IC/86/115

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

GLOBAL ANOMALIES IN SIX DIMENSIONS

E. Bergshoeff

T.W. Kephart

Abdus Salam

and

E. Sezgin

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1986 MIRAMARE-TRIESTE

IC/86/115

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

GLOBAL ANOMALIES IN SIX DIMENSIONS *

E. Bergshoeff
International Centre for Theoretical Physics, Trieste, Italy,

T.W. Kephart
Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37275, USA,

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy,
and
Imperial College, London, England,

and

E. Sezgin
International Centre for Theoretical Physics, Trieste, Italy.

MIRAMARE - TRIESTE
June 1986

* To be submitted for publication.

Abstract

Applying Witten's formula for global gauge and gravitational anomalies to six dimensional supergravities, we find: (a) The perturbatively anomaly free N=4 chiral supergravity coupled to 21 tensor multiplets is global anomaly free for any choice of space-time manifold with vanishing third Betti number (b_3). (b) The perturbatively anomaly free matter coupled N=2 chiral supergravities with arbitrary number of tensor multiplets, whose Yang-Mills gauge groups do not include G_2, SU(2), or SU(3) are free of global anomalies if the theory is formulated on S^6. In the case of 9 tensor multiplets coupled to supergravity, this result holds for any spacetime with vanishing b_3. (c) The N=6 chiral supergravity has perturbative gravitational anomalies, and therefore the global anomalies need not be considered in this case.

1. Introduction

Supergravity theories are candidate low energy limits of superstring theories. Consistency of these theories in four or higher dimensions requires absence of anomalies [1]. The chiral N=2, d=10 supergravity [2] was shown by Alvarez-Gaume and Witten to be free of perturbative gravitational anomalies [3] †. In the case of Yang-Mills coupled N=1, d=10 supergravity [6], as was shown by Green and Schwarz [7], cancellations of perturbative gauge and Lorentz anomalies is possible only for SO(32) [7], $E_8 \times E_8$, $E_8 \times U(1)^{248}$ and $U(1)^{496}$ [7][8].

The cancellation of perturbative anomalies is necessary but not sufficient to provide a consistent theory. One must also face the issue of nonperturbative (i.e. global) anomalies. For example, as was pointed out by Witten [9], an SU(2) gauge theory with an odd number of Weyl doublets in four dimensions is perturbatively anomaly free; however, the theory is mathematically inconsistent since it suffers from a global gauge anomaly. In addition to the global gauge anomalies, global gravitational anomalies can arise [3][10][11][12].

Witten has shown that in ten dimensions global anomalies are absent, both in the chiral N=1 and N=2 supergravity theories [10]. In the context of supergravity theories the only other global anomalies studied so far are the world-sheet anomalies in two dimensions. They have been shown to cancel [11] in the two dimensional models corresponding to the

† A potentially dangerous U(1) σ-model anomaly [4] has also been shown to cancel by Marcus [5].

heterotic string in ten dimensions [13].

In this note we discuss the perturbative as well as global gauge and gravitational anomalies for N=2,4,6 and 8 , d=6 supergravities. In fact, although the N=8 supergravity of [14] is chiral with respect to the automorphism group SO(5)xSO(5), it is vectorlike with respect to the gravitational interactions. Therefore this theory is anomaly free, and will not be discussed further.

The next theory to consider is the chiral N=6 supergravity. Its automorphism group is USp(4)xUSp(2), and its field content is [15]:

Field	$g_{\mu\nu}$	$\psi_{\mu L}$	$\psi_{\mu R}$	$B_{\mu\nu}^{+}$	$B_{\mu\nu}^{-}$	χ_L	χ_R	A_μ	ϕ
Irreps.	(1,1)	(1,2)	(4,1)	(1,1)	(5,1)	(5,2)	(4,1)	(4,2)	(5,1)

It is easy to show that the d=6, N=6 supergravity with this field content suffers from perturbative gravitational anomalies incurable by Green-Schwarz mechanism. Therefore we shall not consider this theory any futher. We devote the rest of the paper to the N=4 and N=2 supergravities.

There exists only one perturbatively anomaly free chiral N=4 supergravity in six dimensions. It arises from the K_3 compactification of the chiral N=2, d=10 supergravity [16]. The field equations and the supersymmetry transformations, though not the lagrangian, of the d=6 theory have been constructed by Romans [17]. We will show that this theory is free from global gravitational anomalies.

The chiral d=6, N=2 supergravity theories have a much richer structure. They allow couplings of tensor, Yang-Mills and scalar multiplets [18]. Examples of perturbatively anomaly free chiral matter coupled N=2 supergravities have been given in [19], [20] and [21]. Here we point out that these examples are also free from global gravitational anomalies. We also provide new examples of perturbatively as well as globally anomaly free theories. In the following we shall discuss the anomaly cancellations in the N=4 and N=2 theories, respectively.

2. Perturbative anomalies in N=4, d=6 supergravity

There exist two basic N=4 chiral multiplets given by [16]

$$\begin{array}{ll} \text{Supergravity multiplet} & g_{\mu\nu},\quad \Psi_\mu^{\ i},\quad B_{\mu\nu}^{+[ij]} \\ \text{Tensor multiplet} & B_{\mu\nu}^{-},\quad \chi^i,\quad \phi^{[ij]} \end{array} \tag{1}$$

where the gravitino is left-handed and the spinor of the tensor multiplet is right-handed (symplectic) Majorana-Weyl. The field strength of $B_{\mu\nu}^{+}$ is selfdual. The automorphism group is Usp(4). The perturbative anomalies can be obtained by descent equations [22] from an 8-form polynomial P. The contributions of a complex left-handed spinor, a complex left-handed Rarita-Schwinger (RS) spinor and a real self-dual tensor field are respectively given by [3]

$$\begin{aligned} I_{1/2} &= \hat{A}(R) \\ &= 1/5760 \left[\operatorname{tr} R^4 + 5/4(\operatorname{tr} R^2)^2 \right] \\ I_{3/2} &= \hat{A}(R) \left[\operatorname{tr}(\cos R - 1) + d - 1 \right] \\ &= \hat{A}(R)(\operatorname{tr}\cos R - 1) - 2\hat{A}(R) \\ &= 1/5760 \left[245 \operatorname{tr} R^4 - 215/4(\operatorname{tr} R^2)^2 \right] \\ I_1 &= - L(R)/8 \\ &= 1/5760 \left[28 \operatorname{tr} R^4 - 10(\operatorname{tr} R^2)^2 \right] \end{aligned} \tag{2}$$

Here $\hat{A}(R)$ is the Dirac genus, L(R) is the Hirzebruch polynomial, d is the dimension of spacetime (d=6 in our case). Thus for 4 left-handed symplectic Majorana-Weyl gravitini, 4 k right-handed symplectic Majorana-Weyl spinors χ, and k tensor multiplets the total anomaly polynomial is

$$\begin{aligned} P_{total} &= 2\, I_{3/2} - 2k I_{1/2} - (k-5)\, I_1 \\ &= (k-21)/\,5760 \left[-30 \operatorname{tr} R^4 + 15/2(\operatorname{tr} R^2)^2 \right] \end{aligned} \tag{3}$$

Therefore for k=21 the theory is perturbatively anomaly free [16]. Note that with this value of k, eq. (3) implies the relation

$$\sigma = - \text{index}\,(RS) + 23\ \text{index}\,(D) \tag{4}$$

which is valid for any closed 8 dimensional spin manifold. In (4) we have used the standard definitions

$$\text{Index}(D) = \int \hat{A}(R)$$

$$\text{Index}(RS) = \int \hat{A}(R)\,(\text{tr}\cos R - 1) \qquad (5)$$

$$\sigma = \int L(R)$$

3. Global anomalies in N=4, d=6 supergravity

The global anomaly is defined by the change in the effective action under the diffeomorphism π of spacetime M, which is not continously connected to the identity [10]:

$$\Delta I = I(g_{\mu\nu}^{\pi}) - I(g_{\mu\nu}) \qquad (6)$$

Global anomaly freedom requires that $\Delta I = 0 \bmod 2\pi i$.

Following Witten [10], for a six dimensional supergravity theory with net N_D left-handed Majorana-Weyl spinors, net N_R left-handed gravitini and net N_S self-dual tensor fields, for ΔI we use the formula

$$\Delta I = \pi i/2\,[\,N_D\,\eta_D + N_R(\eta_R - \eta_D) - N_S\,\eta_S/2\,] \qquad (7)$$

where the η-invariant is defined by

$$\eta = \lim_{\varepsilon \to 0} \sum_{\lambda_i \neq 0} (\text{sign}\,\lambda_i)\exp{-\varepsilon\lambda_i} \qquad (8)$$

Here λ_i are the eigenvalues of the Dirac operator on the mapping cylinder $(MxS^1)_\pi$. The latter is a closed manifold defined by multiplying M by the unit interval $I=[0,1]$ and gluing together the top and bottom of MxI by

identifying (x,0) with $(\pi(x),1)$ for any $x \in M$ †. The η-invariants associated with a Majorana-Weyl spinor, a Majorana-Weyl gravitino and a self-dual tensor field are respectively given by the Atiyah-Patodi-Singer theorem [23] as follows

$$1/2\,\eta_D = \text{index}(D) - \int_B I_{1/2}$$

$$1/2\,\eta_R = \text{index}(RS) - \text{index}(D) - \int_B (I_{3/2} + I_{1/2}) \qquad (9)$$

$$\eta_S = \sigma - \int_B L(R)$$

Although the indices, the Hirzebruch signature σ and the integrals are defined on a manifold B with boundary $(MxS^1)_\pi$, the η-invariants depend only on the boundary ∂B of B. Note that η vanishes on a manifold with no boundary. In that case from (9) one recovers the standard formulae for the index theorems and the Hirzebruch signature for a closed manifold.

Substituting (9) into (7) and recalling that in the N=4, d=6 supergravity coupled to 21 tensor multiplets we have $N_D=-4x21$, $N_R=4$ and $N_S=(5-21)$, we find the result

† To avoid subtleties associated with the zero modes of the self-dual tensor fields, we take M to be any six dimensional spacetime with vanishing third Betti number [10].

$$\Delta I = 2\pi i \left[-46 \text{ index}(D) + 2 \text{ index}(RS) + 2\sigma \right]$$

$$- 2\pi i \int_B \left[-42\, I_{1/2} + 2\, I_{3/2} - 16\, I_1 \right] \qquad (10)$$

The integral term contains precisely the combination of terms which arise in the perturbative anomaly [see (3)], and therefore it vanishes. Moreover, the terms in the first bracket in (10) are evidently multiples of integers. Therefore the action is invariant mod $2\pi i$, so we conclude that the N=4, d=6 chiral supergravity is free from global gravitational anomalies. We emphasize that this result is valid for any 6-dimensional space-time M with vanishing third Betti number. This is in contrast with the case of global anomalies in 10-dimensional N = 2 chiral supergavity where M is taken to be S^{10}. There ΔI is proportional to $\sigma/8$, and so far, only for $M = S^{10}$ it has been shown that $\sigma = 8$ mod 16, and hence $\Delta I = 0$ mod $2\pi i$ [10].

4. Perturbative anomalies in N=2, d=6 supergravity theories

In six dimensions the following N=2 supermultiplets exist [16]:

Supergravity	$g_{\mu\nu}$, $\psi_\mu^{\ A}$, $B_{\mu\nu}^{\ +}$	
Tensor	$B_{\mu\nu}^{\ -}$, χ^A , ϕ	(11)
Yang-Mills	A_μ , λ^A	
Hypermatter	ψ^a , ϕ^α	

The scalars ϕ^α parametrize a quaternionic manifold [24] of the form G/[HxSp(1)]. The possible G and H are listed in Table 1. All spinors are symplectic Majorana-Weyl. Specifically, $\psi_\mu^{\ A}$ and λ^A are left-handed, while ψ^a and χ^A are right-handed. The index A=1,2 labels Sp(1), and the index a labels one of the representations of H listed in Table 1.

Consider the coupling of k tensor multiplets, n hypermultiplets and a Yang-Mills multiplet with the gauge group $\mathbf{G} = G_1 \times G_2 \times \ldots \times G_r$ to supergravity. $\mathbf{G}$ may contain the automorphism group Sp(1) or its U(1) subgroup. Using (2)†, the vanishing of the leading perturbative gravitational anomaly (proportional to trR^4) requires the condition [20]

$$\dim \mathbf{G} - n - 29k + 273 = 0 \qquad (12)$$

Upon the use of this constraint the gravitational anomaly becomes

$$(k-9)/128 \ (trR^2)^2 \qquad (13)$$

This means that if $k \neq 9$, then the anomaly can be cancelled only by the Green-Schwarz mechanism.

Next we consider the pure gauge anomaly which is given by $tr\,\mathbf{F}^4/4!$ (d-1= 5 times this for the gravitino contribution), where $\mathbf{F}$ is the Yang-Mills curvature corresponding to $\mathbf{G}$, and the trace is over all the

---

† In (2) the expressions for $I_{1/2}$ and $I_{3/2}$ now must be multiplied by $tr\, e^{\mathbf{F}}$.

Table 1

$G/[H \times Sp(1)]$	H- Representation of ψ^a
$Sp(n,1)/Sp(n)\times Sp(1)$	$2n$
$SU(n,2)/SU(n)\times U(1)\times Sp(1)$	$n_q + n_{-q}$
$SO(n,4)/SO(n)\times SO(3)\times Sp(1)$	$(n,2)$
$E_8 / E_7 \times Sp(1)$	56
$E_7 / SO(12) \times Sp(1)$	32
$E_6 / SU(6) \times Sp(1)$	20
$F_4 / Sp(3) \times Sp(1)$	14
$G_2 / SU(2) \times Sp(1)$	4

irreducible representations of the fermions in the theory. For anomaly cancellation either $tr\, \mathfrak{F}^4$ must vanish or it must factorize as

$$tr\, \mathfrak{F}^4 = \sum_i^r tr\, F^{2(i)}\, tr\, F^{2(i)} \tag{14}$$

to allow the Green-Schwarz mechanism [7]. This factorization occurs for all the irreps of E_8, E_7, E_6, F_4, G_2, SU(3), SU(2), U(1), and for the $\underline{28}$ of Sp(4), the $\underline{28}$ of SU(8) and all the irreps of SO(2n) with the highest weight $(f_1, f_2, f_1-f_2, 0, 0, \ldots, 0)$ in the Gel'fand-Zetlin basis. Assuming that (14) is satisfied, and noting that the mixed anomaly is given by

$$(1/96)\, tr\, R^2\, tr\, \mathfrak{F}^2 \tag{15}$$

(19 times this for the gravitino contribution), the total anomaly polynomial, which is the sum of (13), (14), and (15) reads [19]

$$P_{total} = \sum_i^r \beta_{ij}\ tr\, F^{2(i)}\, tr\, F^{2(j)} \tag{16}$$

where β_{ij} is an $(r+1)\times(r+1)$ symmetric matrix, and $F^{(0)}$ is the Lorentz algebra valued Riemann curvature 2-form. In (16) the traces are in the fundamental representations of the gauge group. The criteria for anomaly cancellation *a la* Green-Schwarz are that the β-matrix has <u>only</u> two non-zero eigenvalues, and moreover the product of these two eigenvalues must be negative or zero [20]. In that case β_{ij} can be written as

$$\beta_{ij} = 1/2(\alpha_i \gamma_j + \alpha_j \gamma_i) \tag{17}$$

This fixes the two $(r+1)$-dimensional constant vectors α_i and γ_i. In order to cancel the anomaly one adds the counter term [19]†

$$\Delta\mathcal{L} = \sum_i^r \gamma_i\ tr\, F^{2(i)}\, B\ -\ 1/2 \sum_i^r \alpha_i\, \gamma_j\, \omega_i^0\, \omega_j^0 \tag{18}$$

where the ω_i^0 are the Chern-Simons forms, and B is the combination of

† Note that for $k \neq 1$, though very complicated, and not manifestly Lorentz invariant (modulo total derivative), a lagrangian $\mathcal{L}$ to which this counter term can be added, in principle, does exist.

$B_{\mu\nu}^{+}$ and any one of the k $B_{\mu\nu}^{-}$ tensor fields. We then take the remaining (k-1) $B_{\mu\nu}^{-}$ tensor fields to be inert under Yang-Mills gauge transformations, while we modify those of B as [19]

$$\delta B = -\sum_{i}^{r} \alpha_i \, \omega_i^{1} \qquad (19)$$

The 2-forms ω_i^{1} are defined by $\delta \omega_i^{0} = d\,\omega_i^{1}$. From (19) we are led to the definition of the gauge invariant field strength $H = dB + \sum_{i}^{r} \alpha_i \omega_i^{0}$. This H satisfies

$$dH = \sum_{i}^{r} \alpha_i \; \mathrm{tr}\, F^{2(i)} \qquad (20)$$

We now present several examples of theories which obey the criteria stated above. Some of these examples are new.

(a) **k = 1, G = $E_7 \times E_6 \times U(1)$, n = 456** [19]. Here U(1) is the subgroup of the Sp(1) automorphism group. The 456 complex hyperinos fit nicely into the pseudoreal <u>912</u> irrep of E_7.

(b) **k = 1, G = $E_6 \times E_6 \times E_6 \times SU(3)$, n = 486**. The hyperinos are in the [(27,1,1,3) + (1,27,1,3) + (1,1,27,3) + (3↔$\bar{3}$)] of **G**.

(c) **k = 5, G = $E_7 \times U(1)^5$, n = 266**. The hyperinos fit into 2 Weyl <u>133</u> of E_7

(d) **k = 9, U(1) x Any G, n=13 + dim G** [20].

(e) **k = 17, G = E_8, n = 28**. The hyperinos are in the pseudoreal 56 of E_7. In this case one can take the hyperscalar manifold to be $E_8/E_7 \times Sp(1)$.

5. Global anomalies in N=2, d=6 supergravity theories

We now turn to the global anomalies in the matter coupled d=6, N = 2 supergravities. Here, as Witten has shown [10], the global anomaly receives two more contributions in addition to the right hand side of (7). Denoting the contributions given in (7) by $(\Delta I)_{det}$, with $N_D = 2(\dim G - n - k)$, $N_R = 2$ and $N_S = (1-k)$, from (2), (7), (9), (12) and (16) one obtains

$$(\Delta I)_{det} = 2\pi i \left[\text{index}\,(R) + (28k - 274)\,\text{index}\,(D) + (k-1)\,\sigma/8 \right] - 2\pi i \int_B \sum \beta_{ij}\, \mathrm{tr}\, F^{2(i)}\, \mathrm{tr}\, F^{2(j)} \qquad (21)$$

The variation of the Green-Schwarz counterterm (18) under π, and the contribution of the Pauli-Villars regulator fields [10] yield the total result

$$(\Delta I)_{G.S} + (\Delta I)_{reg} = -2\pi i \int_{\partial B} H \sum_{i}^{r} \gamma_i \, \mathrm{tr}\, F^{2(i)} \qquad (22)$$

The global π transformation in this case refers to coordinate and/or gauge transformation. The total anomaly is given by

$$(\Delta I) = (\Delta I)_{det} + (\Delta I)_{G.S} + (\Delta I)_{reg} = 2\pi i \left[(k-1)\,\sigma/8 + \mu\,(B,\partial B) \right] \bmod 2\pi i \qquad (23)$$

where the topological invariant μ is defined by

$$\mu(B,\partial B) = \int_B \Sigma\, \beta_{ij}\, tr\, F^{2(i)}\, tr\, F^{2(j)} - \int_{\partial B} H\, \Sigma\, \gamma_i\, tr\, F^{2(i)} \quad (24)$$

It is worth noting that ΔI is independent of the choice of B. This can be seen by considering another choice B' and evaluating ΔI over B-B' which is closed.

Global gauge anomalies can arise only when $\pi_6(G) \neq 0$ [9]. The only such cases are: $\pi_6(SU(2)) = \pi_6(Sp(1)) = Z_{12}$; $\pi_6(SU(3)) = Z_6$; $\pi_6(G_2) = Z_6$. Assuming that G does not contain SU(2), SU(3) or G_2, the topological invariant μ given in (24) becomes

$$\mu(B,\partial B) = (k-9)/128\, [\int_B (tr\, R^2)^2 - \int_{\partial B} H\, tr\, R^2\,] \qquad (25)$$

Recall that $\partial B = (M\times S^1)_\pi$. We observe that *if k=9, then the total anomaly is $\Delta I = 0 \bmod 2\pi i$ automatically.* Therefore in this case the theory has no global gravitational anomaly for any 6-manifold M with $b_3=0$. To facilitate the computation of ΔI for $k \neq 9$, we choose M to be S^6 [10]. In that case $\partial B = (S^6 x S^1)_\pi = (S^6 x S^1) \# (S^7)_\pi$, where $(S^7)_\pi$ is any one of the 27 exotic 7-spheres [25], and the # refers to connected sum [10] i.e. cut out and discard a 7-disk from both $S^6 x S^1$ and $(S^7)_\pi$, then paste the manifolds together along these boundaries. To calculate ΔI we choose

$$B = B_{Milnor} \# S^6 x D_2 \ , \qquad (26)$$

where D_2 denotes the 2-disk, and according to Milnor's theorem [25], B_{Milnor} is a parallelizable 8 dimensional spin manifold which is bounded by $(S^7)_\pi$. On B_{Milnor} one can choose a connection such that R=0. Therefore with the choice (26) for B, we see that $\mu(B,\partial B)=0$. In ΔI there remains a term proportional to

$$\sigma(B_{Milnor} \# S^6 x D_2) = \sigma(B_{Milnor}) + \sigma(S^6 x D_2) \qquad (27)$$

One can show that $\sigma(S^6 x D_2)=0$. Moreover, due to Milnor's theorem [25] $\sigma(B_{Milnor})$ is divisible by 8. Therefore we conclude that for any choice of k, $\Delta I=0 \bmod 2\pi i$, and hence there are no global anomalies in any perturbatively anomaly free matter coupled d=6, N=2 supergravity with a Yang-Mills group other then Sp(1), SU(3) and G_2. In particular, of the examples we have listed below (20), the theories (a), (c), (d) and (e) are anomaly free, while the theory (b) is anomaly free for global SU(3). The case of local SU(3) requires further investigation.

6. Comments

It is important to note that the total global anomaly ΔI, which is defined to be the sum of (21) and (22), does not depend on the choice of B, and if the index terms are discarded, it changes only by a multiple of $2\pi i$. For example, consider an 8-manifold B', with the same boundary as

B, which is a connected sum of B and p copies of HP_2 each of which has signature 1. One can show that the signature of B' is (8+p). For $p \neq 0$ mod 8, this signature is not divisible by 8. However, on B' the topological invariant μ is no longer zero, and is such that ΔI is again 0 mod $2\pi i$.

If the gauge group G contains Sp(1), SU(3) or G_2, a very elegant way to cancel the global anomaly is to show that an H obeying $dH = \sum_{i}^{r} \gamma_i \, \mathrm{tr} \, F^{2(i)}$ can be defined not just on ∂B but also on B [12]. (In that case, using Stokes' theorem one easily shows that $\mu(B,\partial B)=0$). Alternatively, since the nonvanishing of π_6 for G= Sp(1), SU(3) or G_2 implies the existence of a G instanton over $(M\times S^1)_\pi$, one may then evaluate the integrals in ΔI for these Yang- Mills configurations [10].

For groups with nonvanishing π_5, such as SU(2) or SU(3), another way of analyzing the global gauge anomaly in d=6 is to study the zero modes of fermions in the presence of instantons in spacetime M [10].

We have not been able to rule out any of the perturbatively anomaly free d=6, N=2 theories. An interesting problem is to see whether the global anomalies in d=6, N=2 theories compactified on $\mathrm{Minkowski}_4 \times S^2$ [26] can do so, or whether conditions for their absence can lead to Dirac-like quantization of the field strength of the antisymmetric tensor field [10].

Acknowledments

We have greatly benefitted from discussions with J. Seade, J. Strathdee, and A. Verjovsky to whom we express our appreciation. One of us (T.W.K.) thanks the members of the particle theory group at the I.C.T.P. for generous hospitality while this work was in progress, and the Vanderbilt URC for support.

References

[1] S. Adler,Phys. Rev. 177(1969)2426;
J. Bell and R. Jackiw, Nuovo Cimento, 60A (1969)47;
W. A. Bardeen,Phys. Rev. 184(1969)1848;
P.H. Frampton and T.W.Kephart, Phys. Rev. Lett. 50(1983)1343;1347;
P.K. Townsend and G. Sierra, Nucl.Phys. B222 (1983) 493;
for reviews see L. Alvarez-Gaume and P. Ginsparg, Ann. Phys. 161(1985)423, and L. Alvarez-Gaume," An introduction to anomalies", Lectures given at Erice and The Ecole Normale Summer Workshop (1985)

[2] M.B. Green and J.H. Schwarz, Phys. Lett. 122B (1983) 143;
J.H. Schwarz and P.C. West, Phys. Lett. 126B (1983) 301;
P. Howe and P.C. West, Nucl. Phys. B238 (1984) 181;
J.H. Schwarz, Nucl. Phys. B226 (1983) 269.

[3] L. Alvarez-Gaume and E. Witten, Nucl.Phys. B234 (1983) 269.

[4] G. Moore and P. Nelson, Phys. Rev. Lett. 53 (1984) 1519.

[5] N. Marcus, Phys. Lett. 157B (1985) 383.

[6] A.H. Chamseddine, Nucl. Phys. B185 (1981) 403;
E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys. B195 (1982) 97,
G. Chapline and N.S. Manton,Phys. Lett. 120B (1983) 105.

[7] M.B. Green and J. Schwarz, Phys. Lett. 149B (1984) 117.

[8] J. Thierry-Mieg, Phys. Lett. 156B (1985) 199.
L. Dixon, J. Harvey and E. Witten, unpublished.

[9] E. Witten, Phys. Lett. 117B (1982) 324.

[10] E.Witten, Commun. Math. Phys. 100 (1985) 197.

[11] E. Witten, "Anomalies, Geometry and Topology", World Scientific ,eds. W.A. Bardeen & A.R. White.

[12] E. Witten, Int. J. Mod. Phys. A1 (1986) 39.

[13] D.J. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1984) 502.

[14] Y. Tanii, Phys. Lett. 145B (1984) 197.

[15] J. Strathdee, "Extended Poincare Supersymmetries", to appear in Int. Journ. Mod. Phys. A.

[16] P.K. Townsend, Phys. Lett. 139B (1984) 283.

[17] L.J. Romans,"Self-duality for interacting fields: Covariant field equations for six dimensional chiral supergravities", Santa Barbara Preprint (1986).

[18] H. Nishino and E. Sezgin, Phys. Lett. 144B (1984) 187; "The complete N=2, d=6 Supergravity with matter and Yang-Mills couplings", University of Maryland preprint.

[19] S. Randjbar-Daemi, Abdus Salam, E. Sezgin and J.Strathdee, Phys. Lett. 151B (1985) 351.

[20] Abdus Salam and E. Sezgin, Physica Scripta 32 (1985) 283.

[21] M.B. Green, J.H. Schwarz and P.C. West, Nucl. Phys. B254 (1985) 327.

[22] R. Stora, Lecture at the 1983 Cargese Summer School, in *Progress in Gauge theory* Ed. G. 't Hooft *et al.* (Plenum Press, N.Y., 1984);
B.Zumino, Lectures at the 1983 Les Houches Summer School, *Relativity, Groups and topology II*, Eds. B.S. DeWitt and R. Stora (North Holland, 1984);
L. Baulieu, Nucl. Phys. B241, 557 (1984).

[23] M.F. Atiyah, V.K. Patodi and I.M. Singer, Proc. Camb. Philos. Soc., 77 (1975) 43; 78 (1975) 405; 79 (1976) 71.

[24] J. Bagger and E. Witten, Nucl. Phys. B222 (1983) 1.

[25] See for instance, J. Milnor in *Lectures in Modern Mathematics* Vol II, ed. T. L. Saaty, Wiley (NY) 1964.

[26] Abdus Salam and E. Sezgin, Phys. Lett. 147B (1984).