations are the long-range and low-frequency stochastic excitations of the physical system with respect to stationary state of this system. It is assumed that for typical wave vector \( q \) and frequency \( \omega \) of fluctuations the following conditions are satisfied

\[
\omega T \ll 1, \quad qL \ll 1
\]

(1.1)

where \( L \) and \( \tau \) are microscopic characteristic space and time parameters of system.

In this case the physical system behaves as a continuous medium and can be described by a system of macroscopic equations \[1\]. Nevertheless that does not mean that description of fluctuations by such system of macroscopic equations is complete. The sources of all fluctuations are determined by microscopic random events, i.e. collisions of the particles in the system with each other and with thermal bath. The evolution of these fluctuative excitations has two stages [2].

At the first stage the fast relaxation of initial excitation to local stationary distribution takes place. This stage of relaxation occurs as a relaxation in momentum space and is controlled by Boltzmann-Langevin type equation \[3\]. The local stationarity is established after microscopic random events, i.e. collisions of the particles in momentum space. At times greater than \( \tau \), this distribution takes its usual form in momentum space. In general case this distribution depends not only on momenta of particles but also on some macroscopic parameters of physical system. These macroscopic parameters are the functions of coordinates. As far as the random flows in momentum space at the first stage of relaxation process lead to local flows in coordinate space giving rise to the dependence of the macroscopic parameters of system on the coordinates.

At the second stage of evolution the further relaxation of initial fluctuative excitation to final uniform steady state is ruled by equations of hydrodynamical type for macroscopic parameters. This process specifies the evolution of hydrodynamical fluctuations including space transfer of electron density, energy and so on.

As it follows from above, in spite of the possibility to divide the fluctuative process to the two stages – fast (kinetical) relaxation and more slow (hydrodynamical) one, these two stages are in fact very closely related. The first stage, above all, forms initial conditions for the second stage of the space-time evolution of the fluctuations and, secondly, the behaviour of the fluctuation during first stage is ruled by the Boltzmann-Langevin equation. In consequence, from the same equation the system of stochastic equations of hydrodynamical type for second stage arises. The second stage determines the particular form of space-time dependences of macroscopic parameters of fluctuations for physical system. These macroscopic parameters play a role of integrals of motion in the expressions which describe the kinetical stage of fluctuative process.

As a result, the detailed description of hydrodynamical fluctuations in general case requires also the consideration of kinetical stage of evolution.

In present work the hydrodynamical fluctuations of semiconductor nonequilibrium electron gas will be investigated.

We suppose that the state of electron gas for the stationary conditions is specified by electron temperature \( T \) that differs from the temperature \( T_0 \) of the thermal bath. The introduction of the electron temperature requires the validity of following conditions:

\[
\tau_T \ll \tau_e \ll \tau_i
\]

(1.2)

Here \( \tau_T \) and \( \tau_e \) are respectively the momentum \( p \) and energy \( E \) relaxation times for electrons interacting with thermostat, \( \tau_i \) is the electron-electron scattering time.

Electron fluctuations in such system were studied previously in several publications, here we shall mention references \[3\], \[4\] and \[5\], \[6\].

In the first two works the equations of kinetic type were derived for the two-particle correlation function of fluctuations of the distribution functions. Both the electron-phonon scattering and the electron-electron interaction were taken into account. The formalism developed in \[3\], \[4\] can be also called as a method of correlation moments. The existence of the intensive interparticle collisions leads to arising of the additional correlation for fluctuation. This extra correlation leads to violation of the well-known fluctuation-diffusion Price's relation that determines the proportional relationship between diffusion coefficient and spectral density of space-independent current fluctuations. The effect of the violation of diffusion-noise relation takes place at nonequilibrium steady state of physical system only. Then in \[3\], \[4\] it was found the equations of fluctuation hydrodynamics starting from the kinetic equations for fluctuations. Low-frequency and long-range fluctuations near a nonequilibrium uniform steady state were investigated as well. In the situation considered in \[3\], \[4\] it was only one macroscopic characteristic that ultimately determined the all hydrodynamical fluctuations completely. Such characteristic was fluctuation of electron density \( \delta n(\vec{r}, t) \) which depends on the coordinate of point \( \vec{r} \) and on the time \( t \). Correspondingly, the electron density fluctuations were described in \[3\], \[4\] by only one equation – the stochastic continuity equation.

But if conditions (1.2) are valid, the steady state electron distribution in semiconductor is specified not only by electron density but also by additional macroscopic parameter, that is electron temperature. Thus, in general case, electron gas possesses two fluctuational degrees of freedom and hydrodynamical fluctuations in this case are described by the system of two stochastic equations. Though, the situation is possible where the intensity of electron density fluctuations is much greater than that of electron temperature fluctuations. In this case we can restrict ourselves by results of works \[3\], \[4\] for considering of hydrodynamical fluctuations.

The dynamics of electron gas under consideration is specified by two time scales related to two macroscopic parameters \( n(\vec{r}, t) \) and \( 1/T(\vec{r}, t) \). These scales are: the Maxwell’s relaxation time \( \tau_M \) and time of relaxation of electron temperature \( \tau_T \). Corresponding with the time scales
there are also two characteristic space scales: the screening length \( L_S = \sqrt{D_{TS}} \) and the cooling length of electrons \( L_T = \sqrt{D_{TU}} \), where \( D(T) \) is the diffusion coefficient.

Then if the following conditions

\[
\tau_T < \tau_m, \quad \omega L_T < 1, \quad q^2 L_T^2 < 1 \tag{1.3}
\]

are satisfied the electron density fluctuations are of the only importance. But this does not mean that it is possible to neglect the electron temperature fluctuations completely. As it will be shown further if the inequality (1.3) takes place the electron temperature fluctuations lead to renormalization of kinetic coefficients for hydrodynamical electron density mode under non-equilibrium.

In the works [5],[6] the theory of kinetic and hydrodynamical fluctuations on the base of Langevin's formalism was developed. This method directly deals with the distribution function fluctuations around a nonequilibrium steady state and is based on the introduction of random forces to the linearized Boltzmann kinetic equation for the occupancy of particle state. The correlation function of occupation numbers of one-particle states ultimately is expressed in terms of correlators of the random forces. These correlators were calculated in [5],[6] both for case of the electron interacting with thermal bath and for interaction between electrons. On the base of these correlators it is possible to calculate the spectral density of fluctuations for arbitrary physical quantities if their mean values can be found from stationary electron distribution.

The results of calculation of fluctuation spectra within the method of correlation moments [3],[4] and the Langevin method [5],[6] coincide. That indicates the equivalency of these methods [7].

On the base of electron temperature approximation the system of stochastic transport equations was derived for general case [7] where the low-frequency and long-range fluctuations are presented as a fluctuation of electron density and electron temperature. However, the investigation of hydrodynamics of fluctuations for such general situation has not been carried out in the previous works.

Similar to the case studied in [3],[4] there are conditions where electron temperature fluctuations are important while electron density fluctuations contribute only to renormalization of kinetic coefficients. It requires the following inequalities

\[
\tau_T < \tau_T^*, \quad \omega L_T < 1, \quad q^2 L_T^2 < 1 \tag{1.4}
\]

to be satisfied. For the case of (1.4) within the approach of [5],[6] the intensity of electron temperature fluctuations integrated over \( \omega \) has been obtained in (8). This value was employed in calculate of the differential cross-section for quasielastic scattering of unpolarized radiation by electron plasmas.

One should note that there is a wide range of experimental situations where conditions (1.3) or (1.4) are too hard to be satisfied. Besides the relaxation parameters \( \tau_m \) and \( \tau_T \) can have different dependences on the external electric field. For example, if the scattering of electrons occurs by the acoustical phonons then the Maxwellian time \( \tau_T^* \) increases and the cooling time \( \tau_T \) decreases when the electric field grows. Such different behaviour of \( \tau_m \) and \( \tau_T \) can lead to change of sign of the first inequalities from (1.3) and (1.4). If the parameters of semiconductor \( \tau_m \) and \( \tau_T \) have the same order of magnitude then the results of works [3],[4] and [7] are completely not applicable for the investigation of hydrodynamical fluctuations even for the equilibrium steady state. Moreover, as it will be shown further there are special reasons for the investigation of hydrodynamical fluctuations in such conditions when the inequalities (1.3) and (1.4) do not hold. In particular, as it was indicated above, in the case with intensive interparticle interaction the fluctuation-diffusion Price's relation becomes invalid. The value of such violation is determined by so-called correlation tensor [3]. This correlation tensor gives essential contribution to the fluctuation of physical quantities just in the discussed intermediate case.

Among the new peculiarities of fluctuation hydrodynamics in such intermediate case we also mention the effects of mutual correlation of electron density and electron temperature fluctuations. Investigation of these effects gives additional information about microscopic processes in physical systems especially in nonequilibrium steady state.

The treatment of fluctuations of electron gas in semiconductors will be carried out in this work for the general case where the validity of inequalities (1.3) and (1.4) is not required. The theory of electromagnetic waves scattered by the low-frequency and long-range fluctuations of electron gas of semiconductor is developed. The expressions for differential cross-section of the scattering of light will be obtained for arbitrary relation between space-time characteristics \( q \) and \( \omega \) of fluctuation and parameters \( \tau_m, \tau_T, L_m, L_T \) of semiconductor. As it will be shown the investigation of light scattering in such conditions extends the potentialities of fluctuations spectroscopy of semiconductors and their nonequilibrium steady state.

In our next work the problem of one-time and long range fluctuations will be solved for general case with the main purpose to derive the correlation functions for the fluctuations of occupation numbers of one-particle states in the momentum and coordinate space at the same moment. It is necessary to mark the following important point: The same correlation functions cannot be obtained in general case by integrating of corresponding low-frequency and long-range correlators over frequency \( \omega \). The reason is the divergence of such integrals for lower and upper limits [3]. Then the problem of one-time and long-range fluctuations of occupation numbers cannot be solved within Langevin's procedure for hydrodynamical limit [5],[6]. Whereas the developed in [4] formalism of correlational moments gives a possibility to deal with equation for one-time two-particle correlator straight away. It is the solution of this equation for long-range limit which gives functions of correlation.

In earlier works the problem of simultaneous space-dependent fluctuations was investigated only for case (1.3) in [3],[4].
The Boltzmann-Langevin equation for fluctuation $\delta F_\rho(\vec{r}, t)$ is

$$\dot{\delta F}_\rho(\vec{r}, t) = \sum_{\rho'} \left[ \frac{\partial}{\partial \vec{r}} + \vec{v} \frac{\partial}{\partial \rho} + \hat{L}_{\rho} \right] \delta F_{\rho'}(\vec{r}, t) + e \delta E(\vec{r}, t) \frac{\partial}{\partial \rho} \delta F_\rho(\vec{r}, t) = \delta y_\rho(\vec{r}, t), \quad (2.3)$$

where $y_\rho(\vec{r}, t)$ is the stochastic microscopic force with the known correlation property [5], $\vec{v}$ is the electron velocity, $\hat{L}_{\rho}$ is the linearized operator of the Boltzmann equation:

$$\hat{L}_{\rho} = e \vec{E}_0 \frac{\partial}{\partial \rho} + \int_{\rho'} \hat{I}_{\rho'}^{le}(\hat{P}_\rho)\, d\rho'.$$

Here $\vec{E}_0$ is external d.c. electric field, $\hat{I}_{\rho'}^{le}(\hat{P}_\rho)$ is the integral operator describing the statistical interaction with thermal bath, $\hat{I}_{\rho'}^{le}(\hat{P}_\rho)$ is the linearized electron-electron collision operator and $\delta \vec{E}(\vec{r}, t)$ is the self-consistent electric field arising from the space charge caused by the spatial redistribution of electrons. This electric field is determined by Poisson equation

$$\text{div} \delta \vec{E}(\vec{r}, t) = \frac{4 \pi e}{\varepsilon_0} \sum \delta F_\rho(\vec{r}, t), \quad (2.5)$$

where $\varepsilon$ is the lattice dielectric constant, $e$ is charge of electron.

It is convenient to use the Langevin transforms for equations (2.3) and (2.5). As a result we have

$$\dot{\delta F}_\rho(\vec{q}, \omega) \equiv i \omega \delta F_\rho(\vec{q}, \omega) = \{ -i \omega + i \vec{v} \vec{q} + \hat{L}_{\rho} \} \delta F_\rho(\vec{q}, \omega) - i q U(\vec{q}) \frac{\partial}{\partial \rho} \delta F_\rho(\vec{q}, \omega) \times \sum \delta F_{\rho'}(\vec{q}, \omega) = \gamma_\rho(q, \omega), \quad (2.6)$$

where

$$\delta \dot{E}(\vec{q}, \omega) = - \frac{1}{\varepsilon} \gamma U(q) \sum \delta F_{\rho}(\vec{q}, \omega), \quad (2.7)$$

$$U(q) = \frac{4 \pi e^2}{\varepsilon_0 \varepsilon q^2}. \quad (2.8)$$

We shall look for the solution of the equation (2.6) in the low-frequency and long-range region limit (1.1) using the Chapman-Enskog method [9]. This approach was used in [3] but only with special modification under limitations (1.3). Such modification does not permit to apply the procedure from [3] in our case directly. It could be used the solution that was taken in [6]. Further for calculation of one-time two-particle correlator in general case it will be necessary to apply the results of this section which we are going to obtain by the Chapman-Enskog procedure.

For the electron temperature formalism the nonequilibrium steady state distribution functions is

$$\tilde{F}_\rho = \tilde{F}_0(\rho_0) + \tilde{F}_1(\rho_1), \quad (2.9)$$

$$\tilde{F}_0(\rho_0) = - \frac{\rho_0}{N_e(T)} \exp \left( - \frac{\epsilon_\rho}{T} \right), \quad \tilde{F}_1(\rho_1) = - \epsilon_0 \tau_\rho \frac{\partial}{\partial \rho} \delta F_0, \quad (2.10)$$

Here $\rho_0$ is the concentration of electrons, $\tau_\rho$ is the momentum relaxation time, $N_e(T)$ is effective density of electron states

$$N_e(T) = \int_0^\infty g(\epsilon) \exp \left( - \frac{\epsilon}{T} \right) d\epsilon = 2 \left( \frac{\text{m}T}{2 \pi \hbar^2} \right)^{1/2}, \quad g(\epsilon) = \sqrt{2m} \frac{\epsilon^{3/2}}{\pi^2 \hbar^3 e^{1/2}}, \quad (2.11)$$

$m$ is effective mass of electron.

As it follows from (2.9) and (2.10) the function $\tilde{F}_0$ depends on two parameters: $\rho_0$ and $T$. These parameters are external in respect to the operator of kinetic equation (nonlinearized). That means they are not included to this operator directly but ones are determined from boundary
conditions for distribution function $F_F$. Other words the parameters $\tau_0$ and $T$ are integrals of motion for kinetic operator. The boundary conditions for their determination are the normalization condition and the macroscopic equation for energy balance. In this case linearized operator $\hat{L}_p$ (2.4) has two eigenfunctions with zero eigenvalue:

$$\hat{L}_p \frac{\partial F_F}{\partial \tau_0} = 0, \quad \hat{L}_p \frac{\partial F_F}{\partial T} = 0. \tag{2.12}$$

That can be simply verified by the using of explicit form for operator $\hat{L}_p$ from (2.4) and expression for two-particle collision integral

$$\hat{L}_p \{ F,F \} = \sum_{p,q} \left[ W_{pq} \frac{\partial F_{pq}}{\partial \tau_0} - W_{qp} \frac{\partial F_{pq}}{\partial \tau_0} \right]. \tag{2.13}$$

Here $W_{pq} / \tau_0$ is the transition probability for electron-electron collision which results the transfer of two electrons from occupied states $p$ and $q$ to the empty states $\tilde{p}$ and $\tilde{q}$.

If the operator $\hat{L}_p$ has "zeros" (2.12), it means that the result its action on arbitrary function $\psi_F$ remains invariable respect to following transformation

$$\hat{L}_p \psi_F = \hat{L}_p \left[ \psi_F + C_1 \frac{\partial F_F}{\partial \tau_0} + C_2 \frac{\partial F_F}{\partial T} \right], \tag{2.14}$$

where $C_1$ and $C_2$ are arbitrary constants.

Taking into account the property (2.14) for operator $\hat{L}_p$, we can rewrite the equation (2.6) in the form that is more convenient for iterative procedure

$$\hat{L}_p \left[ \delta F_{p} (\tilde{q}, \omega) - \frac{\delta n (\tilde{q}, \omega)}{\tau_0} F_{p} - \delta T (\tilde{q}, \omega) \frac{\partial F_{p}}{\partial \tau} \right] =$$

$$\{ i \omega - i q \tilde{q} \} \delta F_{p} (\tilde{q}, \omega) + i q U (\tilde{q}) \frac{\partial F_{p}}{\partial \tau} \sum_{p} \delta F_{p} (\tilde{q}, \omega) + \psi_F (\tilde{q}, \omega). \tag{2.15}$$

Here we introduce a new parameters $\delta n (\tilde{q}, \omega)$ and $\delta T (\tilde{q}, \omega)$. As it will be clearly further these parameters determine the fluctuations of electron density and electron temperature. In (2.15) we also utilize the equality $\sum_{p} \delta F_{p} (\tilde{q}, \omega) = \sum_{p} \delta F_{p} (\tilde{q}, \omega)$ for distribution function (2.9)-(2.10).

Let us present the function $\delta F_{p}$ as a power series in parameters (1.1)

$$\delta F_{p} (\tilde{q}, \omega) = \delta F^{(0)}_{p} (\tilde{q}, \omega) + \delta F^{(1)}_{p} (\tilde{q}, \omega) + \ldots \tag{2.16}$$

Because of the order of magnitude of the operator $\hat{L}_p$ is given by quantity $1 / \tau_p$ and $L \approx \Omega \tau_p$ ($\Omega$ is averaged electron velocity) the left-hand of equation (2.15) is a main part of one and the first iteration gives

$$\delta F^{(0)}_{p} (\tilde{q}, \omega) = \frac{\delta n (\tilde{q}, \omega)}{\tau_0} F_{p} + \delta T (\tilde{q}, \omega) \frac{\partial F_{p}}{\partial \tau}. \tag{2.17}$$

The function $\delta F^{(0)}_{p} (\tilde{q}, \omega)$ contains both spherically-symmetric and antisymmetric parts.

Substituting (2.16) and (2.17) to equation (2.15) we obtain the equation for calculation $\delta F^{(1)}_{p} (\tilde{q}, \omega)$:

$$\hat{L}_p \delta F^{(1)}_{p} (\tilde{q}, \omega) = Z_p \{ \delta F^{(0)}_{p} (\tilde{q}, \omega) \}, \tag{2.18}$$

where

$$Z_p \{ \delta F^{(0)}_{p} (\tilde{q}, \omega) \} = \{ i \omega - \tilde{q} \tilde{q} \} \delta F^{(0)}_{p} (\tilde{q}, \omega) + i q U (\tilde{q}) \frac{\partial F_{p}}{\partial \tau} \sum_{p} \delta F_{p} (\tilde{q}, \omega) + \psi_F (\tilde{q}, \omega). \tag{2.19}$$

Since the property (2.14) for operator $\hat{L}_p$ was taken into account on the first step of iteration procedure then, firstly, the function $\delta F^{(1)}_{p} (\tilde{q}, \omega)$ (and all following terms of a series (2.16)) no longer contains of "zeros" of operator $\hat{L}_p$ and, secondly, the function $\delta F^{(0)}_{p} (\tilde{q}, \omega)$ (2.17) completely defines spherically-symmetric part of the solution of initial equation (2.15). The last assertion follows from the fact that the final term in operator $\hat{L}_p$ (2.4) is the main term for each step of iteration procedure of solution of equation for spherically-symmetric part of function. In the approximation (1.2) this term gives, as a result, the function in the form

$$F_{p} = a \cdot F_0 (\epsilon_p) + b \frac{\partial F_0 (\epsilon_p)}{\partial \tau} \tag{2.20}$$

But similar functions was been taken into consideration in (2.17).

Thus from equation (2.18) it is necessary to determine only antisymmetric function. Taking into account conditions (1.2) we obtain

$$\delta F^{(1)}_{p} (\tilde{q}, \omega) = \tau_p \cdot Z_p \{ \delta F^{(0)}_{p} (\tilde{q}, \omega) \}. \tag{2.21}$$

In the right-hand of expression (2.21) it is necessary to discard the term that contains frequency $\omega$ in comparison with other that contains wave vector $\tilde{q}$. For these two terms we have from (2.19) and (2.17)

$$i \{ (i \omega - \tilde{q} \tilde{q} \} \delta F^{(0)}_{p} (\tilde{q}, \omega) \} \sim \{ i \omega F^{(1)}_{p} (\epsilon_p) - \tilde{q} \tilde{q} F_0 (\epsilon_p) \} \sim \{ i \omega F^{(1)}_{p} - \tilde{q} \tilde{q} F_0 \} \frac{\partial F_0 (\epsilon_p)}{\partial \tau} \tag{2.21}$$

taking (2.9) and (2.10) into account. Since inequality $\delta F^{(1)}_{p} \ll 1$ takes place it is possible to neglect term with $\omega$. As a result the solution of equation (2.18) can be obtained in the following form

$$\delta F^{(1)}_{p} (\tilde{q}, \omega) = -i q U (\tilde{q}) \frac{\partial F_0 (\epsilon_p)}{\tau_0} - \tau_p \frac{\partial F_0 (\epsilon_p)}{\partial \tau} +$$

$$+ i q U (\tilde{q}) \frac{\partial F_0 (\epsilon_p)}{\tau_0} \tau_p \frac{\partial F_0 (\epsilon_p)}{\partial \tau} + \tau_p \frac{\partial F_0 (\epsilon_p)}{\partial \tau}, \tag{2.22}$$

taking into account (2.17) and (2.21).
The expressions (2.16), (2.17) and (2.22) define the solution of initial equation (2.6) in the low-frequency and long-range limit. The parameters $\delta n(q,\omega)$ and $\delta T(q,\omega)$ which appear into this solution can be obtained from conditions of solvable of equation (2.6). These conditions are the continuity equation and energy transfer equation for the fluctuations:

$$\sum_{\ell} \delta g(q,\omega) \delta F(q,\omega) = 0,$$

$$\sum_{\ell} \varepsilon_{\ell} q \delta g(q,\omega) \delta F(q,\omega) = \sum_{\ell} \varepsilon_{\ell} q \delta g(q,\omega).$$

Here we used the following property of the random force:

$$\sum_{\ell} g(q,\omega) = 0,$$

that corresponds with the conservation of particle number during collisions. Substituting explicit form of operator $\hat{\delta} q(q,\omega)$ from (2.23) to (2.24) we obtain following equations:

$$-i\omega \delta n(q,\omega) + i\omega \delta T(q,\omega) = 0,$$

$$-i\omega \delta \delta (q,\omega) + i\omega \delta T(q,\omega) = e B_0 \delta T(q,\omega) - e \eta_0 \delta \delta (q,\omega) + \delta P(q,\omega) = \hat{\delta} T(q,\omega).$$

Here $\delta \delta (q,\omega)$ and $\delta T(q,\omega)$ are fluctuations of densities of particle flow and energy flow respectively, $\hat{\delta} T(q,\omega)$ is fluctuation of electron energy density, $\delta P(q,\omega)$ is fluctuation of power transmitted from electrons to lattice, $\hat{\delta} T(q,\omega)$ is the Langevin's source of energy fluctuations.

By using expressions (2.16), (2.17) and (2.22) one can obtain for these quantities:

$$\delta \delta (q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega) = W' \cdot \delta n(q,\omega) + \eta_0 \frac{\partial W'}{\partial T} \delta T(q,\omega) + \hat{\delta} T(q,\omega),$$

$$\delta T(q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega),$$

$$\frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta \delta \delta (q,\omega) = \frac{\partial W'}{\partial T} \delta n(q,\omega) + \eta_0 \frac{\partial W'}{\partial T} \delta T(q,\omega) + \hat{\delta} T(q,\omega).$$

Here we introduced the following designation for “complex velocities” of excitation

$$W' = \frac{1}{N} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega),$$

$$W_1 = \delta \delta \delta (q,\omega).$$

and for diffusion coefficient

$$D_0 = \frac{1}{N} \sum_{\ell} \frac{1}{3} \varepsilon_{\ell} q \delta F(q,\omega).$$

and $N = n_0 V_0$. Coefficient $B_T$ is equal

$$B_T = \frac{1}{N} \sum_{\ell} \left( \varepsilon_{\ell} q \delta F(q,\omega) = T^{1/2} \frac{\partial}{\partial T} [D_0 T^{1/2}] \right).$$

For the fluctuation $\delta \delta (q,\omega)$ and $\delta T(q,\omega)$ we have

$$\delta \delta (q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega) = \frac{3}{2} \left( T \delta n(q,\omega) + \eta_0 \delta T(q,\omega) \right),$$

$$\delta P(q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega) = P_0(T) \delta n(q,\omega) + \eta_0 \frac{\partial P_0(T)}{\partial T} \delta T(q,\omega),$$

where

$$P_0(T) = \frac{1}{N} \sum_{\ell} \frac{1}{3} \varepsilon_{\ell} q \delta F(q,\omega).$$

is the power transmitted by one electron to the thermal bath in steady state.

The introduced above Langevin's sources of energy fluctuations $\delta \delta (q,\omega)$, particle flow density $\delta n(q,\omega)$ and energy flow density $\delta T(q,\omega)$ are

$$\delta \delta (q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta F(q,\omega),$$

$$\delta n(q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta \delta \delta (q,\omega),$$

$$\delta T(q,\omega) = \frac{1}{V_0} \sum_{\ell} \varepsilon_{\ell} q \delta P(q,\omega).$$

The spectral densities of fluctuations of hydrodynamical electron gas parameters $\delta n(q,\omega)$ and $\delta T(q,\omega)$ will be main of interest to us. However, there is the reason to analyze the system of equation (2.26)-(2.27) for fluctuations $\delta n(q,\omega)$ and $\delta T(q,\omega)$ in some limiting cases.

First of all, let us consider the situation when conditions (1.3) takes place. In equilibrium steady state they should be added still by following inequality

$$\gamma_{Tn} \ll 1.$$
Combining of equation (2.26) and (2.27) we obtain expression for $\delta T(\vec{q}, \omega)$

$$\delta T(\vec{q}, \omega) = \frac{2}{3} A \left\{ \left( \vec{U}(\vec{q}, \omega) + e B_0 \vec{I}(\vec{q}, \omega) \right) - \frac{1}{2} B_0 \frac{\partial T}{\partial \tau} \right\} - 2 \delta \Xi(\vec{q}, \omega) - T - \frac{3}{2} \frac{\delta n(\vec{q}, \omega)}{\tau_M}, \quad (2.42)$$

where

$$\frac{1}{\tau_M} = \frac{4 \pi e n_0 \mu_0(T)}{e} \quad (2.43)$$

is the time of Maxwell's relaxation.

The first term between figured brackets in (2.42) is a main one. This term describes the fluctuations of electron temperature for homogeneous nonequilibrium steady state. From (2.42) and inequalities (1.3) and (2.40) one can see that relative magnitude of electron temperature fluctuation $\delta T(\vec{q}, \omega)/T$ is smallest than one for electron density fluctuations $\delta n(\vec{q}, \omega)/n_0$ and dispersive part of fluctuation of $\delta T(\vec{q}, \omega)$. But in spite of that $\delta T(\vec{q}, \omega)$ is small it cannot be neglected. Substitution of expression (2.42) to (2.28) and (2.29) leads to expressions for fluctuation of flows with renormalized kinetical coefficients $\tilde{D}(T)$, $\tilde{\mu}_0(T)$ and Langevin's sources as well. For example, for $\tilde{D}(T)$ we obtain

$$\tilde{D}_{ab}(T) = D_0(T) \left[ 1 + \frac{2}{3} \gamma e E_0 \frac{\partial \mu_0}{\partial T} \left( \frac{B_0}{D_0 T} - \frac{1}{2} \right) \delta_{ab} \right]. \quad (2.44)$$

Using (2.42) from the continuity equation (2.26) we obtain the expression for fluctuations of electron density

$$\delta n(\vec{q}, \omega) = - \frac{1}{\tau_M} \frac{\delta U(\vec{q}, \omega)}{\tau_M} + \frac{2}{3} \gamma e E_0 \left[ \tilde{U}(\vec{q}, \omega) + e B_0 \tilde{I}(\vec{q}, \omega) \right] \frac{\partial \mu_0}{\partial T}, \quad (2.46)$$

where the following designations were introduced

$$\delta U(\vec{q}, \omega) = \tilde{U}(\vec{q}, \omega) + \frac{2}{3} \gamma e E_0 \left[ \tilde{U}(\vec{q}, \omega) + e B_0 \tilde{I}(\vec{q}, \omega) \right] \frac{\partial \mu_0}{\partial T}. \quad (2.47)$$

The last term of (2.42) does not give contribution to Doppler's shift of fluctuation frequency in (2.46) due to inequality $\gamma e / \tau_M \ll 1$.

The expression (2.46) has been earlier obtained in [3], [4] under the modified Chapman-Enskog procedure. This procedure has been based on the shortened transformation (2.14), when only the first equation from (2.12) has been taken into account. Thus as it follows from above the modified algorithm proposed in [3], [4] for investigation of hydrodynamical fluctuations demands of validity of conditions (1.3) and (2.40), which determine the region of its application. Physically it means the neglecting by the electron heat conduction and the cross-over effects in fluctuation flows of particles and their energy.

Let us consider the opposite limit when besides the conditions (1.4) the inequality

$$\gamma e \tau_M \ll 1 \quad (2.49)$$

takes place. Then we can also simplify the task. One can neglect the fluctuation of electron density $\delta n(\vec{q}, \omega)$ in all expressions for $\delta \tilde{E}(\vec{q}, \omega)$ (2.16), (2.17) and also in balance equations (2.26) and (2.27) and also in expressions for $\delta T(\vec{q}, \omega)$, $\delta \tilde{I}(\vec{q}, \omega)$ and $\delta \tilde{T}(\vec{q}, \omega)$. But at the same time it is possible to neglect contribution of self-consistent electric field $\delta \tilde{E}(\vec{q}, \omega)$ in spite of proportionality of $\delta \tilde{E}(\vec{q}, \omega)$ and $\delta n(\vec{q}, \omega)$ (see (2.7)). The reason for that is that the contribution of self-consistent terms in balance equation (2.27) is proportional to $\delta n(\vec{q}, \omega)/\tau_M$ under small $\tau_M$ value. The fluctuation $\delta n(\vec{q}, \omega)$ determined from the continuity equation (2.26) is also proportional to small parameter $\tau_M$ which divide out under substitution to (2.27). From equation (2.26) it can be found the expression for $\delta n(\vec{q}, \omega)$:

$$\delta n(\vec{q}, \omega) = - \frac{1}{\tau_M} \left[ \frac{1}{2} \tau_M \left( \psi - \frac{\partial \tilde{U}(\vec{q}, \omega)}{\partial T} \right) \right] \delta T(\vec{q}, \omega). \quad (2.50)$$

The relative magnitude of $\delta n(\vec{q}, \omega)/n_0$ is less than $\delta T(\vec{q}, \omega)/\tau_M$ if one takes into account the inequalities (1.4) and (2.49). That means that, on the contrary to previous case, here the electron density fluctuations follow to electron temperature fluctuations.
and \( \kappa_0 \) is the electron heat conductivity per unit particle \([11]\):

\[
\kappa_0 = D_0 \left( \frac{3}{2} + \gamma_0 + (\gamma_0 - 1) \frac{\partial \ln \gamma_0}{\partial q^2} \right),
\]

\( \gamma_0 = \frac{\partial \ln D_0(T)}{\partial q^2} \)  

\[
(2.52)
\]

Expressions (2.46) and (2.51) completely define the hydrodynamical fluctuations for two limiting cases which are specified by inequalities (1.3), (2.40) and (1.4), (2.49). Low-frequency and long range fluctuations of other physical quantities for such approaches can be obtained by using expressions (2.16), (2.17) and (2.27) for \( \delta F^q(q,\omega) \).

In general case for arbitrary \( \omega \) and \( \omega' \) in equations (2.26)-(2.29) the contribution of cross-over effects plays important role. Consequently, the simple relation between \( \delta T(q,\omega) \) and \( \delta T(q',\omega) \) of the kind of (2.42) or (2.50) does not hold. The results do not reduce only to renormalization of kinetical coefficients in expressions for fluctuating quantities. For such case the fluctuation spectra show the new qualitative peculiarities already in equilibrium steady state.

By using the relation \([12]\) between correlation function \( \langle \delta A(q,\omega) \delta B^*(q',\omega') \rangle \) of Fourier transform of fluctuations for arbitrary quantities \( \delta A(q,\omega) \) and \( \delta B(q',\omega') \) and spectral density \( \langle \delta A \delta B \rangle \omega \) of fluctuation process:

\[
\langle \delta A(q,\omega) \delta B^*(q',\omega') \rangle = \frac{16 \pi^4}{V_0} \langle \delta \delta A \delta B \rangle \omega \delta (\omega - \omega') \delta (q - q'),
\]

\( 2.55 \)

and remind the property of arbitrary linearized operator \( \hat{T}(q,\omega) \) acting on fluctuating quantities

\[
(\hat{T}(q,\omega) \delta A(q,\omega) \delta \hat{T}^*(q',\omega')) = \hat{T}(q,\omega) \delta A(q,\omega) \delta \hat{T}^*(q',\omega') \times
\]

\[
(2.55')
\]

Then we can write the equations for spectral density of hydrodynamical fluctuations:

\[
(\delta T^2)_{\omega \omega} = \nu^2 - 2 \text{Re} \delta \delta T \omega \omega + \nu^2 [q_1 + q_2] +
\]

\[
+ 2 \text{Im} \delta \delta T \omega \omega \{q_1 + q_2 + (\delta T^2)_{\omega \omega} \} + (T^2)_{\omega \omega},
\]

\( 2.56 \)

\[
(\delta T^2)_{\omega \omega} = \nu^2 - 2 \text{Re} \delta \delta T \omega \omega + \nu^2 [q_1 + q_2] +
\]

\[
+ 2 \text{Im} \delta \delta T \omega \omega \{q_1 + q_2 + (\delta T^2)_{\omega \omega} \} + (T^2)_{\omega \omega},
\]

\( 2.57 \)

Spectral densities of Langevin’s sources on the right-hand of equations (2.56)-(2.59) can be expressed via spectral densities of initial kinetical random sources (2.37)-(2.39). The latter can be obtained using the known correlation functions of random forces \([6]\). As a result we have

\[
(T^2)_{\omega \omega} = \frac{2}{N} D_0 q^2,
\]

\( 2.65 \)

\[
(\nu^2)_{\omega \omega} = \frac{4\nu_\omega}{3N},
\]

\( 2.66 \)

\[
(\nu^2)_{\omega \omega} = \frac{4\nu_\omega}{3N},
\]

\( 2.67 \)

\[
(\nu^2)_{\omega \omega} = \frac{4\nu_\omega}{3N},
\]

\( 2.68 \)

where

\[
\nu_\omega = \nu + \frac{1}{\tau_\omega} - \frac{1}{\tau},
\]

\( 2.69 \)

\( T_0 \) is temperature of thermal bath.

First of all, let us pay attention to the spectral density of the cross-over fluctuations \( \delta \delta T \omega \omega \). It is complex quantity. Its real and imaginary part can be obtained using property of symmetry of correlation functions \([12]\). This approach gives the expressions:

\[
\text{Re} \delta \delta T \omega \omega = \frac{1}{2} \{ (\delta \delta T)^2 \omega \omega + (\delta T^2)_{\omega \omega} \},
\]

\( 2.70 \)

\[
\text{Im} \delta \delta T \omega \omega = \frac{1}{2} \{ (\delta \delta T)^2 \omega \omega - (\delta T^2)_{\omega \omega} \}.
\]

\( 2.71 \)

Solution of system of equations (2.56)-(2.59) is presented in the Appendix. Here we are investigating this solution.
3. FLUCTUATIONS IN THERMAL EQUILIBRIUM CONDITIONS

Setting $\vec{E} = 0$ and $T = T_0$ we find from (A1)-(A12) that $\tilde{D}_q = D_0$, $\sum_q(0) = 0$ since $\nu_T = \nu_i$ and $R(T) = 0$. Thus we obtain following expressions for spectral densities of fluctuations of electron density and electron temperature:

$$
(\delta n^2)_\omega = \frac{2}{N} D_0 q^2 \frac{\omega^2 + \nu_T^* \nu_T}{(\omega^2 - \nu_M^2)^2 + \omega^2 (\nu_M + \nu_T^*)^2},
$$

$$
(\delta T^2)_\omega = \frac{4}{3 N} \frac{1}{\omega^2 - \nu_M^2 - \nu_T^2},
$$

$$
Re(\delta n \delta T)_\omega = 0,
$$

$$
Im(\delta n \delta T)_\omega = 0.
$$

From (3.1)-(3.4) it follows that existence of the cross-correlation of the fluctuations $\delta n(q, \omega)$ and $\delta T(q, \omega)$ leads to non-Lorentz-like of spectra. The critical parameter for this effect is coefficient of $\gamma_0$. If the diffusion coefficient $D_0$ is independent on temperature, i.e. $\gamma_0 = 0$, the fluctuations spectra have the forms

$$
(\delta n^2)_\omega = \frac{2}{N} D_0 q^2 \frac{1}{\omega^2 + \nu_T^* \nu_T},
$$

$$
(\delta T^2)_\omega = \frac{4}{3 N} \frac{1}{\omega^2 - \nu_M^2 - \nu_T^2},
$$

$$
Im(\delta n \delta T)_\omega = 0.
$$

Thus condition $\gamma_0 = 0$ is equivalent to the absence of the cross-correlation. It is of interest to note that the spectra (3.6) and (3.7) can be obtained also from general expressions (3.1) and (3.2), limits (1.3), (2.40) and (1.4), (2.49) correspondingly.

For space-homogeneous cases ($\vec{q} \rightarrow 0$) the electron density fluctuations disappear and only spectral density of electron temperature fluctuations are different from zero:

$$
(\delta T^2)_\omega = \frac{4}{3 N} \frac{1}{\omega^2 + \nu_T^2},
$$

Another interesting peculiarity of spectra (3.1)-(3.4) follows from the rule of sum, i.e. integral over frequencies. Integrated over frequency spectrum the intensity of fluctuation for arbitrary physical quantity ($\delta A^2$)$_\omega$ is defined by expression

$$
(\delta A^2)_\omega = \frac{1}{2 \pi} \int_{-\infty}^{+\infty} (\delta A^2)_\omega d\omega,
$$

4. SPECTRAL DENSITY OF TRANSVERSE FLUCTUATIONS

Let consider the spectra of nonequilibrium hydrodynamical fluctuations with wave vector $\vec{q}$ that is perpendicular to external heating electric field $\vec{E}_0 : \vec{q} \perp \vec{E}_0$. In this case the general
expressions (A3)-(A7) get simplified essentially and we find from (A1)-(A2):

\[(\delta n^2)_{\nu} = \frac{2}{N} \nu n_0 q_0^2 \left( \frac{\omega^2 + \nu_M \nu_R}{(\omega^2 - \nu_M \nu_R)^2} + \omega^2 (\nu_M + \nu_R)^2 \right) \]  

(4.1)

\[(\delta T^2)_{\nu} = \frac{4}{3N} \left( \frac{\omega^2 - \nu_M \nu_R}{(\omega^2 - \nu_M \nu_R)^2} + \omega^2 (\nu_M + \nu_R)^2 \right) \]  

(4.2)

\[Re(\delta n \delta T)_{\nu} = \frac{4}{3N} \gamma_0 n_0 q_0^2 \left( \frac{\omega^2 - \nu_M \nu_R}{(\omega^2 - \nu_M \nu_R)^2} + \omega^2 (\nu_M + \nu_R)^2 \right) \]  

(4.3)

\[Im(\delta n \delta T)_{\nu} = \frac{4}{3N} n_0 q_0^2 \left( \frac{\nu_M}{(\omega^2 - \nu_M \nu_R)^2} + \omega^2 (\nu_M + \nu_R)^2 \right) \]  

(4.4)

As a rule for fluctuations with \( \mathbf{q} \perp \mathbf{E}_0 \) the similarity of nonequilibrium spectra and equilibrium spectra with arbitrary orientation of wave vector \( \mathbf{q} \) takes place. In our case, as it follows from (4.1)-(4.4), the nonequilibrium state of system leads to radical violation of such similarity. The reason of this change is the extra-correlation that is caused by the electron-electron interaction. In equilibrium steady state \( R(T) = 0 \) and extra correlation disappears.

Depending on sign of quantity \( R(T) \) the extra correlation can both increase (if \( R(T) < 0 \)) and decreases (if \( R(T) > 0 \)) the intensity of fluctuations. It can be shown [3] that sign of \( R(T) \) is determined by following expression:

\[ \text{sign}[R(T)] = \text{sign} \left[ \frac{\nu_M - \nu_R}{\nu \nu_M - \nu_R} \right] \]  

(4.5)

If sign of \( R(T) \) changes the imaginary part of cross-spectral density (4.4) also reverses its sign. Qualitative shape of spectral dependence of this quantity is shown on Fig. 2.

The extra correlation affects also on the integral intensity of fluctuations. From (3.10) and (4.1)-(4.4) we obtain for integral intensities:

\[(\delta n^2)_{\nu} = \frac{2}{N} \nu n_0 q_0^2 \left[ 1 - \frac{2}{3N} \frac{R(T)}{\nu \nu_M - \nu_R} \right] \]  

(5.1)

\[(\delta T^2)_{\nu} = \frac{4}{3N} \frac{R(T)}{\nu \nu_M - \nu_R} \]  

(5.2)

These expressions can be obtained also directly from (2.47) by using (2.45). From (5.1) and (5.2) it follows that for \( \mathbf{q} \perp \mathbf{E}_0 \) nonequilibrium spectra for this limit completely are similar to equilibrium spectra.

5. COMPARISON RESULTS FOR THE LIMITING CASES

The general expressions (A1)-(A7) have especially simple form for limiting case which are given by inequalities (1.3), (2.40) and (1.4), (2.49). These cases correspond to predominance of fluctuations of electron density or electron temperature, respectively. They have been investigated in [3],[4] and [5],[6]. Here we adduce only final expressions for these cases. The comparison of these limiting results with general results allows better to understand the peculiarities of intermediate situations.

For the case (1.3), (2.40) we have:

\[ (\delta n^2)_{\nu} = \frac{2}{N} \left( \frac{\omega^2 - \nu_M \nu_R}{(\omega^2 - \nu_M \nu_R)^2} + \omega^2 (\nu_M + \nu_R)^2 \right) \]  

(5.1)

\[ (\delta T^2)_{\nu} = \frac{4}{3N} \frac{R(T)}{\nu \nu_M - \nu_R} \]  

(5.2)

These expressions can be obtained also directly from (2.46) by using (2.55). From (5.1) and (5.2) it follows that for \( \mathbf{q} \perp \mathbf{E}_0 \) nonequilibrium spectra for this limit completely are similar to equilibrium spectra. These spectra have the Lorentz-like shape in opposite to the general case.
For the case (1.4), (2.49) we can obtain results from general expressions or directly from (2.51) and (2.52):

\[
(\delta^2 \sigma) = \frac{4}{3N} \left[ \frac{\mu_0}{\nu} - \frac{\nu}{\nu + \frac{x}{2}} \left( \mu_0^2 + \frac{1}{2} \left( \frac{\mu_0^2}{\nu + \frac{x}{2}} \right)^2 \right) \right] \tag{5.3}
\]

\[
(\delta^2 \sigma) = \frac{2}{3N} \left[ \frac{\mu_0}{\nu} - \frac{\nu}{\nu + \frac{x}{2}} \left( \mu_0^2 + \frac{1}{2} \left( \frac{\mu_0^2}{\nu + \frac{x}{2}} \right)^2 \right) \right] \tag{5.4}
\]

The extra correlation reveals itself in space-uniform spectrum is Lorentz-like and it is independent on the relationship between \( \nu, \nu \), and \( \omega \).

6. THE LIGHT SCATTERING BY THE HYDRODYNAMICAL FLUCTUATIONS

Developed in present work theory of low-frequency and long-range fluctuations can be applied to investigation of light scattering by nonequilibrium electron gas of semiconductors. There is the wide literature on the subject. That can be explained both fundamental and applied significances of phenomenon. The more fully discussion of the problem concerned with the light scattering has been done in well-known series of monographs [14]-[18].

Early, as far as we know, investigation of electron light scattering has not been carried out of the conditions when both the fluctuation of electron density and electron temperature are important.

The quantitative characteristics of scattering medium is differential cross-section \( \delta^2 \sigma(\omega) \) defined as ratio of scattered intensity to incident one by unit volume \( V_0 \) of illuminated material, per unit solid angle \( \Omega \) and unit frequency region \( \omega \). This value can be introduced as well, as a ratio of energy \( dE(\omega) \) radiated per unit time into solid angle \( d\Omega \) in the interval of frequencies \( d\omega \) along a given direction \( \Pi \) to the modulus of Poynting’s vector (\( \vec{S} \)) averaged over the time [19]:

\[
\frac{\partial^2 \sigma(\omega)}{\partial \Omega \partial \omega} = \frac{dE(\omega)}{||\vec{S}||d\Omega d\omega} \tag{6.1}
\]

If the plane monochromatic electromagnetic wave

\[
\vec{B}(\vec{r}, t) = \vec{B}_0 \exp(-i(\omega t - \vec{k}_0 \vec{r})) \tag{6.2}
\]

is incident in the scattering medium, then

\[
||\vec{S}|| = \frac{c}{8\pi} E_0^2 \tag{6.3}
\]

Here \( \omega \) and \( \vec{k}_0 \) are frequency and wave vector of wave, respectively, \( c \) is its velocity, \( E_0 \) is amplitude of wave.

The energy \( dE(\omega) \) can be calculated by using of the method of retard potentials [19]. The electric field of incident wave (6.2) produces the fluctutive electric current in the electron gas with fluctuating density \( n \) and temperature \( T \):

\[
\delta \vec{A}_s(\vec{r}, t) = \left[ \frac{\partial \sigma(\omega)}{\partial n} \delta n(\vec{r}, t) + \frac{\partial \sigma(\omega)}{\partial T} \delta T(\vec{r}, t) \right] \vec{E}(\vec{r}, t) \tag{6.4}
\]

where \( \sigma(\omega) \) is the high-frequency conductivity of electron gas. Fluctuation response (6.4) induces the fluctuations of the vector potential \( \delta \vec{A}_s \), that for wave zone of radiation has the form

\[
\delta \vec{A}_s(\vec{r}, t) = \frac{1}{2R_0} \int \delta \vec{J}(\vec{r}, t) - \frac{\vec{r}}{c^2} \vec{E}(\vec{r}, t) d^3 \vec{r} \tag{6.5}
\]

Here \( R_0 \) is the distance between the system of charges and the point with detector of radiation (\( R_0 \gg r \)). Going to the Fourier components in (6.5) we obtain

\[
\delta \vec{A}_s(\vec{r}, t) = \frac{\exp(i\vec{k}_0 \vec{r})}{R_0 c} \int \delta \vec{J}(\vec{r}, \omega) \exp(-i\vec{k}_0 \vec{r}) d^3 \vec{r} \tag{6.6}
\]

where \( \omega \) and \( \vec{k}_0 \) are respectively the frequency and the wave vector of the scattered light. Moreover, the following relationship between frequency of incident \( \omega \) and scattered \( \omega \) waves, and fluctuation frequency \( \omega \) takes place:

\[
\omega = \omega_i - \omega_s \tag{6.7}
\]

Using (6.5) it can be found the Fourier components of fluctuations of electric and magnetic fields for scattered waves [20]:
We average the expression (6.12) over all possible directions of the polarization vector and \( \phi \) is the angle between the directions of propagation of incident and scattered waves. For the wave vectors of incident \( \mathbf{k}_i \) and scattered \( \mathbf{k}_s \) waves and the wave vector of fluctuation \( \mathbf{q} \) the relation similar to (6.7) is valid:

\[
\mathbf{q} = \mathbf{k}_i - \mathbf{k}_s.
\]  

The expression (6.12) can be used under the experimental investigation of the light scattering by electron density and temperature fluctuations. In the limiting cases, that was discussed above, the specific partial expressions follows from (6.12) and they coincide with one investigated earlier by other authors.

In the intermediate situation as it follows from (6.12), the cross-over terms are significant. Moreover, so far as the real and imaginary parts of high-frequency conductivity have different dependences on frequency of electromagnetic wave \( \omega \), it is possible to separate the contributions from the real and imaginary parts to the cross section.

7. CONCLUSION

Hydrodynamical fluctuations of the electron gas are the low-frequency and long-range stochastic excitation over steady state of the system. These fluctuations are responsible for the set of the physical phenomena which occur for both equilibrium and nonequilibrium conditions (for example, the current noises, the light scattering, etc.)

We have investigated the hot electron plasma that can be characterized by two time parameters — the electric charge decay time \( \tau_{\text{e}} \) and the electron energy relaxation time \( \tau_{\text{T}} \). The two spatial lengths — screening length \( L_{\text{s}} \) and the energy relaxation length \( L_{\text{T}} \) — correspond to these times and are important for the space-nonuniform effects. The hydrodynamical fluctuations of such a plasma were the subject of the numerous works. However, the most advanced results have been formulated in two limit cases (1.3) and (1.4).

In present work we have developed the theory of the hydrodynamical fluctuations of the hot electron plasma for more general case when it was possible to drop the limitation (1.3) and (1.4). Our consideration was based on the Boltzmann–Langevin kinetic equation for the fluctuations of the electron distribution function \( \delta f_{\text{e}} \). The solution of this equation was found under typical criteria for the hot electron plasma (1.2). It was shown that the fluctuation \( \delta f_{\text{e}} \) can be expressed via the two fluctuating parameters: \( \delta n(\mathbf{q}, \omega) \) and \( \delta T'(\mathbf{q}, \omega) \) and via the initial steady state distribution function. The fluctuating parameters \( \delta n(\mathbf{q}, \omega) \) and \( \delta T'(\mathbf{q}, \omega) \) mean the fluctuations of the magnitudes of the electron density and temperature. For them the hydrodynamic equations were deduced and, as a result, the following correlation functions were calculated

\[
\langle \delta n\delta n \rangle_{\text{e}}, \quad \langle \delta T'\delta T' \rangle_{\text{e}}, \quad \langle \delta n\delta T' \rangle_{\text{e}}.
\]  

The analysis of these correlators showed the following features of the general results which were not restricted by criteria (1.3) or (1.4):

a) There exist cross-over correlations of \( \delta n \) and \( \delta T' \), which means the mutual influence of electron density fluctuations and electron temperature.

b) The time and space dependences of the fluctuations strongly differ from that under the limit (1.3) and (1.4). In particular, the frequency dependences of the correlation functions no longer are of Lorentz-like forms.

c) It is important that the mentioned above features are characteristic, as well, for the equilibrium electron plasma under arbitrary relation between \( \tau_{\text{e}} \) and \( \tau_{\text{T}} \).

The above results were applied to the calculation of the light scattering by the electron plasma fluctuations. It was shown that the cross-correlation effect gives the contribution into the cross-section of the light scattering. This cross-correlation contribution has the same order of magnitude with that from the density and temperature fluctuations. The cross-effects bring about, as well, the additional anisotropy of the light scattering. It was found that the measurement of this cross-section for two directions of the light propagation relative to the applied electric field allows to have additional information about the extra-correlations, which are caused by electron-electron interaction.

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Spectral densities as low-frequency long-range fluctuations of electron density and temperature are completely defined by a system of equations (2.56)-(2.59). In spite of that for their solution it is necessary to deal with determinants of fourth order, the solution can be presented after some transformation in a simple and convenient for analysis form:

$$
\begin{align*}
(\delta^2 \omega)_{\omega} &= \frac{\Delta_\phi(\omega, \omega)}{\Delta(\omega, \omega)},
(\delta^2 \omega)_{\nu} &= \frac{\Delta_{\phi \nu}(\omega, \omega)}{\Delta(\omega, \omega)},
\text{Re}(\delta \delta \nu \omega)_{\omega} &= \frac{\Delta_{\text{Re}}(\omega, \omega)}{\Delta(\omega, \omega)},
\text{Im}(\delta \delta \nu \omega)_{\omega} &= \frac{\Delta_{\text{Im}}(\omega, \omega)}{\Delta(\omega, \omega)}.
\end{align*}
$$

Here the corresponding determinants of system are equal:

$$
\Delta(\omega, \omega) = [\omega - \frac{1}{\tau_T} (1 + \frac{\kappa_0}{D_0} + \gamma_0^2)] - \nu_M (\frac{1}{\tau_T} + \frac{1}{\tau_Q} + \frac{3}{4} \kappa_0 q^2) +
\frac{2}{3} \left[ \frac{(\mu_0 D_0)}{q^2} + \frac{2}{3} \gamma_0^2 \right] (1 + \frac{1}{\tau_T} + \frac{2}{3} \kappa_0 q^2) -
\frac{2}{3} \gamma_0 \left[ 1 + \frac{2}{3} \gamma_0 \left( \frac{D_0}{q^2} \right) \right] + [\omega - \frac{1}{\tau_T} (1 + \frac{\kappa_0}{D_0} + \gamma_0^2)]^2 .
$$

The reason of such violation is the extra correlation due to interelectron collisions. The measure of the extra correlation is the quantity $R(T)$ obtained in [4],[13], and which can be presented as:

$$
R(T) = \sum_{R} \epsilon_R \epsilon_{R'} \left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle = 3 NT^2 \left[ \frac{1}{\tau_T} - \frac{1}{\tau_\gamma} \right] .
$$

Correlation flow $\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle$ is defined by operator (2.13) without one summation over $\phi_k$:

$$
\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle = \sum_{\phi} \left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle .
$$

Here following designation for projection of tensor $B_{\phi0}$ on direction of vector $\phi$ is introduced:

$$
B_{\phi0} = \frac{1}{\nu_0} (\gamma_0 - 1)^2 \frac{2}{3} \frac{\gamma_0}{D_0} R(T) \delta_{\phi0} \delta_{00} .
$$

This correlation tensor violates the well-known Pr's diffusion-noise relation:

$$
(\delta \delta \omega)_{\omega} = \frac{\gamma_0}{D_0} \left[ P_{\omega0}(\omega) + D_{\omega0}(-\omega) - \sum_{\omega} \left( \omega \right) - \sum_{\omega} \left( -\omega \right) \right] .
$$

Correlation flow $\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle$ is defined by operator (2.13) without one summation over $\phi_k$:

$$
\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle = \sum_{\phi} \left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle .
$$

Here following designation for projection of tensor $B_{\phi0}$ on direction of vector $\phi$ is introduced:

$$
B_{\phi0} = \frac{1}{\nu_0} (\gamma_0 - 1)^2 \frac{2}{3} \frac{\gamma_0}{D_0} R(T) \delta_{\phi0} \delta_{00} .
$$

This correlation tensor violates the well-known Pr's diffusion-noise relation:

$$
(\delta \delta \omega)_{\omega} = \frac{\gamma_0}{D_0} \left[ P_{\omega0}(\omega) + D_{\omega0}(-\omega) - \sum_{\omega} \left( \omega \right) - \sum_{\omega} \left( -\omega \right) \right] .
$$

The reason of such violation is the extra correlation due to interelectron collisions. The measure of the extra correlation is the quantity $R(T)$ obtained in [4],[13], and which can be presented as:

$$
R(T) = \sum_{R} \epsilon_R \epsilon_{R'} \left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle = 3 NT^2 \left[ \frac{1}{\tau_T} - \frac{1}{\tau_\gamma} \right] .
$$

Correlation flow $\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle$ is defined by operator (2.13) without one summation over $\phi_k$:

$$
\left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle = \sum_{\phi} \left\langle \hat{F}_R, \hat{F}_{R'} \right\rangle .
$$
REFERENCES


Figure Captions

Fig.1  Schematic representation of real part of cross-correlation of electron density and electron temperature fluctuations in equilibrium.

Fig.2  Frequency dependence of imaginary part of cross-correlation for nonequilibrium steady state of fluctuations with $q \perp \vec{B}_0$.

Fig.3  Qualitative shape of nonequilibrium spectrum $\text{Re}(\delta \tilde{n} \delta T)$ for $R(T) > 0$. 

\[ \text{Re} \left( \frac{\delta \tilde{n} \delta T}{q \omega} \right) \]

\[ -\omega_2, -\omega_1, \omega_1, \omega_2, 0 \]

\[ \frac{4}{3N} \gamma_0 \frac{D_0 q^2}{\nu_m \nu_f} \]