ATTRACTOR IN A SUPERSTRING MODEL

The Einstein Theory, The Friedmann Universe and Inflation

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ABSTRACT
In a superstring model, the minimum of the SUSY breaking potential
through a gluino condensation provides a no-scale model, which may explain
the hierarchy problem, permit inflation and give zero cosmological constant
naturally. It is shown that the above minimum is an attractor in the dynamical
system. This minimum also guarantees the 4-dimensional Einstein gravity theory,
rather than the Brans-Dicke theory, in the low energy limit. Using this model,
a preferential scenario of evolution towards \[ \text{the 4-dimensional Friedmann universe} \times \text{a constant internal space} \] via inflation is discussed.

A series of phase transitions due to a quantum tunnelling of the quantized
antisymmetric field \( H_{\mu
\nu} \) may occur, forming a hierarchical bubble structure.

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§ 1 Introduction

Superstring theory is a promising candidate for a final unified
theory including gravity. It is also very successful from the phenomenological
point of view. Its application to cosmology is certainly important and
interesting. Since there is no kinetic term of a "metric of 10-dimensional
background space-time" in a tree-level string action, one does not yet know
the dynamics of a "space-time" at the superstring level. It is, however,
known in the field-theory limit, which is the 10-dimensional, N=1 supergravity
theory coupled to SO(32) or \( E_8 \times E_8 \) - Yang-Mills fields. This theory may be
valid at least in the later stage of the universe, e.g. after the Planck era.

In higher-dimensional cosmology, one of the important problems is how
the present universe is naturally realized. Our universe now must be \( \text{the 4-
dimensional Friedmann universe} \times \text{a constant internal space} \). In
the conventional 4-dimensional theory, the isotropy and the homogeneity of space-
time, which may be deduced from the cosmological principle or from an inflationary
scenario, guarantee that our universe is a Friedmann space-time. In a higher-
dimensional theory, however, that is not true, because the \( F \times K \) space-time is
not isotropic at all in higher-dimensions. Then, the above question is not
trivial. The idea of an attractor may explain why our present universe is \( F \times K \) instead of the cosmological principle.

In the case of the 6-dimensional, N=2 supergravity with Yang-Mills and
matter fields, the \( F \times K \) space-time is a unique attractor in the dynamical
system and all the space-times (apart from the time-reversal ones) approach \( F \times K \)
asymptotically in the later stage. Therefore, the above question is naturally
explained in the 6-dimensional, N=2 supergravity model.
Since a superstring theory is more realistic, we should investigate whether or not the \( F \times K \) space-time is an attractor in the 10-dimensional, \( N=1 \) supergravity model. In the previous paper, it is shown that \( F \times K \) is always (apart from time reversal solutions) the attractor in our dynamical system and then the present space-time can be naturally realized in the later stage of the universe, assuming a Calabi-Yau compactification and adding the curvature squared term

\[
\frac{1}{2} \left( \nabla \phi \right)^2 - A \phi R - V R^2
\]  

(1.1)

which is proposed by Zwiebach and Zuming.

However, the Ricci-flatness of the internal space gives rise to other serious problems. First, the effective 4-dimensional gravity theory is not the Einstein theory but the Brans-Dicke theory because the radius of the internal space behaves like a massless Brans-Dicke scalar in 4-dimensions. Secondly, it may be rather difficult to have a natural inflation because of the absence of a "potential". The curvature squared term does not seem to help. Both difficulties require a "potential" for their resolution, and the potential must also explain the zero cosmological constant naturally. Is there any suitable potential which can appear in the superstring model?

Dine et al. have proposed a breaking mechanism of local supersymmetry through vacuum expectation values (VEVs) of the antisymmetric tensor field \( H_{\mu \nu \rho} \) and through gluino condensation. The associated potential is positive semi-definite and exhibits the remarkable property that zero cosmological constant is obtained naturally, at least at the tree level. At the minimum of the potential, one can find one of the no-scale supergravity models, which may also solve the hierarchy problem.

As for the VEVs of \( H_{\mu \nu \rho} \), Polch and Witten have shown that those must be quantized through the Wess-Zumino interaction in the string action like Dirac string singularities and that those can tunnel quantum-mechanically from one value to the next due to an 'instanton' effect through the coupling of \( H_{\mu \nu \rho} \) to the Yang-Mills Chern-Simons term. The potential may change during the evolution of the universe, if the quantum tunnelling process is taken into account.

In this paper, assuming the above potential, we shall discuss how the universe can reach the potential minimum and consider the above-mentioned problems. We present the 4-dimensional Lagrangian equivalent to the original 10-dimensional one, assuming a Calabi-Yau compactification, in §2. Fixing the VEV of \( H_{\mu \nu \rho} \) (which means that the tunnelling process is neglected in the evolution) in §3, we show that the potential minimum is the attractor in this model if the 3-space is expanding. The 4-dimensional Einstein theory is obtained at the potential minimum. If the tunnelling probability is enough high, the universe changes its history completely before reaching the minimum of the potential. We consider such cases in §4 and show that a series of phase transitions occur and the universe settles down to one of the potential minima. If the transition rate is comparable with a Hubble expansion rate, a hierarchical bubble structure may be formed and each parts of the universe (bubbles) approach some of the potential minima. A preferential scenario of evolution towards \( F \times K \) via inflation will be discussed in §5.
5.2 The 4-dimensional Lagrangian

Assuming a Ricci-flat (Calabi-Yau) compactification, we shall derive the effective 4-dimensional Lagrangian equivalent to that of the 10-dimensional $N=1$ supergravity coupled to a Yang-Mills field. We assume that the metric of 10-dimensional space-time is given by

$$ds_{10}^2 = g_{MN} dx^M dx^N$$

and

$$ds_{CY}^2 = -b(x) dx dx + b(x) ds_{CY}$$

(2.1)

with

$$ds_{4}^2 = R_{MN}(x) dx^M dx^N$$

and

$$ds_{CY}^2 = G_{MN}(y) dy dy.$$  (2.2)

Here, we have factorized out the conformal factor $b^{-6}$ in order to obtain the proper Einstein action in 4-dimensions. $g_{MN}$ is the metric of a static Calabi-Yau manifold, whose explicit form is unknown.

The bosonic part of the 10-dimensional, $N=1$ supergravity Lagrangian consists of

$$L_R + L_F + L_Y + L_H$$

(2.3)

with

$$L_R = - \frac{1}{4 \kappa_4^2} \sqrt{g} \left( \bar{F} \gamma_5 \right)^2$$

and

$$L_F = - \frac{1}{4 \kappa_4^2} \sqrt{g} \left( \bar{F} \gamma_5 \right)^2$$

$$L_Y = - \frac{1}{4 \kappa_4^2} \sqrt{g} \left( \bar{F} \gamma_5 \right)^2$$

and

$$L_H = - \frac{1}{4 \kappa_4^2} \sqrt{g} \left( \bar{F} \gamma_5 \right)^2$$

(2.4)

where $\kappa_4^2 = R_{MN} \gamma_5$, and $\kappa_4$ are the 10-dimensional gravitational constant and the 10-dimensional gauge coupling constant, respectively, whilst $\bar{F}$ and $\bar{F}$ are the scalar curvature and the covariant derivative with respect to $\bar{F}$.

In order to obtain a Calabi-Yau compactification, we need the Riemann curvature squared term, which is derived in the field-theory limit of a superstring theory. Here, we assume the special combination of curvature squared terms (1.1) in order to have a ghost-free theory. Through the vacuum expectation values (VEVs) of Yang-Mills field,

$$F_{\mu\nu} = \begin{cases} \partial^\mu A^\nu - \partial^\nu A^\mu & \text{for } \mu, \nu = M, N \\ \partial^\mu \partial^\nu \phi & \text{for } \mu, \nu = 1, 2, \ldots, 4 \\ 0 & \text{otherwise,} \end{cases}$$

we obtain a Calabi-Yau compactification in a 4-dimensional-coordinate dependent background (2.1) as well. Here,

$$\tilde{F}_{\mu\nu} = \begin{cases} \partial^\mu A^\nu & \text{for } \mu, \nu = M, N \\ \partial^\mu \partial^\nu \phi & \text{for } \mu, \nu = 1, 2, \ldots, 4 \\ 0 & \text{otherwise,} \end{cases}$$

(2.5)

in a "Calabi-Yau" solution of the Yang-Mills equations in a static background

$$ds_{10}^2 = \eta_{MN} dx^M dx^N + ds_{CY}^2$$

and

$$ds_{10}^2 = \eta_{MN} dx^M dx^N + ds_{CY}^2.$$  (2.10)

The curvature squared term and $L_{\tilde{F}}$ are described as

$$L_{\tilde{F}} = - \frac{1}{4 \kappa_4^2} \sqrt{g} \left( \bar{F} \gamma_5 \right)^2$$

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(2.5)
where $\tilde{\nabla}$ is the Ricci tensor with respect to $\tilde{\rho}_{MN}$. The first term of the r.h.s. of Eq. (2.11) vanishes because of the Calabi-Yau compactification.

If the time scale or length scale of changes in $g_{\alpha\beta}$ and $\hat{b}$ is much smaller than the Planck scale (e.g. in the later stage of the universe), then the $I$-term in Eq. (2.11) can also be neglected as compared with the other terms such as $\lambda_{R}^{2}$. It is worth noting that this may be true for some combination of curvature squared terms because a structure of a dynamical system may change completely if higher-order time derivatives appear.

The Einstein action can be reduced in 4-dimensions as

$$\int d^{10} \sqrt{\tilde{\rho}} = \frac{1}{2 e^{4}} \int d^{4} \sqrt{\tilde{\rho}} \left\{ R - 2R \left[ \frac{1}{2} \rho \right]^{2} \right\},$$

(2.12)

where $\rho = \rho_{(\tilde{\rho})}$ and $R$ and $\rho$ are the scalar curvature and the covariant derivative with respect to $\tilde{\rho}_{MN}$.

The VEVs of $H_{\alpha\beta\gamma}$ through Dirac string singularities and of $\bar{\chi} \gamma_{\mu} \gamma_{5} X$ through a gluino condensation provide the effective 4-dimensional SUSY breaking potential, which is

$$V = \frac{4 \lambda^{3}}{\lambda^{4}} \left( b - \frac{e^{3}}{2} \right)^{2},$$

(2.13)

where constants $c$ and $b$ are defined by

$$H_{\alpha\beta\gamma} = c \rho_{PL} \epsilon_{\alpha\beta\gamma} \text{ and } \bar{\chi} \gamma_{\mu} \gamma_{5} X = \sqrt{2} \epsilon_{\mu

(2.14)

respectively. $\rho_{PL}$ and $\mu$ are the Planck mass and the energy scale of condensation, $\hat{b}$ is a constant of order unity and is fixed by a gauge group. The effective 4-dimensional gauge coupling constant $g_{4}$ is given by $g_{4}^{2} = b - e^{3}/2$ and $b_{0}$ is an axion.

The potential is also derived from the Kahler potential

$$V = - \ln (S+\bar{S}) - 3 \ln (T+\bar{T}) + \ln |\omega|^{2},$$

(2.15)

with

$$S = b \epsilon + \exp(-3S/2b),$$

(2.16)

where $S$ and $T$ are chiral superfields defined by

$$S = b \epsilon + \frac{3}{2} \ln b$$

(2.17)

and

$$T = \frac{1}{2} \epsilon^{2} + \frac{3}{2} \ln b,$$

(2.18)

with $\epsilon$ being another axion.

The term in $T$ is responsible for the no-scale structure and guarantees the positive semi-definiteness of the potential.

As was discussed by Rohm and Witten, the VEV of $H_{\alpha\beta\gamma} \epsilon$, is quantized through the Wess-Zumino interaction in the string action as

$$c = n c_{0},$$

where $n$ is an integer and $c_{0}$ is some constant determined by the geometry of internal space. $n$ can be changed by quantum mechanically $H_{\alpha\beta\gamma}$ to the Yang-Mills Chern-Simons term.

Neglecting the $I$-term, one can obtain the 4-dimensional Lagrangian as

$$L \sim \sqrt{\tilde{\rho}} \left\{ \frac{1}{2} \rho \left[ \frac{1}{2} \rho \right]^{2} \right\} - \frac{1}{2} \ln b.$$  

(2.19)

where

$$\rho = \frac{1}{2} \ln \left[ \frac{\text{Re} (S)}{\text{Re} (T)} \right] - \frac{1}{2} \frac{1}{2} \left( 6 \ln b - \frac{e^{3}}{2} \right),$$

(2.20)

and

$$\text{Re} (S) = \frac{1}{2} \ln \left[ \frac{\text{Re} (S)}{\text{Re} (T)} \right] + \frac{1}{2} \left( 6 \ln b - \frac{e^{3}}{2} \right),$$

(2.21)

and

$$\text{Re} (S) = \frac{1}{2} \ln \left[ \frac{\text{Re} (S)}{\text{Re} (T)} \right] - \frac{1}{2} \left( 6 \ln b - \frac{e^{3}}{2} \right),$$

(2.22)

Here, we have neglected the imaginary parts of $S$ and $T$, i.e. the axions, although those may play important roles in the early stage, especially in the case that $H_{\alpha\beta\gamma}$ appears at the compactification scale and the gluino condensation occurs at some scale much smaller than the Planck scale. Because in that case the potential minimum locates at $\alpha = 0$, then the universe seems to
roll over to CO before the gluinos condense. The universe cannot reach a preferential minimum discussed later. The imaginary part of the complex chiral field $S$, however, provides a suitable barrier in the potential for its real part ($\phi$-field) and prevents $\sigma$ from going away to $\infty$, if the kinetic energy is of order of the Planck scale at compactification. This barrier will disappear later as a $-6$. The details on effects of imaginary parts of chiral complex fields will be discussed elsewhere.

The Lagrangian (2.19) is just the same as that of the 4-dimensional gravity theory with two scalar fields whose potential is $V(\sigma,\chi)$. The potential $V$ is shown in Fig.1.

Before SUSY is broken ($c = h = 0$), the theory is the Brans-Dicke theory because massless scalar field $\ln b$ is coupled with the metric (2.1). Analytic solutions with $c = h = 0$ are discussed in Ref.9.

In the next section, first fixing a constant $c$ we will show that the minimum of the potential ($\sigma = \sigma_0$), which provides the no-scale model, is an attractor.

§.3 The Friedmann universe and the Einstein gravity as an attractor

In this section, we assume that $c$ is constant during the evolution of the universe. This assumption is justified if the tunnelling probability is low enough, i.e. if the time scale of transition is much larger than the dynamical one (~ a Hubble expansion time). $c$ can also be constant during a period between two phase transitions even if quantum transitions cannot be neglected in the whole history of the universe.

We consider time-dependent homogeneous cosmological solutions of the system given by the Lagrangian (2.19). The ansatz of the metric form and scalar fields are the following:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu - dt^2 + a(t)^2 \eta_{ij} dx^i dx^j \] (3.1)

\[ \phi = \phi(t) \quad \sigma = \sigma(t) \]

(3.2)

From Eq.(7.19), we find the basic equations as follows:

(1) the Einstein equations

\[ 3H^2 = \kappa^2 \left[ \frac{\dot{\phi}^2}{2} + V(\sigma,\chi) + \frac{1}{2} (\dot{\sigma}^2 + \dot{\chi}^2) \right] \] (3.3)

(2) the dynamical equation

\[ 3H = \kappa^2 (\dot{\phi} \dot{\phi} - \dot{\sigma}^2 - \dot{\chi}^2) \] (3.4)

(2) the equations for scalar fields $\sigma$ and $\chi$

\[ \dot{\sigma} + 3H \sigma = -\frac{3V}{\dot{\phi}} \] (3.5)

\[ \dot{\chi} + 3H \chi = -\frac{3V}{\dot{\chi}} \] (3.6)

where $H = \dot{a}/a$ and an overdot denotes the derivative with respect to $t$. In the above equations, we have also added a 4-dimensional matter fluid, whose energy density and pressure are given by $\rho$ and $p$.

At the minimum of the potential $V$, given by

\[ \sigma = \sigma_0 = \frac{1}{\sqrt{3}} \ln \left( \frac{2\Lambda}{3} \ln \left( \frac{b_0}{c_0} \right) \right) \] (3.7)

and

\[ \chi = \chi_0 \quad \text{(arbitrary constant)} \] (3.8)

Eqs.(3.5,6) are satisfied automatically, and the cosmological constant vanishes, giving a Friedmann solution of the Einstein equations (3.3,4).

We shall prove that the Friedmann solution with Eqs.(3.7,8) is always one of the attractors if the 3-space is expanding. We introduce a new time
Define the "energy" \( E \) and the "potential" \( U(\sigma) \) of the dynamical system for \( \sigma \) by
\[
E = \frac{1}{2} \left( \frac{d\sigma}{dt} \right)^2 + U(\sigma),
\]
(3.10)
and
\[
U(\sigma) = \frac{4}{m^2} \sigma^2 \exp \left(-\frac{3}{2b} \xi \sigma^2 \right) \left( \frac{d\sigma}{dt} \right)^2.
\]
(3.11)

Then, Eq. (3.5) can be rewritten in terms of \( E \) and \( \dot{\sigma} \) as
\[
\frac{dE}{dt} = -3 \left( \frac{da}{a} - \kappa \frac{d\xi}{\xi} \right) \left( \frac{d\sigma}{dt} \right)^2.
\]
(3.12)

From the constraint equation (3.3),
\[
|H| > \kappa |\dot{\xi}|/\xi^2.
\]
(3.13)

Therefore, if the 3-space is expanding \( H > 0 \) or \( H(t) > 0 \) at some epoch \( t = t_0 \),
\[
\left( \frac{da}{a} - \kappa \frac{d\xi}{\xi} \right) \left( \frac{d\sigma}{dt} \right)^2
\]
is always positive, then the system is always dissipative \( (dE/dt < 0) \). The potential \( U(\sigma) \), which is a cross-
section of the potential \( V(\sigma, \xi) \) for a fixed \( \xi \), is shown in Fig.2(a). Therefore, once the universe is trapped in the shadow region \( T_0 \) in Fig.2 at any value of \( \xi \), then the universe always approaches the minimum at \( \xi = \xi_0 \) along the dotted line \( A \) in Figs.1 and 2(a), or along the spiral lines \( A \) or \( A' \) in the phase diagram Fig.2(b). On the other hand, if the universe goes into the region \( T_{\xi=0} \) in Fig.2 at any value of \( \xi \), then \( \sigma \) always goes to infinity \( (H > 0) \), which gives the Brans-Dicke theory even in the low energy limit. Therefore, the potential minimum \( \sigma = \sigma_0 \) fixes only \( \sigma \) but not \( \xi \). In the \( \xi \)-direction, the potential minimum is degenerate, i.e. \( \xi \) is still the massless scalar field.

It is worth noting that the "energy" of the system for \( \sigma \) and \( \xi \) decreases with time in the potential \( V(\sigma, \xi) \) if the 3-space is expanding \( (H > 0) \). That is to say, if we define the energy by
\[
E = \frac{1}{2} \left( \dot{\sigma}^2 + \xi^2 \right) + V(\sigma, \xi),
\]
(3.14)
then, from Eqs. (3.5, 6), we obtain
\[
E = -3\kappa \left( \sigma^2 + \xi^2 \right) < 0.
\]
(3.15)

In that case, by losing energy \( E \), the universe may reach either the region \( T_0 \) or the region \( T_{\xi=0} \) at some value of \( \xi \) (see Fig.2). Therefore, the minimum \( \sigma = \sigma_0 \), which represent the no-scale model, is one of the attractors in this model. In Fig.2(b), one can see that the minimum point \( (\sigma_0, 0) \) is a nice attractor.

We shall show, next, that this minimum also guarantees the Einstein gravity in the low energy limit. The world interval in 4-dimensions is defined by
\[
d\tilde{s}_{10}^2 = b^{-6} ds^2_{10} = b^{-6} g_{mn} dx^m dx^n,
\]
(3.16)
\[
= \exp(\sqrt{2/3} \sqrt{1 + \xi} \sigma) g_{mn} dx^m dx^n.
\]

The potential minimum \( \sigma = \sigma_0 \) fixes only \( \sigma \) but not \( \xi \). In the \( \xi \)-direction, the potential minimum is degenerate, i.e. \( \xi \) is still the massless scalar field. Therefore, the theory at this minimum does not seem to be the Einstein gravity.

However, if we assume that the world interval is defined in the field-theory limit of the superstring theory, i.e. by the world interval before the Weyl-rescaling in terms of the dilaton field, then the conformal factor \( e^{\Phi/2} \) must multiply the world interval \( ds_{10}^2 \) in the 10-dimensional supergravity theory.

Accordingly, the proper world interval is
\[
d\tilde{s}_{10}^2 = e^{\Phi/2} \left( b^{-6} ds^2_{10} + b^2 \frac{d\sigma^2}{\xi^2} \right)
\]
\[
= \exp(-\sqrt{2/3} \sqrt{1 + \xi} \sigma) g_{mn} dx^m dx^n + e^{(2/3) \sigma} \tilde{g}_{10} \tilde{dx}^2.
\]
(3.17)

Then, the world interval in 4-dimensions is essentially given only by \( g_{mn} \) at the potential minimum \( \sigma = \sigma_0 \) because the conformal factor is fixed in the low energy limit. The theory in 4-dimensions becomes just the Einstein theory.
The radius of the internal space is given by \( r = \frac{\kappa t}{\rho} \).

In this picture, before the SUSY breaking potential appears, i.e., before the local supersymmetry is broken, we have a Brans-Dicke theory \( \omega = -1 \) and \( \Phi \) is the Brans-Dicke scalar. Afterwards, in the low energy limit, we have the Einstein gravity. If the universe evolves towards the minimum at infinity rather than towards the minimum at \( \sigma = \sigma_0 \), we have still a Brans-Dicke theory, in the low energy limit, which must be excluded.

§.4. Quantum Tunnelling and Creation of a Hierarchical Bubble Structure

In this section, we consider the case that the tunnelling process from one value of \( c \) to the other is fast enough that this process cannot be neglected in the evolution of the universe. We can imagine two cases: one is that the transition time is extremely short compared with a dynamical time \( \sim H \) (a Hubble expansion time). The other case is that both time scales are of same order of magnitude. We consider the evolution of the universe in both cases separately.

First, define potentials \( \phi_n \) by

\[
\phi_n(c) = \frac{\kappa}{c^a} \exp(-\frac{1}{2}c^2\sigma^2), \quad n = 0, 1, \ldots, n_{\text{max}},
\]

where \( \kappa = h/c_0 \).

\( \phi_n \) is the cross section of the total potential \( V \) with \( c = c_0 \) at \( \sigma = 0 \). The schematic shapes of \( U \) and \( U_{n+1} \) are shown in Fig.3 to explain what happens at the phase transition.

When the universe is in the region \( I \) in Fig.3, \( U \) is smaller than \( U_{n+1} \), then no phase transition from the n-vacuum to the \((n+1)\)-vacuum occurs. We call a state with \( c = c_0 \), \( \omega = n \)-vacuum. We assume here that the tunnelling occurs only for the case that the potential in the present vacuum is higher than that in the other vacuum (e.g. \( n\)-vacuum in the region \( I_{n+1} \) in Fig.3).

As the universe evolves beyond a point \( A \) in Fig.3, new potential \( U_{n+1} \) becomes smaller than \( U_n \), then the phase transition takes place at some point after \( A \). Since the transition probability for the whole universe to change uniformly the value of \( n \) by \( \pm 1 \) is zero, we expect bubble formation. \( 16,23 \)

\((n+1)\)-vacuum bubbles are formed in old \( n \)-vacuum phase. The bubble may expand with light velocity \( 23,24 \).

First, we consider the case that the transition time is very much shorter than the dynamical time \( \sim H \). The phase transition occurs immediately after the point \( A \). Many small bubbles are formed and collide with each other. The whole universe evolves into new \((n+1)\)-vacuum phase, just as the quark-hadron phase transition. Bubble structure may be destroyed. In new phase, the universe evolves under the influence of new potential \( U_{n+1} \), because the value \( c \) does not change until next phase transition. The 'effective' potential for this system is given by the minimum of \( \{U_n(\sigma)\} \) defined by

\[
U_{\text{eff}}(\sigma) = \min \{ U_n(\sigma) \ | \ n = 0, 1, \ldots, n_{\text{max}} \}
\]

where \( n_{\text{max}} = \left[ \frac{h}{c_0^2} \right] \). \( U_n \) for \( n > n_{\text{max}} \) has no extremum and does not take the smallest value for any fixed value of \( \sigma \). We need not take into account \( U_n \) with \( n > n_{\text{max}} \).

The evolution of the universe after the gluino condensation is influenced by the potential \( U_{\text{eff}} \) in this case. At each cusp of the potential, the universe changes its phase from some integer-vacuum by \( \pm 1 \) (see Fig.4). We can repeat similar discussions using new potential \( U_{\text{eff}} \) instead of \( U_n \) or \( V \) as in §.3.

Since the universe cannot escape from trapped regions (shaded regions in Fig.4) as discussed in §.3, the universe experiences new phase transition after...
Either the universe settles down to one of the potential minima, where the Friedmann universe is realized and the 4-dimensional Einstein gravity is guaranteed; or, it misses all the minima and go away to \( r = \infty \), in which case one finds a different type of universe and the Brans-Dicke gravity.

Next, we consider the case that the transition time is comparable with the Hubble expansion time, in which case things are much more complicated.

We assume that when the gluinos condense, the universe is first in an \( n_0 \)-vacuum state \( ( \epsilon - n_0 ) \). As the universe evolves as is shown in Fig.5(a), it reaches a point A and experiences a phase transition. Many \((n+1)_0\)-vacuum bubbles are formed in the \( n_0 \)-vacuum during the evolution after the point A. Since two time scales are almost same, it is likely that the phase transition does not finish and then a bubble structure is not destroyed in contrast with the previous case. If we can neglect the boundary effect of bubble surface on the Einstein equations, the evolution of the universe after transition can be described by Eqs.(3.3-6) with the potential given by

\[
V = \begin{cases} 
 e^{\frac{\sqrt{2} \mathcal{G}}{G^*}} u_n(\sigma) & \text{for old phase} \\
 e^{\frac{\sqrt{2} \mathcal{G}}{G^*}} u_{n_0}(\sigma) & \text{for new bubble phase} 
\end{cases}
\]

(4.3)

One can discuss the evolution of each phase independently. The bubble phase evolves along the dotted line in Fig.5 (a) under the influence of the potential \( u_{n+1} \), whilst the old phase does along the solid line under the old potential \( u_{n_0} \). After some period, the bubble universe may reach again new phase transition point B. New \((n+1)_0\)-vacuum bubbles are formed in \( (n_0)_{-1} \)-vacuum bubbles in the same way as the above. In old \( n_0 \)-vacuum, creation of bubbles is going on. However, if this phase does not disappear before this part of the universe reaches a new transition point B, then a similar phase transition occurs, i.e. new \((n_0-1)\)-vacuum bubbles are formed in \( n_0 \)-vacuum. Repeating these phase transitions, the universe finally finds a hierarchical bubble structure as is shown in Fig.5(b). Some part of the universe (e.g. \( n \)-vacuum bubble) reaches one of the trapped regions of the potential \( u_{n_0}^{\text{eff}} \). The phase transition in this bubble (or old \( n_0 \)-vacuum) ceases and the bubble approaches the minimum in the trapped region.

In this scenario, some bubbles can reach some of the minima of \( u_{n_0}^{\text{eff}} \) and some other bubbles may miss all minima and go away to \( r = \infty \). In each bubble, \( G_0 \) and \( \kappa' \) take different values (because both fundamental constants depend only on \( \phi \)-field), and these are determined by a given integer \( n \). This hierarchical bubble structure, if it remains, may give rise to the so-called 'domain wall' problem. In that case, inflation after reaching the potential minimum will be desirable.

If inflation occurs at the potential minimum but not at \( r = \infty \), most of the universe is in one of other of potential minima at present. The probability that we find ourselves in a preferential minimum becomes much higher.

Now, we shall discuss the whole story of the universe, taking into account discussions in §§3 and 4., in next section.
§.5. A preferential scenario in a higher-dimensional cosmology

Towards the Friedmann universe via inflation

In this section, we briefly discuss a preferential scenario of the universe in our higher-dimensional theory. As previously mentioned in Ref.9, it may be difficult to have any inflation (even including a Kaluza-Klein type) which occurs before the SUSY breaking because of the lack of a 'potential'. Since inflation is very desirable in modern cosmology, we should look for inflation in the present model too. It seems that no-scale inflation (primordial or chaotic inflation) could be possible because the present model contains the no-scale structure. Then, we might consider the following scenario (see Fig.6).

(1) The era before the gluino condensation

Which initial condition we should imply on the value of $c$? When the internal space is compactified, $c$ may appear through non-trivial configuration of $B^\mu$. If the tunneling probability is small enough, $c$ may not change before the gluino condense. In this case, in order for $\sigma$-field to stay around the preferential minima when the gluino condensation appears, the imaginary part of the chiral field $S$ (axion) must be taken into account as briefly discussed before, and the details will be given elsewhere (see also Ref.21).

On the other hand, if the transition probability is high enough, $c$ vanishes soon even if it appeared at compactification. Because the potential $V_{\text{non-min}}$ (i.e. $c=0$) takes the smallest value for any fixed value of $\sigma$. Then, $c=0$ before the gluino condensation in this case. $\sigma$ must reappear at the gluino condensation through the quantum tunnelling process. We discuss the behaviour of the universe in this case, i.e. $c=\sigma=0$, here.

If the universe starts in a high-temperature thermal equilibrium state, then it may be initially supersymmetric. The real initial state might be at the string-theory level, rather than in the field-theory limit. In this epoch, if the universe contains the 4-dimensional radiation ($\rho_4$, $\omega_4$), $a$ approaches the scale factor of the Friedmann universe ($a_4 \propto t^{2/3}$) and $b$ becomes some constant. One can find similar behaviours of the universe, i.e. all solutions approach $[F^4$ with $P=([1-\delta])P_0]X$, for more general cases in which the equation of state is $P=([1-\delta])P_0$ ($0<\delta<2$). If the universe contains the 10-dimensional radiation ($P=\rho/9$), both scale factors $a=b^{-3}$ and $b$ expand similarly (i.e. $a,b \propto t^{1/5}$). If the temperature is sufficiently high that massive modes of strings becomes important, then the equation of state must be changed. If $a\gg b$, the equation of state is the 4-dimensional one, then the behaviour of $a$ is the same as described in Ref.28 and $b$ approaches some constant asymptotically. If $a\sim b$, the equation of state is the 10-dimensional one, and both scales $a$ and $b$ expand similarly as in the 10-dimensional Friedmann universe. At this stage, we have the Brans-Dicke theory.

If the gluino condensation occurs at almost the Planck scale, an alternative scenario from the superstring era to the SUSY breaking might be possible as follows. (There is no Brans-Dicke era). Since quantum gravity effects may still not be negligible at the SUSY breaking stage, we may predict an initial state of the universe using the idea of a quantum creation of the universe. For large negative values of $\sigma$ and $\xi$, the (classical) field-theory limit is not valid. Then, if one can assume that the universe is created from a 'string region', the classical universe may appear with a large probability in a higher part of the potential but not in the 'string region',
i.e., near the points $1_1$ or $1_2$ in Fig. 7 (a). The universe may reach our potential minimum ($\sigma^2 \sigma'_0$) much more easily. Our present universe might be almost unique. The details will be discussed elsewhere.

(2) Towards the Friedmann universe via inflation

When the temperature drops below some critical value ($\sim H_*$), the gluinos condense and the local supersymmetry is broken. After this SUSY breaking potential appears, the universe may approach its minimum as discussed in §3 and 4. A hierarchical bubble structure might be created. The minimum furnishes a no-scale model with zero cosmological constant at least at tree level. A potential for $\mathcal{T} (\sim \ln|\text{Re}(T)|)$ appears due to non-gravitational radiative corrections of the matter fields. It determines the present value of $\text{Re}(T)$, at which the cosmological constant vanishes, i.e., the absolute minimum. (It is marked by $X$ in Fig. 1). When the universe reaches the minimum of the tree SUSY breaking potential ($\sigma^2 \sigma'_0$) it is not necessary at the absolute minimum. The VEV of some field, which might be $\text{Re}(T)$ itself or another inflaton, may provide a non-zero cosmological constant, and then the universe experiences 'no-scale' inflation, rolling down its flat potential towards the absolute minimum ($X$ in Fig. 1) along the line $\sigma^2 \sigma'_0$, then we find the (nearly) flat present Friedmann universe.

Since $\mathcal{T}$ (or $\text{Re}(T)$) is Polyakov field, it is worth noting that inflation must take place after $\mathcal{T}$-field is fixed or $\mathcal{T}$-field itself is responsible for inflation. Otherwise there is an 'entropy crisis' after inflation. Using $\mathcal{T}$-field, the possibility of inflation is discussed in Ref. 33. Another consequence of the necessity of inflation at this stage is that if the hierarchical bubble structure is formed one may find a 'domain wall' problem unless each bubble expands to one universe by inflation.

We should investigate further whether sufficient inflation really can occur in the context of the superstring model, and whether the inflaton interacts with matter fields in such a way that reheating can occur with the generation of the baryon asymmetry. Also, we have not estimated the transition probability in this paper, although such a calculation is very important and should be done in near future because the history of the universe depends much on this value as we have seen here.
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The no scale model was first proposed by J. Ellis, A.B. Leanchor, D.V. Nanopoulos


17. The indices $x,y,...$ run from 0 to 3 and $5$ to 10, while $m,n,...$ from
    0 to 3 and $N,...$ from $5$ to 10. Our signature and notations are the
    same as those of “Gravitation” by C.W. Misner, K.S. Thorne and J.A.
    Wheeler (Freeman, San Francisco, 1973).

18. $G = \frac{\hbar G}{\mathfrak{R}}$, where $G$ is the Newtonian gravitational constant. We normalize
    the scale of the internal space by this equation.


21. The inflationary model was first proposed by J. Ellis, A.B. Leanchor, D.V. Nanopoulos


24. So far, there are a few discussions on inflation in a superstring.


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The SUSY breaking potential due to a gluino condensation. The universe can reach its minimum \( \mathcal{V} = \mathcal{V}_0 \) along the dotted line A if the 3-space is expanding. The radiative corrections determines the present value of \( \mathcal{T} \), which is marked by \( X \). The universe may reach its value after enough inflation.

The potential of the dynamical system for \( \mathcal{V} \). If the universe is trapped in the region \( T_w \), losing its energy, the universe can reach the potential minimum \( \mathcal{V} = \mathcal{V}_0 \), which provides no-scale model and guarantees the Einstein gravity theory. (The dotted line A). If the universe is trapped in \( T_{BD} \), we have still a Brans-Dicke theory. (The dotted line B).

The schematic phase diagram of \( Q_{CT} \). When the universe is trapped in the region \( T_{E} \), the universe approaches the point \( (Q_{CT}, 0) \) along spiral lines \( A \) or \( A' \), and then the \( F \times K \) space-time. If the universe is trapped in \( T_{BD} \), \( C \to \infty \) along lines \( B \) or \( B' \). The dot-dashed line is the separator of the no-scale model and guarantees the Einstein gravity theory.

Schematic shapes of potentials \( U_n \) and \( U_{n+1} \). When the universe is in the region \( I_n \), the \( n \)-vacuum state is stable, and no phase transition occurs before the point \( A \). As the universe evolves along the solid line, it reaches the point \( A \). At some point (marked by \( n \)), \( (n+1) \)-vacuum bubbles are formed in the \( n \)-vacuum state. Bubbles evolve along the solid line, whilst the old phase does along the dotted line.

The evolution of the universe in the case that the transition probability is extremely high. (The case of \( n = 4 \) is shown). The universe evolves along the solid line. Phase transitions occur at each cusp of the potential \( U_{\text{eff}} \). Eventually the universe which starts with \( n = 1 \) settles down to the \( n = 3 \)-vacuum phase. Once the universe is trapped in one of the shaded regions, the phase transition does not occur any more.

The evolution of the universe and creation of bubble structure in the case that the transition time is comparable with the dynamical time. The universe starts with uniform \( n \)-vacuum phase and evolves beyond the point \( A \). The \( (n+1) \)-vacuum bubbles are formed in the \( n \)-vacuum phase. The bubbles (the dotted line) and the old \( n \)-vacuum phase (the solid line) evolve under the influence of the potentials \( U_{n+1} \) and \( U_n \). When the bubbles reach the point \( B \), new bubble creation takes place. New \( (n+2) \)-bubbles evolve along the dashed line. The similar creation phenomena happen when the old \( n \)-vacuum phase reaches the point \( B' \). Each bubble or the old \( n \)-vacuum phase approaches the potential minima. (The case of \( n = 4 \) and \( n = 2 \) is shown).

A hierarchical bubble structure. The hatched, the cross-hatched, the black-shaded, the dot-hatched, and the dotted regions denote \( (n+1) \)-, \( (n+2) \)-, \( (n+3) \)-, \( (n-1) \)-, and \( (n-2) \)-vacuums, respectively.
Fig. 6  One preferential scenario towards the Friedmann universe via inflation.
Before the SUSY breaking, there are several possibilities in the evolution of the universe (see the text). During an inflationary stage, the radius of the internal space is also changing along the line $\sigma = \sigma_0$. After reheating of the universe, the Friedmann universe is coming out in 4-dimensions while the radius of the internal space settles down to be constant.
potentials $U_1, U_2, U_3$
Fig. 6