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E. Bergshoeff

Abdus Salam

and

E. Sezgin

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# A SUPERSYMMETRIC $R^2$-ACTION IN SIX DIMENSIONS AND TORSION *

E. Bergshoeff
International Centre for Theoretical Physics, Trieste, Italy

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy,
and
Imperial College, London, England

and

E. Sezgin
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We give the superconformal extension of $(R_{\mu\nu ab})^2$ in six dimensions. We show that in a superconformal gauge the 3-form field $H_{\mu\nu\rho}$ has a natural torsion interpretation. We also give partial results on the superconformal extension of the Gauss-Bonnet combination: $R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2$ .

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Supersymmetrization of an $R^2$-action in $d = 10$ is an important issue which arises in the study of the low energy limit of superstrings. There have been a few attempts in this direction. In Refs.1 and 2 the starting point is $N = 1, d = 10$ supergravity with 2-form field $B_{\mu\nu}$. In Ref.3, transformation rules are considered in the dual theory with a 6-form field $B_{\mu_1 \ldots \mu_6}$. So far these attempts have not led to a complete result. One of the difficulties is the mixing of $R$ and $R^2$ actions, and the necessity of modifying the transformation rules, both of which are due to the fact that the theory is on-shell. In $d = 4$ an off-shell formulation of $N = 1$ supergravity is known, and supersymmetrization of the ghost-free [4] Gauss-Bonnet combination $\phi(R^2_{\mu\nu ab} - 4R_{\mu a}^{\ 2} + R^2)$ has been fully worked out in [5]. The only other supersymmetric $R^2$-actions that we know of are the $d = 4, N = 1$ [6] and $N = 2$ [7] superconformal actions containing the (Weyl tensor)$^2$, i.e. $(R^2_{\mu\nu ab} - 2R_{\mu a}^{\ 2} + \frac{1}{3} R^2)$.

In this note we give the first example of a supersymmetric (up to quartic fermion terms) $R^2$-action in a higher dimension by constructing *the off-shell superconformal extension of the (Riemann tensor)*$^2$ in $d = 6$. We note that an $R + R^2$ supergravity action in $d = 6$ may be the low energy limit of a superstring theory [8]. In this context we recall that an anomaly free [9] $d = 6$ supergravity theory [10] which is not, in any known way, related to the $d = 10$ theory does exist. Moreover, such a theory does admit a $(\text{Minkowski})_4 \times S^2$ compactification [11,9].

In this note we also briefly discuss the supersymmetrization of the Gauss-Bonnet combination of $R^2$-terms, both in 6 and 10 dimensions. A detailed derivation of our results will be given in a separate paper [12].

The off-shell chiral $N = 2$, $40 + 40$ conformal multiplet containing the 2-form field $B_{\mu\nu}$ has the following transformation rules [13]:

$$\delta e_\mu^a = \frac{1}{2} \bar{\varepsilon} \gamma^a \psi_\mu - \Lambda_D e_\mu^a$$

$$\delta \psi_\mu^i = D_\mu \varepsilon^i + \frac{1}{48} \phi^{-1} \gamma^{\rho\sigma\tau} \gamma_\mu \varepsilon^i H_{\rho\sigma\tau} + \gamma_\mu \eta^i - \frac{1}{2} \Lambda_D \psi_\mu^i$$

$$\delta V_\mu^{ij} = -4 \bar{\varepsilon}^{(i} \phi_\mu^{\ j)} - 2 \phi^{-1} \bar{\varepsilon}^{(i} \gamma_\mu \hat{\not{D}} \lambda^{j)} + \frac{1}{12} \phi^{-2} \bar{\varepsilon}^{(i} \gamma_\mu \gamma^{\rho\sigma\tau} \lambda^{j)} H_{\rho\sigma\tau} - 4 \bar{\eta}^{(i} \psi_\mu^{\ j)}$$

$$\delta B_{\mu\nu} = -\phi \bar{\varepsilon} \gamma_{[\mu} \psi_{\nu]} - \bar{\varepsilon} \gamma_{\mu\nu} \lambda$$

$$\delta \lambda^i = \frac{1}{4} \hat{\not{D}} \phi \varepsilon^i + \frac{1}{48} \gamma^{\rho\sigma\tau} \varepsilon^i H_{\rho\sigma\tau} - \phi \eta^i + \frac{5}{2} \Lambda_D \lambda^i$$

$$\delta \phi = \bar{\varepsilon} \lambda + 2 \Lambda_D \phi \ , \qquad (1)$$

where $\Lambda_D$, $\epsilon^i$ and $\eta^i$ are the parameters of dilatation, supersymmetry and conformal supersymmetry, respectively. All the derivatives are supercovariant, and $H_{\mu\nu\rho}$ is the supercovariant field strength of $B_{\mu\nu}$. As usual the gauge field, $\phi_\mu$ , of conformal supersymmetry is not an independent field. For its explicit expression and for the rest of our notation and conventions see Ref.13.

An important observation which greatly simplifies the construction of the $R^2$-action is the fact that in the gauge $\phi = 1$ (fixing $\Lambda_D$) and $\lambda = 0$ (fixing $\eta$) the transformation rules (1) become [14,12]

$$\begin{aligned}
\delta e^a_\mu &= \tfrac{1}{2}\ \bar{\epsilon}\ \gamma^a\ \psi_\mu \\
\delta\psi^i_\mu &= D_\mu(\omega_+)\ \epsilon \\
\delta V^{ij}_\mu &= \bar{\epsilon}^{(i}\ \gamma^\lambda\ \psi_{\lambda\mu}{}^{j)} - \tfrac{1}{6}\ \bar{\epsilon}^{(i}\ \gamma^{\rho\sigma\tau}\ \psi_\mu{}^{j)}\ H_{\rho\sigma\tau} \\
\delta B_{\mu\nu} &= -\ \bar{\epsilon}\ \gamma_{[\mu}\ \psi_{\nu]} \qquad , \qquad\qquad (2)
\end{aligned}$$

where

$$\omega_{\mu ab\,\pm} = \omega_{\mu ab}\ (e,\psi) \pm \tfrac{1}{2}\ H_{\mu ab} \qquad (3a)$$

$$\psi_{\mu\nu} = D_\mu(\omega_+)\psi_\nu - \mu \longleftrightarrow \nu \qquad . \qquad (3b)$$

We see that, in the gauge we are considering, the field strength $H_{\mu\nu\rho}$ has a natural torsion interpretation. Since (2) is much simpler than (1), our strategy will be to first constuct an $R^2$-action invariant under (2) and then undo the gauge conditions, ($\phi = 1$, $\lambda = 0$), by performing the following field redefinitions:

$$\begin{aligned}
e^a_\mu &\rightarrow \phi^{1/2}\ e^a_\mu \\
\psi^i_\mu &\rightarrow \phi^{1/4}(\psi^i_\mu + \phi^{-1}\ \gamma_\mu\ \lambda^i) \\
V^{ij}_\mu &\rightarrow V^{ij}_\mu - 4\ \phi^{-1}\ \bar{\lambda}^{(i}\ (\psi_\mu{}^{j)} + \phi^{-1}\ \gamma_\mu\lambda^{j)}) \qquad\qquad (4)
\end{aligned}$$

Let us now consider the supersymmetrization of an arbitrary combination of $R^2$-terms with respect to (2). It turns out that the $R^2$-terms must be of the form

$$R^2_{\mu\nu ab} + \alpha(R^2_{\mu\nu ab} - 4R^2_{\mu a} + R^2) \quad . \qquad (5)$$



We find this result already at the stage of cancelling the terms which are bilinear in the fields.

In the case $\alpha = 0$, applying the standard Noether procedure we find that the action which is invariant under (2) is given by

$$\begin{aligned}
I = \int d^6x\ e\Big[&R_{\mu\nu ab}(\omega_-)\ R^{\mu\nu ab}(\omega_-) + 2\ \bar{\psi}_{\mu\nu}\ \gamma^\lambda\ D_\lambda(\omega_+,\Gamma)\ \psi_{\mu\nu} \\
&-V_{\mu\nu}{}^{ij}\ V^{\mu\nu}{}_{ij} - R^{\mu\nu ab}(\omega_+)\ \bar{\psi}_\lambda\ \gamma_{ab}\ \gamma^\lambda\ \psi_{\mu\nu} \\
&+ 4\ V_{\mu\nu}{}^{ij}\ \bar{\psi}_{\mu i}\ \gamma^\lambda\ \psi_{\lambda\nu j} - \tfrac{1}{3}\ H_{\mu\nu\rho}\Big(\bar{\psi}_{\lambda\tau}\ \gamma^{\mu\nu\rho}\ \psi_{\lambda\tau} + 12\ \bar{\psi}_{\mu\lambda}\ \gamma_\rho\ \psi_{\nu\lambda} \\
&+ V_{\lambda\tau}{}^{ij}\ \bar{\psi}_{\lambda i}\ \gamma^{\mu\nu\rho}\ \psi_{\tau j}\Big) - \tfrac{1}{4}\ e^{-1}\ \epsilon^{\mu\nu\rho\sigma\lambda\tau}\ R_{\mu\nu}{}^{ab}(\omega_-)\ R_{\rho\sigma}{}^{ab}(\omega_-)\ B_{\lambda\tau} \\
&+ \text{quartic fermions}\Big] \qquad\qquad (6)
\end{aligned}$$

where $V_{\mu\nu}{}^{ij}$ is the field strength of the SU(2) gauge field $V_\mu{}^{ij}$. To obtain the superconformal version of (6), one simply performs the re-definitions (4), which in particular will yield the term $e\phi R^2_{\mu\nu ab}$ . The following comments are in order:

(1) All the terms containing DH and $H^2$ have been absorbed into $R_{\mu\nu ab}(\omega_\pm)$ .

(2) The $R\wedge R\wedge B$ term in (6) is invariant under $\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$ thanks to the Bianchi identity $(D_\mu(\omega_-)\ R_{\nu\rho ab}(\omega_-) + \text{cyclic}) = 0$. Note that this term is the analogue of a $F\wedge F\wedge B$ term which arises in the supersymmetrization of the Yang-Mills action [13].

(3) In the absence of the $R^2$-invariant, one can dualize the Poincaré theory to obtain a field strength for the dual 2-form which contains the Yang-Mills Chern-Simons, coming from the $F\wedge F\wedge B$ term. However, in the presence of the $R^2$-invariant, due to the $H\Box H$, $H^4$ and $H^2\psi^2$, etc. terms implicit in (6), the duality transformation which would give rise to Yang-Mills and Lorentz Chern-Simons form is far from trivial, and if impossible it would suggest inequivalence between the two formulations.

(4) We expect that this action has ghosts (certainly in d = 6 Minkowski background) which arrange themselves into a supermultiplet [15].

(5) Since our theory is off-shell, the transformation rules, (1), are fixed. Moreover, as the auxiliary field $V_\mu^{ij}$ propagates, it cannot be eliminated. Hence no corrections in the transformation rules which would have arisen if one could have eliminated $V_\mu^{ij}$.

(6) There exists another chiral $N = 2$, 40 + 40 conformal multiplet [13] which contains an anti-self dual 3-form field $T^-_{abc}$ (with no gauge invariance). We find that using this multiplet one cannot construct an action containing $R^2_{\mu\nu ab}$.

(7) Concerning the Gauss-Bonnet combination of $R^2$-terms †) we find that it is not difficult to cancel all variations which are independent of $H_{\mu\nu\rho}$. (In this context we note that in Refs.1,2 and 18 only such variations are considered). More explicitly, we find that all H-independent terms cancel in the variation of the following action [12]:

$$I = \int d^6x\, e\Big[R_{[ab}{}^{ab}(\omega)\, R_{cd]}{}^{cd}(\omega) - \frac{5}{6} R_{[ab}{}^{ab}(\omega)\, \bar\psi_c \gamma^{cde} \psi_{de]}$$
$$+ \frac{1}{12} V^{ij}_{ab}\, \bar\psi_{ci}\, \gamma^{abcde}\, \psi_{dej} + \frac{i}{24} e^{-1} \varepsilon^{\mu\nu\rho\sigma\lambda\tau} V^{ij}_{\mu\nu} V_{\rho\sigma ij} B_{\lambda\tau}$$
$$- \frac{i}{24} e^{-1} \varepsilon^{\mu\nu\rho\sigma\lambda\tau} R_{\mu\nu}{}^{ab}(\omega)\, R_{\rho\sigma}{}^{ab}(\omega)\, B_{\lambda\tau} + \ldots\Big] \qquad (7)$$

where ... refers to quartic fermions and to terms which are to be determined by demanding the cancellation of all the $H_{\mu\nu\rho}$ dependent variations of the action.

It turns out that the complete supersymmetrization of the action is very involved. This is mainly due to the fact that it is not clear to us how to absorb all the DH and $H^2$ dependent terms needed for supersymmetry into the terms containing the Riemann tensor.

(8) Note the presence of the $R\wedge R\wedge B$ term and $V\wedge V\wedge B$ term in the action (7). These terms are related to Lorentz-Chern-Simons terms, and Chern-Simons terms for the connection $V^{ij}_\mu$, respectively.

(9) The action (7) does not contain a higher derivative kinetic term for any of the fields of the conformal multiplet like $\bar\psi_{\mu\nu} \partial\!\!\!/ \psi_{\mu\nu}$ etc. Hence this action has no ghosts. This generalizes the result of Zwiebach [4] to the supersymmetric case. Moreover the absence of a $H \Box H$ term indicates that a duality transformation might be possible in this case.

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†) For some recent discussions of Gauss-Bonnet $R^2$-actions in the context of strings see Refs.4, 16 and 17.



(10) Results similar to those found in this paper are expected to hold in d = 10. A significant difference is that the off-shell conformal multiplet in d = 10 [19] contains a 6-form field as opposed to the 2-form field $B_{\mu\nu}$. As was shown in Ref.19 in the conformal gauge $\phi = 1$, $\lambda = 0$ the transformation rules are given by

$$\delta e^a_\mu = \frac{1}{2} \bar\varepsilon \gamma^a \psi_\mu$$
$$\delta\psi_\mu = D_\mu \varepsilon + \frac{1}{6} (\gamma_\mu \gamma^{(7)} - 3\gamma^{(7)} \gamma_\mu) H_{(7)} \varepsilon$$
$$\delta B_{\mu_1 \ldots \mu_6} = \frac{3}{4.6!} \bar\varepsilon \gamma_{[\mu_1 \ldots \mu_5} \psi_{\mu_6]} , \qquad (8)$$

where $H_{(7)}$ is the field strength of $B_{\mu_1 \ldots \mu_6}$. From (8) we deduce that the 6-form field does not lead to any torsion interpretation in the sense of (2) and (3a). An unusual feature in d = 10 is that the zehnbein and the gravitino satisfy a differential constraint [19]

$$R = -4.7!\, H^{(7)} H_{(7)} + 9\, \bar\psi_\mu \gamma_\nu \psi^{\mu\nu}$$
$$\gamma^{\mu\nu} \psi_{\mu\nu} = 0 \qquad (9)$$

Requiring the cancellation of all terms bilinear in the fields we find that the $R^2$ – terms must be of the form

$$R^2_{\mu\nu ab} + \alpha(R^2_{\mu\nu ab} - 4 R_{\mu a}{}^2) \qquad (10)$$

Note that higher order cancellations may require that $(R_{\mu\nu ab}{}^2 - 4 R_{\mu a}{}^2)$ be extended to the Gauss-Bonnet combination $(R_{\mu\nu ab}{}^2 - 4 R_{\mu a}{}^2 + R^2)$ since $R^2 \sim H^4$. To extend the result (10) to higher order in the fields is not easy. This is mainly due to the presence of the differential constraints (9) and the fact that there is no torsion interpretation for $H_{(7)}$ †). Also in d = 10 the transition from conformal to Poincare supergravity is non-trivial [19]. One needs a compensating scalar superfield with a large number of components.

(11) In this paper we have constructed a superconformal $R^2$-action in d=6. In order to obtain an (off-shell) Poincaré Einstein-Yang Mills plus $R^2$-action, one adds the separately superconformal piece containing $L(\Box + R)L + \phi F^2_{\mu\nu}$, where L is the scalar of a compensating linear multiplet [13], so as to obtain

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†) Several authors have suggested a torsion interpretation of $H_{(3)}$ in the low-energy limit of the D = 10 supersymmetric string [20,1].

$$I = \int d^6x\, e\left[\phi\, R^2_{\mu\nu ab} + \ldots + L(\Box + R)L + \alpha\phi\, F^2_{\mu\nu} + \ldots\right] , \qquad (11)$$

where $\alpha$ is an arbitrary constant. Next, one imposes the conformal gauge $L = 1$. This yields the off-shell Poincaré action

$$I = \int d^6x\, e\left[R + \phi(R^2_{\mu\nu ab} + \alpha F^2_{\mu\nu}) + \ldots\right] \qquad (12)$$

in which the $R^2$ and $F^2$ part of the action are still superconformal invariant! This follows from the fact that the $R^2$ and the $F^2$ part of the action (11) does not contain the fields of the compensating multiplet, such as L.

In Ref.10 (see also [21]) it is observed that the $(R + \alpha\phi F^2_{\mu\nu})$ action has a global scale invariance characterized by

$$e^a_\mu \to -c\, e^a_\mu , \qquad \phi \to -2c\phi$$

$$e\mathcal{L} \to -4c\mathcal{L} , \qquad (13)$$

where c is a constant parameter. This is to be contrasted with the local scale invariance of the $\phi F^2$ part of the action [19] characterized by

$$e^a_\mu \to -c(x)\, e^a_\mu , \qquad \phi \to +2c(x)\phi$$

$$\mathcal{L}(F) \to \mathcal{L}(F) . \qquad (14)$$

Assuming that this local scale invariance is the principal which will prevail in the higher order terms of the low energy limit of the string theory, we then conjecture that the off-shell Poincaré Lagrangian in that limit will take the form

$$e^{-1}\mathcal{L} \sim R + \sum_{n=2}^{\infty} \phi^{3-n}\, (R^n_{\mu\nu ab} + \alpha_n F^n_{\mu\nu} + \ldots) , \qquad (15)$$

where $\alpha_n$ are arbitrary coefficients not determined by supersymmetry, ... refers to all possible contractions allowed by supersymmetry, and the action of (15), excluding the R-term, is locally superconformal invariant.

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