



**INTERNATIONAL CENTRE FOR
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ABSTRACT

We analyse the spectrum of the eleven dimensional supermembrane quantized in $AdS_4 \times S^7$ background. The classical membrane lives at the boundary of AdS_4 which is $S^2 \times S^1$, and has $OSp(8,4)$ symmetry. We find that the spectrum contains, in addition to the $N=8$ supersymmetric (massive) singletons (which may possibly be the ultimate preons), also massless states of all higher integer and half-integer spin. These states fill the irreducible representations of $OSp(8,4)$ with highest spin $s_{max}=2,4,6,\dots$. The $s_{max} = 2$ multiplet corresponds to the states of the de Wit-Nicolai's $N=8$ gauged supergravity in four dimensions.

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1. INTRODUCTION

The quantum theory of the eleven dimensional supermembrane [1] in a background which is the product of the four dimensional anti de Sitter spacetime (AdS_4) with the seven-sphere (S^7) was studied recently [2]. In particular the four-index field strength was taken to be proportional to the Levi-Civita tensor in AdS_4 [3], and the membrane world-volume was identified with the boundary of AdS_4 which is $S^2 \times S^1$ [4]. This background has $OSp(8,4)$ symmetry. One of the main results of ref. [2] is that in a physical gauge where all the bosonic excitations in the direction of the world-volume coordinates (r, σ, ρ) , and half of the fermionic excitations are set equal to zero, the theory becomes exactly solvable. The physical excitations which consist of eight bosons and eight fermions are described by an $N=8$ supersymmetric free singleton* action defined on $S^2 \times S^1$ [2]. The singletons described by such an action may in principle serve as preons [7][8]. It has been shown that [2] the normal ordered quantum $OSp(8,4)$ generators of the theory (which are the analogs of the super Poincare generators of the usual superstring theories) obey the full $OSp(8,4)$ algebra. Thus we have a candidate for a quantum consistent theory of the supermembrane. The purpose of this letter is to analyse the spectrum of this theory.

Our main result is to construct all the massless states. By acting once with a bosonic or fermionic creation operator on the Fock vacuum we generate the $N=8$ singleton states, which are rather unusual objects with no Poincare analogs [9]. Acting twice with the creation operators we obtain all the massless states. They are the $OSp(8,4)$ supermultiplets with highest spin $s=2,4,6,\dots$. Thus, unlike in the string theory where the highest spin of the massless states is two, here we find that the supermembrane produces massless states of all integer and half integer spin. The action of three or higher number of creation operators on the vacuum gives only massive states.

The massless higher spin states have been encountered before in the interesting work of Flato and Fronsdal [10] in the context of a singleton field theory in AdS_4 . However, it is important to note that while their massless states are bound states of two singletons, in our model they correspond to excitations of the supermembrane. We will comment further on this in Sec. 4.

In the next section, we give some preliminaries. In Sec. 3 we analyse the spectrum. In Sec. 4 we discuss the implications of our results, and point out open problems.

* The singletons are the most fundamental representations of the anti de Sitter group $SO(3,2)$, first discovered by Dirac [5]. He referred to them as "A remarkable representation ...". The name singleton was coined later [6] due to the fact that their weight space consists of a single Regge trajectory.

2. PRELIMINARIES

The fields which occur in the supermembrane action are the coordinates of the eleven dimensional superspace; i.e. eleven bosonic coordinates, and thirty two component Majorana spinors. After fixing the world-volume reparametrizations and the Siegel symmetry, we are left with 8 bosonic degrees of freedom, φ^I , and 8 fermionic degrees of freedom, λ_-^α . The bosons are in the 8_v , and the fermions in the 8_s of $SO(8)$. In ref. [2] it was shown that the supermembrane action in the background described above reduces to the following N=8 supersymmetric singleton action

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}(h^{ij}\partial_i\varphi^I\partial_j\varphi^I + \frac{1}{4}\varphi^I\varphi^I - i\bar{\lambda}_-^\alpha\gamma^i\nabla_i\lambda_-^\alpha). \quad (1)$$

The N=8 supersymmetry transformation rules are

$$\begin{aligned} \delta\varphi^I &= \bar{\epsilon}_+^{\dot{\alpha}}\Sigma_{\dot{\alpha}\alpha}^I\lambda_-^\alpha, \\ \delta\lambda_-^\alpha &= -i\gamma^i\partial_i\varphi^I\Sigma_{\alpha\dot{\alpha}}^I\epsilon_+^{\dot{\alpha}} - \frac{i}{2}\Sigma_{\alpha\dot{\alpha}}^I\varphi^I\epsilon_-^{\dot{\alpha}}, \end{aligned} \quad (2)$$

The action is also invariant under the following bosonic symmetry transformations

$$\begin{aligned} \delta\varphi^I &= \xi_{AB}^i\partial_i\varphi^I + \Omega_{AB}\varphi^I + \Lambda_{ij}^I\varphi^j, \\ \delta\lambda_-^\alpha &= \xi_{AB}^i\nabla_i\lambda_-^\alpha + \frac{1}{4}\gamma^{ij}(\nabla_i\xi_{jAB})\lambda_-^\alpha + 2\Omega_{AB}\lambda_-^\alpha + \frac{1}{4}\Lambda^{IJ}(\Sigma_{IJ})^\alpha_\beta\lambda_-^\beta, \end{aligned} \quad (3)$$

where Λ^{IJ} is constant, $(\xi^i, \Omega)_{AB} = -(\xi^i, \Omega)_{BA}$ ($A, B=0, 1, 2, 3, 5$) are the 10 generators of $SO(3, 2)$ transformations, and $\Sigma_{IJ} \equiv \Sigma_{[I}\Sigma_{J]}^\dagger$. The explicit form of the (conformal) Killing vectors (ξ^i, Ω) can be found in [2]. These vectors satisfy the following equation

$$\nabla_i\xi_{jAB} + \nabla_j\xi_{iAB} = 4\Omega_{AB}h_{ij}. \quad (4)$$

Since $\Omega_{05} = \Omega_{\hat{m}\hat{n}} = 0$ ($\hat{m}, \hat{n} = 1, 2, 3$), it follows that $\xi_{05}, \xi_{\hat{m}\hat{n}}$ are the Killing vectors which generate the $SO(3) \times SO(2)$ transformations, while $\xi_{0\hat{m}}, \xi_{5\hat{m}}$ are conformal Killing vectors which generate the remaining $SO(3, 2)$ transformations.

The field equations which follow from (1) have the solutions [11][2]

$$\varphi^I = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} (a_{\ell m}^I \varphi_{\ell m} + a_{\ell m}^{I\dagger} \varphi_{\ell m}^*), \quad (5)$$

$$\lambda_-^\alpha = \sum_{j=\frac{1}{2}}^{\infty} \sum_{m=-j}^{+j} (d_{jm}^\alpha \lambda_{jm} + d_{jm}^{\alpha\dagger} \lambda_{jm}^*), \quad (6)$$

with

$$\varphi_{\ell m} = \frac{1}{\sqrt{4\pi}} e^{-i(\ell+\frac{1}{2})t} D_{0m}^\ell(L^{-1}), \quad (7)$$

and

$$\lambda_{jm} = \begin{pmatrix} u_{jm} \\ iu_{jm} \end{pmatrix}, \quad u_{jm} = e^{-\frac{t}{4}} \left(\frac{2j+1}{8\pi}\right)^{\frac{1}{2}} e^{-i(j+\frac{1}{2})t} \begin{pmatrix} D_{-\frac{1}{2}m}^j(L^{-1}) \\ D_{+\frac{1}{2}m}^j(L^{-1}) \end{pmatrix}. \quad (8)$$

$L(\theta, \phi)$ is the representative of the coset $SO(3)/SO(2)=S^2$ [12], and $D_{mm'}^j(L^{-1})$ denotes the unitary representation matrix of $SU(2)$ for angular momentum j . The single valuedness of the scalar field requires that we work on double (more generally even) covering of AdS_4 .

The model is quantized with the following (anti) commutation relations:

$$[a_{\ell m}^I, a_{\ell' m'}^{I\dagger}] = \delta^{IJ} \delta_{\ell\ell'} \delta_{mm'}, \quad \ell = 0, 1, \dots, \quad -\ell \leq m \leq \ell, \quad (9)$$

$$\{d_{jm}^\alpha, d_{j' m'}^{\beta\dagger}\} = \delta^{\alpha\beta} \delta_{jj'} \delta_{mm'}, \quad j = \frac{1}{2}, \frac{3}{2}, \dots, \quad -j \leq m \leq j. \quad (10)$$

Other (anti)commutators vanish. The $a_{\ell m}$ and d_{jm} are now operators in a Fock space whose vacuum $|0\rangle$ is defined by

$$a_{\ell m}|0\rangle = d_{jm}|0\rangle = 0. \quad (11)$$

In ref. [2] an oscillator representation was found for the Noether charges corresponding to the $SO(3, 2) \times SO(8)$ symmetries (3), and N=8 supersymmetries (2). These charges form the basis of our calculation of the mass spectrum. Therefore we reproduce them here. In a self-explanatory notation, the result is [2]*

$$M_{05} = \sum_{\ell, m} (\ell + \frac{1}{2}) a_{\ell m}^{I\dagger} a_{\ell m}^I + \sum_{j, m} (j + \frac{1}{2}) d_{jm}^{\alpha\dagger} d_{jm}^\alpha, \quad (12)$$

$$M_{12} = \sum_{\ell, m} m a_{\ell m}^{I\dagger} a_{\ell m}^I + \sum_{j, m} m d_{jm}^{\alpha\dagger} d_{jm}^\alpha, \quad (13)$$

$$\begin{aligned} M_{23} + iM_{31} &= \sum_{\ell, m} [(\ell - m)(\ell + m + 1)]^{\frac{1}{2}} a_{\ell m+1}^{I\dagger} a_{\ell m}^I + \\ &+ \sum_{j, m} [(j - m)(j + m + 1)]^{\frac{1}{2}} d_{j m+1}^{\alpha\dagger} d_{jm}^\alpha, \end{aligned} \quad (14)$$

$$\begin{aligned} iM_{03} + M_{53} &= \sum_{\ell, m} [(\ell - m + 1)(\ell + m + 1)]^{\frac{1}{2}} a_{\ell+1 m}^{I\dagger} a_{\ell m}^I + \\ &+ \sum_{j, m} [(j - m + 1)(j + m + 1)]^{\frac{1}{2}} d_{j+1 m}^{\alpha\dagger} d_{jm}^\alpha, \end{aligned} \quad (15)$$

* A normal ordering ambiguity occurring in M_{05} , which is the sum of the zero point energies of the fermi and bose oscillators, vanishes. This has been shown by using the zeta function regularization [2][13]. In fact, the fermi and bose contributions vanish separately.

$$\begin{aligned}
(iM_{01} + M_{51}) + i(iM_{02} + M_{52}) &= \\
& - \sum_{\ell, m} [(\ell + m + 2)(\ell + m + 1)]^{\frac{1}{2}} a_{\ell+1, m+1}^{\dagger I} a_{\ell m}^I - \\
& - \sum_{j, m} [(j + m + 2)(j + m + 1)]^{\frac{1}{2}} d_{j+1, m+1}^{\dagger \alpha} d_{j, m}^{\alpha}, \quad (16)
\end{aligned}$$

$$\begin{aligned}
(iM_{01} + M_{51}) - i(iM_{02} + M_{52}) &= \\
& \sum_{\ell, m} [(\ell - m + 2)(\ell - m + 1)]^{\frac{1}{2}} a_{\ell+1, m-1}^{\dagger I} a_{\ell m}^I + \\
& + \sum_{j, m} [(j - m + 2)(j - m + 1)]^{\frac{1}{2}} d_{j+1, m-1}^{\dagger \alpha} d_{j, m}^{\alpha}. \quad (17)
\end{aligned}$$

$$T^{IJ} = 2i \sum_{\ell n} a_{\ell n}^{\dagger I} a_{\ell m}^J + \frac{i}{2} \sum_{j m} d_{j m}^{\dagger \alpha} \Sigma_{\alpha \beta}^{IJ} d_{j m}^{\beta}, \quad (18)$$

$$Q^{\dot{\alpha}1} = \sum_{\ell, m} [(\ell + m + 1)]^{\frac{1}{2}} a_{\ell m}^{\dagger I} d_{\ell+\frac{1}{2}, m+\frac{1}{2}}^{\beta} + (\ell + m)^{\frac{1}{2}} d_{\ell-\frac{1}{2}, m-\frac{1}{2}}^{\dagger \beta} a_{\ell m}^I (\Sigma_I)_{\beta}^{\dot{\alpha}}, \quad (19)$$

$$Q^{\dot{\alpha}2} = \sum_{\ell, m} [(\ell - m + 1)]^{\frac{1}{2}} a_{\ell m}^{\dagger I} d_{\ell+\frac{1}{2}, m-\frac{1}{2}}^{\beta} + (\ell - m)^{\frac{1}{2}} d_{\ell-\frac{1}{2}, m+\frac{1}{2}}^{\dagger \beta} a_{\ell m}^I (\Sigma_I)_{\beta}^{\dot{\alpha}}. \quad (20)$$

It has been shown [2] that these charges obey the following $\text{OSp}(8,4)$ algebra:

$$[M_{AB}, M_{CD}] = -i(\eta_{BC}M_{AD} - \eta_{AC}M_{BD} - \eta_{BD}M_{AC} + \eta_{AD}M_{BC}), \quad (21)$$

$$[T_{IJ}, T_{KL}] = i(\delta_{JK}T_{IL} - \delta_{IK}T_{JL} - \delta_{JL}T_{IK} + \delta_{IL}T_{JK}), \quad (22)$$

$$\{Q^{\dot{\alpha}}, Q^{\dot{\beta}}\} = \delta^{\dot{\alpha}\dot{\beta}} \ell^{AB} M_{AB} + \frac{1}{2} (\Sigma_{IJ})^{\dot{\alpha}\dot{\beta}} T^{IJ}, \quad (23)$$

$$[M_{AB}, Q^{\dot{\alpha}}] = \frac{i}{2} \ell_{AB} Q^{\dot{\alpha}}, \quad (24)$$

$$[T^{IJ}, Q^{\dot{\alpha}}] = -\frac{i}{2} (\Sigma^{IJ})_{\dot{\beta}}^{\dot{\alpha}} Q^{\dot{\beta}}. \quad (25)$$

Here $\ell^{rs} = \tilde{\gamma}^r, \ell^{rs} = \tilde{\gamma}^{rs}$ ($r, s = 0, 1, 2, 3$), where we have used the following γ -matrix notation

$$\tilde{\gamma}_0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \tilde{\gamma}_i = \begin{pmatrix} 0 & -i\sigma^i \\ i\sigma^i & 0 \end{pmatrix}, \quad \tilde{C} = \tilde{\gamma}_0 \tilde{\gamma}_2. \quad (26)$$

This representation is chosen so that the Majorana spinor $Q^{\dot{\alpha}}$ has the simple form

$$Q^{\dot{\alpha}} = \begin{pmatrix} Q^{\dot{\alpha}1} \\ Q^{\dot{\alpha}2} \\ (Q^{\dot{\alpha}2})^{\dagger} \\ (-Q^{\dot{\alpha}1})^{\dagger} \end{pmatrix}, \quad (27)$$

We are now ready to compute the spectrum of the theory.

3. THE SPECTRUM

The physical states are obtained by the repeated action of the creation operators $a_{\ell m}^{\dagger}, d_{j m}^{\dagger}$ on the vacuum $|0\rangle$ of the Fock space which has been defined in (11). All these states should arrange themselves into irreducible representations of $\text{OSp}(8,4)$. For each such representation there must exist a unique highest weight multiplet in a given $\text{SO}(3) \otimes \text{SO}(8)$ representation with degeneracy $(2s + 1) \otimes D$, where s denotes the total angular momentum of the lowest energy state of a given $\text{SO}(3,2)$ representation and D is the dimension of the $\text{SO}(8)$ representation of that lowest energy state. This vacuum multiplet has the property that

$$Q^{\dot{\alpha}1}|\psi\rangle = Q^{\dot{\alpha}2}|\psi\rangle = 0. \quad (28)$$

As a consequence of this, from (23) it follows that

$$M_i^-|\psi\rangle \equiv (iM_{0i} - M_{5i})|\psi\rangle = 0, \quad i = 1, 2, 3. \quad (29)$$

Furthermore, the operators M_{04}, J_k and T^{IJ} leave the vacuum multiplet invariant, while the operators $M_i^{\pm} \equiv -(M_i^-)^{\dagger}$ act as $\text{SO}(3,2)$ energy boosts. They are the raising operators which build up a given $\text{SO}(3,2)$ representation with the vacuum multiplet as highest weight. The action of $(Q^{\dot{\alpha}1})^{\dagger}$ and $(Q^{\dot{\alpha}2})^{\dagger}$ and appropriate combinations of these operators transforms the vacuum multiplet to a $\text{SO}(3) \otimes \text{SO}(8)$ state belonging to another $\text{SO}(3,2)$ representation which is part of the same supermultiplet [14][15]. The following observation is useful. Since all generators are of the form $a^{\dagger}a, d^{\dagger}d, a^{\dagger}d$ or ad^{\dagger} a given supermultiplet can only consist of states which are obtained by the action of a fixed number of creation operators a^{\dagger}, d^{\dagger} on the Fock space vacuum.

Let us first consider the Fock space vacuum $|0\rangle$. Since there are no zero modes the vacuum must be in the singlet representation of $\text{OSp}(8,4)$. Such a vacuum has no particle interpretation. We next consider the 1-oscillator states:

$$a_{\ell m}^{\dagger I}|0\rangle, \quad d_{j m}^{\dagger \alpha}|0\rangle. \quad (30)$$

In order to find all supermultiplets which are described by these states it is necessary and sufficient to find the most general solution of (28). Using the explicit oscillator representation of the supercharges given in (19), (20) one can easily verify that the only solution of (28) is given by

$$|\psi\rangle = a_{00}^{\dagger I}|0\rangle. \quad (31)$$

This vacuum multiplet is a singlet of $\text{SO}(3)$ and forms a 8_v representation of $\text{SO}(8)$. Acting with the raising operators $(Q^{\dot{\alpha}1})^{\dagger}$ and $(Q^{\dot{\alpha}2})^{\dagger}$ on it one obtains the $N=8$ Dirac supermultiplet [16], which consists of a $N=8$ singleton Rac in the 8_v representation and a $N=8$ singleton Di in the 8_s representation:

$$[D(\frac{1}{2}, 0) \otimes 8_v] \oplus [D(1, \frac{1}{2}) \otimes 8_s], \quad (32)$$

where $D(E_0, s)$ denotes an irreducible unitary representation of $SO(3,2)$ for which E_0 is the minimum eigenvalue of the energy operator M_{05} , and s is the maximum eigenvalue of the spin operator M_{12} (the total angular momentum) in the lowest energy state. As pointed out in the work of Flato and Fronsdal [9], all these singletons are not locally observable.

Let us now consider the 2-oscillator states. A general 2-oscillator state is given by:

$$a_{\ell m}^{\dagger I} a_{\ell' m'}^{\dagger J} |0\rangle, \quad a_{\ell m}^{\dagger I} d_{j n}^{\dagger \alpha} |0\rangle, \quad d_{j n}^{\dagger \alpha} d_{j' n'}^{\dagger \beta} |0\rangle, \quad (33)$$

which corresponds to taking the tensor product of two singletons. According to Flato and Fronsdal [10] such a product contains only massless representations of $SO(3,2)$. Such representations are characterized by the relation $E_0 = s + 1$ [17]. Flato and Fronsdal give three reasons for this [9]. For reader's convenience we reproduce them here: (1) Each $D(s+1, s)$ is the restriction to $SO(3,2)$ of a massless UIR of the conformal group $SO(4,2)$ for $2s = 1, 2, 3, \dots$ [For $s = 0$, the restriction is $D(1,0) \oplus D(2,0)$]. (2) Each of them (for $s > 0$, and $D(1,0) \oplus D(2,0)$ for $s = 0$) is smoothly contractable to a massless UIR of the Poincare group in the flat space limit. (3) Each of them (for $s \geq 1$) is closely associated with gauge fields, just as the representations the Poincare group with zero mass and discrete helicity [18]. Thus the representations $D(s+1, s)$ are massless for all s , though the cases $s = 0, \frac{1}{2}$ are somewhat exceptional.

Our aim is now to find the most general solution of (28) involving only 2-oscillator states. It is clear that we can consider the $a^\dagger d^\dagger$ states independently from the $a^\dagger a^\dagger$ and $d^\dagger d^\dagger$ states. It is sufficient to consider the 2-oscillator states with maximum angular momentum. Only such states can be the highest weight states of a massless $SO(3,2)$ representation of the form $D(s+1, s)$ [17]. The exceptional massless representation $D(2,0)$ deserves special care. The highest weight state of that representation is given by the state $d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\alpha)} d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\beta)} |0\rangle$.

To simplify the calculation we will not consider the whole vacuum multiplet but instead only the part with lowest eigenvalue of M_{12} . In other words instead of taking $-m \leq \ell \leq m$ in (33) we will only consider states with $m = -\ell$. Similarly we will restrict the indices m', n and n' in (33) to be $m' = -\ell', n = -j$ and $n' = -j'$. Of course this does not apply to the special state $d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\alpha)} d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\beta)} |0\rangle$. An advantage of these restrictions is that $Q^{\alpha 1}$ acting on these states gives zero automatically. This is not true for the special state $d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\alpha)} d_{\frac{1}{2} - \frac{1}{2}}^{\dagger(\beta)} |0\rangle$. Consequently this state can not be the highest weight of a supermultiplet and hence need not be considered further.

In view of the above for our purposes the most general $a^\dagger d^\dagger$ state with minimum eigenvalue of M_{12} to consider is given by:

$$|\psi_1\rangle = \sum_{\ell+j=s} C_{\ell j}^{\alpha I} a_{\ell - \ell}^{\dagger I} d_{j - j}^{\dagger \alpha} |0\rangle, \quad s = \frac{1}{2}, \frac{3}{2}, \dots$$

$$Q^{\alpha 1} |\psi_1\rangle = 0, \quad (34)$$

where summation over ℓ, j, I and α is understood. One can verify that the equation $Q^{\alpha 2} |\psi_1\rangle = 0$ has no solution. We next consider the $a^\dagger a^\dagger$ and $d^\dagger d^\dagger$ states. The most general such state with minimum eigenvalue of M_{12} is given by:

$$|\psi_2\rangle = \sum_{\ell+j=s} C_{\ell j}^{IJ} a_{\ell - \ell}^{\dagger I} a_{j - j}^{\dagger J} |0\rangle + \sum_{j+j'=s} C_{j j'}^{\alpha \beta} d_{j - j}^{\dagger \alpha} d_{j' - j'}^{\dagger \beta} |0\rangle, \quad s = 0, 1, \dots, \quad (35)$$

where the constant coefficients have the symmetries $C_{\ell \ell'}^{IJ} = C_{\ell' \ell}^{JI}$ and $C_{j j'}^{\alpha \beta} = -C_{j' j}^{\beta \alpha}$ which follow from the fact that the a^\dagger 's commute, while the d^\dagger 's anticommute. The action of $Q^{\alpha 2}$ on $|\psi_2\rangle$ gives rise to the following linear combination of states:

$$Q^{\alpha 2} |\psi_2\rangle = \sum_{n=1}^s \left[(2n)^{\frac{1}{2}} C_{n, s-n}^{IJ} (\Sigma_J)_\alpha^\alpha + (2s - 2n + 1)^{\frac{1}{2}} C_{s-n+\frac{1}{2}, n-\frac{1}{2}}^{\beta \alpha} (\Sigma^I)_\beta^\alpha \right] \times$$

$$\times a_{s-n, -s+n}^{\dagger I} d_{n-\frac{1}{2}, -n+\frac{1}{2}}^{\dagger \alpha} |0\rangle. \quad (36)$$

From this we deduce that for $s \geq 1$ the most general solution of the equation $Q^{\alpha 2} |\psi_2\rangle = 0$ is given by:

$$C_{n, s-n}^{IJ} = - \left(\frac{2s - 2n + 1}{2n} \right)^{\frac{1}{2}} \delta^{IJ} C_{n, s-n}, \quad s = 2, 4, 6, \dots,$$

$$C_{s-n+\frac{1}{2}, n-\frac{1}{2}}^{\beta \alpha} = \delta^{\beta \alpha} C_{n, s-n}, \quad s = 2, 4, 6, \dots \quad (37)$$

It is important to realize that only the even values of s are allowed. From (37) and the symmetry properties given below (35) one finds that all the $C_{n, s-n}$ are related to each other by the following recurrency relations:

$$C_{n, s-n} = -C_{s-n+1, n-1} = \left[\frac{2(s-n)(2s-2n+1)}{2n(2n+1)} \right]^{-\frac{1}{2}} C_{s-n, n}. \quad (38)$$

For $s = 0$ the r.h.s. of (36) vanishes identically which means that all states $a_{00}^{\dagger I} a_{00}^{\dagger J} |0\rangle$ are highest weight states. These states correspond to the representation $D(1,0) \otimes (35+1)$.

To summarize, we have found that the highest weight states of all supermultiplets are given by the highest weight of the $SO(3,2) \otimes SO(8)$ representations $D(1,0) \otimes 35$ and the series $D(s+1, s) \otimes 1$ ($s = 0, 2, 4, \dots$). They are the highest weights of the following massless supermultiplets:

$$[D(1,0) \otimes 35] \oplus [D(2,0) \otimes 35] \oplus [D(\frac{3}{2}, \frac{1}{2}) \otimes 56] \oplus [D(2,1) \otimes 28]$$

$$\oplus [D(\frac{5}{2}, \frac{3}{2}) \otimes 8] \oplus [D(3,2) \otimes 1], \quad (39)$$

$$[D(1,0) \otimes 1] \oplus [D(2,0) \otimes 1] \oplus [D(\frac{3}{2}, \frac{1}{2}) \otimes 8] \oplus [D(2,1) \otimes 28]$$

$$\oplus [D(\frac{5}{2}, \frac{3}{2}) \otimes 56] \oplus [D(3,2) \otimes (35+35)] \oplus [D(\frac{7}{2}, \frac{5}{2}) \otimes 56]$$

$$\oplus [D(4,3) \otimes 28] \oplus [D(\frac{9}{2}, \frac{7}{2}) \otimes 8] \oplus [D(5,4) \otimes 1], \quad (40)$$

$$\begin{aligned}
& D|(s+1, s) \otimes 1| \oplus |D(s + \frac{3}{2}, s + \frac{1}{2}) \otimes 8| \oplus |D(s+2, s+1) \otimes 28| \\
& \oplus |D(s + \frac{5}{2}, s + \frac{3}{2}) \otimes 56| \oplus |D(s+3, s+2) \otimes (35+35)| \oplus |D(s + \frac{7}{2}, s + \frac{5}{2}) \otimes 56| \\
& \oplus |D(s+4, s+3) \otimes 28| \oplus |D(s + \frac{9}{2}, s + \frac{7}{2}) \otimes 8| \oplus |D(s+5, s+4) \otimes 1| \\
& \qquad \qquad \qquad s = 2, 4, 6, \dots \qquad (41)
\end{aligned}$$

This concludes our description of the massless supermultiplets that arise in the 2-oscillator sector. We have not worked out the representation content of the n -oscillator states for $n \geq 3$. In all these sectors only massive supermultiplets arise. This is so because of the following. Consider a general n -oscillator state $a_{\ell_1 m_1}^\dagger \dots a_{\ell_n m_n}^\dagger |0\rangle$. (Replacing a 's by d 's does not change the argument). On the one hand side the total energy E of such a state is given by $E = (\sum_{i=1}^n \ell_i) + \frac{n}{2}$, while the maximum value of the total angular momentum s_{max} is given by $s_{max} = (\sum_{i=1}^n \ell_i)$. On the other hand side the condition that a state is the highest weight of a massless representation is that such a state satisfies $E = s + 1$. Given the values of E and s_{max} above it is clear that this condition can only be satisfied for $n = 2$. Hence only the 2-oscillator sector contains massless supermultiplets. All the next n -oscillator sectors ($n \geq 3$) only contain massive supermultiplets.

4. DISCUSSION

The singleton states which occur in the spectrum of our model are rather unconventional. As Dirac noted first [5], their wave function has a fixed ψ -dependence (the radial coordinate of AdS_4). Thus there is no principal quantum number associated with this radial direction. For a recent review of singletons, see ref. [6] and further references therein.

The massless states occurring in our model, like all the other states, correspond to excitations of the supermembrane. This is to be contrasted with the interpretation of similar states as bound states of singletons, by Flato and Fronsdal [8]. In that case the interactions between ordinary particles are induced by elementary interactions of singletons [8], and this is possible only for the $N=1$ supersymmetric singleton theory [11][19]. In the $N=8$ supersymmetric singleton theory, although interactions on the world-volume are not possible, we can still talk about interactions of spacetime fields. To this end one constructs the membrane propagator and vertex operators. Using them, one can build spacetime amplitudes in an operator formalism in much the same way it is done for string theories.

Our model contains extra gravitons (1+35+35 of them, to be precise) as well as higher spin massless states. Before deciding whether this is physically acceptable, we must first analyse the coupling of these states to the usual massless particles of spin ≤ 2 that we "observe" in nature. Strictly speaking, this should be done in a version of the theory in

which the energy scale is extrapolated from the Planck scale to the usual low energy scale (100GeV). Such an extrapolation ideally would have to give masses to the extra gravitons and all the higher spin massless particles. To see whether a Higgs mechanism is feasible at least kinematically, we observe that the spectrum of massless states has the following pattern:

$L \setminus S$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$...	$2s$	$2s + \frac{1}{2}$	$2s + 1$	$2s + \frac{3}{2}$...
-1	70	56	28	8	1									
0	2	8	28	56	70	56	28	8	...					
1					1	8	28	56	...					
\vdots														
$s-2$...	1				
$s-1$...	70	56	28	8	...
s										1	8	28	56	...
\vdots														

In this table S denotes the spin of the massless state, and L is the label of the supermultiplet in which a given massless state occurs. The representations (39) and (40) have labels -1 and 0, respectively, while the ones given in (41) have label 1,2,3,... The table clearly shows that there is a regular pattern which repeats itself modulo units of spin 2 (except for spin 0 states which are special); i.e. the $SO(8)$ content of spin s states is the same as that of spin $(s+2)$ states. In particular, the even spin states are in $(1+70+1)$ of $SO(8)$ coming from three adjacent supermultiplets, while the odd spin states are in $(28+28)$ and the half integer spin states are in $(56+8)$ of $SO(8)$, both of which come from two adjacent supermultiplets. We believe that this periodicity of the $SO(8)$ content of the spectrum is rather suggestive of a Higgs mechanism (appropriately generalized to anti de Sitter space) in which spin s and spin $(s+2)$ states are involved for all s , and which leaves the usual graviton massless.

The occurrence of infinitely many massless higher spin particles in our model implies the existence of infinitely many (local) gauge symmetries which are analogous to the Maxwell, general coordinate and local supersymmetries, associated with spin 1,2 and 3/2 respectively. It may be a relatively easy matter to analyse these symmetries at the linearized level. An interesting possibility is that these linearized transformations correspond to an infinite dimensional super Lie algebra which contains the $OSp(8,4)$ as a finite subalgebra, in a fashion suggested by the work of Fradkin and Vasiliev [20] and Vasiliev [21]. If that were the case, our model would provide a field theoretic realization of their superalgebras. The nonlinear extension of the infinitely many linearized gauge transformations mentioned above is an interesting open problem which would probably require the analysis of membrane amplitudes.

Another interesting open problem is to find the analog of the super-Virasoro algebra for the supermembrane. Since we are working in a physical gauge, the generators of such an algebra will act on the physical states and transform them into each other, i.e. they will belong to a spectrum generating algebra. In the gauge unfixed version of our theory, a "covariant" super Virasoro-like algebra is expected to arise. However, the relation of such an algebra to the higher spin superalgebras of Fradkin and Vasiliev [20] is not clear.

Using the result of this paper one could contemplate computing the vertex operators in our model, in an attempt to understand the nature of the higher spin massless fields, e.g. the coupling of massless spin 5/2 or 3, to the usual graviton. The study of the vertex operators is also important for the purpose of computing certain tree diagrams, or one loop diagrams in an operator formalism, the way it was done several years ago for strings. This could possibly shed some light on the divergence structure of the theory (e.g. the role of the higher spin massless states). One might expect that the couplings among particles which will survive in a limit in which α' goes to zero while keeping κ constant, would be those described in the de Wit-Nicolai's $N=8$ supergravity [22]. The question of how the higher spin massless states couple to the membrane is an interesting open problem. If one keeps any higher spin state in the theory, it is believed that consistency of its couplings will require the presence of all higher spin states [20][21]. Furthermore, it is believed that the couplings of the higher spin states will involve higher derivatives [20][23][24]. An illustration of these properties in our model remains to be investigated.

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