ON INFLATION IN THE HETEROTIC SUPERSTRING MODEL

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ABSTRACT

We consider the possibility of achieving inflation in the field-theory limit of the $E_8 \times E_8$ superstring model, which is an $N = 1$ supergravity theory possessing a 'no-scale' $SU(n,1)/SU(n) \times U(1)$ structure. We show that neither type I inflation (due to higher-derivative terms $G(R)$), nor inflation due to a SUSY-breaking gaugino-condensation potential, is possible, essentially because of the absence of free dimensionless parameters. Kaluza-Klein type inflation is ruled out because the internal space is Ricci flat. The occurrence of type II inflation (due to some gauge singlet 'inflaton' field $\phi$) depends upon the form of the superpotential $F$ and of the Kahler potential $G$, but this also seems not to be possible, unless the $SU(n,1)$ symmetry can be broken in a particular way. Hence, some new type of compactification scheme may be called for, or a different type of inflation.

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Much interest has recently been focused on superstring- [1] since the discovery of anomaly cancellations [2]. A valid theory of quantum gravity may be at hand, which also makes acceptable phenomenological predictions, especially in the $E_8 \times E_8$ heterotic string [3]. A low-energy approximation to the superstring theory is the zero-slope or field-theory limit, valid at mass scales $\ll m_s$ where $m_s$ is the string mass and $(2\pi)^{1/2}$ the string tension $\alpha'$. It has been identified with two-dimensional, $N = 1$ chiral supergravity [4] and has been found to possess a no-scale $SU(n,1)/SU(n) \times U(1)$ structure, represented by the Kahler potential $G$ [5]:

$$G = -3\ln (S + S^*) - 3\ln (T + T^* - 2\phi \phi^* + \ln |F|^2).$$

Here, $\phi^0$ are the observable gauge non-singlet fields, $F$ is the superpotential and the complex chiral superfields $S$ and $T$ are defined in terms of the dilaton $\phi$, the scale length of the internal space $e^S$ and pseudo-scalar fields which arise upon compactification from ten dimensions to four. The ten-dimensional interval is written as

$$ds^2_{10} = e^{-2\phi} \left[ e^{2B} g_{\mu\nu}(x^5) dx^\mu dx^\nu + e^{2B} \delta_{mn}(y^k) dy^m dy^n \right],$$

where $g_{\mu\nu}(x^5)$ describes the four-dimensional space-time and $\delta_{mn}(y^k)$ the six-dimensional internal space. Then, we have

$$S = e^{B-2\phi} - \frac{1}{3} i b_5 \phi_5,$$

and

$$T = e^{B-2\phi} + \phi \phi^* + i \phi^*,$$

where $\phi_5$ and $\phi^*$ represent two axion fields (the invisible superpartners of $\phi$ and $\bar{S}$) and $b_5$ is the coefficient of the one-loop $\delta$ function of $Q$, which is some convenient subgroup of the second $E_8$, assumed to be the 'hidden sector' of the low-energy supergravity model. Local supersymmetry is broken by the process of gaugino condensation [6], and this defines the superpotential as described below.

It is necessary to examine the superstring theory from a cosmological point of view as well. The first obvious question is whether the field-theory limit is capable of naturally reproducing the Friedmann universe in which we live. This issue has been studied in [7], using the potential
V that is derivable from expression (1) when one assumes on \( V \) arising from gaugino condensation (additional arguments in favour of which have been put forward in [8]). It was found, for certain initial conditions, that the Friedmann universe is an 'attractor'. But if the universe starts off on the wrong side of the barrier in \( V \), then it ends up as a Brans-Dicke universe with \( w = -1 \), in conflict with the observational constraint [9] \( \omega \gg 500 \). This latter feature reflects the fact that the 'dilaton field' \( \text{Re}(S) \) (which plays the role of the Brans-Dicke scalar) does not acquire a mass.

The next important question to ask is whether inflation [10] can occur, and that is the subject of the present paper (for reviews of inflation, see [11]). There are various possibilities. One might expect, for example, that inflation could take place while the internal space is rapidly contracting, i.e. the so-called Kaluza-Klein inflation [12]. This cannot happen as usually in Kaluza-Klein theories, however, because the internal space is Ricci flat [7]. (A compactification which is not Ricci flat [13] may make this type of inflation possible. It may be difficult, however, to obtain our Friedmann universe via Kaluza-Klein inflation in a consistent fashion [14].) Hence, we are led to consider the properties of the four-dimensional space-time obtained after compactification, including the effects of terms of order \( R^2 \) and of a gaugino-condensation potential. And since we are dealing with a no-scale supergravity model [15], then we also discuss the possibility of no-scale inflation ([16],[17] and references therein).

In order to have a compactification onto the Calabi-Yau manifold, a term proportional to the square of the Riemann tensor must be included [18]. And to ensure stability of the eventual Friedmann universe, if this is reached, further higher-derivative terms proportional to \( R^2 \) and \( R_{\mu
u\rho\sigma}R_{\mu
u\rho\sigma} \) must be present [7],[19]. Whilst the gaugino condensation mechanism [6] implies the superpotential

\[
F = m_p^3 \left[ c + h \exp(-3S/2h) \right]
\]

where the complex constant \( c = 0(1) \) is related to the field strength \( H_{\mu\nu\rho} \) as

\[
\langle H_{ijk} \rangle = M_p^3 \epsilon_{ijk}
\]

\( h \) is a constant and \( m_p \) is the Planck mass.

Introducing the fields \( \sigma \) and \( \tau \), defined by

\[
\sigma = \frac{1}{2k} \ln \left[ \text{Re}(S) \right] \quad \text{and} \quad \tau = \frac{1}{4k} \ln \left[ \text{Re}(T) \right],
\]

where \( \nu^2 = 8\pi m_p^2 \), we write the four-dimensional effective Lagrangian in the form

\[
\mathcal{L} \equiv \mathcal{L}_0 + \epsilon' \mathcal{L}' + \epsilon \mathcal{L}
\]

where \( \epsilon = (-\det(g_{\mu\nu}))^{1/2} \) and \( \alpha_1 \) and \( \alpha_2 \) are some constants. The contribution \( \mathcal{L}' \) represent all the terms which appear explicitly as a result of the space-time dependence of the conformal factors \( e^{\phi_B} \) and \( e^{2\phi} \) and contain higher-derivatives of \( B \) plus derivatives of \( B \) coupled to \( \epsilon \) and \( \epsilon' \). It has the form

\[
\mathcal{L}' = \frac{d^2}{4k} e^{2\phi} \left\{ [a_0 (\partial B + a_1 V B)^2] R^0 \right. \\
+ [a_3 (\partial B + a_2 V B)(\partial B)(V B)] R^0 + b_1 (V B)^2 + b_2 (\partial V B)^2 \\
+ b_3 (\partial V B)(\partial V B)(\partial V B) + b_4 (V B)^2 + c (V B)^4 \}
\]

where the coefficients \( a_1, b_i \) (i = 1-4) and \( c \) depend upon \( \alpha_1 \) and \( \alpha_2 \) but are all \( \lesssim 10^2 \) for \( \alpha_1, \alpha_2 \lesssim 1 \). (We shall not require their exact values, which are straightforward to obtain.) According to [19], the coefficients \( \alpha_1 \) and \( \alpha_2 \) must obey the inequalities

\[
9 \alpha_1 > 10 \alpha_2 > 0 \quad \text{or} \quad \alpha_1 = \alpha_2 = 0,
\]

if the Friedmann universe is to be stable at the low-energy scale. The point \( \alpha_1 = \alpha_2 = 0 \) gives a ghost-free theory, as discovered in [20].
The potential \( V(\sigma, \tau) \) can be derived from the Kähler potential \( G \) and the superpotential \( F \) as

\[
V = e^{\frac{G}{4}} \left( \frac{\partial F / \partial \Phi_1}{2} - \frac{3}{2} IF^2 \right)
\]  

(10)

Substitution from expressions (1) and (4) leads to the result [6]:

\[
V = \frac{4\pi^2}{k_0} e^{-\frac{1}{2} \text{Re} \Phi} \left[ C + \frac{\text{Re} \Phi}{k_0} \left( 1 + \frac{3}{2} \text{Re} \Phi \right) \exp \left( -\frac{3}{2} \frac{\text{Re} \Phi}{k_0} \right) \right]^{\frac{1}{2}}
\]  

(11)

This potential is positive semi-definite and automatically vanishes at the minimum point \( \sigma = \sigma_0 \), given by

\[
e^{\frac{3}{k_0}} \Phi_0 = \frac{2}{3} k_0 \Phi_0 \left[ \frac{1}{k_0} \left( 3 e^{\frac{3}{k_0}} \Phi_0 + 1 \right) \right].
\]  

(12)

The equations of motion for the fields \( \sigma, \tau \) and \( \Phi_0 \) are obtained by setting the variations \( \delta V / \delta \sigma, \delta V / \delta \tau \) and \( \delta V / \delta \Phi_0 \) to zero, where

\[
S = \int d\tau d^3x \mathcal{L}.
\]  

Here, we assume a Friedmann-Robertson-Walker (FRW) space-time \( \eta = \frac{a}{H} \). Since we primarily seek a quasi-exponentially, expanding universe, for which \( |H| \ll H \) (where \( H = a/\dot{a} \) and \( a \) denotes differentiation with respect to time \( t \)), it is sufficient to consider only one field equation, which is most conveniently taken to be the trace equation \( \delta \mathcal{E}_t = 0 \). Even then, the presence of the term \( \delta \mathcal{E}_t \) in expression (7) makes the exact equations of motion for \( \delta \mathcal{E}_t \), \( \sigma \) and \( \tau \) very difficult to treat.

Our approach, therefore, is first to ignore \( \delta \mathcal{E}_t \), solve the resulting simplified equations of motion, and then check the self-consistency of our solutions. (We shall, in fact, find that the terms arising from \( \delta \mathcal{E}_t \) cannot always be ignored.) Accordingly, we write the equations of motion for the equations of motion \( \delta \mathcal{E}_t \), \( \sigma \) and \( \tau \) in the approximate forms

\[
\ddot{\sigma} + 3H \dot{\sigma} - \partial (e^{\frac{3}{2} \Phi_0}) / \partial \sigma \approx 0,
\]  

(14)

\[
\ddot{\tau} + 3H \dot{\tau} - \partial (e^{\frac{3}{2} \Phi_0}) / \partial \tau \approx 0
\]  

(15)

and

\[
\ddot{\tau} + 3H \dot{\tau} + m^2(\sigma) \left[ R - \kappa (\text{Tr} \nabla^2 - \sigma^2 + \tau^2 + 4V) \right] \approx 0,
\]  

(16)

where

\[
m^2(\sigma) = 2 \left( 4 \dot{\tau}^2 - 3 \dot{\tau} \right) \ddot{\tau} \left( 4 \dot{\tau}^2 - 3 \dot{\tau} \right)^{-1} e^{\frac{3}{2} \Phi_0}
\]  

(17)

and the trace anomaly is of the form [21]

\[
\text{Tr } \nabla^2 = - \frac{2}{3} \left( 4 \dot{\tau}^2 - 3 \dot{\tau} \right) \ddot{\tau} \left( 4 \dot{\tau}^2 - 3 \dot{\tau} \right)^{-1} e^{\frac{3}{2} \Phi_0}
\]  

(18)

The coefficients \( k_i (i = 1-3) \) depend upon the number and types of fields in the theory, and \( \kappa_{uvp} \) is the Weyl tensor (which vanishes in a FRW space-time).

An early version of the inflationary universe was constructed by Starobinsky [22], based upon the term in \( k_2 \) in expression (18), for which a self-consistent de Sitter solution exists [23]. The associated vacuum energy density is much too high, however, for compatibility with the observed anisotropy in the dipole [24], [25] and quadrupole [26] components of the microwave background radiation. An improved version of the model [22] can be constructed [27], provided that \( k_2 > 0, k_3 < 0 \) and \( |k_3| >> (\pi/18)^2 k_2 \). This gives a quasi-de Sitter space-time with \( H = H_0 \sqrt{2} k_2 / \sqrt{k_3} \). The trouble is, however, that one usually has \( |k_2| \sim |k_3| \). Although \( k_2 \) can vanish in some supergravity theories [28], this is only true if \( N > 2 \), whereas we are dealing with an \( N = 1 \) theory here. And even then, one would require \( k_2 \gg 10^{11} \), so that the difficulty mentioned [26] re-emerges (namely, the number of fields available is insufficient by a large factor).

In order to overcome this further problem, a term \( \frac{1}{2} \eta^2 \rho_{\text{vac}} R^2 \) was added to \( e^{\frac{3}{2} \Phi_0} \) by hand, where \( \rho_{\text{vac}} = \frac{m_0^2}{2} \) [27]. The resulting theory is ghost-free, and the Ricci scalar obeys the differential equation

\[
\Box R - m_0^2 R = 0
\]  

(19)

Assuming that this equation admits a quasi-exponential solution with \( R \sim \frac{1}{2} H^2, \quad |H| \ll H \) and \( H \ll 3H \), then that solution is \( H = - \frac{k}{C} \).
which remains self-consistent until $H^2 \approx \frac{n_0}{96\pi^2} - \frac{2H^3}{3}$, at which point
the slow-rolling approximation is no longer valid and the inflation effectively ceases [15].
(Note that we must have $n_0 > 0$ in order to exclude a run-away solution.)
The constraint [24] on the vacuum energy density then means that we must have
$n_0/n^2 < 6 \times 10^{-6}$ if density fluctuations are
$k_0 < 3 \times 10^{-4}$ [25].

It is important to notice that this implies a very large value
$n_0/96\pi^2 > 10^8$ for the dimensionless coefficient of $H^2$, which may not
be easy to obtain in a realistic model. On the other hand, the initial value of $H$ is now unrestricted, and one may have $H_0 < n_1$. Inflation of this kind, based upon higher-order corrections to the Einstein equations,
has been called type I inflation [29].

At very large values of the curvature, such that $R \gg \kappa^2(4\pi - \sigma^2 - \gamma^2)$,
Eq. (16) reduces to

$$
\Box H = n^2(\sigma)R \approx 0
$$

which coincides with Eq. (19), apart from the $\sigma$-dependence of the mass. This
$\sigma$-dependence puts the required very small value of $n^2$ within reach, suggesting
that inflation may now work. Unfortunately, however, this turns out not to
be true, as we shall shortly see.

The values of $\sigma_1$ and $\sigma_2$ must be chosen such that $\gamma \approx 4\sigma_2 - 3\sigma_1 > 0$,
or there will be a run-away instability. The final value of $\sigma$ should be given by expression (12) and presumably $\sqrt{2}\nu_1 \approx 1$. Therefore, a necessary
(but not sufficient) condition for type I inflation in the superstring model is that some initial condition $\exp(\sqrt{2}\nu_1) \gg 10^5$ (i.e., $\sqrt{2}\nu_1 \approx 18$) is permitted, and also that the mass $m(\sigma)$ increases sufficiently slowly
that inflation can continue for long enough.

This hypothesis can be tested by obtaining solutions of Eqs. (14) and (16), assuming the slow-rolling hypothesis to hold, so that we have

$$
H_{\text{slow}} \approx 2\sqrt{2} \left[ (\sigma_1 - \sigma_2) + 1 \right] \delta \epsilon^\sigma \frac{\sqrt{2}k_0}{H^4}
$$

and

$$
\dot{H} \approx -\frac{1}{6} m^2(\sigma).
$$

But the necessary condition $H^2 > m^2$ then means that $|m/\dot{m}| > 1$, showing
that $m^2$ changes much too fast, and the inequalities (9) imply that $\sigma$
increases, rather than decreases, since $\delta(\sigma_1 - \sigma_2) + 1 \geq 1 + 2\sigma_2 > 0$.

This universe cannot reach our present vacuum state ($\sigma = n_0$). Further, the solution (21) implies that the neglected terms in $\Delta^2$ are actually extremely large. Taking them into account, we find that a typical term
in $e^{-a}\Delta^2$ is then $(m^2/96\pi^2(\sigma))^2(\epsilon^2)^2 < (m^2/16\pi)(\delta(\sigma)/\sigma^2) < (m^2/16\pi)$. Heuristically, the effect of such a term is to replace the coefficient $m^2(\sigma)$ in Eq. (21) by a coefficient $\nu^2 \delta(\sigma)/\sigma^2 < m^2(\sigma)$.

What this means is that inflation (quasi-exponential expansion) even if possible at all, would stop after about one e-folding. We do not attempt any exact treatment of the non-linear part of $\Delta^2$ of the theory, but it would be very surprising if this changed the conclusion. Further, if it turns out that $\sigma_1$ and $\sigma_2$ have to vanish, so as to give a ghost-free theory [20] (as also results naturally if one requires a supersymmetric extension of the four-dimensional theory [30]), then $\gamma = 0$ anyway, and the term $\epsilon^\sigma$ is absent from Eq. (16). Unfortunately, it therefore appears very difficult to obtain type I inflation in the superstring theory.

As for inflation due to a gaugino-condensation potential, it is
easy to show that it is not possible. Calculations of the principal
curvatures $m^2 = \partial^2 V/\sigma^2$ and $\sigma^2 = \partial^2 V/\sigma^2$ yield the results
$m^2 \approx 30$ and $\sigma^2 \approx 4$. But it is known [31] that
no slow roller can take place unless $m^2 \geq 3\pi^2 / 2$, and that inflation
will not continue for a sufficient period of time unless, at some starting
point, we have $m^2 \leq 3\pi^2 / 2$. Neither of these conditions is satisfied
by the above result, which shows that inflation associated with a gaugino-
condensation potential at tree level is ruled out.

Both of the above difficulties in obtaining inflation are essentially
due to the absence of free dimensionless parameters in the superstring
theory. Hence, we turn to the possibility of no-scale inflation, due
to some inflaton field, as proposed in [7]. The conditions that must
be placed upon the Kahler potential $G$ in order to realize inflation have
been discussed in [16] and models constructed in [17]. Either the $SU(n,1)$
symmetry has to be broken, or else the superpotential must be chosen carefully,
if the requirements [3] for inflation are to be satisfied.
In the first case, it was found necessary to choose a Kähler potential of the form [16]

\[ G = -3 \ln \left( \frac{Z + Z^* - K(\Phi, \Phi^* - 2 \Phi \Phi^*)}{a K(\Phi, \Phi^*) + \delta} \right)^2 \]  

(24)

where \( Z \) is the Polonyi field responsible for supersymmetry breaking and \( K(\Phi, \Phi^*) \) describes the gauge-singlet 'inflaton' sector. The term \(-aK(\Phi, \Phi^*)\) breaks the \( SU(n,1) \) symmetry. Its presence leads to a minimum of \( V \) with non-zero vacuum energy which can drive inflation.

But the introduction of the parameter \( a \) essentially involves new dimensionless parameters, which are almost certainly forbidden by the superstring theory. Indeed, the simple truncation procedure does not furnish such a term [5]. This truncation does not exactly describe compactification onto a Calabi-Yau manifold, on which one might expect an additional scalar field associated with deformation of its shape. The question is whether the difference between expression (1) and this precise expression can manifest itself as a term such as \(-aK(\Phi, \Phi^*)\) in expression (3). This does not a priori seem very likely.

Similar arguments suggest that the superpotential cannot be chosen in an arbitrary way, in accordance with inflationary requirements. Rather, the compactification mechanism fixes the field content and functional forms of \( G \) and \( F \) precisely. We do not know, at present, whether or not a suitable superpotential exists in a superstring model. It should be kept in mind, however, that the enforced approximate equality of the compactification mass \( m_c \) and the string mass \( m_s \) sheds some doubt on the validity of the field theory limit [32] and no dynamical compactification mechanism is yet known in a superstring theory.

Is there any other way in which one might conceivably obtain inflation? Most inflationary models in the literature assume a long flat plateau of the potential, so as to avoid the problem that the initial state of the universe is not on that plateau as the result of thermal fluctuations [33]. Consideration of the required initial conditions suggests that it is generally likely, both in chaotic inflation [34] and for normal inflation [35] that the characteristic scale of the field is of the order of \( m_s \). This immediately takes us into the high-energy realm. On the other hand, superstring theories predict a limiting maximum temperature \( T_0 \) \( \approx m_s /10 \) and it has been suggested that the universe may undergo a phase transition at that temperature, comparable in nature to the confinement-deconfinement phase transition. In that case, a new type of inflation might be possible at that phase transition [36], although the details still remain to be worked out.

The above considerations reveal a serious difficulty in the superstring theory from the cosmological point of view (unless one believes, for example, that the horizon problem is in some way spurious, finding an explanation at the pre-classical level of the theory). It may be necessary to seek a new type of inflation, or some alternative 'stringy' compactification scheme before we have a full understanding.

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REFERENCES


A.S. Goncharov and A.D. Linde, Phys. Lett. 139B, 27 (1984);
Classical and Quantum Gravity 1, 175 (1984).

