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E. Bergshoeff

Abdus Salam

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E. Sezgin

INTERNATIONAL ATOMIC ENERGY AGENCY

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SUPERSYMMETRIC R^2-ACTIONS, CONFORMAL INVARIANCE AND LORENTZ CHERN-SIMONS TERM IN 6 AND 10 DIMENSIONS *

E. Bergshoeff
International Centre for Theoretical Physics, Trieste, Italy

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy
and
Imperial College, London, England

and

E. Sezgin
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We consider the superconformal extension of R^2-actions in 6 and 10 dimensions. We show that the fields of the conformal multiplet alone admit a one parameter family of R^2-actions of the form $\phi[R^2_{\mu\nu ab} + \alpha(R^2_{\mu\nu ab} - 4\,R^2_{\mu a} + R^2)]$. In $d = 6$ we give the supersymmetric action for $\alpha = 0$, while for $\alpha \neq 0$ we give partial results. We show that for $\alpha = 0$ and in a conformal gauge the 3-form field $H_{\mu\nu\rho}$ has a natural torsion interpretation. In $d = 10$ we find similar results (except the torsion interpretation) up to variations containing the 7-form field strength.

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1. INTRODUCTION

The original interest in R^2-supergravity theories was the fact that they have better quantum behaviour. However, it has also been widely recognized that these theories are nonunitary. A class of bosonic $R + R^2$ theories with $(\text{torsion})^2$ terms in $d = 4$ are known to be unitary; however they are non-renormalizable [1].

Over the last year there has been a revival of interest in R^2-supergravity theories [2] due to the fact that they arise naturally in the low energy limit of superstring theories [3] and moreover they play an important role in their compactification [4]. Since the superstring theories are presumably finite, the R^2-actions they give rise to in the low energy effective action need no longer be renormalizable. On the other hand, given a low energy effective action from which one computes the S-matrix elements, of course, it must be unitary on physical grounds. Therefore, it is of considerable interest to search for ghost-free $R + R^2$ actions in $d = 10$, in the same way it was done in Ref. [1]. One such action has been found so far, which is the Gauss-Bonnet combination of R^2-terms [5]: $R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2$. Whether the (unitary) superstring theory gives rise to this combination is an important issue which has not been resolved yet.

Concerning the local supersymmetry of the low energy limit of superstrings, this is a property which must hold, again, on physical grounds. This is so because the theory contains the gravitino field whose field equation is consistent provided that local supersymmetry holds. We recall [6] that the supersymmetric R-action is consistent because the divergence of the gravitino field equation is proportional to the Einstein equation obtained from that action, and therefore vanishes on-shell. Obviously the addition of R^2-terms alone to this action will destroy this property, thus making the theory inconsistent.

There have been a few attempts towards supersymmetrization of R^2-actions in $d = 6$ and $d = 10$. In particular, it has been shown that the supersymmetrization of the Gauss-Bonnet invariant, $R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2$, [7] as well as $R^2_{\mu\nu ab}$ [8],[9] is compatible with the Lorentz Chern-Simons form occurring in the field strength of the 2-form field $B_{\mu\nu}$. In Ref. [10] the transformation rules of the $d = 10$ theory are considered in the dual theory which contains a 6-form field $B_{\mu_1 \dots \mu_6}$. In the dual theory the connection between R^2-terms and the Lorentz Chern-Simons term is the same as the one between the Yang-Mills F^2-terms and the Yang-Mills Chern-Simons term *) [11]. To see how this works consider the following bosonic terms

*) We thank B. de Wit for pointing this out to us.

$$\mathcal{L} = H^2_{(7)} + F\wedge F\wedge B_{(6)} + R\wedge R\wedge B_{(6)} \tag{1.1}$$

where $H_{(7)} = dB_{(6)}$ and F is the Yang-Mills field strength. To dualize (1.1), one adds to it a total derivative term $H_{(7)}\wedge G_{(3)}$, where $G_{(3)} = dB_{(2)}$. Solving for $H_{(7)}$, and substituting the solution back into (1.1), one finds the result

$$\mathcal{L} = (G_{(3)} + \omega_{3Y} + \omega_{3L})^2 , \tag{1.2}$$

where ω_{3Y} and ω_{3L} are the Yang-Mills and Lorentz Chern-Simons 3-forms, respectively.

So far the attempts towards the supersymmetrization of the R^2-action in $d = 10$ or $d = 6$ have not led to complete results. One of the difficulties is the mixing of the R and R^2 actions, and the necessity of modifying the transformation rules. Both of these features are due to the fact that the theory is on-shell. In $d = 4$ the off-shell formulation of $N = 1$ supergravity, of course, is well known. This has facilitated the fully supersymmetric extension of the Gauss-Bonnet combination $\phi\,(R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2)$ [12].

In this note we give the first example of a supersymmetric (up to quartic fermion terms) R^2-action in a higher dimension by constructing the off-shell superconformal extension of (Riemann tensor)^2 in six dimensions. We have considered $d = 6$ because (a) matter and Yang-Mills coupled $N = 2$, $d = 6$ supergravity [13] is known off-shell [14]. Therefore, without changing the supersymmetry transformation rules, we can construct the R and R^2-invariants separately. (b) the theory resembles the $N = 1$, $d = 10$ Yang-Mills coupled supergravity a great deal [15] and (c) the $N = 2$, $d = 6$ theory may itself be the low energy limit of a heterotic superstring theory [16].

The reason for considering the conformal supersymmetry as opposed to the Poincaré supersymmetry is the following. The most general R^2-action in $d = 6$ which can admit a superconformal extension is

$$\mathcal{L} \sim \phi(c_1R^2_{\mu\nu ab} + c_2R^2_{\mu a} + c_3R^2) + \phi^{\alpha}(c_4R^2_{\mu a} + c_5R^2)L^{\frac{-\alpha+1}{2}} \equiv \mathcal{L}_1 + \mathcal{L}_2 \tag{1.3}$$

where ϕ is the dilaton of the conformal multiplet, L is the scalar of the compensating linear multiplet [14] and α is a real parameter different from 1. Upon fixing the conformal gauge, $L = 1$, we see that (1.3) reduces to

$$\mathcal{L} \sim \mathcal{L}_1 + \phi^{\alpha}(c_4R^2_{\mu a} + c_5R^2), \quad \alpha \neq 1 \tag{1.4}$$

This means that, if we want to supersymmetrize an action of the form $\mathcal{L}_1$ then, since $\alpha \neq 1$, the second term in (1.4) should not be considered. On the other hand, since $\mathcal{L}_1$ contains <u>only</u> the fields of the conformal multiplet, and <u>not</u> the fields of the compensating multiplet, it follows that the off-shell super-Poincaré extension of $\mathcal{L}_1$, even in the gauge $L = 1$, has still a residual superconformal invariance. Of course, the gauge condition $L = 1$ does change the transformation rules of the fields of the conformal multiplet. However, these changes are merely field-dependent dilatation, conformal supersymmetry and conformal boost transformations [17].

From the above argument we see that an <u>off-shell</u> super-Poincaré action of the form $R + \mathcal{L}_1$ is such that although the R term is not conformal invariant, $\mathcal{L}_1$ is. In fact, if one assumes that the higher order R^n terms arising in the low energy limit of the superstring have a common factor ϕ^{α_n} then it seems to follow that <u>all the corrections to the R-action will be conformally invariant</u>! In any event, it is clear that there are advantages in working directly in a conformal framework. We recall that in this framework the R-action is easily obtained from $L(\Box + R)L$ in the gauge $L = 1$ [18].

We have also considered the superconformal extension of the R^2-actions in $d = 10$. Our results for the superconformal R^2-actions in $d = 6$ and $d = 10$ can be summarized as follows. In $d = 6$, we find that the fields of the conformal multiplet, containing $B_{\mu\nu}$, admit a one-parameter family of R^2-actions of the form

$$\phi\left[R^2_{\mu\nu ab} + \alpha(R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2)\right] \tag{1.5}$$

We point out that the alternative conformal multiplet containing an anti-self-dual 3-form field T^-_{abc}, instead of $B_{\mu\nu}$, [14] does not allow an R^2-action. Our results are presented in the gauge $\phi = 1$, since in that gauge the construction of the action is particularly simple. To undo this gauge condition one performs appropriate field redefinitions, which we give, at the end.

In $d = 6$, we find that in the gauge $\phi = 1$ the gravitino transformation rule reads

$$\delta\psi_\mu = D_\mu(\omega_+)\epsilon \quad , \tag{1.6}$$

where

$$\omega_{\mu\ \pm}^{\ ab} = \omega_\mu^{\ ab}(\epsilon,\psi) \pm \frac{1}{2} H_\mu^{\ ab} \quad , \tag{1.7}$$

$H_{\mu\nu\rho}$ being the field strength of $B_{\mu\nu}$. Evidently, this means that $H_{\mu\nu\rho}$ can be interpreted as bosonic torsion. The remarkable simplicity of (1.6) allows us to formulate the action in terms of curvatures defined with respect to the torsionful connections $\omega_{\mu\ \pm}^{\ ab}$ defined in (1.7). For example, in the case $\alpha = 0$, the action contains the terms $R(\omega_-)\wedge {}^*R(\omega_-) + R(\omega_-)\wedge R(\omega_-)\wedge B$. In the case $\alpha \neq 0$, although one can still use $\omega_\pm$, it turns out that it is not particularly advantageous to do so here. For this reason, we have considered only the variations of the action which are bilinear in the fields, and all the variations containing the auxiliary field $V_{\mu i}^{\ \ j}$.

In $d = 10$, we find similar results as in $d = 6$. Again, the conformal multiplet admits a one parameter family of actions as in (1.5). As the conformal multiplet contains a 6-form field $B_{\mu_1 \ldots \mu_6}$ [11], instead of a 2-form field $B_{\mu\nu}$, there is no natural torsionful connection in this case. Consequently, the construction of the R^2-action is much more involved here, and we give the result which is invariant up to variations containing $H_{\mu_1 \ldots \mu_7}$.

This paper is organized as follows. In Sec.2, we show how in the presence of a dilaton field ϕ, any R^2-action can be written in a conformally invariant way. In Sec.3, we discuss the conformal supermultiplet, containing $B_{\mu\nu}$, in $d = 6$ and show that in the superconformal gauge $\phi = 1$ and $\lambda = 0$, (λ is a spinor in the multiplet), the transformation rules take up a simple form allowing a torsion interpretation of $H_{\mu\nu\rho}$. In Sec.4, we construct the $d = 6$ R^2-actions invariant under variations which are bilinear in the fields. Already at that level we show that R^2-terms in these actions must be of the form (1.5). In Sec.5, we describe the construction of the $R^2_{\mu\nu ab}$ action in $d = 6$ up to quartic fermion terms. The supersymmetrization of the Gauss-Bonnet combination of R^2-terms is discussed in Sec.6, while Sec.7 contains our results for R^2-actions in $d = 10$. Some of the open problems are pointed out in the conclusions. In App. A we collect some useful identities for curvatures with torsion. The result for the supersymmetric $R^2_{\mu\nu ab}$ action is given in App. B, while App. C contains our results on the supersymmetrization of $R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2$ in $d = 6$.

2. CONFORMAL INVARIANCE AND R^2 ACTIONS IN d DIMENSIONS

It is well known that local conformal supersymmetry [19] is a very convenient framework to study general matter couplings to supergravity theories. The same superconformal techniques can be applied to construct general supersymmetric R^2 actions as well. At first sight one might think that the high degree of gauge invariance of conformal gravity restricts the variety of R^2 actions. However, this is not quite so since R^2 terms do not only arise from (Weyl tensor)^2 but they can also arise from higher derivative Brans-Dicke type Lagrangians such as $\mathcal{L} = (c_1 R^2_{\mu\nu} + c_2 R^2)\phi^2 + \ldots$, where ϕ is a compensating field.

An example of a conformally invariant R^2 Lagrangian that contains a compensating scalar field ϕ is the generalization of the $d = 4$ conformal gravity action to d dimensions [11]:

$$\mathcal{L} = e\,\phi^{(d-4)}\, C_{\mu\nu}^{\ \ ab} C_{\mu\nu}^{\ \ ab}$$

$$= e\,\phi^{(d-4)} \left\{ R_{\mu\nu}^{\ \ ab} R_{\mu\nu}^{\ \ ab} - \frac{4}{d-2} R_\mu^{\ a} R_\mu^{\ a} + \frac{2}{(d-1)(d-2)} R^2 \right\} , \tag{2.1}$$

where

$$C_{\mu\nu}^{\ \ ab} = R_{\mu\nu}^{\ \ ab} - \frac{4}{d-2}\, e^{[a}_{[\mu} R_{\nu]}^{\ b]} + \frac{2}{(d-1)(d-2)}\, e^{a}_{[\mu} e^{b}_{\nu]} R$$

is the Weyl tensor in d dimensions. We use the following definitions:

$$R_{\mu\nu}^{\ \ ab} = 2\,\partial_{[\mu}\omega_{\nu]}^{\ ab} + 2\,\omega_{[\mu}^{\ ac}\,\omega_{\nu]}^{\ cb} \quad , \quad e = \det e_\mu^{\ a} \quad ,$$

$$R_\mu^{\ a} = R_{\mu\nu}^{\ \ ab}\, e_b^{\ \nu} \quad , \quad R = R_{\mu\nu}^{\ \ ab}\, e_a^{\ \mu} e_b^{\ \nu} \quad , \tag{2.2}$$

where $e_\mu^{\ a}$ is the vielbein, $e_a^{\ \mu}$ its inverse and $\omega_\mu^{\ ab}(e)$ the spin connection field. Under dilatation $e_\mu^{\ a}$ and ϕ transform according to

$$\delta_D\, e_\mu^a = -\Lambda_D\, e_\mu^a$$

$$\delta_D\, \phi = +\Lambda_D\, \phi \quad . \tag{2.3}$$

Note that in $d = 4$ the dependence of the conformal gravity action on ϕ disappears. The Lagrangian (2.1) can also be expressed in terms of a redefined vielbein which is inert under dilatations

$$\hat{e}_\mu^a = \phi\, e_\mu^a \quad . \tag{2.4}$$

In this case (2.1) no longer depends on ϕ. Equivalently one can directly impose the gauge condition

$$\phi = 1 \tag{2.5}$$

that breaks D symmetry.

The Lagrangian (2.1) is not the only conformally invariant R^2 Lagrangian that one can write down. In fact any R^2 Lagrangian in terms of the inert vielbein $\hat{e}_\mu{}^a$

$$\mathcal{L} = \hat{e}\left\{ c_1\, \hat{R}_{\mu\nu}^{ab}\, \hat{R}_{\mu\nu}^{ab} + c_2\, \hat{R}_\mu^a\, \hat{R}_\mu^a + c_3\, \hat{R}^2 \right\} \tag{2.6}$$

with c_1, c_2 and c_3 arbitrary coefficients can be made conformal invariant by reintroducing the compensating scalar ϕ as in (2.4). Upon making the redefinition (2.4) the Riemann tensor $\hat{R}_{\mu\nu}{}^{ab}$ can be expressed in terms of the Weyl tensor $C_{\mu\nu}{}^{ab}$ plus derivatives of the dilaton field ϕ according to

$$\hat{R}_{\mu\nu}{}^{ab} = \phi^{-2}\, C_{\mu\nu}{}^{ab} + e_{[\mu}^{[a}\left(4\, \phi^{-1} \mathcal{D}^{b]} \partial_{\nu]}\, \phi^{-1} - 2\, e_{\nu]}^{b]} (\partial_\lambda \phi^{-1})^2 \right) \tag{2.7}$$

with the second conformal covariant derivative $\mathcal{D}_\mu \partial_\nu \phi^{-1}$ of ϕ^{-1} defined by

$$\mathcal{D}_\mu \partial_\nu \phi^{-1} = \partial_\mu \partial_\nu \phi^{-1} - \Gamma_{\mu\nu}{}^\rho\, \partial_\rho \phi^{-1} + \frac{1}{2}\left(\frac{2}{d-2} R_\mu{}^a - \frac{1}{(d-1)(d-2)}\, e_\mu{}^a R \right) \phi^{-1} e_{a\nu} \,. \tag{2.8}$$

Using (2.7) one can easily show that the Lagrangian (2.6) can be rewritten in the following conformally covariant way:

$$\begin{aligned} e^{-1}\mathcal{L} = {} & c_1\, \phi^{d-4}\, C_{\mu\nu}{}^{ab} C_{\mu\nu}{}^{ab} \\ & + \left[c_1 + \frac{d-2}{4}\, c_2 \right] (d-2)\, \phi^{d-2}\, (\mathcal{D}_\mu \partial_\nu \phi^{-1})(\mathcal{D}_\mu \partial_\nu \phi^{-1}) \\ & + \left[4\, c_1 + (3d-4)\, c_2 + 4(d-1)^2\, c_3 \right] \phi^{d-2}\, (\mathcal{D}^\lambda \partial_\lambda \phi^{-1})^2 \\ & - 4\,(d-1)\left[2\, c_1 + (d-1)\, c_2 + d(d-1)\, c_3 \right] \phi^{d-1} (\mathcal{D}^\lambda \partial_\lambda \phi^{-1})(\partial_\mu \phi^{-1})^2 \\ & + d(d-1)\left[2\, c_1 + (d-1)\, c_2 + d(d-1)\, c_3 \right] (\partial_\nu \phi^{-1})^2 (\partial_\nu \phi^{-1})^2 \,. \end{aligned} \tag{2.9}$$

Note that if we had started in (2.6) from the Weyl tensor squared, i.e. $\mathcal{L} = \hat{e} c_1 \hat{C}_{\mu\nu}{}^{ab} \hat{C}_{\mu\nu}{}^{ab}$, then all the $\partial\phi$ terms in (2.9) would vanish. This reflects the fact that the Weyl tensor is scale covariant. Of course we can always obtain the original Lagrangian (2.6) from (2.9) by making the inverse redefinition $e_\mu{}^a = \phi^{-1} \hat{e}_\mu{}^a$ or, equivalently, by imposing the gauge condition $\phi = 1$.

Comparing formulae (2.6) and (2.9) it is clear that the R^2 actions in terms of the inert vielbein $\hat{e}_\mu{}^a$ are much simpler. This is why in this paper we will construct supersymmetric R^2 actions in terms of $e_\mu{}^a$. Once we have obtained this result we can always perform the redefinition (2.4) and thus obtain the supersymmetric generalization of (2.9).

As has been explained in the introduction we will restrict ourselves in this paper to R^2 actions which are constructed out of the fields of the conformal multiplet alone. Furthermore we will only consider R^2 actions of the form (2.6) with $c_1 \neq 0$, since it turns out that only supersymmetric R^2 actions with $c_1 \neq 0$ contain the Lorentz Chern-Simons related term $R \wedge R \wedge B$.

3. SUPERCONFORMAL INVARIANCE IN SIX DIMENSIONS AND TORSION

The $N = 2$ conformal supergravity theory in six dimensions has been constructed in [14] *). In this reference it was shown that there exist two different formulations of the $N = 2$, $d = 6$ Weyl multiplet, both with 40 (bosonic) + 40 (fermionic) components. Their field content is given by

$$\text{formulation I:} \quad (e_\mu^a, \psi_\mu^i, V_\mu^{ij}, T^-_{abc}, \chi^i, D)$$

$$\text{formulation II:} \quad (e_\mu^a, \psi_\mu^i, V_\mu^{ij}, B_{\mu\nu}, \lambda^i, \phi) \quad , \tag{3.1}$$

where e_μ^a is the sechsbein, ψ_μ^i $(i = 1,2)$ a positive chiral gravitino (i.e. $\psi_\mu^i = +\gamma_7\psi_\mu^i$), $V_\mu^{ij} = V_\mu^{ji}$ an SU(2) gauge field, T^-_{abc} an anti-selfdual tensor field (i.e. $T^-_{abc} = -\frac{i}{6}\varepsilon_{abcdef}T^{-def}$), $B_{\mu\nu}$ an antisymmetric tensor gauge field, χ^i a positive chiral spinor (i.e. $\chi^i = +\gamma_7\chi^i$), ψ^i a negative chiral spinor (i.e. $\psi^i = -\gamma_7\psi^i$) and D, ϕ real scalars. The spinors ψ_μ^i, χ^i / ψ and ψ^i are SU(2)-Majorana. We use the same notation and conventions as in [14] (the scalar ϕ and the spinor λ were called σ and ψ, respectively in [14]).

In the next section we will show that one cannot construct R^2 actions for formulation I of the $d = 6$ Weyl multiplet. Therefore we only give here the transformation rules of the second formulation:

$$\delta e_\mu^a = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu - \Lambda_D e_\mu^a$$

$$\delta\psi_\mu^i = D_\mu \varepsilon^i + \frac{1}{48}\phi^{-1}\gamma\cdot H\gamma_\mu\varepsilon^i + \gamma_\mu\eta^i - \frac{1}{2}\Lambda_D\psi_\mu^i$$

$$\delta V_\mu^{ij} = -4\bar{\varepsilon}^{(i}\phi_\mu^{j)} - 2\phi^{-1}\bar{\varepsilon}^{(i}\gamma_\mu\hat{\not{D}}\lambda^{j)} + \frac{1}{12}\phi^{-2}\bar{\varepsilon}^{(i}\gamma_\mu\gamma\cdot H\lambda^{j)} - 4\bar{\eta}^{(i}\psi_\mu^{j)}$$

*) For earlier related work see [20].

$$\delta B_{\mu\nu} = -\phi\bar{\varepsilon}\gamma_{[\mu}\psi_{\nu]} - \bar{\varepsilon}\gamma_{\mu\nu}\lambda$$

$$\delta\lambda^i = \frac{1}{4}\hat{\not{D}}\phi\varepsilon^i + \frac{1}{48}\gamma\cdot H\varepsilon^i - \phi\eta^i + \frac{5}{2}\Lambda_D\lambda^i$$

$$\delta\phi = \bar{\varepsilon}\lambda + 2\Lambda_D\phi \quad , \tag{3.2}$$

where Λ_D, the positive chiral ε^i and the negative chiral η^i are the parameters of a dilatation, a supersymmetry and a special supersymmetry, respectively. The derivatives $D_\mu\varepsilon^i$, $\hat{D}_\mu\lambda$ and $\hat{D}_\mu\phi$ and the field strength tensor $H_{\mu\nu\rho}$ of $B_{\mu\nu}$ are defined as follows:

$$D_\mu\varepsilon^i = \partial_\mu\varepsilon^i + \frac{1}{2}b_\mu\varepsilon^i + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon^i - \frac{1}{2}V_\mu{}^i{}_j\varepsilon^j$$

$$D_\mu\phi = (\partial_\mu - 2b_\mu)\phi - \bar{\psi}_\mu\lambda$$

$$\hat{D}_\mu\lambda^i = (\partial_\mu - \frac{5}{2}b_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab})\lambda^i - \frac{1}{2}V_\mu{}^i{}_j\lambda^j - \frac{1}{48}\gamma\cdot H\psi_\mu^i - \frac{1}{4}(\hat{\not{D}}\phi)\psi_\mu^i + \phi\phi_\mu^i$$

$$H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + 3\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda + \frac{3}{2}\phi\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} \tag{3.3}$$

Here b_μ is the gauge field for dilatations. The gauge field ϕ_μ^i of special supersymmetry is not an independent field but given in terms of ψ_μ^i and λ^i (see Eq. (2.25) of [14]).

As has been stressed in the previous section, the construction of R^2 actions for a conformal multiplet with a compensating scalar ϕ is enormously simplified by working with redefined fields that are inert under the conformal transformations. Once one has constructed and R^2 action for these redefined fields one can always reintroduce the compensating fields by making the inverse redefinition and hence obtain an R^2 action for the full conformal multiplet.

The conformal multiplet (3.2) contains two compensating fields: the scalar ϕ (compensator for the dilatations) and the spinor λ (compensator for the special supersymmetry transformations). These two fields can be eliminated from the conformal multiplet by making the following redefinitions, which are the supersymmetric analog of the redefinition (2.4) (note that the ϕ in (2.4) has scale weight 1 whereas the ϕ here has scale weight 2):

$$\hat{e}_\mu^a = \phi^{1/2}\, e_\mu^a$$

$$\hat{\psi}_\mu^i = \phi^{1/4}\, \psi_\mu^i + \phi^{-3/4}\, \gamma_\mu\, \lambda^i$$

$$\hat{V}_\mu^{ij} = V_\mu^{ij} - 4\, \phi^{-1}\, \bar{\lambda}^{(i}\, \psi_\mu^{j)} - 4\, \phi^{-2}\, \bar{\lambda}^{(i}\, \gamma_\mu\, \lambda^{j)}$$

$$\hat{B}_{\mu\nu} = B_{\mu\nu}$$

$$\hat{\varepsilon}^i = \phi^{1/4}\, \varepsilon^i \tag{3.4}$$

In terms of these redefined fields $(\hat{e}_\mu{}^a, \hat{\psi}_\mu{}^i, \hat{V}_\mu{}^{ij}, \hat{B}_{\mu\nu})$ the transformation rules (3.2) are given by (for simplicity we omit from now on the hats):

$$\delta\, e_\mu^a = \tfrac{1}{2}\, \bar{\varepsilon}\, \gamma^a\, \psi_\mu$$

$$\delta\, \psi_\mu^i = (\partial_\mu\, \varepsilon^i + \tfrac{1}{4}\, \omega_\mu{}^{ab}\, \gamma_{ab}\, \varepsilon^i - \tfrac{1}{2}\, V_\mu{}^i{}_j\, \varepsilon^j) + \tfrac{1}{8}\, H_\mu{}^{ab}\, \gamma_{ab}\, \varepsilon^i$$

$$\delta V_\mu^{ij} = \bar{\varepsilon}^{(i}\, \gamma^\lambda\, \psi_{\lambda\mu}^{j)} - \tfrac{1}{6}\, \bar{\varepsilon}^{(i}\, \gamma \cdot H\, \psi_\mu^{j)}$$

$$\delta B_{\mu\nu} = -\, \bar{\varepsilon}\, \gamma_{[\mu}\, \psi_{\nu]} \tag{3.5}$$

where the supercovariant curvatures $\psi_{\mu\nu}$ and $H_{\mu\nu\rho}$ are defined by

$$\psi_{\mu\nu}^i = 2\, D_{[\mu}\, \psi_{\nu]}^i + \tfrac{1}{4}\, H_{[\mu}{}^{ab}\, \gamma_{ab}\, \psi_{\nu]}^i$$

$$H_{\mu\nu\rho} = 3\, \partial_{[\mu}\, B_{\nu\rho]} + \tfrac{3}{2}\, \bar{\psi}_{[\mu}\, \gamma_\nu\, \psi_{\rho]} \tag{3.6}$$

Of course an alternative way to derive the transformation rule (3.5) from (3.2) is to impose the gauge conditions $\phi = 1$ and $\lambda = 0$ instead of making the redefinitions (3.4). To preserve the gauge $\lambda = 0$ one must perform a compensating S transformation with parameter:

$$\eta^i = \tfrac{1}{48}\, \gamma \cdot H\, \varepsilon^i - \tfrac{1}{2}\, \not{D}\, \varepsilon^i \tag{3.7}$$

In order to simplify the construction of the R^2 actions even further it is now crucial to make the following observation. We see that in the transformation rule for the gravitino in (3.5) the field strength $H_\mu{}^{ab}$ occurs as a fully antisymmetric bosonic torsion part of a spin connection field $\omega_\mu^{ab}(H)$. More explicitly, we can write the gravitino transformation rule in the following suggestive way:

$$\delta\, \psi_\mu^i = \mathcal{D}_\mu(\omega_+)\, \varepsilon^i \tag{3.8}$$

with

$$\mathcal{D}_\mu\, \varepsilon^i = \partial_\mu\, \varepsilon^i + \tfrac{1}{4}\, \omega_{+\mu}{}^{ab}\, \gamma_{ab}\, \varepsilon^i - \tfrac{1}{2}\, V_\mu{}^i{}_j\, \varepsilon^j$$

$$\omega_\mu{}^{ab}{}_\pm \equiv \omega_\mu{}^{ab}\,(e,\psi) \pm \tfrac{1}{2}\, H_{\mu\nu}{}^{ab}\ , \tag{3.9}$$

where $H_\mu{}^{ab}$ is the bosonic torsion and $B_{\mu\nu}$ the corresponding torsion potential. From the sechsbein postulate

$$\mathcal{D}_\mu\, (\omega_+, \Gamma)\, e_\nu^a \equiv \partial_\mu\, e_\nu^a + \omega_{+\mu}{}^{ab}\, e_\nu^b - \Gamma_{\mu\nu}{}^\rho\, e_\rho{}^a = 0 \tag{3.10}$$

it follows that the antisymmetric part of the Christoffel symbol is given by

$$\Gamma_{[\mu\nu]}{}^\lambda = -\, \tfrac{1}{2}\, H_{\mu\nu}{}^\lambda\ . \tag{3.11}$$

In the next sections we will see that the construction of R^2 actions can sometimes be dramatically simplified by working immediately with the torsionful curvature tensor. For our purposes it appears to be convenient to introduce both a curvature tensor $R_{\mu\nu}{}^{ab}(\omega_+)$ with positive torsion and a curvature tensor $R_{\mu\nu}{}^{ab}(\omega_-)$ with negative torsion:

$$R_{\mu\nu}{}^{ab}(\omega_\pm) \equiv 2\, \partial_{[\mu}\, \omega_{\nu]}{}^{ab}{}_\pm + 2\, \omega_{[\mu}{}^{ac}{}_\pm\, \omega_{\nu]}{}^{cb}{}_\pm \qquad (3.12)$$

The curvatures $R_{\mu\nu}{}^{ab}(\omega_+)$ and $R_{\mu\nu}{}^{ab}(\omega_-)$ are related to each other by the interchange of pair indices:

$$R_{\mu\nu,ab}(\omega_-) = R_{ab,\mu\nu}(\omega_+) + \text{bil. ferm.} \qquad (3.13)$$

This relation turns out to be very useful in actual calculations. Under supersymmetry the connection fields $\omega_\mu{}^{ab}{}_+$ and $\omega_\mu{}^{ab}{}_-$ transform as follows:

$$\delta\omega_\mu{}^{ab}{}_+ = \bar\varepsilon\, \gamma^{[a}\, \psi^{b]}{}_\mu + \frac{1}{2}\, \bar\varepsilon\, \gamma^\lambda\, \psi_\mu\, H^{ab}{}_\lambda$$

$$\delta\omega_\mu{}^{ab}{}_- = -\frac{1}{2}\, \bar\varepsilon\, \gamma_\mu\, \psi^{ab} \quad . \qquad (3.14)$$

Curvature tensors with torsion satisfy different identities than ones without torsion. We have listed all relevant identities which we will need in the following in Appendix A..

We finally mention that with the torsion interpretation of $H_\mu{}^{ab}$ one can write the gravitino curvature $\psi^i_{\mu\nu}$ defined in (3.6) as follows:

$$\psi^i_{\mu\nu} = 2\, \mathcal{D}_{[\mu}\, (\omega_+)\, \psi^i_{\nu]} \quad . \qquad (3.15)$$

This curvature satisfies the following Bianchi identity:

$$\mathcal{D}_{[\mu}(\Gamma,\omega_+)\, \psi^i_{\nu\rho]} = \frac{1}{4}\, R_{[\mu\nu}{}^{ab}(\omega_+)\, \gamma_{ab}\, \psi_{\rho]}$$

$$- \frac{1}{2}\, V_{[\mu\nu}{}^i{}_j\, \psi_{\rho]}{}^j + H_{[\mu\nu}{}^\lambda\, \psi^i_{\lambda\rho]} + \text{tril. ferm.} \;, \qquad (3.16)$$

where $V^{ij}_{\mu\nu}$ is the curvature tensor of V^{ij}_μ:

$$V^{ij}_{\mu\nu} \equiv 2\, \partial_{[\mu}\, V^{ij}{}_{\nu]} + V_{[\mu}{}^{k(i}\, V_{\nu]}{}^{j)}{}_k \qquad (3.17)$$

Under supersymmetry $\psi^i_{\mu\nu}$ transforms according to

$$\delta\psi^i_{\mu\nu} = \frac{1}{4}\, R_{\mu\nu}{}^{ab}\, (\omega_+)\, \gamma_{ab}\, \varepsilon^i - \frac{1}{2}\, V_{\mu\nu}{}^i{}_j\, \varepsilon^j + \text{bil. ferm.} \qquad (3.18)$$

4. R^2 ACTIONS FOR THE $d = 6$ CONFORMAL MULTIPLET INVARIANT UNDER VARIATIONS BILINEAR IN FIELDS

In order to investigate what kind of R^2 actions we can construct for the $d = 6$ conformal multiplet (3.5) we first make the following most general ansatz for the part of the action which is bilinear in the fields.

$$\mathcal{L}(\text{bilinear}) = c_1 R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} + c_2 R_\mu{}^a R_\mu{}^a + c_3 R^2$$

$$+ c_4\, \bar\psi_{\mu\nu}\, \gamma^{\mu\tau\lambda}\, \partial_\lambda\, \psi_{\tau\nu} + c_5\, \bar\psi_{\mu\nu}\, \partial\!\!\!/\, \psi_{\mu\nu}$$

$$+ c_6\, V^{ij}_{\mu\nu}\, V_{\mu\nu ij} + c_7\, H^{\mu\nu\rho}\, \Box\, H_{\mu\nu\rho} \;, \qquad (4.1)$$

where $c_1,\ldots,c_7$ are arbitrary coefficients and $R_{\mu\nu}{}^{ab}$ is the torsionless curvature. The requirement that in the variation of (4.1) under the transformations (3.5) with constant ε all terms bilinear in the fields cancel leads to relations between the coefficients $c_1,\ldots,c_7$. We give these relations below, where we have also indicated which type of terms vanish in the variation of $\mathcal{L}$ if the corresponding relation is imposed:

$4\, c_1 + c_2 + c_4 - 2\, c_5 = 0$	$\bar\varepsilon\, \gamma^a\, \psi_{\mu\nu}\, \partial_\mu\, R_\nu{}^a$
$-\frac{1}{2}\, c_2 - 2\, c_3 + \frac{1}{2}\, c_4 = 0$	$\bar\varepsilon\, \gamma^\lambda\, \psi_{\lambda\rho}\, \partial_\rho\, R$

$$c_4 = 0 \qquad \bar{\epsilon}^i \gamma^{\mu\tau\lambda} \partial_\lambda \psi^j_{\tau\nu} V_{\mu\nu ij}$$

$$\frac{1}{2} c_5 + c_6 = 0 \qquad \bar{\epsilon}^i \gamma^\lambda \psi^j_{\lambda\nu} \partial_\mu V_{\mu\nu ij}$$

$$c_4 = 0 \qquad \bar{\epsilon} \gamma^{\mu\nu\rho} \psi_{\mu\lambda} \Box H_{\nu\rho\lambda}$$

$$\frac{1}{6} c_5 + c_7 = 0 \qquad \bar{\epsilon} \gamma^\mu \psi^{\nu\rho} \Box H_{\mu\nu\rho} \tag{4.2}$$

In the derivation of the first two equations in (4.2) we have used the Bianchi identity for $R_{\mu\nu}{}^{ab}$ and its consequences

$$D_{[\mu} R_{\nu\rho]}{}^{ab} = 0 \quad , \quad D_b R_{\mu\nu}{}^{ba} = 2 D_{[\mu} R_{\nu]}{}^a \quad , \quad D_a R_\mu{}^a = \frac{1}{2} \partial_\mu R \tag{4.3}$$

From (4.2) we see that $c_4 = 0$ and that all other coefficients can be expressed in terms of c_1 and c_2:

$$c_3 = -\frac{1}{4} c_2 \quad , \quad c_5 = -2c_6 = -6 c_7 = \frac{1}{2} (4c_1 + c_2) \tag{4.4}$$

Substituting (4.4) into (4.1) we can rewrite the Lagrangian in the following way

$$\mathcal{L}(\text{bilinear}) = \alpha \left\{ R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} + 2\psi_{\mu\nu} \not{\partial} \psi_{\mu\nu} - V^{ij}_{\mu\nu} V_{\mu\nu ij} - \frac{1}{3} H^{\mu\nu\rho} \Box H_{\mu\nu\rho} \right\}$$

$$+ \beta \left\{ R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} - 4 R_\mu{}^a R_\mu{}^a + R^2 \right\} \quad , \tag{4.5}$$

where $\alpha = c_1 + \frac{1}{4} c_2$ and $\beta = -\frac{1}{4} c_2$. From (4.5) we see that the Lagrangian can be written as the sum of the Riemann tensor squared together with kinetic terms for ψ_μ, V^{ij}_μ and $B_{\mu\nu}$ plus the Gauss-Bonnet combination

$(R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} - 4R_\mu{}^a R_\mu{}^a + R^2)$ each with independent coefficients α and β respectively. Note that the variation of the Gauss-Bonnet combination does not contain terms bilinear in the fields. This is easily seen by writing it in the form

$$\mathcal{L}(\text{Gauss-Bonnet}) = \beta \, (R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} - 4 R_\mu{}^a R_\mu{}^a + R^2)$$

$$= 6\beta \, R_{[ab}{}^{ab} R_{cd]}{}^{cd} \tag{4.6}$$

and in its variation to use the Bianchi identity $D_{[c} R_{ab]}{}^{ab} = 0$.

In the next two sections, without loss of generality, we will separately consider the supersymmetrization of the Riemann tensor squared and the Gauss-Bonnet combination by applying a Noether procedure. For the supersymmetrization of such actions a torsion interpretation of $H_{\mu\nu\rho}$ turns out to be very useful. Note that the α and β terms in (4.5) can be written in the following suggestive way

$$\mathcal{L}(\text{Riemann-tensor}) = \alpha \left\{ R_{\mu\nu}{}^{ab}(\omega_\pm) R_{\mu\nu}{}^{ab}(\omega_\pm) + 2\bar{\psi}_{\mu\nu} \not{D} \psi_{\mu\nu} - V_{\mu\nu}{}^{ij} V_{\mu\nu ij} \right\}$$

$$\mathcal{L}(\text{Gauss-Bonnet}) = 6\beta \, R_{[ab}{}^{ab}(\omega_\pm) R_{cd]}{}^{cd}(\omega_\pm) \quad , \tag{4.7}$$

where $R_{\mu\nu}{}^{ab}(\omega_\pm)$ is the torsionful curvature defined in (3.12). The sign of H in $R_{\mu\nu}{}^{ab}(\omega_\pm)$ in the final nonlinear answer should follow from the Noether procedure.

5. SUPERSYMMETRIZATION OF $R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab}$ IN $d = 6$

We will now supersymmetrize the Riemann tensor squared by applying a Noether procedure. The reader who is not interested in the technical details given in this section can find our final results in Appendix B. It turns out that the construction of the action is dramatically simplified by using a torsion interpretation for $H_{\mu\nu\rho}$. Such a torsion interpretation has been proposed earlier in the low energy limit of the $d = 10$ superstring [7],[21],[22]. The usefulness of this torsion interpretation is only justified after having proven the invariance of the action. A priori it is not clear whether this torsion interpretation will be useful for the construction of other actions as well. Whether or not a torsion interpretation is useful depends on the following. In a torsion interpretation of $H_{\mu\nu\rho}$ one makes the assumption that in the action all $R_{\mu\nu}{}^{ab}$ terms and $\mathcal{D}_\mu H_\nu{}^{ab}$ terms occur in the combination $R_{\mu\nu}{}^{ab}(\omega_\pm)$, i.e. the R and $\mathcal{D}H$ terms are on equal footing. This is only true if in showing the invariance of the action one does not use identities which are valid for $R_{\mu\nu}{}^{ab}$ but not for $R_{\mu\nu}{}^{ab}(\omega_\pm)$. A typical example of such identities are the first three relations in (A.3). For instance, $R_{\mu[\nu,\rho\sigma]}(\{\}) = 0$ but $R_{\mu[\nu,\rho\sigma]}(\Gamma(H))=-\frac{1}{3}\mathcal{D}_\mu H_{\nu\rho\sigma}$. If in showing the invariance of the action one has to use many identities like this, then it just means that the $R_{\mu\nu}{}^{ab}$ terms and $\mathcal{D}_\mu H_\nu{}^{ab}$ terms in the action have to be treated differently and will not always occur in the combination $R_{\mu\nu}{}^{ab}(\omega_\pm)$. Clearly, when this happens a torsion interpretation of $H_{\mu\nu\rho}$ is not so useful anymore.

It turns out that the torsion interpretation of $H_{\mu\nu\rho}$ used in this section does not only predict all $\mathcal{D}H$ terms in the action, but also all H^2 terms! This enormously simplifies the calculation. For instance, a priori we can have in the Lagrangian terms of the form

$$\mathcal{L} = a_1 R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab} + a_2 \mathcal{D}H\,\mathcal{D}H + a_3 RH^2 + a_4 H^4 \tag{5.1}$$

in arbitrary combinations and with different possible contractions. However all these terms turn out to occur in the combination

$$\mathcal{L} = a_1 R_{\mu\nu}{}^{ab}(\omega_-) R_{\mu\nu}{}^{ab}(\omega_-) \tag{5.2}$$

The fact that we have taken in (5.2) $R_{\mu\nu}{}^{ab}(\omega_-)$ and not $R_{\mu\nu}{}^{ab}(\omega_+)$ will be explained by the Noether procedure which we will discuss now.

We start from the ansatz given in (4.7):

$$\mathcal{L}\,(\text{Riemann-tensor}) = R_{\mu\nu}{}^{ab}(\omega_\pm) R_{\mu\nu}{}^{ab}(\omega_\pm) + 2\bar\psi_{\mu\nu} \not{\mathcal{D}}\, \psi_{\mu\nu} - V^{ij}_{\mu\nu} V_{\mu\nu ij} \tag{5.3}$$

Note that the sign of H in $R_{\mu\nu}{}^{ab}(\omega_\pm)$ is not yet fixed. In the variation of (5.3) with constant ϵ all terms bilinear in the fields cancel. For local ϵ the same cancellations occur if we add to the Lagrangian (5.3) the following Noether terms.

$$\mathcal{L}\,(\text{Noether}) = -R_{\mu\nu}{}^{ab}(\omega_\pm)\, \bar\psi_\lambda \gamma^{ab} \gamma^\lambda \psi_{\mu\nu} + 4 V^{ij}_{\mu\nu} \bar\psi_{\mu i} \gamma^\lambda \psi_{\lambda\nu j} \tag{5.4}$$

Since we will determine the full action only up to quartic fermion terms we will from now on neglect terms bilinear in the fermions in the variation of the action. Furthermore we will always remove the derivative from ϵ by partial differentiation. For instance when we vary ψ_μ in (5.4) as $\delta\psi_\mu = \mathcal{D}_\mu(\omega_+)\,\epsilon$ we will move the derivative to $R(\omega_\pm)$, $V_{\mu\nu}$ and $\psi_{\mu\nu}$. The type of terms bilinear in the fields which cancel in the variation of (5.3) and (5.4) are given in the first column of Table 1.

The choice of the sign in the $R_{\mu\nu}{}^{ab}(\omega_\pm)$ terms in (5.3) and (5.4) is motivated by the cancellation of trilinear terms of the type $\psi_\mu R^2$. They arise from varying the sechsbein in the R^2 and $\psi_{\mu\nu}$ in the $R\psi_\mu\psi_{\mu\nu}$ term and are given by

$$\frac{1}{2}\bar\epsilon\,\gamma^\lambda\psi_\lambda\, R_{\mu\nu}{}^{ab}(\omega_\pm) R_{\mu\nu}{}^{ab}(\omega_\pm) - 2\,\bar\epsilon\,\gamma^\mu\,\psi^\nu R_{\mu\lambda}{}^{ab}(\omega_\pm) R_{\nu\lambda}{}^{ab}(\omega_\pm) \tag{5.5}$$

and

$$-\frac{1}{2}\bar\epsilon\,\gamma^\lambda\psi_\lambda\, R_{\mu\nu}{}^{ab}(\omega_\pm) R_{\mu\nu}{}^{ab}(\omega_+) + 2\,\bar\epsilon\,\gamma_\mu\psi_\nu\, R_{ab}{}^{\mu\lambda}(\omega_\pm) R_{ab}{}^{\nu\lambda}(\omega_+)$$

$$+\frac{1}{4} R_{ab}{}^{\mu\nu}(\omega_\pm) R_{ab}{}^{\rho\sigma}(\omega_+)\,\bar\epsilon\,\gamma_{\mu\nu\rho\sigma\tau}\,\psi_\tau\,, \tag{5.6}$$

respectively. Note that the sign of H in one of the R's in (5.6) is fixed since we have $\delta\psi_{\mu\nu}\alpha R(\omega_+)\,\epsilon$. We now see that the $\gamma^{(1)}$-terms in (5.5) and (5.6) cancel in a natural way if we take for the kinetic term $R(\omega_-)^2$ (i.e. - sign in (5.5)) and for the Noether term $R(\omega_+)\psi_\mu\psi_{\mu\nu}$ (i.e. + sign in (5.6)) and then use the identity $R_{\mu\nu,ab}(\omega_-) = R_{ab,\mu\nu}(\omega_+)$ + bil. ferm.(see (3.13)).

The $\gamma^{(5)}$-term in (5.6) can be cancelled by adding a term $\mathcal{L}$(Chern-Simons) $\sim R\wedge R\wedge B$ which is related to the Lorentz Chern-Simons form as discussed in the introduction. For this cancellation to occur it is essential that the conformal supergravity multiplet contains an antisymmetric tensor gauge field $B_{\mu\nu}$. It is at this point that we would have failed to write down an R^2-action for formulation I of conformal supergravity (see Eq. (3.1)) which contains the tensor T^-_{abc} instead of $B_{\mu\nu}$. The cancellation occurs in a natural way if we take the R in $R\wedge R\wedge B$ to be $R(\omega_-)$:

$$\mathcal{L}\,(\text{Chern-Simons}) = -\frac{1}{4}\,\varepsilon^{\mu\nu\rho\sigma\lambda\tau}\,R_{\mu\nu}{}^{ab}(\omega_-)\,R_{\rho\sigma}{}^{ab}(\omega_-)\,B_{\lambda\tau}\ . \tag{5.7}$$

To show the gauge invariance of this term under $\delta B_{\lambda\tau} = \partial_{[\lambda}\Lambda_{\tau]}$ we use the Bianchi identity for $R_{\mu\nu}{}^{ab}(\omega_-)$ which can be written as

$$\mathcal{D}_{[\mu}(\omega_-)\,R_{\nu\rho]}{}^{ab}(\omega_-) = 0 \tag{5.8}$$

and the fact that no explicit sechsbein occur in $\mathcal{L}$(Chern-Simons). This allows us to write $\delta B_{\lambda\tau} = \mathcal{D}_{[\lambda}(\omega_-)\Lambda_{\tau]}$ and to partial differentiate with $\mathcal{D}_\lambda(\omega_-)$. Note that the Bianchi-identity (5.8) is an exact identity (i.e. not modulo bil. ferms): Our result up to now can be summarized as follows. All bilinear terms and all trilinear terms of the form $\psi_\mu R^2$ vanish in the variation of the Lagrangian

$$\begin{aligned}\mathcal{L} &= R_{\mu\nu}{}^{ab}(\omega_-)\,R_{\mu\nu}{}^{ab}(\omega_-) + 2\bar{\psi}_{\mu\nu}\not{\mathcal{D}}\,\psi_{\mu\nu} - V_{\mu\nu}^{ij}\,V_{\mu\nu ij}\\ &- R_{\mu\nu}{}^{ab}(\omega_+)\,\bar{\psi}_\lambda\,\gamma^{ab}\gamma^\lambda\psi_{\mu\nu} + 4\,V_{\mu\nu}^{ij}\,\bar{\psi}_{\mu i}\,\gamma^\lambda\,\psi_{\lambda\nu j}\\ &- \frac{i}{4}\,\varepsilon^{\mu\nu\rho\sigma\lambda\tau}\,R_{\mu\nu}{}^{ab}(\omega_-)\,R_{\rho\sigma}{}^{ab}(\omega_-)\,B_{\lambda\tau}\end{aligned} \tag{5.9}$$

In (5.9) it is understood that the covariant derivative $\mathcal{D}_\mu$ is with respect to ω_+ and Γ (for the definition of $\mathcal{D}_\lambda\psi_{\mu\nu}$ for instance, see (B.3)).

Our strategy is now as follows. First we determine the full variation of (5.9). Whenever we have to partial differentiate in this variation we do this with respect to the torsionful connection ω_+ and Γ. Because of the

sechsbein postulate (3.10) we can always move the derivative $\mathcal{D}_\mu(\omega_+,\Gamma)$ through a sechsbein. The variation of (5.9) leads to the terms marked with x indicated in Table 2. In this table we have not indicated the bilinear terms and the $\psi_\mu R^2$ terms since they where already cancelled in the variation of (5.3), (5.4) and (5.7). The same cancellations occur in the variation of (5.9), but only up to higher order terms. A typical example is the cancellation of the $V\mathcal{D}\psi_{\mu\nu}$ terms coming from the kinetic term V^2 and the Noether term $V\psi_\mu\psi_{\mu\nu}$. The two contributions cancel upon using the Bianchi identity of $\psi_{\mu\nu}$ (see (3.16)). This then leads to trilinear terms of the form $\psi_{\mu\nu}HV$, $\psi_\mu RV$ and $V^2\psi_\mu$. Also the terms arising from such manipulations have been indicated in Table 2 (these are the terms marked with 0).

We now first consider all trilinear terms in the variation which contain V. The terms of the type $\psi_\mu V^2$ and $\psi_\mu RV$ cancel. The term of the form $\psi_\mu H\ V$ coming from varying the V^2 kinetic term is cancelled by adding to the Lagrangian the following form:

$$\mathcal{L}\,(VH\psi_\mu^2) = -\frac{1}{3}\,V_{\mu\nu}^{ij}\,\bar{\psi}_{\mu i}\,\gamma\cdot H\,\psi_{\nu j}\ . \tag{5.10}$$

This term gives new contributions to the variation of the type $\psi_\mu V\mathcal{D}H$, $\psi_{\mu\nu}HV$ and $\psi_\mu H^2V$ (see Table 2). The $\psi_\mu V\mathcal{D}H$ term cancels against a term which did arise in the cancellation of the $\psi_\mu RV$ terms (see Table 2). The $\psi_{\mu\nu}HV$ terms which do arise from the variation of (5.9) and (5.10) and from the cancellation of the $V\mathcal{D}\psi_{\mu\nu}$ terms are cancelled by adding to the Lagrangian the following terms of the type $\psi_{\mu\nu}^2$ H:

$$\mathcal{L}\,(\psi_{\mu\nu}^2\,H) = -\frac{1}{3}\,\bar{\psi}_{\mu\nu}\,\gamma\cdot H\,\psi_{\mu\nu} - 4\,H^{\mu\nu\rho}\,\bar{\psi}_{\mu\lambda}\gamma_\rho\psi_{\nu\lambda}\ . \tag{5.11}$$

This concludes the cancellation of all trilinear terms in the variation that contain V.

The remarkable thing is that the result up to now, i.e. the Lagrangian given by the sum of (5.9), (5.10) and (5.11) turns out to be the final answer for the $R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab}$ - action (up to quartic fermions of course)! In other words just looking to variations proportional to V gives us the whole answer. We have given an explicit expression for the $R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab}$ - action in Appendix B.

It is now straightforward to check that all the remaining variations, i.e. $\psi_{\mu\nu}HR$, $\psi_{\mu\nu}H\mathcal{D}H$, $\psi_\mu H^2V$ and $\psi_\mu H^3$ cancel. We have indicated the different contributions to these variations in Table 2. Note that the $\psi_{\mu\nu}H^3$ terms only arise from cancelling the $\psi_{\mu\nu}HR$ terms and they turn out to cancel by themselves.

It is amazing to see that a whole set of structures which could arise in the variation of the Lagrangian (i.e. the terms of the type $\mathcal{D}\psi_{\mu\nu}\mathcal{D}H$, $\psi_{\mu\nu}\mathcal{D}\mathcal{D}H$, $\psi_\mu H\mathcal{D}R$, $\psi_\mu R\mathcal{D}H$, $\psi_\mu\mathcal{D}H\mathcal{D}H$, $\psi_\mu H^2R$, $\psi_\mu H^2\mathcal{D}H$ and $\psi_\mu H^4$ in Table 1) simply do not explicitly arise once we use a torsion interpretation for $H_{\mu\nu\rho}$ everywhere. Without this torsion interpretation the construction of a supersymmetric $R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab}$ action would be significantly more difficult.

6. THE SUPERSYMMETRIZATION OF $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_\mu{}^aR_\mu{}^a + R^2)$ IN $d = 6$

In this section we will discuss the supersymmetrization of the Gauss-Bonnet combination $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_\mu{}^aR_\mu{}^a + R^2)$ in six dimensions. In particular we will investigate whether a torsion interpretation for $H_{\mu\nu\rho}$ can simplify the construction of the action.

We start from the ansatz given in (4.7):

$$\mathcal{L}(\text{Gauss-Bonnet}) = R_{[ab}{}^{ab}(\omega_\pm)\,R_{cd]}{}^{cd}(\omega_\pm) \quad . \tag{6.1}$$

Note that in (6.1) the $(\partial H)^2$ terms cancel. The sign of H in (6.1) is not yet fixed. Since the variation of (6.1) does not give rise to terms bilinear in the fields (the Gauss-Bonnet combination is a topological invariant) we are only allowed to add Noether like terms to (6.1) whose variation under $\delta\psi_\mu = \mathcal{D}_\mu\epsilon$ does not give rise to bilinear terms. The most general such terms one can write down are given by:

$$\mathcal{L}(\text{Noether}) = c_1\,R_{[ab}{}^{ab}(\omega_\pm)\,\bar\psi_c\,\gamma^{cde}\,\psi_{de]} + c_2\,V^{ij}_{ab}\,\bar\psi_{ci}\,\gamma^{abcde}\,\psi_{dej} \,, \tag{6.2}$$

with c_1, c_2 arbitrary.

We first consider in the variation of (6.1) and (6.2) the trilinear terms of the type $\psi_\mu R^2$. They arise from varying the sechsbein in the R^2 and ψ_μ, $\psi_{\mu\nu}$ in the $R\psi_\mu\psi_{\mu\nu}$ term and are given by

$$\frac{5}{2}\,\bar\epsilon\,\gamma^a\,\psi_{[a}\,R_{bc}{}^{bc}(\omega_\pm)\,R_{de]}{}^{de}(\omega_\pm) \tag{6.3}$$

and

$$3c_1\,\bar\epsilon\,\gamma^a\,\psi_{[a}\,R_{bc}{}^{bc}(\omega_\pm)\,R_{de]}{}^{de}(\omega_+)$$

$$-\frac{1}{2}\,c_1\,R_{[ab}{}^{ab}(\omega_\pm)\,R_{cd}{}^{gh}(\omega_+)\,\bar\epsilon\,\gamma^{cdegh}\,\psi_{e]} \quad , \tag{6.4}$$

respectively. Note that the sign of H in one of the R's in (6.4) is fixed since we have $\delta\psi_{\mu\nu} \propto R(\omega_+)\epsilon$. The cancellation of the $\gamma^{(1)}$-terms in (6.3) and (6.4) occurs only in a natural way if one takes for the Gauss-Bonnet combination and the Noether term $R(\omega_+)R(\omega_+)$ and $R(\omega_+)\psi_\mu\psi_{\mu\nu}$ (i.e. + signs in (6.3) and (6.4)) <u>or</u> $R(\omega_+)R(\omega_-)$ *) and $R(\omega_-)\psi_\mu\psi_{\mu\nu}$ (i.e. + and - sign in (6.3) and - sign in (6.4)) respectively. The γ^5-term in (6.4) can be rewritten in the following form (we have substituted the value of c_1):

$$\frac{5}{12}\,R_{[ab}{}^{ab}(\omega_\pm)\,R_{cd}{}^{gh}(\omega_+)\,\bar\epsilon\,\gamma^{cdegh}\,\psi_{e]}$$

$$= \frac{1}{24}\,R_{ab}{}^{gh}(\omega_+)\,R_{cd}{}^{gh}(\omega_+)\,\bar\epsilon\,\gamma^{abcde}\,\psi_e$$

$$+\ R\mathcal{D}H\,\psi_\mu,\ RH^2\,\psi_\mu \ \text{terms} \tag{6.5}$$

The trilinear term of the type $\psi_\mu R^2$ in (6.5) can be cancelled by adding to the Lagrangian the following Chern-Simons related term:

$$(\text{Chern-Simons}) = -\frac{i}{24}\,\epsilon^{\mu\nu\rho\sigma\lambda\tau}\,R_{\mu\nu}{}^{ab}(\omega_+)\,R_{\rho\sigma}{}^{ab}(\omega_+)\,B_{\lambda\tau} \tag{6.6}$$

We will not discuss the cancellation of the $R\,H\psi_\mu$ and $RH^2\psi_\mu$ terms in (6.5) here. Note that such terms did not occur in the cancellation of the $\gamma^{(5)}$ terms of the type $\psi_\mu R^2$ in the previous section (see the text after (5.6)).

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*) In [21] it has been shown that the combination $R(\omega_+)\,R(\omega_-)$ should arise in the low energy limit of strings.

The variation of the Gauss-Bonnet combination (6.1) does give rise to one other trilinear variation which is of the type $\psi_{\mu\nu}RH$. For the combinations $R(\omega_+)R(\omega_+)$ and $R(\omega_-)R(\omega_-)$ these variations are given by

$$- 10 \, R_{[ab}{}^{ab}(\omega_+) \, H_{cd}{}^{c} \, \bar{\varepsilon} \, \gamma^{d} \, \psi^{e}{}_{e]} \qquad (6.7)$$

and

$$- 5 \, R_{[ab}{}^{ab}(\omega_+) \, H_{c}{}^{cd} \, \bar{\varepsilon} \, \gamma^{e} \, \psi_{de]} + \text{tril. ferm.} \qquad (6.8)$$

respectively, whereas the combination $R(\omega_+)R(\omega_-)$ leads to a more complicated expression, which we shall not give here. It turns out that we are only able to cancel the $RH\psi_{\mu\nu}$ terms in (6.8) in a natural way. They are cancelled by adding the following $\psi^2_{\mu\nu}H$ term to the action:

$$\mathcal{L}\,(\psi^2_{\mu\nu}\, H) = + \frac{5}{3}\, \bar{\psi}_{[ab}\, \gamma^{abc}\, \psi_{cd}\, H_{e]}{}^{de} \quad . \qquad (6.9)$$

Unfortunately we have seen above that it was exactly this $R(\omega_-)R(\omega_-)$ combination which did give rise to a $\psi_\mu R^2$ variation which could not be cancelled in a natural way.

We have thus come to the formulation of our main problem with the supersymmetrization of the Gauss-Bonnet combination. Unlike to the supersymmetrization of the Riemann-tensor squared in the previous section we here cannot find a magic combination for the signs of the bosonic torsion in the curvatures $R(\omega_\pm)$ such that we do find natural cancellations everywhere. This does not mean of course that such a supersymmetric Gauss-Bonnet action does not exist. It only indicates that a torsion interpretation for $H_{\mu\nu\rho}$ seems to be less natural in this case. This would mean that the actual construction of such a Gauss-Bonnet action probably will require much more work than was the case in the previous section.

Of course one can restrict oneself to first try to cancel all terms arising in the variation of (6.1) and (6.2) containing V. Remember that in the previous section this sector did give us the final expression for the $R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab}$ action. The cancellations occur in a natural way if we take + signs in both (6.1) and (6.2). In that case we are indeed able to cancel all variations containing V by adding appropriate terms to the action. We have given the result in Appendix C. Unfortunately the result given in (C.1) is not the final answer. Starting from this expression the main problem would be to cancel the $RH\psi_{\mu\nu}$ variation given in (6.7). We hope to give the final result in a future publication.

7. THE SUPERSYMMETRIZATION OF R^2 ACTIONS IN d = 10

In this section we investigate what kind of R^2 actions one can write down for the d = 10 conformal multiplet. The conformal supergravity theory in ten dimensions with 128 (bosonic) + 128 (fermionic) components has been constructed in Ref. [11]. Its field content is given by

$$(e_\mu{}^a,\ \psi_\mu,\ B_{\mu_1 \dots \mu_6},\ \lambda,\ \phi) \quad , \qquad (7.1)$$

where e_μ^a is the zehnbein, ψ_μ a positive chiral gravitino (i.e. $\psi_\mu = +\gamma_{11}\psi_\mu$), $B_{\mu_1 \dots \mu_6}$ a 6-index antisymmetric tensor gauge field, λ a negative chiral spinor (i.e. $\lambda = -\gamma_{11}\lambda$) and ϕ a real scalar. In the gauge $\phi = 1$, $\lambda = 0$ the transformation rules are given by

$$\delta e_\mu^a = \frac{1}{2}\, \bar{\varepsilon}\, \gamma^a\, \psi_\mu$$

$$\delta\psi_\mu = D_\mu \varepsilon + \frac{1}{6}\,(\gamma_\mu \gamma^{(7)} - 3\, \gamma^{(7)}\, \gamma_\mu)\, \varepsilon\, H_{(7)}$$

$$\delta B_{\mu_1 \dots \mu_6} = \frac{3}{4\cdot 6!}\, \bar{\varepsilon}\, \gamma_{[\mu_1 \dots \mu_5}\, \psi_{\mu_6]} \quad , \qquad (7.2)$$

where the covariant derivative $D_\mu \varepsilon$ and the supercovariant curvature $H_{(7)}$ is given by

$$D_\mu \varepsilon = \partial_\mu \varepsilon - \frac{1}{4}\, \omega_\mu{}^{ab}\, \gamma_{ab}\, \varepsilon$$

$$H_{\mu_1 \dots \mu_7} = \partial_{[\mu_1} B_{\mu_2 \dots \mu_7]} - \frac{3}{8\cdot 6!}\, \bar{\psi}_{[\mu_1}\, \gamma_{\mu_2 \dots \mu_6}\, \psi_{\mu_7]} \qquad (7.3)$$

We use the same notation and conventions as in [11]. An unusual feature is that the zehnbein and the gravitino both satisfy a differential constraint:

$$R = - 4\cdot 7!\, H^{(7)} H_{(7)} + 9\, \bar{\psi}_\mu\, \gamma_\nu \psi^{\mu\nu}$$

$$\gamma^{\mu\nu}\, \psi_{\mu\nu} = 0 \qquad (7.4)$$

Here R is the Ricci scalar and $\psi_{\mu\nu}$ the supercovariant gravitino curvature

$$R = (\partial_{[\mu}\,\omega_{\nu]}{}^{ab} - \omega_{[\mu}{}^{ac}\,\omega_{\nu]}{}^{cb})\, e^{\mu}{}_a\, e^{\nu}{}_b$$

$$\psi_{\mu\nu} = D_{[\mu}\,\psi_{\nu]} + \frac{1}{6}\,(\gamma_{[\mu}\,\gamma^{(7)} - 3\,\gamma^{(7)}\,\gamma_{[\mu})\,\psi_{\nu]}\,H_{(7)} \tag{7.5}$$

In order to investigate what kind of R^2 actions one can construct for the $d = 10$ conformal multiplet (7.2) we first make the following most general ansatz for the part of the action which is bilinear in the fields:

$$\mathcal{L} = c_1\,R_{\mu\nu}{}^{ab}\,R_{\mu\nu}{}^{ab} + c_2\,R_\mu{}^a\,R_\mu{}^a$$

$$+ c_3\,\bar\psi_{\mu\nu}\,\not\partial\,\psi_{\mu\nu} + c_4\,H^{(7)}\,\Box\,H_{(7)} \quad , \tag{7.6}$$

where $c_1, \ldots, c_4$ are arbitrary coefficients. Note that we have not written down an R^2 term since $R^2 = 0$ up to terms quadrilinear in the fields. Also we have not written down a $\bar\psi_{\mu\nu}\gamma^{\mu\tau\lambda}\partial_\lambda\psi_{\tau\nu}$ term since this term can be rewritten as $\bar\psi_{\mu\nu}\not\partial\psi_{\mu\nu}$ by using the Bianchi identity for $\psi_{\mu\nu}$ and the constraint $\gamma^{\mu\nu}\psi_{\mu\nu} = 0$.

The requirement that in the variation of (7.6) under the transformation rules (7.2) with constant ε all terms bilinear in the fields cancel leads to the following relations between $c_1, \ldots, c_4$:

$$4\,c_1 + c_2 - 2\,c_3 = 0 \qquad \bar\varepsilon\,\gamma^a\psi_{\mu\nu}\,\partial_\mu\,R_\nu{}^a$$

$$-42\,c_3 - \frac{3}{2\cdot 6!}\,c_4 = 0 \qquad \bar\varepsilon\,\gamma^{\mu_1\ldots\mu_5}\,\psi^{\mu_6\mu_7}\Box H_{\mu_1\ldots\mu_7} \tag{7.7}$$

In (7.7) we have indicated which type of terms vanish in the variation of $\mathcal{L}$ in the corresponding relation is imposed. In the derivation of the second equation of (7.7) we have used the identity

$$D_\mu\gamma_\nu\psi^{\mu\nu} = 0 \tag{7.8}$$

which follows from the constraint $\gamma_{\mu\nu}\psi^{\mu} = 0$ and the Bianchi identity $D_{[\mu}\psi_{\nu\rho]} = 0$. From (7.7) it follows that $c_4 = -\frac{28}{6!}$, $c_3 = \frac{1}{2}(4c_1 + c_2)$. Substituting this into (7.6) we can rewrite the Lagrangian in the following way:

$$\mathcal{L} = \alpha(R_{\mu\nu}{}^{ab}\,R_{\mu\nu}{}^{ab} + 2\,\bar\psi_{\mu\nu}\,\not\partial\,\psi_{\mu\nu} - \frac{14}{6!}\,H^{\mu_1\ldots\mu_7}\Box H_{\mu_1\ldots\mu_7})$$

$$+ \beta(R_{\mu\nu}{}^{ab}\,R_{\mu\nu}{}^{ab} - 4\,R_\mu{}^a\,R_\mu{}^a) \tag{7.9}$$

where $\alpha = c_1 + \frac{1}{4}c_2$ and $\beta = -\frac{1}{4}c_2$. From (7.9) we see that the Lagrangian can be written as the sum of the Riemann tensor squared together with kinetic terms for ψ_μ and $B_{\mu_1,\ldots,\mu_6}$ plus the combination $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_\mu{}^aR_\mu{}^a)$ each with independent coefficients α and β respectively. Note that in the full action $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_\mu{}^aR_\mu{}^a)$ may be extended to the Gauss-Bonnet combination $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_\mu{}^aR_\mu{}^a + R^2)$ since in that case $R^2 \sim$ quartic terms (see (7.4)).

It is not easy to extend the result (7.9) to the nonlinear case by applying a Noether procedure. This is mainly due to the following two reasons. First of all it is clear that unlike in $d = 6$ we cannot give the six-index antisymmetric tensor gauge field of the $d = 10$ conformal multiplet the interpretation of a torsion potential. This means that one has to consider separately variations containing $R_{\mu\nu}{}^{ab}$ and $H_{(7)}$, which complicates the calculation enormously. Secondly one must apply the Noether procedure in the presence of the constraints (7.4). The effect of these constraints is that a priori independent variations become dependent which is again a complicating factor in the calculation. Consequently it could well be that in $d = 10$ it is easier to immediately construct R^2 actions for the $d = 10$ Poincaré supergravity theory. On the other hand, a disadvantage of that strategy is of course that one then has to deal with an additional spinor λ and dilaton ϕ. Giving the two index gauge potential $B_{\mu\nu}$ of the Poincaré theory the interpretation of a torsion potential does not seem to lead to dramatic simplificactions since by doing this one cannot avoid the explicit torsion dependent variations of the type $\delta\lambda = H\varepsilon$.

It is not difficult to extend the result (7.9) to actions in whose variation all H-independent terms cancel. For completeness we have given this result below.

$$e^{-1}\mathcal{L}(\text{Riemann-tensor}) = R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} + 2\,\bar{\psi}_{\mu\nu}\not{D}\,\psi_{\mu\nu} - \frac{14}{6!}H^{\mu_1\ldots\mu_7}\Box H_{\mu_1\ldots\mu_7}$$

$$+ R_{\mu\nu}{}^{ab}\bar{\psi}_\lambda\gamma^{ab}\gamma^\lambda\psi_{\mu\nu} \tag{7.10}$$

$$-\frac{i}{2}e^{-1}\varepsilon^{\mu_1\cdots\mu_{10}}R_{\mu_1\mu_2}{}^{ab}R_{\mu_3\mu_4}{}^{ab}B_{\mu_5\cdots\cdots\mu_{10}} + \cdots$$

$$e^{-1}\mathcal{L}(\text{Gauss-Bonnet}) = R_{[ab}{}^{ab}R_{cd]}{}^{cd}$$

$$+\frac{5}{6}R_{[ab}{}^{ab}\bar{\psi}_c\gamma^{cde}\psi_{de]} \tag{7.11}$$

$$-\frac{i}{3}e^{-1}\varepsilon^{\mu_1\cdots\mu_{10}}R_{\mu_1\mu_2}{}^{ab}R_{\mu_3\mu_4}{}^{ab}B_{\mu_5\cdots\mu_{10}} + \cdots$$

Note in both (7.10) and (7.11) the presence of the Lorentz Chern-Simons related term $R\wedge R\wedge B$. The dilaton ϕ and the spinor λ of the conformal multiplet are easily reintroduced into the R^2 actions by performing the field redefinitions

$$e_\mu^a \rightarrow \phi^{1/2}e_\mu^a$$

$$\psi_\mu \rightarrow \phi^{1/4}(\psi_\mu + \gamma_\mu\lambda) \tag{7.12}$$

thereby undoing the gauge conditions $\phi = 1$ and $\lambda = 0$.

8. CONCLUSIONS

In this paper we have supersymmetrized various R^2-actions in 6 and 10 dimensions which are closely related to the Lorentz Chern-Simons 3-form, and we have emphasized the importance of the superconformal invariance. In addition to their conformal invariance, the R^2-actions constructed here, differ in two more respects from the ones discussed in the literature [7],[8],[9].

Firstly, our theories are off-shell. Moreover the auxiliary fields are propagating and therefore cannot be eliminated to go on-shell, unless one allows infinitely many terms arising in an iterative solution. This prompts us to conjecture that the supersymmetrization of the on-shell R^2-action may require infinitely many corrections in the action as well as in the transformation rules.

Secondly, the duality transformation in the presence of R^2-actions does not seem to be possible due to the fact that the action contains derivatives of H and/or its higher powers. This is in accordance with the fact that the usual on-shell Poincaré actions containing the $B_{\mu\nu}$ field cannot be made off-shell.

In order to obtain a Poincaré Einstein/Yang-Mills plus R^2-action in $d = 6$, we add a separately superconformal invariant action containing $L(\Box + R)L + \phi F^2_{\mu\nu}$, where L is the scalar of the compensating linear multiplet [14]. Hence the total action becomes

$$I = \int d^6x\, e\left[\phi\, R^2_{\mu\nu ab} + \cdots + L(\Box + R)L + \alpha\,\phi\, F^2_{\mu\nu} + \cdots\right] \tag{8.1}$$

where α is an arbitrary constant. Imposing the conformal gauge $L = 1$ one finds the off-shell Poincaré action

$$I = \int d^6x\, e\left[R + \phi\,(R^2_{\mu\nu ab} + \alpha\, F^2_{\mu\nu}) + \cdots\right] \tag{8.2}$$

Since the F^2 and R^2 part of the action (8.1) did not contain the fields of the compensating multiplet, it follows that the F^2 and R^2 parts of the action (8.2) are still superconformal invariant!

It has been observed in Ref. [13] (see also (23]) that the $(R + \phi F^2_{\mu\nu})$ action has the following <u>global scale covariance</u>

$$e_\mu^a \rightarrow -c\, e_\mu^a \;,\quad \phi \rightarrow -2c\,\phi$$
$$e\mathcal{L} \rightarrow -4c\, e\mathcal{L} \;, \tag{8.3}$$

with c constant. This must be compared with the local scale invariance of the $\phi F^2_{\mu\nu}$ part of the same action given by

$$e^a_\mu \to -c(x) e^a_\mu \quad , \quad \phi \to +2\, c(x)\, \phi$$
$$e \mathcal{L}(F) \to e \mathcal{L}(F) \tag{8.4}$$

Assuming that it is this local scale invariance which will hold in the higher order terms of the low energy limit of the string theory, we conjecture that the Lagrangian in that limit will take the form

$$e^{-1} \mathcal{L} \sim R + \sum_{n=2}^{\infty} \phi^{3-n} \left(R^n_{\mu\nu ab} + \alpha_n F^n_{\mu\nu} + \cdots\right) \tag{8.5}$$

where α_n are arbitrary coefficients not determined by supersymmetry, ... refers to all possible contractions allowed by supersymmetry, and the action of (8.5), excluding the R-term, is locally superconformal invariant.

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APPENDIX A

IDENTITIES FOR CURVATURES WITH TORSION

In this appendix we have collected all identities for curvatures with torsion which have been used in this paper. All these identities can easily be derived by first expressing the torsion-ful curvature $R_{\mu\nu,\rho\sigma}(\Gamma)$ into the torsionless curvature $R_{\mu\nu,\rho\sigma}(\{\})$ by using $\Gamma_{\mu\nu}{}^{\rho} = \left\{ {\rho \atop \mu\nu} \right\} - \frac{1}{2} H_{\mu\nu}{}^{\rho}$:

$$R_{\mu\nu,\rho\sigma}(\Gamma) \equiv \partial_\mu \Gamma_{\nu\rho}{}^{\sigma} + \cdots$$
$$= R_{\mu\nu,\rho\sigma}(\{\}) - \mathcal{D}_{[\mu}(\Gamma) H_{\nu]\rho\sigma} + \frac{1}{2} H_{[\mu}{}^{\rho\lambda} H_{\nu]\lambda}{}^{\sigma} + \frac{1}{2} H_{\mu\nu}{}^{\lambda} H_{\lambda\rho\sigma} \tag{A.1}$$

and then using the known identities of $R_{\mu\nu,\rho\sigma}(\{\})$. Note that from the sechsbein postulate (3.10) it follows that

$$R_{\mu\nu,\rho\sigma}(\Gamma) = - R_{\mu\nu}{}^{ab}(\omega_+)\, e_a{}^{\rho}\, e_b{}^{\sigma} \tag{A.2}$$

We will only give the identities in terms of $R_{\mu\nu,\rho\sigma}(\Gamma)$. Those for $R_{\mu\nu,ab}(\omega_\pm)$ can easily be derived by using (A.2) and the relation $R_{\mu\nu,ab}(\omega_-) = R_{ab,\mu\nu}(\omega_+) +$ + bil. ferm.

We stress that some of the identities given below are only valid up to terms bilinear in fermion fields. Therefore these identities can only be used in the variation of an action which is determined up to quartic fermion terms but not in the expression for the action itself.

The identities read as follows. All the curvatures $R_{\mu\nu,\rho\sigma}$ and the covariant derivatives $\mathcal{D}_\mu$ are with respect to the torsionful connection $\Gamma(H)$.

$$R_{\mu\nu,\rho\sigma} - R_{\rho\sigma,\mu\nu} = -\mathcal{D}_{[\mu} H_{\nu]\rho\sigma} + \mathcal{D}_{[\rho} H_{\sigma]\mu\nu}$$

$$R_{[\mu\nu,\rho]\sigma} = -\frac{1}{6} \mathcal{D}_\sigma H_{\mu\nu\rho} - \frac{1}{2} \mathcal{D}_{[\mu} H_{\nu\rho]\sigma} \quad + \text{bil. ferm}$$

$$R_{\mu[\nu,\rho\sigma]} = -\frac{1}{3} \mathcal{D}_\mu H_{\nu\rho\sigma} \quad + \text{bil. ferm}$$

$$\mathcal{D}_{[\mu} R_{\nu\rho]}{}^{\sigma\tau} = - H_{[\mu\nu}{}^{\lambda} R_{\rho]\lambda}{}^{\sigma\tau}$$

$$\mathcal{D}_{[\mu} \mathcal{D}_{\nu]} H_{\lambda\rho\tau} = -\frac{3}{2} R_{\mu\nu,\sigma[\lambda} H_{\rho\tau]}{}^{\sigma} + \frac{1}{2} H_{\mu\nu}{}^{\sigma} \mathcal{D}_{\sigma} H_{\lambda\rho\tau}$$

$$\mathcal{D}_{[\mu} H_{\nu\rho\sigma]} = \frac{3}{2} H_{[\mu\nu}{}^{\lambda} H_{\rho\sigma]\lambda} + \text{bil. ferm.} \qquad \text{(A.3)}$$

APPENDIX B

THE SUPERSYMMETRIC $R_{\mu\nu}{}^{ab} R_{\mu\nu}{}^{ab}$ ACTION IN $d = 6$

In this appendix we give the final result of Sec. 5 where we applied the Noether procedure to supersymmetrize the Riemann tensor squared in $d = 6$. The Lagrangian up to quartic fermion terms is given by:

$$\begin{aligned} e^{-1}\mathcal{L} = {} & R_{\mu\nu}{}^{ab}(\omega_-) R_{\mu\nu}{}^{ab}(\omega_-) + 2 \bar{\psi}_{\mu\nu} \not{\mathcal{D}} \psi_{\mu\nu} - V^{ij}_{\mu\nu} V_{\mu\nu ij} \\ & - R_{\mu\nu}{}^{ab}(\omega_+) \bar{\psi}_{\lambda} \gamma^{ab} \gamma^{\lambda} \psi_{\mu\nu} + 4 V^{ij}_{\mu\nu} \bar{\psi}_{\mu i} \gamma^{\lambda} \psi_{\lambda\nu j} \\ & - \frac{1}{3} V^{ij}_{\mu\nu} \bar{\psi}_{\mu i} \gamma \cdot H \psi_{\mu\nu j} - \frac{1}{3} \bar{\psi}_{\mu\nu} \gamma \cdot H\psi_{\mu\nu} - 4 H^{\mu\nu\rho} \bar{\psi}_{\mu\lambda} \gamma_{\rho} \psi_{\nu\lambda} \\ & - \frac{i}{4} e^{-1} \varepsilon^{\mu\nu\rho\sigma\lambda\tau} R_{\mu\nu}{}^{ab}(\omega_-) R_{\rho\sigma}{}^{ab}(\omega_-) B_{\lambda\tau} \\ & + \text{quartic fermions} \end{aligned} \qquad \text{(B.1)}$$

The action of this Lagrangian is invariant under

$$\begin{aligned} \delta e^{a}_{\mu} &= \frac{1}{2} \bar{\varepsilon} \gamma^{a} \psi_{\mu} \\ \delta \psi_{\mu} &= \mathcal{D}_{\mu}(\omega_+) \varepsilon \\ \delta V^{ij}_{\mu} &= \bar{\varepsilon}^{(i} \gamma^{\lambda} \psi_{\lambda\mu}{}^{j)} - \frac{1}{6} \bar{\varepsilon}^{(i} \gamma \cdot H \psi_{\mu}{}^{j)} \\ \delta B_{\mu\nu} &= - \bar{\varepsilon} \gamma_{[\mu} \psi_{\nu]} \end{aligned} \qquad \text{(B.2)}$$

The torsionful curvatures $R_{\mu\nu}{}^{ab}(\omega_\pm)$ are defined in (3.12). The covariant derivative $\mathcal{D}_{\mu}$ is with respect to the torsionful connection ω_+, Γ. For instance in the gravitino kinetic term we have

$$\mathcal{D}_{\lambda} \psi_{\mu\nu} = \partial_{\lambda} \psi_{\mu\nu} + \frac{1}{4}\omega_{+\lambda}{}^{ab} \gamma_{ab} \psi_{\mu\nu} - 2 \Gamma_{\lambda[\mu}{}^{\rho} \psi_{\rho\nu]} \qquad \text{(B.3)}$$

APPENDIX

RESULTS ON THE SUPERSYMMETRIZATION OF $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_{\mu}{}^{a}R_{\mu}{}^{a} + R^2)$ IN $d = 6$

In this appendix we summarize the results of Sec. 6 where we discussed the supersymmetrization of the Gauss-Bonnet combination $(R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab} - 4R_{\mu}{}^{a}R_{\mu}{}^{a} + R^2)$ in $d = 6$. Starting from $R_{[ab}{}^{ab}(\omega_+)R_{cd]}{}^{cd}(\omega_+)$ (i.e. choosing + signs in (6.1)) we find that in the variation of the following Lagrangian

$$\begin{aligned}
e^{-1}\mathcal{L} &= R_{[ab}{}^{ab}(\omega_+)\, R_{cd]}{}^{cd}(\omega_+) \\
&- \frac{5}{6} R_{[ab}{}^{ab}(\omega_+)\, \bar{\psi}_c\, \gamma^{cde}\, \psi_{de]} + \frac{1}{12} V^{ij}_{ab}\, \bar{\psi}_{ci}\, \gamma^{abcde}\, \psi_{dej} \\
&- \frac{10}{9}\, H^{abc}\, H_{[abc}\, \bar{\psi}_d\, \gamma^{def}\, \psi_{ef]} \\
&- \frac{1}{3}\, \bar{\psi}^a{}_{[a}\, \gamma^{bcdef}\, \psi_{bc}\, H_{def]} \\
&- \frac{i}{24}\, e^{-1}\, \varepsilon^{\mu\nu\rho\sigma\lambda\tau}\, R_{\mu\nu}{}^{ab}(\omega_+)\, R_{\rho\sigma}{}^{ab}(\omega_+)\, B_{\lambda\tau} + \frac{i}{24}\, e^{-1}\, \varepsilon^{\mu\nu\rho\sigma\lambda\tau}\, V^{ij}_{\mu\nu}\, V_{\rho\sigma ij} B_{\lambda\tau} \\
&+ \text{quartic fermions} \qquad \text{(C.1)}
\end{aligned}$$

under the supersymmetry transformation rules given in (B.2) all H-independent terms and all variations containing V^{ij}_{ab} cancel.

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TABLE 1

All possible terms arising in the variation of the $R_{\mu\nu}{}^{ab}R_{\mu\nu}{}^{ab}$ action in d = 6.

bilinear terms	trilinear terms	quartic terms	quintic terms
$\mathcal{D}\psi_{\mu\nu}V$	$\psi_\mu V^2$	$\psi_\mu H^2 V$	
$\psi_{\mu\nu}\mathcal{D}V$	$\psi_\mu RV$		
	$\psi_\mu H\mathcal{D}V$		
	$\psi_\mu V\mathcal{D}H$		
	$\psi_{\mu\nu}HV$		
$\mathcal{D}\psi_{\mu\nu}R$	$\psi_\mu R^2$	$(\psi_\mu H^2 R)$	
$\psi_{\mu\nu}\mathcal{D}R$	$\psi_{\mu\nu}HR$		
	$(\psi_\mu H\mathcal{D}R)$		
	$(\psi_\mu R\mathcal{D}H)$		
$(\mathcal{D}\psi_{\mu\nu}\mathcal{D}H)$	$\psi_{\mu\nu}H\mathcal{D}H$	$\psi_{\mu\nu}H^3$	$(\psi_\mu H^4)$
$(\psi_{\mu\nu}\mathcal{D}\mathcal{D}H)$	$(\psi_\mu\mathcal{D}H\mathcal{D}H)$	$(\psi_\mu H^2\mathcal{D}H)$	

We use the abbreviations $V = V_{\mu\nu}^{ij}$ and $R = R_{\mu\nu}{}^{ab}$. The terms between brackets denote structures that are automatically cancelled if we use a torsion interpretation for $H_{\mu\nu\rho}$, i.e. use both in the action and its variation $R_{\mu\nu}{}^{ab}(\omega_\pm)$ instead of $R_{\mu\nu}{}^{ab}$.

TABLE 2

Sources of contributions to the variation of the $R_{\mu\nu}^{ab} R_{\mu\nu}^{ab}$ action

variation / source	$\psi_\mu V^2$	$\psi_\mu RV$	$\psi_\mu H \not{D} V$	$\psi_\mu V \not{D} H$	$\psi_{\mu\nu} HV$	$\psi_{\mu\nu} HR$	$\psi_{\mu\nu} H \not{D} H$	$\psi_\mu H^2 V$	$\psi_{\mu\nu} H^3$
R^2						x			
$\bar{\psi}_{\mu\nu} \not{D} \psi_{\mu\nu}$									
V^2	x		x		x			x	
$R\psi_\mu \psi_{\mu\nu}$		x				0	0		0
$V\psi_\mu \psi_{\mu\nu}$	0	0		0	0				
$R^2 B$						x			
$VH\psi_\mu{}^2$			x	x	x			x	
$\psi_{\mu\nu}{}^2 H$					x	x	0		0

The terms marked with 0(x) arise from the variations which do (do not) involve the Bianchi identities given in Appendix B and Eq. (3.16).