IC/85/231
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SUPERUNIFICATION, PHASE TRANSITIONS AND COSMOLOGY *

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MIRAMARE – TRIESTE
October 1985

* To be submitted for publication.

ABSTRACT: In this article, we survey the main features behind the idea of grand unification, both without and with (local) supersymmetry. We then study the high temperature phase transitions in the theories so realized, and their relevance to the cosmology of the early universe. In particular, we review the basic ingredients of (super) grand unified models and we give the basic tools needed for the study of their phase transitions. After a short introduction to cosmology, we focus on the interplay between unified particle physics models and cosmology, with particular emphasis on the inflationary universe scenario. In the same perspective, new research directions, in the context of higher dimensional theories, are also discussed.

I. INTRODUCTION

It is now accepted that the experimental evidence gives a strong support to the so-called standard model: strong interactions are described by quantum chromodynamics (QCD), based on the group $SU(3)_C$, and electroweak interactions by the Glashow-Weinberg-Salam (G-W-S) model, based on the group $SU(2)_L \times U(1)_Y$.[1] This $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard gauge model has passed the most crucial tests so far, the culmination being the discovery of the intermediate

$W^{\pm}$, Z vector bosons.[2] Despite this, however, and although the standard model is a theoretically consistent theory, there is a number of issues, which point to the direction that the standard model is physically limited and must be only an effective theory, obtained from an underlying more fundamental one.[3] On the phenomenological side, the unexplained mysteries include the quantum numbers, the mass spectrum and the family replication of the fermions (quarks and leptons) and the arbitrary character of the scalar (Higgs) sector of the theory. The latter leaves unanswered the tremendous difference of the mass scales present in microphysics, namely the Fermi scale $G_F^{-1/2} \approx 250$ GeV and the Planck scale $G_N^{-1/2} \equiv M_P \approx 10^{19}$ GeV.

In the past years, there have been many attempts to remedy the above problems of the standard model. One way pursued was the road to compositeness [4]: use preons as more fundamental constituents and QCD-like dynamics to describe their interactions. The other way was grand unification [5]: postulate a larger set of gauge interactions, based on a group $G \supset SU(3)_C \times SU(2)_L \times U(1)_Y$, with a unique gauge coupling constant g_G. Supersymmetry (SUSY) [6], which transforms fermions into bosons and vice-versa, has been used in both approaches. The compositeness framework, with a compositeness scale Λ_C not far away from $G_F^{-1/2}$, where supersymmetry is mainly used to supply supersymmetric partners of Goldstone bosons, to be identified with the known fermions, has not yet provided a realistic unified picture and will not be discussed here.

What we will consider, in this article, is the grand unification approach towards the ultimate dream of a synthesis of all fundamental interactions. Supersymmetry

serves a double purpose in this approach. First, through the no renormalization theorems [7], it can help in the solution of the much discussed gauge hierarchy problem, by stabilizing tree level relations and keeping light scalars really light, by preventing them from acquiring radiatively large masses. As a matter of fact, that was exactly the motivation of introducing supersymmetry into grand unification [8]. Second, and what is much more important, there is the possibility of incorporating gravity in the unification picture, since supersymmetry, realized locally, leads to supergravity [9]. Supersymmetric grand unified theories (SUSY GUT's), based on N = 1 supergravity (SUGRA)[10], have been discussed extensively during the past three years [11] and we will give here their general characteristic features.

Grand unification physics takes place at extremely high energies (possibly at $E \lesssim M_P$). So, its natural arena is the early universe, in its first moments. Here is where particle physics confronts cosmology.[12] At that early moments, the universe was extremely hot and, so, the behaviour of the unified theories at high temperatures becomes an essential part of their study. Accordingly, we will give the main relevant concepts and discuss the phase transitions in GUT's. Then, after a brief introduction to standard cosmology and its problems, we will give the cosmological implications of these phase transitions. The main focus will be the inflationary universe scenario [13], i.e. the proposal that, early in its history, the universe has passed an exponentially expanding (de Sitter phase) period. That was one of the most influential proposals in the recent years, given that it can provide a natural solution to many of the cosmological problems in the standard big bang theory coupled to particle physics.

At the end, some new exciting research directions, based on the higher-dimensional approach [14], will be discussed, within the same perspective.

II. ORDINARY GUTS

We are convinced that the so-called standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model must represent an intermediate step in the ladder towards a truly unified picture of the fundamental forces of Nature. The Glashow-Weinberg-Salam $SU(2)_L \times U(1)_Y$ model [15] correlates the weak and electromagnetic interactions, but it is not a unification of them. True, it predicts the weak neutral currents whose structure is of the form $T_{3L} - \sin^2\theta_W Q_{em}$, T_{3L} being the third generator of the $SU(2)_L$, Q_{em} the electric charge generator and $\sin^2\theta_W = g_Y^2/(g_2^2 + g_Y^2)$ the relative strength of $SU(2)_L$ (coupling constant g_2) and $U(1)_Y$ (coupling constant g_Y). This prediction has been confirmed experimentally in various leptonic, semileptonic and hadronic interactions [2]. More spectacularly, the G-W-S model has received an even more strong support from the discovery of the gauge bosons $W^{\pm}$ and Z by the UA1 [16] and UA2 [17] collaborations, at CERN, within the predicted range of values. On the other hand, there is also excellent evidence that hadrons are built of coloured quarks, bound by vector gluons exchanges, which leads to the $SU(3)_C$ gauge theory of strong interactions, the quantum chromodynamics (QCD) [18]. The evidence for three colours comes from various sources: the spin-statistics problem, the lifetime of π^0, the cross-section measurements of $R = (e^+e^- \to \text{hadrons})/(e^+e^- \to \mu^+\mu^-)$, the cancellation of

anomalies for the $SU(2)_L \times U(1)_Y$ theory [19]. Note, however, that $SU(3)_C$ itself has a triangle anomaly, with an arbitrary parameter θ (strong CP violation) [20], but this can find a natural solution, e.g. through the Peccei-Quinn mechanism [21]. Most importantly, $SU(3)_C$ is asymptotically free [22], which leads to the understanding of scaling in high-energy scattering experiments. The last success of the standard model is the absence of flavour changing neutral currents [23].

However, the standard model contains 18 independent parameters (3 gauge couplings g_3, g_2, g_Y and 15 elementary particle masses and quark mixing angles) and the hope is that by embedding it in a grand unified theory (GUT)[24-26], we will reduce the number of independent parameters. In fact, the property of asymptotic freedom of non-Abelian gauge theories permits grand unification of $SU(3)_C \times SU(2)_L \times U(1)_Y$. This means that the g_3, g_2 and g_Y coupling constants merge into one single coupling constant g_G. This is possible, since coupling constants are different at different mass scales and, so, although the presently observed couplings are different $g_3(M_W) > g_2(M_W) > g_Y(M_W)$, it is possible that they are equal at another mass scale. The evolution of the coupling constant $\alpha(\mu) \equiv g^2(\mu)/4\pi$ is governed by the Callan-Symanzik β-function:

$$\beta(\alpha) = \mu \frac{d}{d\mu} \alpha(\mu) \tag{2.1}$$

For small α, $\beta(\alpha)$ can be evaluated perturbatively

$$\beta(\alpha) = -b \frac{\alpha^2}{2\pi} + O(\alpha^3) \tag{2.2}$$

where b is the one-loop contribution. Then

$$\sin^2\theta_w(\mu) = \frac{3}{8}\left[1 - \frac{\alpha_{em}(\mu)}{2\pi}\frac{5(b_2 - b_1)}{3}\ln\frac{M_G}{\mu}\right]$$
$$= \frac{3}{8} - \frac{5(b_2 - b_1)}{(8b_3 - 3b_2 - 5b_1)}\left[\frac{3}{8} - \frac{\alpha_{em}(\mu)}{\alpha_3(\mu)}\right] \qquad (2.14)$$

Eq. (2.13) and (2.14) can now be used to predict the grand unification scale M_G and $\sin^2\theta_w$, once we take conveniently $\mu \approx M_w$, where $\alpha_{em}(M_w) = 1/128$, $\alpha_3(M_w) \simeq 0.11$. The above are one loop results. One has also to take into account one-loop radiative corrections at $\mu \approx M_w$, two-loop corrections to the evolution between M_w and M_G and threshold effects at $\mu \approx M_G$. Then [28], for a GUT with the "great desert" hypothesis and, in particular, for the minimal SU(5) with three families and one light Higgs doublet, the GUT scale M_G is approximately proportional to the QCD scale $\Lambda_{\overline{MS}}$:

$$M_G = (1.5 \pm 0.5) \times 10^{15}\,\Lambda_{\overline{MS}} \qquad (2.15)$$

With $\Lambda_{\overline{MS}} = (150 \pm 50)$MeV, we have

$$M_G = (1 - 4) \times 10^{14}\ \text{GeV} \qquad (2.16)$$

The precise value of M_G is important for the proton decay lifetime prediction (τ_p scales as M_G^4, see later). Once M_G is known, the coupling constant $\alpha(M_G)$ at the unification point M_G can be obtained from any of the Eqs.(2.12) and it is

$$\alpha(M_G) = 0.024 \qquad (2.17)$$

On the other hand, within the same approximations, we find

$$\sin^2\theta_w(M_w) = 0.214^{+0.004}_{-0.003} \qquad (2.18)$$

in a remarkably good agreement with the experimental value, including radiative corrections,

$$\sin^2\theta_w(M_w)_{exp} = 0.217 \pm 0.014 \qquad (2.19)$$

The search for the grand unified symmetry G is dictated by some general characteristic features: (i) The representation of the gauge group G must reproduce the correct particle content of the observed fermion spectrum. This means, in particular, that G must possess complex representations, in order to give rise to massless left-handed fermions, upon spontaneous symmetry breakdown to SU(3) × SU(2) × U(1). All fermions transforming according to a self-conjugate (real) representation of G acquire a super-heavy mass of order M_G, in the course of spontaneous G symmetry breakdown ("survival hypothesis" [29]). Moreover, the representation content must be such that there are no Adler-Bell-Jackiw [30] anomalies. (ii) G must contain the gauge group SU(3) × SU(2) × U(1) of the standard model, which means that it must be (at least) of rank 2 + 1 + 1 = 4.

The first requirement restricts the possible GUT gauge group to SU(n), $n \geq 3$, SO(4n + 2), $n \geq 1$ and E_6 [31]. The second one makes the further restriction SU(n), $n \geq 5$, SO(4n + 2), $n \geq 2$ and E_6. In fact, models based on the gauge groups $SU(5) \subset SO(10) \subset E_6$ have been proposed as possible candidates for a grand unified model [26]. At this point, we have to mention that the recent higher (ten-) dimensional theories [32], based on superstrings [33] and attempting to provide a consistent unified theory of everything (including gravity), upon compactification

and dimensional reduction [34], can actually give rise to the above mentioned GUT gauge groups, a very encouraging and welcome result.

In the following, we will give some pertinent features of the SU(5), SO(10) and E_6 GUT models, giving more details for the SU(5), which will serve as our prototype in the subsequent sections.

1. SU(5)

The SU(5) [25] model is that GUT model which incorporates the $SU(3) \times SU(2) \times U(1)$ as its maximal subgroup. Hence, its uniqueness and its stringent predictions. It has 24 generators

$$G^a = \frac{\lambda^a}{2}, \quad a = 1,\ldots,24 \tag{2.20}$$

where λ^a are 5×5 traceless Hermitian matrices normalized as $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$. Associated with them, there are 24 gauge bosons A^a_μ, which can be represented in a 5×5 traceless adjoint matrix $A_\mu = \frac{1}{\sqrt{2}}\sum_a \lambda^a A^a_\mu$:

$$A_\mu = \left(\begin{array}{ccc|cc} G^1_1 + \frac{2B}{\sqrt{30}} & G^1_2 & G^1_3 & \bar{X}_1 & \bar{Y}_1 \\ G^2_1 & G^2_2 + \frac{2B}{\sqrt{30}} & G^2_3 & \bar{X}_2 & \bar{Y}_2 \\ G^3_1 & G^3_2 & G^3_3 + \frac{2B}{\sqrt{30}} & \bar{X}_3 & \bar{Y}_3 \\ \hline X_1 & X_2 & X_3 & \frac{W_3}{\sqrt{2}} - \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_3}{\sqrt{2}} - \frac{3B}{\sqrt{30}} \end{array}\right) \tag{2.21}$$

The 8 gluons G^i_j are in the top diagonal 3×3 submatrix, the 3 weak bosons W's in the bottom diagonal 2×2 submatrix and the hupercharge boson B corresponds to the traceless diagonal generator

$$\frac{1}{\sqrt{15}}\left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & 0 \\ & & 1 & & \\ \hline & 0 & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{array}\right)$$

Finally, the off-diagonal (X,Y) bosons are in a 3×2 submatrix (they are (3,2) under $SU(3)_C \times SU(2)_L$) and mediate baryon number violating interactions (see below).

The correct fermion assignment is to a $\bar{5} + 10$ representation of SU(5)

$$\chi \equiv \bar{5} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ \hline e^- \\ -\nu_e \end{pmatrix}_L \qquad \psi \equiv 10 = \left(\begin{array}{ccc|cc} 0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\ -u^c_3 & 0 & u^c_1 & -u_2 & -d_2 \\ u^c_2 & -u^c_1 & 0 & -u_3 & -d_3 \\ \hline u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{array}\right)_L \tag{2.22}$$

That this is an anomaly free assignment for the fermions of each generation comes from the general formula [31] for the anomaly, characterized by $Tr(\{T^aT^b\}T^c) = A([m])d^{abc}$, of a totally antisymmetric representation [m] of the group SU(n) (the anomally of the fundamental representation is normalized to 1): $A([m]) = \frac{(n-3)!\,(n-2m)}{(n-m-1)!\,(m-1)!} = -A([n-m])$

So: $A(\underline{\bar{5}} = [4]) = -1$, $A(\underline{10} = [2]) = +1$ and thus the combination $[4] + [2] = \underline{\bar{5}} + \underline{10}$ is anomaly free.

Finally, the scalar fields needed to implement the two stages of spontaneous symmetry breaking[35] are $SU(5) \xrightarrow{\Phi=\underline{24}} SU(3)_C \times SU(2)_L \times U(1) \xrightarrow{H=\underline{5}} SU(3)_C \times U(1)_{em}$; Φ is a $\underline{24}$ of Higgs, represented by a 5×5 traceless Hermitian matrix, with potential and vacuum expectation value (vev) given by

$$V(\Phi) = -\frac{\mu^2}{2} \mathrm{Tr}\, \Phi^2 + \frac{a}{4} (\mathrm{Tr}\, \Phi^2)^2 + \frac{b}{2} \mathrm{Tr}(\Phi^4)$$

$$\langle\Phi\rangle = V \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ \hline & 0 & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{array}\right) \qquad (2.23)$$

$\langle\Phi\rangle$ is a singlet in the $SU(3)_c$ and $SU(2)_L$ spaces and also conserves $U(1)$. It gives the superheavy masses $M_X^2 = M_Y^2 = \frac{25}{8} g^2 V^2$. H is a $\underline{5}$ of Higgs with potential and vev

$$V(H) = -\frac{\nu^2}{2} (H^+ H) + \frac{\lambda}{4} (H^+ H)^2$$

$$\langle H\rangle = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ \nu \end{array}\right) \qquad (2.24)$$

$\langle H\rangle$ carries T_3 into/out of the vacuum and breaks $SU(2)_L$, giving masses to the weak intermediate vector bosons.

$\langle H\rangle$ also gives fermion masses through the couplings

$$\lambda_1 H^{\alpha +} \chi^\beta \psi_{\alpha\beta}, \quad \lambda_2 \varepsilon^{\alpha\beta\gamma\delta} \psi_{\alpha\beta} \psi_{\gamma\delta} H_\varepsilon \qquad (2.25)$$

The first gives equal masses to the $Q = -\frac{1}{3}$ quarks and $Q = -1$ leptons and the second to the $Q = +\frac{2}{3}$ quarks. Of course, these tree level predictions are subject to renormalization corrections[35], by a calculable amount, according to the renormalization group equation

$$\left[\mu \frac{\partial}{\partial\mu} - \gamma_f(\alpha(\mu))\right] m_f(\mu) = 0 \qquad (2.26)$$

where $\gamma_f(\alpha(\mu))$ are the corresponding anomalous mass dimensions. Summing the leading contribution to all orders gives[35,36]

$$\frac{m_{Q=-\frac{1}{3}}(\mu)}{m_{Q=-1}(\mu)} = \left(\frac{\alpha_3(\mu)}{\alpha_3(M_G)}\right)^{\frac{4}{11 - \frac{4}{3} n_f - \frac{1}{3} n_H}} \times \left(\frac{\alpha_1(\mu)}{\alpha_1(M_G)}\right)^{\frac{1}{\frac{4}{3} n_f - \frac{1}{3} n_H}} \qquad (2.27)$$

For $n_f = 3$ and $n_H = 1$, Eq.(2.27) gives at $\mu \approx 10$ GeV

$$\frac{m_{Q=-\frac{1}{3}}}{m_{Q=-1}} \approx 3 \qquad (2.28)$$

This prediction is quite successful for the third family, relatively good for the second one and certainly bad for the first one. However, since the first family involves masses of a few MeV, this bad prediction can be corrected by additional assumptions, e.g. through induced non-renormalizable terms [37].

It would be interesting to make here the following remark: the (minimal) SU(5) Lagrangian has a global $U(1)_Z$ symmetry

$$U(1)_Z: \quad \begin{array}{l} \chi \longrightarrow e^{-3i\phi}\chi \\ \psi \longrightarrow e^{i\phi}\,\psi \\ H \longrightarrow e^{-2i\phi}\,H \end{array} \tag{2.29}$$

and everything else unchanged. This symmetry is spontaneously broken by <H>. However, since the neutral component of H has a non-zero hypercharge under $U(1)_Y$, then one can find a linear combination of $U(1)_Z$ and $U(1)_Y$: $\frac{1}{5}(Z + 4Y)$, which is not broken by <H>. It is easy to see that this is the B-L symmetry, which thus remains as a global symmetry of SU(5). It acts as a selection rule for proton decay in the simple SU(5) model (the final state must contain an antilepton). It also means that neutrinos cannot get a mass (except if one introduces extra Higgses or a right handed component ν_R or non-renormalizable terms [38]). In groups bigger than SU(5) (see below), B-L corresponds to a local symmetry and we expect small violations of it.

Finally, from the covariant derivatives, one gets, besides the usual $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions, the interactions involving the superheavy X,Y gauge bosons (charges $\frac{4}{3}$, $\frac{1}{3}$, respectively) and the current eigenstates of the fermions

$$\mathcal{L}_{X,Y} = \frac{g}{\sqrt{2}}\Big[-\bar{e}_L\gamma^\mu \bar{X}_\mu^\alpha d^c_{\alpha L} + \bar{\nu}_L\gamma^\mu \bar{Y}_\mu^\alpha d^c_{\alpha L} + \bar{d}_{\alpha L}\gamma^\mu \bar{X}_\mu^\alpha e^c_L + \varepsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma^\mu \bar{X}_\mu^\alpha \bar{u}^\beta_L - \bar{u}_{\alpha L}\gamma^\mu \bar{Y}_\mu^\alpha e^c_L + \varepsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma^\mu \bar{Y}_\mu^\alpha d^\beta_L + h.c.\Big] \tag{2.30}$$

These are the new interactions, which violate B- and L-numbers. They are responsible for "the rise and fall" of the baryon number of the universe: generation of the baryon asymmetry of the universe, at first place (see cosmology, below), and, then, nucleon decay.

Nucleon decay [39], mediated by superheavy gauge boson exchanges, as given by Eq.(2.30), is mostly sensitive on $M_{X,Y} \simeq M_G$. From (2.30), one writes down the effective Lagrangian $\mathcal{L}_{eff}$ for B-violating d=6 four Fermi $qqq\ell$ interactions, responsible for nucleon decay, and then scales them down to energies ∿1 GeV, which will give an enhancement factor A: $\mathcal{L}_{eff}(1\text{ GeV}) = A\,\mathcal{L}_{eff}$, where A can be calculated using the renormalization group and the anomalous dimensions of the $qqq\ell$ operators. However, the most uncertain part of the computation lies in the calculation of hadronic matrix elements of the effective Lagrangian, which interpolate between the initial nucleon state and the various meson final states. These depend on the model of hadronic structure employed, and several methods have been applied [39]. The partial lifetime for the SU(5) dominant decay mode $p \to e^+\pi^0$ thus turns out to be

$$\tau(p \to e^+\pi^0) \simeq (0.06 - 240) \times 10^{29}\ \text{yr} \tag{2.31}$$

to be compared with the experimental bound [40]

$$\tau \gtrsim 5 \times 10^{32}\ yr \qquad (2.32)$$

which appears to rule out the minimal SU(5) model or any simple GUT model with the "great desert" between M_W and M_G.

One can, of course, consider non-minimal extensions of the SU(5), with additional Higgses and/or fermions, which are not ruled out. But these are ad hoc suggestions, without any other justification. Another possibility would be the existence of non-renormalizable interactions, e.g. $\varepsilon \frac{\Phi}{M_p} F_{\mu\nu}F^{\mu\nu}$, which modify the gauge boson kinetic terms [41]. These can lead to an increase of M_G and τ_p, while keeping $\sin^2\theta_w$ acceptable. And, of course, higher than SU(5) unification groups are still compatible with proton decay experiments, due to a possible higher unification scale M_G.

Note, also, that proton decay can take place through exchanges of Higgs bosons, e.g. the colour triplet in the $\underline{5}$ representation H, which has therefore to be superheavy ($\gtrsim 10^{10}$ GeV), to be compatible with experiment. Hence, the need to split the $\underline{5}$ - plet of Higgses (doublet-triplet separation), another gauge hierarchy problem.

2. SO(10)

The next bigger grand unified model is the rank 5 $SO(10) \supset SU(5) \times U(1)$ [42]. It has 45 generators $T^{\alpha\beta} = -T^{\beta\alpha}$, $\alpha,\beta = 1,\ldots 10$, with commutation relations

$$[T_{\alpha\beta},T_{\gamma\delta}] = i(\delta_{\alpha\gamma}T_{\beta\delta} - \delta_{\alpha\delta}T_{\beta\gamma} - \delta_{\beta\gamma}T_{\alpha\delta} + \delta_{\beta\delta}T_{\alpha\gamma}) \qquad (2.33)$$

These can be constructed from the 32 × 32 generalized Dirac matrices Γ_i, generating a Clifford algebra

$$\{\Gamma_\alpha,\Gamma_\beta\} = 2\delta_{\alpha\beta}\,\mathbb{1}_{32},\ \alpha,\beta = 1,\ldots 10 \qquad (2.34)$$

The matrices $\Gamma^+_{\alpha\beta}$:

$$\Gamma^+_{\alpha\beta} = \frac{\mathbb{1} + \Gamma_{11}}{2}\,\Gamma_{\alpha\beta},$$

$$\Gamma_{\alpha\beta} = \frac{1}{2}[\Gamma_\alpha,\Gamma_\beta],\ \Gamma_{11} = i\Gamma_1\Gamma_2\ldots\Gamma_{10} \qquad (2.35)$$

are the generators of the chiral spinor representation of SO(10). Associated with them, there are 45 gauge bosons $A^{\alpha\beta}_\mu = -A^{\alpha\beta}_\mu$, which can be classified properly by their $SU(3)_C \times SU(2)_L \times SU(2)_R$ subgroup content:

$$\begin{array}{lllll} 45 = & (8,1,1) & + (1,3,1) & + (1,1,3) & + (1,1,1) \\ & G^i_j & W^{1,2,3}_L & W^{1,2,3}_R & B' \\ & + (\bar{3},2,2) & + (3,2,2) & + (3,1,1) & + (\bar{3},1,1) \\ & \begin{pmatrix} X & \bar{Y}' \\ Y & \bar{X}' \end{pmatrix} & \begin{pmatrix} X' & \bar{Y} \\ Y' & \bar{X} \end{pmatrix} & X_S & \bar{X}_S \end{array} \qquad (2.36)$$

The B' gauge boson is associated with the U(1)' generator $Y' = 2(Y - T_{3R})$, which, for fermions, coincides with B-L. Besides the gauge bosons known from SU(5), there are the new X', Y' bosons with charge $\frac{2}{3}$ and $-\frac{1}{3}$, which also contribute to B- and L- violating interactions, and, also, the X_S boson with charge $\frac{2}{3}$, which does not contribute to such interactions, except through mixing with X'. Such processes violate B-L, but they are enormously suppressed by additional inverse powers of M_G.

The fermions of each generation are assigned to the spinor representation $\underline{16}$ of SO(10) [42,43], which under SU(5) is decomposed as

$$\underline{16} = \underline{\bar{5}} + \underline{10} + \underline{1} \qquad (2.37)$$

Clearly, the reducible $\overline{5} + 10$ representation of SU(5) is contained in an irreducible representation of SO(10) (hence the cancellation of the anomalies), and, in addition, there is a SU(5) singlet that has the quantum numbers of an anti-neutrino ν_L^C (hence neutrinos are, in general, massive in SO(10) [44]).

There are many patterns of symmetry breaking, and the various renormalization effects (for coupling constants and mass scales) vary analogously [45].

3. $\underline{E_6}$

The next higher group is the rank 6 exceptional[46] group $E_6 \supset SO(10) \times U(1)$. It has 78 generators

$$\begin{cases} H_{\alpha\beta}, H_{\alpha\beta} = -H_{\beta\alpha}, \quad \alpha,\beta = 1,\ldots 10 \\ Y \\ X^a, \overline{X}_a, \quad a = 1,\ldots 16 \end{cases} \tag{2.38}$$

whose commutation relations we do not write down explicitly [47]. Associated with them, there are 78 gauge bosons, which under SO(10), are

$$\underline{78} = \underline{45}\,(H_{\alpha\beta}) + \underline{16}(X^a) + \overline{\underline{16}}\,(\overline{X}_a) + 1(Y) \tag{2.39}$$

The fermions are assigned [48] to the fundamental $\underline{27}$ representation, which, under SO(10), is

$$\underline{27} = \underline{16} + \underline{10} + \underline{1} \tag{2.40}$$

There are the 15 ordinary fermions, whereas the ν_L^C, $\underline{10}$ and $\underline{1}$ form a vector-like representation with respect to $SU(3) \times SU(2) \times U(1)$ and they are superheavy ("survival hypothesis").

Recently, the interest in E_6 models has been revived, thanks to developments in superstring theories, which point towards exceptional groups, as one class of possible GUTS. Again, one has to choose from many patterns of symmetry breaking [49,50].

For completeness, we mention some attempts to construct a GUT model, based on the group O(18) [51], which would give an answer to the problem of replication of families. In O(18), all the known fermion families fit into just one representation, the 256-dimensional spinor. Initial phenomenological difficulaties [51], related to mirror fermions, are avoided in a new construction [52], whose phenomenological implications can be tested soon.

III. SUPERSYMMETRIC GUTS

A. N = 1 Globally Supersymmetric GUTS

The main motivation for introducing supersymmetry into grand unification [8] has been the gauge hierarchy problem [26]. This has to do with the fact that the scalar boson masses, being not protected by any symmetry, have the tendency to get radiative contributions, which push them to the highest mass scale present in the problem. This means that light Higgs scalars, associated with the $SU(2)_L \times U(1)_Y$ breaking, will have the tendency to get masses $\sim O(M_G)$ in a conventional GUT, except if one is making a highly accurate tuning of the parameters in the potential, which, however, has to be altered in each order of perturbation theory. Supersymmetry, in which there is no separate scalar mass or scalar self interaction coupling constant renormalization, but only wave function and gauge coupling constant renormalization, can, at least partly, solve the problem.

Supersymmetry is the last possible symmetry of the S-matrix, as a comprehensive analysis of the possible symmetries compatible with a non-trivial S-matrix, in a relativistic quantum theory, reveals. At the beginning, there is the Coleman-Mandula theorem [53], which states that the only possible conserved "charges" are the energy-momentum P_μ and Lorentz scalar "charges" T_ℓ. However, the theorem, by assuming only symmetries involving commutation relations, misses symmetries involving fermionic generators, i.e. involving anticommutation relations. This is remedied by the Haag-Lopuszanski-Sohnius theorem [54], which states that the most general algebra of generators of symmetry transformations of the S-matrix is a graded Lie algebra spanned by the following generators: (i) Bose-type generators: the energy-momentum operator P_μ, the generator of the homogeneous Lorentz transformations $M_{\mu\nu}$ and a finite number of scalar "charges" T_ℓ, taken Hermitian $T_\ell = T_\ell^+$ (internal group). (ii) Fermi-type generators: a set of spinorial "charges" $Q_\alpha^N (N = 1,2,\ldots;\ \alpha = 1,2)$ and their Hermitian conjugates $\overline{Q}_{\dot\alpha}^N = (Q_\alpha^N)^+$, transforming as $Q_\alpha^N \sim (\frac{1}{2},0)$ and $\overline{Q}_{\dot\alpha}^N \sim (0, \frac{1}{2})$ under the group $SL(2,C)$.

The case $N = 1$, characterized by just one fermionic generator Q_a, is called simple supersymmetry. Its algebra is

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \tag{3.1a}$$

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \tag{3.1b}$$

$$[Q_\alpha, P_\mu] = [\overline{Q}_{\dot\alpha}, P_\mu] = 0 \tag{3.1c}$$

$$[Q_\alpha, M_{\mu\nu}] = \frac{1}{2} (\sigma_{\mu\nu})_\alpha^\beta Q_\beta \tag{3.1d}$$

The fermionic generators Q_α and $\overline{Q}_{\dot\alpha}$ have dimensions: $\dim Q_\alpha = \dim \overline{Q}_{\dot\alpha} = \frac{1}{2}$ (see Eq.(3,1a)) and carry spin $\frac{1}{2}$ (see Eq.(3.1d)). As a result, they transform bosonic states $|B\rangle$ into fermionic $|F\rangle$ and vice-versa:

$$Q_\alpha |B\rangle = |F\rangle,\ Q_\alpha |F\rangle = |B\rangle \tag{3.2}$$

So, supersymmetry relates bosons and fermions, requiring equal numbers of bosonic and fermionic degrees of freedom. To every particle, there is associated a supersymmetric partner, obeying different statistics. Moreover, since the mass operator $P_\mu P^\mu = M^2$ is a Casimir invariant of the supersymmetry algebra, supersymmetric partners have degenerate masses. Apparently, this is not the case in nature, so supersymmetry must be broken.

The reasons, we consider only $N = 1$ simple supersymmetry (SUSY) to construct unified models at energies below the Planck mass, are phenomenological, since $N > 2$ SUSY requires fermions with left and right-handed components transforming equivalently. In $N = 1$ SUSY, an irreducible representation, called supermultiplet, contains supersymmetric partners with identical quantum numbers, but spins different by $\frac{1}{2}$ unit. Our theories can then be constructed in terms of only two classes of supermultiplets, both containing two bosonic and two fermionic degrees of freedom:

Supermultiplet	Particle content
Chiral Φ	Z: 2 real scalar fields A,B($Z = A + iB$).
	ψ_α: 2 component Weyl spinor, describing a Majorana fermion.

Vector V $\quad A_\mu$: 2 component vector.

λ_α: 2 component Weyl spinor, describing a Majorana fermion.

Again for phenomenological reasons, we are forced to double the number of elementary particles in the theory, by pairing, in a chiral multiplet, every known fermion (quark lepton) with a scalar fermion (squark, slepton), and, in a vector multiplet, every gauge boson (gluon, W, Z, photon) to a gauge fermion or gaugino (gluino, wino, zino, photino). Also, Higgs bosons are paired, in a chiral multiplet, with Higgs fermions (higgsinos). So, in a supersymmetric SU(5) theory [55], we have the following particle content:

Supermultiplet	SU(5) representation
Vector (A_μ, λ)	$\underline{24}$
Matter chiral (χ, Z_χ)	$\underline{\bar{5}}$
Matter chiral (ψ, Z_ψ)	$\underline{10}$
Higgs chiral (Σ, ψ_Σ)	$\underline{24}$
Higgs chiral (H_1, ψ_{H_1})	$\underline{5}$
Higgs chiral (H_2, ψ_{H_2})	$\underline{\bar{5}}$

(3.3)

The two Higgs scalars $H_1(\underline{5})$ and $H_2(\underline{\bar{5}})$ are necessary in order to give masses to both up and down quarks and leptons. As a result of the doubling of degrees of freedom, the expressions for the b_i coefficients (Eq.(2.6)) now become:

$$b_3 = 9 - 2n_f$$
$$b_2 = 6 - 2n_f - \frac{1}{2} n_H$$
$$b_1 = -2n_f - \frac{3n_H}{10} \qquad (3.4)$$

These are the modified expressions, which have to be used in Eqs. (2.13) and (2.14), for the calculation of M_G and $\sin^2\theta_w$ in SUSY GUTS [56]. They yield a higher GUT scale M_G and a bigger gauge coupling $\alpha(M_G)$:

$$M_G = 6 \times 10^{16} \Lambda_{\overline{MS}} \text{ GeV} = 10^{16} \text{ GeV} \qquad (3.5a)$$

$$\alpha(M_G) = 0.042 \qquad (3.5b)$$

The value (3.5a) pushes beyond experimental limits the proton decay due to X and Y boson exchanges. However, Higgs boson mediated nucleon decays, through $d = 5$ operators [57], are still important and fall within observable limits [39]. The main decay modes, here, are to kaons and antineutrinos, e.g. $p \to K^+\bar{\nu}_\mu$, where the present experimental bound is $\gtrsim 5 \times 10^{31}$ yr. For $\sin^2\theta_w$, one obtains

$$\sin^2\theta_w(M_W) = 0.236 \pm 0.003, \qquad (3.6)$$

relatively higher than the corresponding prediction of ordinary GUTS. Note, however, that these predictions can be changed by complicating a bit the Higgs system of the theory [58]. Remarkably, the successful prediction for m_b/m_τ of the ordinary SU(5) is maintained in the minimal SUSY SU(5) [56b].

The first of Eqs.(3.1) has an important consequence, since it relates the Hamiltonian $H \equiv P_0$ to the supercharges $Q_\alpha, \bar{Q}_{\dot\alpha}$:

$$H \equiv P_0 = \frac{1}{4} (\bar{Q}_1 Q_1 + Q_1\bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2\bar{Q}_2) \qquad (3.7)$$

This implies that $H > 0$.

If SUSY is unbroken, this means that Q_a annihilates the vacuum:

$$Q_\alpha|0\rangle = 0 \tag{3.8}$$

and hence Eq.(3.7) gives

$$E_{vac} = \langle 0|H|0\rangle = 0 \tag{3.9}$$

Supersymmetric ground state has always $E_{vac} = 0$. If, however, SUSY is broken, i.e. if

$$Q_\alpha|0\rangle \neq 0, \tag{3.10}$$

then $E_{vac} > 0$: SUSY can be broken only if the potential is strictly positive.

In discussing supersymmetric models and spontaneous breaking of supersymmetry, it is very convenient to use the superfield formalism [59]. A superfield $\Phi(X,\theta,\bar\theta)$ is defined as a function of the space-time point X_μ and the anticommuting parameters θ_α and $\bar\theta_{\dot\alpha}$, which are elements of a Grassmann algebra:

$$\{\theta_\alpha,\theta_\beta\} = \{\bar\theta_{\dot\alpha},\bar\theta_{\dot\beta}\} = \{\theta_\alpha,\bar\theta_{\dot\beta}\} = 0 \tag{3.11}$$

The superfield is defined in superspace, which is parametrized by

$$S(X,\theta,\bar\theta) = \exp i\,[-X_\mu P^\mu + \theta Q + \bar\theta\,\bar Q\,] \tag{3.12}$$

The superfield $\Phi(X,\theta,\bar\theta)$ is actually a Taylor expansion in θ and $\bar\theta$ with coefficients, which are themselves local fields over Minkowski space. It turns out that, to find superfields, whose components transform irreducibly under supersymmetry transformations, we need constraints. Superfields that satisfy the constraint $\bar D_{\dot\alpha}\Phi = 0\,(D_\alpha\Phi = 0)$, where D's are

covariant derivatives [59], are called left-handed (right-handed) chiral superfields. Their particle content is composed of two physical fields: a scalar and a spin $\frac{1}{2}$ fermion (see chiral supermultiplets above), and an auxiliary (non-propagating) field, called F, which is the highest θ^2 component of the superfield; its SUSY transformation is a total derivative. Superfields, that satisfy the reality constraint $\Phi^+ = \Phi$, are called vector superfields. Their particle content is composed of two physical fields: a spin-1 field (gauge boson) and a spin-$\frac{1}{2}$ fermion (gaugino) (see vector supermultiplets above), and an auxiliary (non-propagating) field, called D, which is the highest $\theta^2\bar\theta^2$ component of the superfield and, again, its SUSY transformation is a total derivative.

One can thus construct the Lagrangian, describing a vector as well as a chiral superfield and their interaction, by using F- and D- terms. By introducing integration over Grassmann variables as

$$\int d\theta = 0,\ \int d\theta\,\theta = \int d^2\theta\,\theta^2 = \int d^2\theta\,d^2\bar\theta\,\theta^2\bar\theta^2 = 1 \tag{3.13}$$

the Lagrangian is expressed as follows:

$$\begin{aligned}\mathcal{L} &= \{\tfrac{1}{4}\int d^2\theta\, W^\alpha W_\alpha + \text{h.c.}\} + \\ &+ \int d^2\theta\, d^2\bar\theta\,\bar\Phi\, e^{2gV}\,\Phi \\ &+ \{\int d^2\theta\, g(\Phi) + \text{h.c}\}\end{aligned} \tag{3.14}$$

In Eq.(3.14), $W_\alpha = \frac{1}{4}\bar D^2 e^{-V} D_\alpha e^V$ and the first term gives the kinetic terms for the components of the vector superfields. The second term gives the kinetic terms for the components of the chiral superfield and describes their gauge interactions. Finally, the third term gives Yukawa interactions and mass terms for the components of the chiral superfield, in terms of the superpotential $g(\Phi)$, which is a function of Φ, up to degree 3, for reasons of renormalizability. In particular, the scalar potential, which is obtained from (3.14), is

$$V = FF^* + \frac{1}{2} D_a^2 \geqslant 0, \tag{3.15}$$

where

$$F = \frac{\partial g}{\partial Z},\ D_a = gZ^+T^aZ, \tag{3.16}$$

with T^a the generators of the gauge group. From Eq.(3.15) (or from the supersymmetric transformation properties for the fermion fields ψ and λ), one can see that SUSY is broken if and only if the auxiliary fields F [60] and/or D [61] develop a non zero vev. The supersymmetry breaking is accompanied by the appearance of a massless Goldstone fermion, which is the fermionic partner of the auxiliary field developing vev.

The above are examples of spontaneous supersummetry breaking. In the F-breaking, F is usually taken as a singlet under the gauge group. In the D-breaking, D is the component of a vector superfield of a fundamental $U(1)$ factor of the gauge group. One can also envisage a dynamical supersymmetry breaking [62], in which case the Goldstone fermion is not a fundamental particle of the theory, but a composite bound state. Finally, supersymmetry breaking can take place explicitly, when there are "soft" SUSY breaking terms in the Lagrangian [63]. This is the case with mass terms for complex scalars and gauge fermions.

For the SUSY SU(5) model (Eq.(3.3)), the full superpotential is [55]

$$g = \lambda_1\left(\frac{1}{3}\Sigma^3 + \frac{1}{2}M\Sigma^2\right) + \lambda_2 H_2(\Sigma + 3M')H_1 + f_{ij}\, H_1\, Z^i_\psi\, Z^j_\psi + f'_{ij}\, H_2 Z^i_\psi Z^j_\chi \tag{3.17}$$

M and M' are of the order of the grand unification scale. The scalar potential, associated with (3.17), is

$$V = \sum_\phi \left|\frac{\partial g}{\partial \phi}\right|^2 + \text{D-terms} \tag{3.18}$$

where ϕ stands for the various scalar fields. This potential is minimized for $\langle H_1\rangle = \langle H_2\rangle = \langle Z_\psi\rangle = \langle Z_\chi\rangle = 0$ and one of the following three vev's for Σ:

$$\langle\Sigma\rangle 0 = 0 \qquad SU(5) \text{ unbroken}$$

$$\langle\Sigma\rangle = \begin{pmatrix} \frac{1}{3}M &&&& \\ & \frac{1}{3}M &&& \\ && \frac{1}{3}M && \\ &&& \frac{1}{3}M & \\ &&&& -\frac{4}{3}M \end{pmatrix} \qquad SU(5) \to SU(4) \times U(1)$$

$$\langle\Sigma\rangle = \begin{pmatrix} 2M &&&& \\ & 2M &&& \\ && 2M && \\ &&& -3M & \\ &&&& -3M \end{pmatrix} \qquad SU(5) \to SU(3) \times SU(2) \times U(1) \tag{3.19}$$

All these are degenerate minima, with $V = 0$, and SUSY is unbroken. For the desired $SU(5) \to SU(3) \times SU(2) \times U(1)$

solution, while the colour triplets of H_1, H_2 are getting masses of the order of M_G, the SU(2) doublets in H_1, H_2 get masses $M - M'$. Since these doublets must be massless, in order to implement the $SU(2) \times U(1)$ breaking, we have to impose the tree-level fine tuning

$$M = M' \tag{3.20}$$

However, because of the exact SUSY, this is stable against radiative corrections [7] and this is the technical advantage compared with ordinary GUTS fine tunings.

In order to discard altogether the fine-tuning (3.20) and obtain naturally massless Higgs doublets, two main mechanisms have been suggested. The first is the "sliding singlet" mechanism [64] and the second, more universally applicable, is the "missing partner" mechanism [65], based on purely group theoretical arguments: one introduces a representation ($\underline{50} + \overline{\underline{50}}$ in SU(5)), which contains colour triplets, but no weak doublets. Then, with a direct mass term $M H_1 H_2$ absent, no Higgs doublet mass term is induced. Neither of these mechanisms is, however, without problems.

To break SUSY, below M_G, one usually invokes either an F-breaking mechanism or an explicit symmetry breaking one. Using the first approach (as in the inverse hierarchy models [66], see later), one makes use of the common property of this type of models that the potential has a flat direction at the minimum. Using the second approach [55] means that we have to introduce, explicitly, mass terms in the potential, e.g. in SU(5)

$$\Delta V_{SU(5)} = M_\Sigma^2 \, Tr \, \Sigma^2 + m_{H_1}^2 \, |H_1|^2 + m_{H_2}^2 \, |H_2|^2 + \\ + m^2 \, |Z_\psi|^2 + m'^2 \, |Z_\chi|^2 \tag{3.21}$$

Now, what is interesting is that such terms arise naturally when we pass from global to local supersymmetry, which is the most natural further step to do. So, we turn to local supersymmetry, i.e. supergravity, GUTS.

B. N = 1 Supergravity GUTS

Given the success of the gauge principle, it is natural to ask what happens when SUSY is realized as a local symmetry with an anticommuting parameter $\xi = \xi(X)$ depending explicitly on X_μ. In that case, since ξ is a spinor and since the gauge field should transform into $\partial_\mu \xi(X)$, the gauge field of local supersymmetry is a spin $\frac{3}{2}$ spinorial vector ψ_μ^α. Furthermore, since the product of two local SUSY transformations leads to local coordinate transformation $\xi_1(X)\sigma_\mu \xi_2(X)\partial_\mu$, one expects a connection with gravity, hence the name supergravity [9]. Anyway, in SUSY models we have the same number of fermionic and bosonic degrees of freedom, so we expect a new bosonic field to be obtained in a locally supersymmetric theory. In fact, the graviton e_μ^m coupled to the energy momentum tensor, turns out to be the SUSY partner of the spin $\frac{3}{2}$ gauge field ψ_μ^α, which is then called gravitino.

So, in N = 1 SUGRA theories, we have the following three kinds of supermultiplets (with the corresponding physical fields): (i) Gravity multiplet : (e_μ^m, ψ_μ); (ii) Gauge multiplet: (A_μ, λ); (iii) Matter multiplets: (Z_i, χ_i).

In global SUSY, generalizing Eq.(3.14), one can write the most general Lagrangian as [11]

$$\frac{1}{4} Re \int d^2\theta f(\Phi) WW + \int d^2\theta d^2\bar{\theta} \, \phi(\bar{\Phi} e^{2gV}, \Phi) + Re \int d^2\theta g(\Phi) \tag{3.22}$$

where f, ϕ and g (the superpotential) are general functions of their arguments. $f(\Phi)$ transforms as the symmetric product of two adjoint representations with respect to the gauge group. The theory is renormalizable only if $f(\Phi)$ is constant, $\phi = \bar{\Phi}\, e^{2gV}\Phi$ and $g(\Phi)$ a polynomial of degree less or equal to three. Giving up these restrictions means giving up renormalizability, which is a logical step to do towards a supergravity theory. We however demand that all nonrenormalizable terms contain the gravitational coupling constant $K^2 \equiv \frac{8\pi}{M_P^2}$, so that, in the flat limit $K \to 0$, the theory becomes renormalizable.

In $N = 1$ supergravity, the kinetic function ϕ and the superpotential g lose their independent meaning and the general coupling [10] of chiral and vector multiplets to $N = 1$ supergravity is specified by the following two independent functions of the complex scalar fields Z_i, contained in the chiral multiplets:

$$f_{ab}(Z_i) = f_{ba}(Z_i)$$

$$G(Z_i, Z_i^*) = -3\ln\left(-\frac{\phi(Z_i, Z_i^*)}{3}\right) + \ln|g(Z)|^2$$

$$\equiv d(Z_i, Z_i^*) + \ln|g(Z)|^2 \tag{3.23}$$

f_{ab} is an analytic function related to the Yang-Mills part of the Lagrangian; $G(Z_i, Z_i^*)$ is a real function, called Kähler potential, because the tensor

$$G^i_j = \frac{\partial^2 G}{\partial Z_i \partial Z^{*j}} \tag{3.24}$$

acts as a metric of the kinetic terms of the scalar fields.

In fact, the bosonic part of the Lagrangian is (we put $K = 1$)

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R - \frac{1}{4}(\mathrm{Re} f_{ab})F^a_{\mu\nu}F^{b\mu\nu} + \frac{i}{4}(\mathrm{Im}\, f_{ab})F^a_{\mu\nu}\tilde{F}^{b\mu\nu}$$
$$- G^i_j D_\mu Z_i D^\mu Z^{*j}$$
$$- e^G(G_i (G^i_j)^{-1} G^j - 3) - \frac{1}{2}(\mathrm{Re}\, f^{-1}_{ab})D^a D^b \tag{3.25}$$

where $G^i = \frac{\partial G}{\partial Z_i}$ etc., and $D^a = gG^i\, T^{aj}_i Z_j$. We do not write down explicitly the fermionic part of the Lagrangian (Refs. 10, 11).

To discuss spontaneous SUSY breaking, one considers, in analogy with global SUSY, the auxiliary field for the chiral fermions

$$F_i = e^{G/2}(G^j_i)^{-1} G_j + \text{fermion dependent terms} \tag{3.26}$$

and the auxiliary field for the gauge fermion

$$\tilde{D}_a = i(\mathrm{Re} f_{ab}^{-1})\, g\, G^i T^{bj}_i Z_j + \text{fermion dependent terms} \tag{3.27}$$

As in global SUSY, local SUSY is broken if at least one of these auxiliary fields receives a vev and the SUSY breaking scale M_S^2 is given by $\langle F\rangle$ or $\langle D\rangle$. Note that, a vev for the fermion dependent terms occurs only in the presence of a strongly interacting gauge force, that leads to a vacuum condensation of bilinear fermion-antifermion states. In the absence of these, it is evident from Eq.(3.26) and (3.27) that the criterion for SUSY breaking is $\langle G^i\rangle \neq 0$. We have seen that spontaneous SUSY breakdown is accompanied by a Goldstone fermion field. What

happens here is that, the Goldstone fermion

$\eta_L = -\langle e^{G/2} G_i \rangle \chi_L^i + \frac{i}{2} \langle D_a \rangle \lambda_L^a$, can be rotated away by a local SUSY transformation. Then the super-Higgs effect [67] is effective and the gravitino acquires a mass (we put back here $\frac{1}{K} = \frac{M_P}{\sqrt{8\pi}} \equiv M = 2.4 \times 10^{18}$ GeV)

$$m_{3/2} = Me^{G/2} \tag{3.28}$$

The potential given from Eq.(3.25) is

$$V = e^G[G_i(G^i_j)^{-1} G^j - 3] + \frac{1}{2} (\mathrm{Re} f_{ab}{}^{-1}) D^a D^b \tag{3.29}$$

The form of the potential (3.29) suggests that there is the possibility of broken supergravity with vanishing vacuum energy. Although this is not a solution to the problem of the cosmological constant, at least there is the possibility to fine tune it to zero. In that case, by redefining the fields such that the kinetic terms are canonical, i.e.

$$G^i_j = \delta^i_j, \ f_{ab} = \delta_{ab} \tag{3.30}$$

(something which can always be done in a well defined theory), it is not difficult to give the relation between the gravitino mass and the scale of SUSY breakdown [67]

$$m_{3/2} = \frac{M_S{}^2}{\sqrt{3}\, M} \tag{3.31}$$

Under these conditions, one can also derive the mass formula (for N chiral superfields) [10]

$$S\,\mathrm{Tr}\, m^2 = \sum_{J=0}^{3/2} (-)^{2J}(2J + 1)\, m_J{}^2 =$$
$$= (N - 1)\,(2m^2_{3/2} - D^a D_a/M^2) - 2g\, D^a\, \mathrm{Tr}\, T^a \tag{3.32}$$

The phenomenological relevance of this formula is immediate: Due to the supergravity correction first term, the "mean squared mass" of the scalar fields is $m^2_{3/2}$ and thus these fields are heavier than the corresponding fermions, a phenomenological must [68]. The low-energy spectrum, however, has to be studied after inclusion of renormalization effects. In fact, one-loop corrections on scalar masses have been shown to be able to induce $SU(2)_L \times U(1)_Y$ symmetry breaking at low energies, even if all scalars have positive tree level "mean squared masses" of order $m^2_{3/2}$.[11]

In general, in realistic models, we distinguish two sectors: a hidden sector, responsible for the spontaneous breakdown of supergravity, and an observable sector, containing all the superfields needed for the construction of the model. These two sectors communicate only through gravitational interactions, which means that the fields of the hidden sector are gauge singlets with no Yukawa couplings to the observable sector. The simplest case for the hidden sector is to consider minimal kinetic energy terms, i.e. $G^i_j = \delta^i_j$, $f_{ab} = \delta_{ab}$ with a Kähler potential

$$G(Z,Z^*) = \frac{ZZ^*}{M^2} + \ln \frac{|g(z)|^2}{M^6} \tag{3.33}$$

from which we obtain

$$V = e^{\frac{ZZ^*}{M^2}} \left\{ \left|\frac{\partial g(Z)}{\partial Z} + \frac{Z^*}{M^2}\, g(Z)\right|^2 - \frac{3}{M^2} \left|g(Z)\right|^2 \right\} + \frac{1}{2} D_a{}^2 \tag{3.34}$$

In the limit $M \to \infty$, we recover the global SUSY result, which is never zero for $\langle g^Z \rangle \equiv \langle \frac{\partial g(Z)}{\partial Z} \rangle \neq 0$, when global SUSY is broken. Now, the signal of broken local SUSY is $\langle G^Z \rangle \neq 0$ or

$$\langle g(Z)G^Z\rangle = \langle g^Z + \frac{Z^*}{M^2}\, g\rangle \neq 0 \qquad (3.35)$$

We see that, in the present case, we have the possibility $V = 0$ (zero cosmological constant), even with $\langle G^Z\rangle \neq 0$, because of the negative term in (3.34), provided we fine-tune the superpotential $g(Z)$.

The simplest example of a hidden sector Kahler superpotential, for one chiral superfield, for which the super-Higgs effect occurs, is the superpotential [69]

$$g(Z) = M_S^2(Z + \beta) \qquad (3.36)$$

where β will be fine-tuned, so that the vacuum energy is zero. With the F component of the z-field being the only source of SUSY breaking, the minimum of the potential V is zero for $\beta = (2 - \sqrt{3})\,M$ and $\langle Z\rangle = (\sqrt{3} - 1)\,M$. The super Higgs effect occurs and the gravitino mass is

$$m_{3/2} = \frac{M_S^2}{M}\exp\frac{(\sqrt{3} - 1)^2}{2} \qquad (3.37)$$

The two real scalars $(Z = A + iB)$ have $(\text{masses})^2$ $2\sqrt{3}\, m_{3/2}^2$ and $2(2 - \sqrt{3})\, m_{3/2}^2$ and the formula $\mathrm{STr}m^2 = 2(N-1)m_{3/2}^2$ is satisfied $(N = 1)$.

The combined superpotential $g(z_i, y_a)$, with z_i the fields inducing the super Higgs effect and y_a the "observable" fields, containing quarks, leptons and Higgs fields and satisfying $\langle y_a\rangle \ll \langle z_i\rangle \sim M$, is most frequently written as [70]

$$g(z_i, y_a) = g(z_i) + g(y_a) \qquad (3.38)$$

with the $g(z_i)$ a Polonyi-type superpotential and $g(y_a)$ an arbitrary gauge invariant superpotential. One can also write down a factorized superpotential [71] or even consider

a more general case [72]. With the form (3.38) for the full superpotential, we can write

$$\langle z_i\rangle = \beta_i M,\ \langle g_i\rangle = \alpha_i^* M_S^2,\ \langle g(z_i)\rangle = M_S^2 M \qquad (3.39)$$

with $\Sigma|\alpha_i + \beta_i|^2 = 3$ from the condition $V = 0$. The gravitino mass is

$$m_{3/2} = \frac{M_S^2}{M}\exp\frac{\sum_i |\beta_i|^2}{2} \qquad (3.40)$$

M_S is usually taken $\sim 10^{11}$ GeV, an intermediate energy scale, sometimes desirable from cosmological considerations. One can now expand the potential in power series in $K \equiv \frac{1}{M}$, take the limit $K \to 0$ with $m_{3/2}$ fixed and get an effective Lagrangian valid at scales lower than M. Then the fields z_i consistently decouple in the effective theory; the interactions of the hidden sector with the observable one are governed by K. We thus obtain the scalar potential

$$V = |\hat{g}_a|^2 + \frac{1}{2}D_u^2 + m_{3/2}^2 \sum_a |y_a|^2 +$$

$$+\, m_{3/2}\,\{\sum_a y_a \hat{g}_a + (A-3)\,\hat{g} + \text{h.c.}\} \qquad (3.41)$$

where $\hat{g} = g \exp\frac{\sum_i|\beta_i|^2}{2}$ and [73]

$$A = \sum_i \beta_i^*(\alpha_i + \beta_i) \qquad (3.42)$$

The first two terms in (3.41) coincide with that in the global SUSY. The third one is a soft breaking term, with a common value for all the y_a's (compare with (3.21)). The effective theory is renormalizable as long as the superpotential $g(y_a)$ is a polynomial at most cubic in fields y_a. We should bear in mind, however, that, in general, nonrenormalizable terms in the superpotential can be present, given that a $N = 1$ SUGRA theory so constructed must be only considered as an effective theory [74]. With non-minimal kinetic terms for the gauge fields $f_{ab} = \delta_{ab}$, one can also introduce mass terms for the gauginos, as well as CP-violating $F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms [10, 74]. With the use of (3.41), various unified models coupled to $N = 1$ supergravity, with or without nonrenormalizable terms, have been constructed, as well as $SU(3) \times SU(2) \times U(1)$ phenomenological models, predicting many superpartners around 100 GeV [11].

Returning now to SU(5), recall that in global SUSY, the different supersymmetric minimal SU(5), SU(4) × U(1) and SU(3) × SU(2) × U(1), were degenerate with $V = 0$. This is no longer true in supergravity. Now, different supersymmetric minima have (from Eq. (3.29))
$V = -3e^G \equiv \frac{-3|g|^2}{M^2} e^{d/M^2}$ and are only degenerate if they have the same value of the superpotential, which is not always the case. We can take the SU(3) × SU(2) × U(1) as the highest of these minima with $g = 0$ and $V = 0$ (zero cosmological constant - Minkowski vacuum solution). The other SUSY minima, however, will, in general, have $V < 0$ (anti-de Sitter vacuum solution). Nevertheless, there are arguments [75] that the presence of gravitation renders the Minkowski minimum stable, even if it does not have the lowest energy (see also later). In general, however, these considerations apply only to the observable sector. In the hidden sector, SUSY is broken, and the question of different SU(5) minima can only be answered in the context of specific models.

This brings us to an interesting class of models, where the cosmological constant naturally vanishes after the breaking of supergravity. These "zero flat potentials" are taken from the Kähler potential

$$G(Z,Z^*) = -3 \ln (Z + Z^*) \qquad (3.43)$$

which leads naturally to $V = 0$ [76]. In that case, the geometry of the Kähler manifold is characterized by

$$R_{ZZ^*} = \frac{\partial}{\partial Z \partial Z^*} \ln G_{ZZ^*} = \frac{2}{3} G_{ZZ^*} \qquad (3.44)$$

which means that the Kähler manifold is an Einstein space (maximally symmetric space). There is an $SU(N,1)$ global symmetry, where N is the number of chiral fields. The gravitino mass is

$$m^2_{3/2} = e^{<G>} = <Z + Z^*>^{-3} \qquad (3.45)$$

and, at the tree level, is undetermined, but non-zero. Upon coupling with the observable sector by writing

$$G = -3 \ln\left(Z + Z^* - \frac{y_a y_a^*}{3}\right) + \ln|g(y_a)|^2 \qquad (3.46)$$

the gravitino mass as well as subsequent mass scales can be obtained via "dimensional transmutation" [77] (see later) from non-gravitational radiative corrections, along one of the flat directions of the scalar potential. This is the program of the no-scale models [78], in which from only one scale, to be identified with M, every other scale below that is dynamically determined. The consistency of the whole scheme relies on the assumption that no quantum gravitational effects affect the radiative corrections.

It is again interesting to point out that such a particular Kähler potential can be actually obtained, under a simple truncation in the process of compactification, from the ten-dimensional superstring/supergravity theories[79].

IV. PHASE TRANSITIONS IN FIELD THEORIES

4.1 The Effective Potential at Zero Temperature

The main concept, in discussing symmetry breaking and phase transitions in field theory is the effective potential [80]. The effective potential includes all quantum corrections to the classical field theory potential. By minimizing it, we have the field configuration with minimal energy, i.e. the vacuum of the theory. Thus, we can study the symmetries of the full theory and not just those of the Lagrangian. This will then reveal phenomena of symmetry breaking.

The generating functional $Z(J)$ for the full Green's functions is defined by

$$Z(J) = \langle 0|\ T \exp\left(i\int d^4x\ \ J(x)\phi(x)\right)|0\rangle \qquad (4.1)$$

where $|0\rangle$ is the physical vacuum and $\phi(x)$ a field, in both cases for the theory without the source term $J(x)\phi(x)$. In functional integral representation

$$Z(J) = N\int d\phi\ \exp\ [iS(\phi,J)] \qquad (4.2a)$$

where S is the action

$$S(\phi,J) = \int d^4x[\mathcal{L}(\phi(x)) + J(x)\phi(x)] \qquad (4.2b)$$

The logarithm of $Z(J)$ is the generating functional $iW(J)$ for the connected Green's functions

$$Z(J) = e^{iW(J)} \qquad (4.3)$$

The Legendre transform of $W(J)$ is the generating functional $\Gamma(\bar{\phi})$ for the one-particle irreducible Green's functions

$$W(J) = [\Gamma(\bar{\phi}) + J\bar{\phi}]_{\frac{\delta\Gamma}{\delta\bar{\phi}} = -J}$$

$$\Gamma(\bar{\phi}) = [W(J) - J\bar{\phi}]_{\frac{\delta W}{\delta J} = \bar{\phi}} \qquad (4.4)$$

where $J\bar{\phi} \equiv \int d^4x\ J(x)\bar{\phi}(x)$ and $\bar{\phi}$ is the average field

$$\bar{\phi}(x) = \frac{\delta W}{\delta J}(x) = \frac{\int d\phi\ \phi\ \exp\ iS(\phi,J)}{\int d\phi\ \exp\ iS(\phi,J)} \qquad (4.5)$$

$\Gamma(\bar{\phi})$ is called the effective action. In a translationary invariant theory, where $\bar{\phi}(x)$ is constant, we define the effective potential $V_{eff}(\bar{\phi})$ by

$$\Gamma(\bar{\phi}) = \int d^4x[-V_{eff}(\bar{\phi})] \qquad (4.6)$$

The physical meaning of the effective potential can be seen if we write

$$\Gamma(\bar{\phi}) = \sum_{n=2}^{\infty} \frac{1}{n!}\ \bar{\phi}^{\,n}\int d^4x_1\ldots d^4x_n\Gamma^{(n)}(x_1,\ldots x_n)$$

$$= \sum_{n=2}^{\infty}\frac{1}{n!}\bar{\phi}^{\,n}\int d^4x_1\ldots d^4x_n\ \frac{d^4k_1}{(2\pi)^4}\ \ldots\ \frac{d^4k_n}{(2\pi)^4}\ e^{i(k_1x_1+\ldots+k_nx_n)}\times$$

$$\times \Gamma^{(n)}(k_1,\ldots k_n)(2\pi)^4\delta^4(\sum_{i=1}^{n} k_i)$$

$$= \sum_{n=2}^{\infty} \frac{1}{n!} \bar{\phi}^{\,n} \int d^4x_1 \; \Gamma^{(n)} \; (0,\ldots 0) \tag{4.7}$$

Then

$$V_{eff}(\bar{\phi}) = -\sum_{n=2}^{\infty} \frac{1}{n!} \bar{\phi}^{\,n} \; \Gamma^{(n)} \; (0,\ldots 0) \tag{4.8}$$

$V_{eff}(\bar{\phi})$ is thus the generating functional for one-particle irreducible graphs with vanishing external momenta. So, we can calculate it by summing all one-particle irreducible graphs and inserting a factor $\bar{\phi}$ for each external line. Minimizing $V_{eff}(\bar{\phi})$ with respect to $\bar{\phi}$ gives the ground state energy of the theory. The value of $\bar{\phi}$, at which the minimum of the $V_{eff}(\bar{\phi})$ is taken on, yields information about spontaneous symmetry breaking. Specifically, if the minimum occurs at $\bar{\phi} \neq 0$, then all the symmetries of $\mathcal{L}(\phi,\partial_\mu\phi)$, which do not leave $\bar{\phi}$ invariant, are spontaneously broken.

To compute the effective potential, many techniques have been proposed. For the one-loop level, the most convenient one is the Coleman-Weinberg combinatorial method [77]. This method cannot be easily extended to higher orders in loop expansion. We note, however, that such an extension is more easy with the functional integral method [81].

Consider a general non-Abelian theory given by

$$\mathcal{L}(A,\psi,\phi) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + i \,\mathrm{Tr}\, \bar{\psi} \not{D} \psi, \; \bar{\psi}\,\Gamma\,\phi\psi - \mathrm{Tr}\,\bar{\psi}\,\Gamma\,\phi\psi + \frac{1}{2}\mathrm{Tr}(D_\mu\phi)^+(D^\mu\phi) - U(\phi) \tag{4.9}$$

where Γ is the matrix of Yukawa coupling constants, $D_\mu = \partial_\mu + ig\, A_\mu^a T_a$ is the covariant derivative and the trace refers to the appropriate representation space. Then, in covariant gauges and to one-loop order, the effective potential is

$$V_{eff}^{(1)}(\bar{\phi}) = U(\bar{\phi}) + V_s^{(1)} + V_f^{(1)} + V_g^{(1)} \tag{4.10}$$

where

$$V_s^{(1)} = \frac{1}{64\pi^2} \;\mathrm{Tr}\; M_s^4(\bar{\phi}) \;\ln \frac{M_s^2}{\mu^2} \tag{4.11}$$

$$V_f^{(1)} = -\frac{1}{64\pi^2} \; 4 \;\mathrm{Tr}\; M_f^{(4)}(\bar{\phi}) \;\ln \frac{M_f^2}{\mu^2} \tag{4.12}$$

$$V_g^{(1)} = \frac{1}{64\pi^2} \; 3 \;\mathrm{Tr}\; M_g^4(\bar{\phi}) \;\ln \frac{M_g^2}{\mu^2} \tag{4.13}$$

In the above formulae, $M_s^2 = \left.\frac{\partial^2 U}{\partial\phi^2}\right|_{\phi=\bar{\phi}}$ is the squared mass matrix for the scalars, $M_f^2 = (\Gamma\bar{\phi})^2$ that for fermions, $M_g^2 = g^2\mathrm{Tr}[(T\bar{\phi})^+(T\bar{\phi})]$ that for the gauge bosons in the broken phase with vev $\bar{\phi}$, μ is the renormalization mass. The factor 4 in Eq.(4.12) stands for the four degrees of freedom of a Dirac fermion and the factor 3 in Eq.(4.13) for the three degrees of freedom of a gauge boson, in the broken phase.

In non-supersymmetric theories, for a Higgs potential $V(\phi) = \lambda P(\phi)$ with $P(\phi)$ a fourth order polynomial and λ small, the dominant contribution to the effective potential will come from the gauge boson part: $V_{eff}^{(1)}(\bar{\phi}) = U(\bar{\phi}) + V_g^{(1)}(\bar{\phi})$. This is the case of the Coleman-Weinberg mode [77], where there is no mass term

$\frac{1}{2}\mu^2 \bar{\phi}^2$ in the potential $U(\bar{\phi})$ and the minimum is at $\bar{\phi} = 0$. In these models, the one-loop result predicts spontaneous symmetry breaking, with the minimum of the effective potential occuring at $\bar{\phi} = \sigma$, while one can solve λ in terms of σ (dimensional transmutation).

Now, for the SU(5) model in the Coleman-Weinberg mode, for most values of the potential parameters [82], we have the energetically favoured symmetry breaking channel $SU(5) \to SU(3) \times SU(2) \times U(1)$, with the vev for the Higgs field in the adjoint representation

$$\langle\Phi\rangle = \bar{\phi} \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hline & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{array} \right) \qquad (4.14)$$

Twelve gauge bosons become massive with mass

$$M_{X,Y}^2 = \frac{25}{8} g^2 \bar{\phi}^2 \qquad (4.15)$$

Then, the dominant contribution to the effective potential will be

$$V_g^{(1)}(\bar{\phi}) = \frac{5625}{1024\pi^2} g^4 \bar{\phi}^4 \ln \bar{\phi}^2 + O(\bar{\phi}^4) \qquad (4.16)$$

and the effective potential $V_{eff}^{(1)}(\bar{\phi}) = U(\bar{\phi})$ will be

$$V_{eff}^{(1)}(\bar{\phi}) = \frac{5625}{1024\pi^2} g^4 \bar{\phi}^4 \left(\ln \frac{\bar{\phi}^2}{\sigma^2} - \frac{1}{2}\right) \qquad (4.17)$$

where σ is the minimum solution $\left.\frac{\partial V_{eff}^{(1)}}{\partial \bar{\phi}}\right|_{\bar{\phi}=\sigma} = 0.$

As another example, let us consider the inverse hierarchy SU(5) SUSY models [66]. The superpotential is taken to be

$$g = \lambda_1 \, \mathrm{Tr}\, A^2 Y + \lambda_2 X(\mathrm{Tr}\, A^2 - m^2) \qquad (4.18)$$

where A,Y are two adjoint representations and is a singlet. The equations $\frac{\partial g}{\partial Y} = 0$ and $\frac{\partial g}{\partial X} = 0$ are inconsistent, so SUSY is broken and minimization of the potential, derived from (4.18), happens with [66]

$$\langle A\rangle = \frac{\lambda_2 m}{\sqrt{\lambda_1^2 + 30\lambda_2^2}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

$$\langle Y\rangle = \frac{\lambda_2 \langle X\rangle}{\lambda_1} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

$\langle X\rangle$ undetermined (4.19)

Then, including one-loop corrections, which will determine $\langle X\rangle$, we get, to leading terms,

$$V_{eff}^{(1)} = \frac{\lambda_1^2 \lambda_2^2 m^4}{\lambda_1^2 + 30\lambda_2^2} \left[1 + \frac{3}{8\pi^2}\left(\frac{\lambda_2^2}{\lambda_1^2 + 30\lambda_2^2}\right)(29\lambda_1^2 - 50g^2) \times \times \ln \frac{X^2}{\mu^2}\right] \qquad (4.20)$$

Now, for $29\lambda^2 < 50g^2$ (g is the SU(5) gauge coupling), X tends to be large compared to m ($\mu = m$ is the natural scale). The potential decreases for increasing X till the region where (4.20) is no longer valid. Due to asymptotic freedom, the potential must turn over at some point and a stable minimum develops at $\langle X\rangle \sim m\, e^{1/g^2}$. In these models, m will be the small input mass scale and the scale $\langle X\rangle \gg m$ obtained by "dimensional transmutation", will be the large one. In realistic models, m is taken at the intermediate range $m \sim 10^{10}$GeV, from which the large mass scale M_G is induced, out of the flat direction. The construction of such realistic models, however, encounters problems both from particle physics [83] and from cosmology (see below).

4.2 The Effective Potential at Finite Temperatures

As we have said in the introduction and seen in the previous sections, unification of all particles forces takes place at extremely high energies and, hence, its natural arena is the early universe. The assumption of an empty space, in which scattering events take place, is totally inapplicable in that case, since one is dealing with high matter and radiation density. There, the scattering events take place in a thermal bath at some finite temperature $T \equiv 1/\beta$. Then, in analogy with the T = 0 temperature, we use the finite temperature generating functional $Z^\beta(J)$

$$Z^\beta(J) = \frac{\mathrm{Tr}[e^{-\beta H}\, T\, \exp(\int i\, J(x)\phi(x)d^4x)]}{\mathrm{Tr}\, e^{-\beta H}} \qquad (4.21)$$

where H is the Hamiltonian. As in the T = 0 theory, we also have

$$W^\beta(J) = -i\, \ln Z^\beta(J)$$
$$\Gamma^\beta(J) = (W^\beta(J) - J\bar{\phi})_{\frac{\delta W^\beta(J)}{\delta J} = \bar{\phi}} \qquad (4.22)$$

Then $\frac{\delta\Gamma^\beta(J)}{\delta\bar{\phi}} = -J$, and $\bar{\phi}(x)$, evaluated at J = 0, is the thermodynamic average of the field $\phi(X)$

$$\bar{\phi}(x)\Big|_{J=0} = \frac{\mathrm{Tr}\, e^{-\beta H}\phi(x)}{\mathrm{Tr}\, e^{-\beta H}} \qquad (4.23)$$

$\Gamma^\beta(\bar{\phi})$ is the finite temperature effective action and, in a translationary invariant theory with $\bar{\phi}(x) = \bar{\phi}$, we define the finite temperature effective potential $V^\beta_{eff}(\bar{\phi})$ by

$$\Gamma^\beta(\bar{\phi}) = \int d^4x[-V^\beta_{eff}(\bar{\phi})] \qquad (4.24)$$

Then, symmetry breaking occurs when $\frac{\partial V^\beta(\bar{\phi})}{\partial\bar{\phi}} = 0$ for $\bar{\phi} \neq 0$.

The important observation [84] is that, in Minkowski space-time, the only changes in the functional integral, compared to zero temperature, are the boundary conditions. This means that in Euclidean space, the finite temperature Green's functions are periodic for Bose fields and anti-periodic for Fermi fields, in Euclidean time, with period β. As a result, the momentum-space Feynman rules, needed to calculate graphs, are the same as in the zero temperature quantum field theory, except that every internal energy is replaced by a quantity iω, satisfying the quantization conditions:

$$\omega = 2n\pi T \qquad \text{for bosons}$$
$$\omega = (2n+1)\pi T \quad \text{for fermions} \qquad (4.25)$$

and all energy integrals are replaced with ω sums:

$$p_o \rightarrow i\omega$$
$$\int d^4p \rightarrow 2i\pi T \sum_\mu \int d^3p$$

$$\delta^4(p - p') \to (2i\pi T)^{-1}\ \delta^3(\vec{p} - \vec{p}')\delta_{\omega\omega'} \qquad (4.26)$$

Using these modifications, we find that to one-loop order the finite temperature effective potential is

$$V^{(1)}_{eff}(\bar\phi, T) = V^{(1)}_{eff}(\bar\phi) + V^T_s + V^T_f + V^T_g \qquad (4.27)$$

where $V^{(1)}_{eff}(\bar\phi)$ is given by Eq.(4.10)-(4.13) and

$$V^T_s = \frac{T^4}{2\pi^2}\,\mathrm{Tr}\int_0^\infty dx\, x^2\, \ell n\left[1 - \exp\left(-x^2 + \frac{M^2_s(\bar\phi)}{T^2}\right)^{1/2}\right] \qquad (4.28)$$

$$V^T_f = -4\,\frac{T^4}{2\pi^2}\,\mathrm{Tr}\int_0^\infty dx\, x^2\, \ell n\left[1 + \exp\left(-x^2 + \frac{M^2_f(\bar\phi)}{T^2}\right)^{1/2}\right] \qquad (4.29)$$

$$V^T_g = 3\,\frac{T^4}{2\pi^2}\,\mathrm{Tr}\int_0^\infty dx\, x^2\, \ell n\left[1 - \exp\left(-x^2 + \frac{M^2_g(\bar\phi)}{T^2}\right)^{1/2}\right] \qquad (4.30)$$

There is a high temperature $T \gg M$ expansion of the above expressions. For bosons and for each degree of freedom

$$V^T_b = \frac{T^4}{2\pi^2}\int_0^\infty dx\, x^2\, \ell n\left[1 - \exp\left(-x^2 + \frac{M^2_b(\bar\phi)}{T^2}\right)^{1/2}\right]$$

$$= -\frac{\pi^2 T^4}{90} + \frac{1}{24}\, M^2_b(\bar\phi) T^2 + \dots \qquad (4.31)$$

whereas, for fermions

$$V^T_f = -\frac{T^4}{2\pi^2}\int_0^\infty dx\, x^2\, \ell n\left[1 + \exp\left(-x^2 + \frac{M^2_f(\bar\phi)}{T^2}\right)^{1/2}\right]$$

$$= \frac{7}{8}\left(-\frac{\pi^2 T^2}{90}\right) + \frac{1}{2}\,\frac{M^2_f(\bar\phi) T^2}{24} + \dots \qquad (4.32)$$

The finite temperature corrections to the one-loop effective potential, in the high temperature limit, generate a temperature-dependent mass term $CT^2\,\bar\phi^{\,2}$. This term converts $\bar\phi = 0$ from a local maximum of V_{eff} to a local minimum. In fact, above some T_c, $\bar\phi = 0$ is the absolute minimum, whereas for $T < T_c$, $\bar\phi = 0$ remains a relative minimum, a metastable false vacuum. This critical temperature T_c, is the temperature, at which the minima are degenerate. We can roughly estimate T_c by assuming that the location $\bar\phi = \sigma$ of the asymmetric minimum is temperature independent. In this approximation, the criterion for T_c is

$$V^{(1)\ T_c}_{eff}(\sigma) = 0 \qquad (4.33)$$

For

$$V^{(1)\ T}_{eff}(\bar\phi) = B\bar\phi^{\,4}\left(\ell n\,\frac{\bar\phi^{\,2}}{\sigma^2} - \frac{1}{2}\right) + C\,\bar\phi^{\,2}\,T^2 \qquad (4.34)$$

where B,C are model dependent constants, the condition (4.33) gives

$$T_c^2 = \frac{B\sigma^2}{2C} \qquad (4.35)$$

The above result is expected in the Coleman-Weinberg type of theories, since σ is the only mass scale present and any critical temperature should be of the same order of magnitude. Thus in the SU(5) model, we have the dominant contribution

$$V^{(1)\ T}_{eff}(\bar\phi) = \frac{5625}{1024\pi^2}\, g^4\,\bar\phi^{\,4}\left(\ell n\,\frac{\bar\phi^{\,2}}{\sigma^2} - \frac{1}{2}\right) + \frac{75}{16}\, g^2\,\bar\phi^2 T^2 \qquad (4.36)$$

In a globally SUSY model, the finite temperature corrections to the one-loop effective potential, in the high temperature limit, are written [85]

$$V_{eff}^{(1)\,T}(\bar\phi) = -\frac{1}{90}\pi^2 T^4 (N_b + \frac{7}{8} N_f) + \frac{1}{24} T^2 \{\mathrm{Tr}\, M_s^2(\bar\phi) + \mathrm{Tr}\, M_f^2(\bar\phi) + 3\,\mathrm{Tr}\, M_g^2(\bar\phi)\} \qquad (4.37)$$

where $N_b(N_f)$ are the operetive (i.e. massless compared to T) bosonic (fermionic) degrees of freedom. Using this result one can easily deduce that, for example, in the inverted gauge hierarchy model (4.18), the critical temperature is $T_c \sim m$ [86], which is the natural scale of the model.

The finite temperature corrections in N = 1 supergravity are more complicated [87]. The potential is given by Eq.(3.29) and the finite temperature corrections arise through loops of properly normalized (i.e. with canonical kinetic terms) scalar fields and their fermionic superpartners, gauge fields and gauge fermions, and gravitinos. In the high temperature limit, we will have

$$V_{eff}^{(1)\,T} = -\frac{\pi^2}{24}(N + N_G + 1)T^4 + \frac{1}{24} T^2 \{\mathrm{Tr}\, M_0^2 + \mathrm{Tr}\, M_{1/2}^2 + 3\,\mathrm{Tr}\, M_1^2 + \mathrm{Tr}\, M_{3/2}^2\} \qquad (4.38)$$

where N is the total number of chiral superfields, N_G is the number of generators of the gauge group G and M_J^2 is the square mass matrix for the spin J fields. Note that, in order to compute the gravitino contribution, one uses the gauge $\gamma^\mu \psi_\mu = 0$, where the gravitino decouples from the would-be goldstino, which is separately included in the

fermion-$\frac{1}{2}$ contribution in (4.38), since it is eaten by gravitino (super-Higgs effect) only at the minimum of the potential. Then the effective action gives a contribution to the one-loop finite-temperature potential proportional to

$$\mathrm{Tr}\, M_{3/2}^2 = -2e^G \qquad (4.39)$$

Let us first consider the minimal supergravity (3.30). Then, the spin-zero contribution is

$$\mathrm{Tr}\, M_0^2 = 2e^G\{(G^{ij} + G^i G^j)(G_{ij} + G_i G_j) + (N-1)G^i G_i - 2N\} + 2g^2 C_2(R)|y_\alpha|^2; \qquad (4.40)$$

where $C_2(R)$ is the Casimir invariant for the representation R and a sum over all representations is implied. The fermion contribution is

$$\mathrm{Tr}\, M_{1/2}^2 = e^G(G^{ij} + G^i G^j)(G_{ij} + G_i G_j) + 4g^2 C_2(R)|y_\alpha|^2 \qquad (4.41)$$

and that of gauge bosons

$$\mathrm{Tr}\, M_1^2 = 2g^2 C_2(R)|y_\alpha|^2 \qquad (4.42)$$

Putting everything together, the thermal corrections are

$$V_{eff}^{(1)\,T} = -\frac{\pi^2}{24}(N + N_G + 1) + \frac{T^2}{24}\{3(G^{ij} + G^i G^j)(G_{ij} + G_i G_j) + 2(N-1)G^i G_i - 2(N+1) + 12g^2 C_2(R)|y_\alpha|^2\} \qquad (4.43)$$

The number of the chiral superfields is expected to be O(100); for example, in the minimal SUSY SU(5) we have

$N = 80$ (the singlet Z for SUSY breaking, 3 families of quarks and leptons, the adjoint $\underline{24}$ Higgs Σ and two $\underline{5}$ and $\overline{\underline{5}}$ Higgses $H,\overline{H}$). Then, in the large N limit, (4.44) reduces to

$$V^{(1)\,T}_{eff} = -\frac{\pi^2}{24} NT^4 + \frac{T^2}{12} e^G N(G^i G_i - 2) \qquad (4.44)$$

Finally, let us consider the non-minimal supergravity, characterized by (see Eq.(3.46))

$$G = -3 \ln(Z + Z^* - \frac{y_\alpha y_\alpha^*}{3}) + F(y_\alpha) + F^*(y_\alpha^*) \qquad (4.45)$$

along with a non-minimal f_{ab}. The zero-temperature potential in that case is

$$V = e^{2/3\, G + F + F^*} F_\alpha F^\alpha + \frac{1}{2} f^{-1}_{ab} D^a D^b \qquad (4.46)$$

One has now to normalize the fields and proceed as before. In the large N limit, we now find [88]

$$V^{(1)\,T}_{eff} = -\frac{\pi^2}{24} N\, T^4 + \frac{T^2}{18} N e^{2/3\, G + F + F^*} F_\alpha F^\alpha \qquad (4.47)$$

One has to remark the following. From (4.46), (4.47), it is evident that, in the large N limit, the global minima of the total potential are given by the SUSY condition $F_\alpha = 0$. However, as in the global SUSY theories, there are often degenerate minima satisfying this condition, and then the non-leading terms in N could break this degeneracy.

4.3 Decay of the False Vacuum

According to the big bang model (see later), the universe started from a very high temperature and, then, cooled down up to the presently observed temperature of 3^oK. In our field theory language at finite temperature, this is equivalently described by saying that, initially $\overline{\phi} = 0$ is the ground state of the theory. Then, as the temperature decreases, at some critical temperature T_c the symmetric vacuum will cease to be stable (false vacuum) and a new energetically favoured ground state appears (true vacuum). At that point, quantum, thermal and gravitational fluctuations will tend to push the theory from the false to the true vacuum. In discussing this transition, we have to evaluate the relevant quantity Γ/V, the decay rate per unit volume. During this phase transition, the universe will expand exponentially and supercool in the false vacuum for a period $(\Gamma/V)^{-1}$, until the phase transition complete itself.

First, let us consider the zero-temperature theory, taking into account only quantum effects. One treats quantum fluctuations in a semiclassical approximation, using functional integral methods. After tunnelling, the further evolution of the fields will be determined by solving the classical field equations. We will not give full details, since there are already excellent review articles [80]. We will simply state the final results. The decay rate per unit volume is

$$\frac{\Gamma}{V} = \Omega^4 e^{-A} \qquad (4.48)$$

where

$$A \equiv S_E(\overline{\phi}) = \int d^4x \, \{\frac{1}{2} \partial_\mu \overline{\phi}\, \partial^\mu \overline{\phi} + V_{eff}(\overline{\phi})\} \qquad (4.49)$$

is the Euclidean action and Ω^4 is a determinental factor with dimensions of $[\text{mass}]^4$. $\bar{\phi}$ is the O(4) symmetric solution (instanton) of the equations (Euclidean space)

$$\partial_\mu \partial_\mu \phi = \frac{\partial V_{eff}(\phi)}{\partial \phi} \qquad (4.50)$$

or equivalently

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = \frac{\partial V_{eff}(\phi)}{\partial \phi}, \quad \rho = \sqrt{x_\mu x_\mu} = \sqrt{|\vec{x}| + \tau^2} \qquad (4.51)$$

This O(4) symmetric solution has the lowest Euclidean action S_E [89] and, thus, gives the dominant contribution to (4.48). The initial conditions are

$$\lim_{\rho \to \infty} \bar{\phi}(\rho) = \phi_{\text{false vacuum}}$$

$$\left.\frac{d\bar{\phi}(\rho)}{d\rho}\right|_{\rho = 0} = 0 \qquad (4.52)$$

(4.51) is the classical equation of motion of a point particle in the potential $-V_{eff}$, subject to a time-dependent damping force $\frac{3}{\rho}\frac{d\phi}{d\rho}$ (ρ plays the role of time). In terms of ρ, the Euclidean action becomes

$$A \equiv S_E(\bar{\phi}) = 2\pi^2 \int_0^\infty \rho^3 d\rho \left[\frac{1}{2}\left|\frac{d\bar{\phi}}{d\rho}\right|^2 + V_{eff}(\bar{\phi})\right] \qquad (4.53)$$

The solution $\bar{\phi}$ must be non-trivial in the sense that we cannot use $\bar{\phi} = \phi_{\text{false vacuum}}$. If more than one such solutions exist, the leading contribution is from the one with the smallest action.

After tunnelling the evolution of the field is described by the classical equations of motion (Minkowski space now)

$$\partial_\mu \phi\, \partial^\mu \phi = -\frac{\partial V_{eff}(\phi)}{\partial \phi} \qquad (4.54)$$

or, equivalently ($\lambda = i\rho$),

$$\frac{d^2\phi}{d\lambda^2} + \frac{3}{\lambda}\frac{d\phi}{d\lambda} = -\frac{\partial V_{eff}(\phi)}{\partial \phi} \qquad (4.55)$$

which is the classical equation of motion for a particle in the potential $V_{eff}(\phi)$, starting at some $\phi = \phi^*$ with a time dependent damping force. The evolution of $\phi(t,\vec{x})$, for $t > 0$, describes the growth of a bubble of the true vacuum in a sea of false vacuum.

Now, for the finite-temperature theory, there are some modifications needed [90]. Quantum statistics at $T \neq 0$ is equivalent to the Euclidean quantum field theory, but with periodicity condition with period $1/T$ in the Euclidean time direction. Therefore, instead of the O(4) symmetric bubble of size $r(0)$ considered previously at zero-temperature, one has now bubbles with centers separated by $r(T) = 1/T$ in the Euclidean time direction. As the temperature increases and the boundary energies become more and more significant, at sufficiently high temperatures $T \gg r^{-1}(0)$ the minimal energy field configuration will be constant in Euclidean time and have an O(3) symmetry. As a result, the action A should be replaced by $\frac{S_3(\bar{\phi})}{T}$, where $S_3(\bar{\phi})$ is the three-dimensional bubble action. To compute $S_3(\bar{\phi})$, one should find the O(3) symmetric solution of

$$\partial_\mu \partial_\mu \phi = \frac{\partial V_{eff}(\phi,T)}{\partial \phi}, \quad \mu = 1,2,3 \qquad (4.56)$$

with boundary condition

$$\lim_{|\vec{x}|^2 \to \infty} \phi(|\vec{x}|^2) = \phi_{\text{false vacuum}} \qquad (4.57)$$

or of

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V_{eff}(\phi,T)}{\partial\phi}, \quad r = \sqrt{|\vec{x}|^2} \qquad (4.58)$$

The decay rate per unit volume is again given by (4.48), but now the action is

$$A = \int_0^{1/T} dt \int d^3x \, \{\tfrac{1}{2}\partial_\mu\bar{\phi}\,\partial_\mu\bar{\phi} + V_{eff}(\bar{\phi},T)\} \equiv$$

$$\equiv \frac{S_3(\bar{\phi})}{T} = \frac{4\pi}{T}\int_0^\infty dr\, r^2 \, \{\tfrac{1}{2}|\frac{d\phi}{dr}|^2 + V_{eff}(\bar{\phi},T)\} \qquad (4.59)$$

In a cosmological context, one has to compare the decay rate per unit volume Γ/V,(computed from (4.48) and (4.59)) with the corresponding natural quantity of cosmology (see next section), i.e. the expansion rate of the universe per unit volume, which from Einstein equations is given by

$$H^4 \equiv \left(\frac{\dot{R}}{R}\right)^4 = \left(\frac{8\pi}{3} G\rho\right)^2 \qquad (4.60)$$

where $R(t)$ is the scale factor of the universe, dot means differentiation with respect to time and $\rho = V(0)$ is the energy density of the symmetric vacuum. For the phase transition to occur, the condition

$$\Gamma/V \gtrsim H^4 \qquad (4.61)$$

must be satisfied.

Usually, it is impossible to obtain an analytic solution to Eqs.(4.58) and (4.59) and, also, to calculate the determinental factor Ω^4 analytically. One can obtain a so-

lution and the corresponding action $S_3(\bar{\phi})$ numerically, using computer calculations. Also, in most cases, it is sufficient to have a rough estimate of the order of magnitude of Ω^4.

Let us consider some cases, where the above considerations have been applied. For the Coleman-Weinberg SU(5) theory, we have seen that, for $T >> \bar{\phi}$,

$$V_{eff}^{(1)T}(\bar{\phi}) = B\bar{\phi}^4\left(\ln\frac{\bar{\phi}^2}{\sigma^2} - \frac{1}{2}\right) + C\bar{\phi}^2T^2 + \frac{1}{2}B\sigma^4 \qquad (4.62)$$

where $B = \frac{5625}{1024\pi^2} g^4$, $C = \frac{75}{16} g^2$ and the last term has been added to cancel the cosmological term, so that the true vaccum state $\bar{\phi} = \sigma$ have zero energy. As a result, the false vacuum state $\bar{\phi} = 0$ has a positive energy $1/2\, B\sigma^4 \equiv V(0)$. The condition (4.61) has been examined, in details, in the present case [91], with the conclusion that the phase transition, due to tunnelling, happens only at temperatures $T << \Lambda_{SU(5)}$, where, for the minimal SU(5), $\Lambda_{SU(5)} \sim 10^6$GeV is the SU(5) Λ parameter. However, already before $\Lambda_{SU(5)}$, the coupling constant becomes large, perturbation theory breaks down and the above analysis no longer applies. In fact [92], one can argue that, due to non-perturbative effects, like condensations of the type $\langle F_{\mu\nu}F^{\mu\nu}\rangle$ or $\langle\bar{\psi}\psi\rangle$ the phase transition will take place before Λ. Also, from another direction [93], taking into account the temperature dependence of the coupling constants of the SU(5) potential, one finds that the phase transition will take place before Λ, when

$$\mu^2(T) = \left.\frac{d^2V_{eff}(\phi,T)}{d\phi^2}\right|_{\phi=0} < 0.$$

For the SUSY SU(5) theory, the superpotential is given by (3.17) and there exists the degenerate vacuum (3.19). In supergravity, these degenerate states get split, but the splitting is very small compared to M^4. Let us now add the temperature effects [94]. Concentrating only on the adjoint Higgs field Σ, the quadratic temperature-dependent term in the potential is

$$V^T = \frac{1}{20}(21\lambda^2 + 25g^2)\, T^2\, \mathrm{Tr}\,\Sigma^2 \tag{4.63}$$

These finite temperature effects produce an additional splitting in the various degenerate states. However, (4.63) is minimized by $\langle\Sigma\rangle = 0$, so the SU(5) phase is preferred at high temperatures $T \gg M$. At $T \sim M$, we cannot make a definite statement, but at low temperatures $T \ll M$, we can use the massless quantum gas approximation, in which the effective potential (equal to the free energy) is given by

$$V = -\frac{\pi^2}{90}\left(N_b + \frac{7}{8} N_f\right)T^4 + \text{F-terms} + \text{D-terms} \tag{4.64}$$

Since the SU(5) phase has the most unbroken generators, because of the first term in (4.64), the SU(5) phase has the lowest energy and so is again preferred for $T \ll M$. However, due to strong coupling effects [94], one would expect a succession of phase transitions $SU(5) \to SU(4) \times U(1) \to SU(3) \times SU(2) \times U(1)$. A closer inspection [95], however, shows that this transition cannot occur: Condition (4.61) is far from satisfied and, so, a rapid phase transition with many bubbles forming and percolating looks impossible in the present model. Supergravity terms are of order $(Mm_{3/2})^2$ and cannot reverse the situation. The only way out is to suppress the barrier between the phases and

this can only be achieved [96] by introducing very small coupling parameters $\sim 10^{-12}$, a severe fine-tuning.

Finally, another important factor in discussing phase transitions in cosmology is the curvature of spacetime. One considers the action [97,98]

$$S = \int d^4x\sqrt{-g}\,\left[\frac{1}{2} g^{\mu\nu}\, \partial_\mu\phi\, \partial^\nu\phi - V(\phi) - \frac{1}{16\pi G} R\right] \tag{4.65}$$

and looks for minimal action stationary points of its analytic continuation to Euclidean space. Under the reasonable assumption that the minimal solutions will have O(4) symmetry (at zero temperature), the Euclidean equations of motion are

$$\phi'' + 3\frac{\rho'}{\rho}\phi' = \frac{dV}{d\phi} \tag{4.66}$$

and

$$\rho' = 1 + \frac{8\pi G}{3}\,\rho^2\left(\frac{1}{2}\phi'^2 - V\right) \tag{4.67}$$

where the length element is $(ds)^2 = (d\xi)^2 + \rho(\xi)^2(d\Omega)^2$, and prime denotes $d/d\xi$. Then, one can construct [97] an explicit solution, in the thin-wall approximation (small energy difference between the vacuum), and show that, in the case in which a state with positive cosmological constant decays into Minkowski space-time, gravity makes vacuum decay more likely (the bubble action decreases and the radius of the bubble at its moment of materialization becomes smaller), compared to the flat space-time result. Note, however, that in decays from a false vacuum with $V(\text{false vacuum}) \leq 0$, things are just the other way around and gravity can stabilize a metastable scalar field configuration [75, 97, 99] (some other recent work on phase transitions in de Sitter or anti-de Sitter space, can be found in Ref. 100).

For sufficiently large curvature, it has been pointed out [101] that the radius of the bubble solution would exceed the de Sitter radius H^{-1}. In this case, the only Euclidean solution of the instanton equations (apart from $\phi = 0$) is the homogeneous solutions $\phi = \phi_1$, where ϕ_1 is the local maximum of the potential. This is interpreted as homogeneous tunnelling of a horizon volume of space from $\phi = 0$ to $\phi = \phi_1$, with tunnelling probability $P \propto e^{-B}$, where $B = \frac{M_p^4}{8}\left(\frac{1}{V(\phi=0)} - \frac{1}{V(\phi=\phi_1)}\right)$. ϕ_1 is unstable and, so, after tunnelling, the field ϕ will classically move towards the global minimum.

V. STANDARD COSMOLOGY

The expansion of the universe can be described by a simple Newtonian argument. Consider a mass m at radius r of a homogeneous and isotropic sphere. Now expand all length scales by a factor $R(t)$. The initial radius vector r_0 will transform to r, where

$$r = R(t) r_0 \tag{5.1}$$

Hence, the velocity of the expansion is

$$\upsilon = \dot{r} = \frac{\dot{R}}{R} r \equiv Hr \tag{5.2}$$

where $H \equiv \dot{R}/R$ is called the Hubble parameter and expresses the expansion rate of the sphere (dot, as usual, means differentiation with respect to time). If the spherical ball represents the universe, then the present value of the expansion rate H_0 is called Hubble's constant. We can write down an energy conservation equation for the motion

of m ($E_{kin} + E_{pot}$ = const.). This yields, if ρ is the average density of the ball of matter,

$$\frac{\dot{R}^2}{2} - \frac{4\pi G}{3} \rho R^2 = \text{const.} \equiv -\frac{k}{2} \tag{5.3}$$

This is just Friedman's equation for the expansion of a homogeneous and isotropically expanding universe, filled with pressure free material, which of course can be obtained rigorously from Einstein's equations, as we will see now.

At the heart of the standard cosmology of our expanding universe [12, 102], there is the cosmological principle: The Universe is, and always was, homogeneous and isotropic. On sufficiently small scales, of course, the universe deviates from isotropy and homogeneity. It is assumed that these smaller scale, more local departures may be treated as perturbations on the homogeneous isotropic cosmological model, in the early universe.

The cosmological principle implies the Robertson-Walker (RW) metric

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right] \tag{5.4}$$

where r, θ, ϕ are comoving coordinates and $k = 1, -1, 0$ corresponds to a closed, open or flat universe. The energy momentum tensor for a perfect fluid applies here

$$T_{\mu\nu}^{M} = \text{diag}(\rho, p, p, p) \tag{5.5}$$

where ρ is the energy density and p is the pressure. Then, (5.4) and (5.5), plagued into Einstein's equations

$$R_{\mu\nu} = -8\pi G \left(T_{\mu\nu}^{M} - \frac{1}{2} g_{\mu\nu} T_\lambda^\lambda\right) \tag{5.6}$$

give for the time-time component

$$3\ddot{R} = -4\pi G(\rho + 3p)R \qquad (5.7)$$

and for the space-space component

$$R\ddot{R} + 2\dot{R}^2 + 2k = 4\pi G(\rho - p)R^2 \qquad (5.8)$$

By eliminating $\ddot{R}$ from Eqs.(5.7) and (5.8), we get the first order differential Friedman's equation

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \qquad (5.9)$$

The expansion rate $H \equiv \dot{R}/R$ sets the characteristic time for the growth of $R(t)$. Presently $H_0 = 100\ h_0$ km $s^{-1}Mpc^{-1}$ $= h_0(10^{10}yr)^{-1}$, where observationally $\frac{1}{2} \leq h_0 \leq 1$ [102a].

In general, the ground state will contribute a non-zero vacuum energy density

$$T_{\mu\nu}^{\ vac} = <0|\ T_{\mu\nu}^{\ op}\ |0> = \rho_0\ g_{\mu\nu} \qquad (5.10)$$

And then Eq.(5.9) is written as

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \qquad (5.11)$$

where $\Lambda \equiv 8\pi G\ \rho_0$ is the so-called cosmological constant. Today, this is extremely small: $\Lambda < 10^{-82}$ GeV^2 $\sim 10^{-120} M_p^{\ 2}$ and there still remains a big mystery why.

In addition to Eq.(5.9) we have the continuity equation (conservation of energy)

$$d(\rho\ R^3) = -p\ d\ R^3 \qquad (5.12)$$

or

$$\dot{\rho} = -3(\rho + p)\ \frac{\dot{R}}{R} \qquad (5.13)$$

Since we have three functions of time: $R(t)$, $\rho(t)$ and $p(t)$, to close the set, we need a third equation. This is provided by the equation of state:

$$p = p(t) \qquad (5.14)$$

Two simple equations of state are of most interest:

(i) Ideal gas of non relativistic particles (i.e. $KT << mc^2$) or "dust", with $p = nKT$, $\rho = nm$ and $p << \rho c^2$, e.g. for galaxies $\upsilon \sim 250$ km/sec $<< c$, $p \sim \rho\upsilon^2 << \rho c^2$. Then, $p = 0$ is an excellent approximation and Eq.(5.13) gives

$$\rho_{NR} \propto R^{-3} \qquad (5.15)$$

(ii) Ideal gas of extremely relativistic particles (i.e. of massless particles $mc^2 << KT$), so

$$p = \frac{\rho}{3},\ \rho = \frac{\pi^2}{30} N(T)T^4 \qquad (5.16)$$

where $N(T) = N_b(T) + \frac{7}{8} N_f(T)$ is the effective number of bosonic and fermionic degrees of freedom at temperature T. Eqs.(5.13) and (5.16) then imply

$$\rho_R \propto R^{-4} \qquad (5.17)$$

In this case, the entropy density is given by

$$s = \frac{2\pi^2}{45} N(T)T^3 \qquad (5.18)$$

whereas the photon density is

$$\rho_\gamma = \frac{2\rho}{N(T)} = \frac{\pi^2}{15} T^4 \qquad (5.19)$$

Note that, from Eqs.(5.15) and (5.17), we have

$$\frac{\rho_{NR}}{\rho_R} \propto R \rightarrow 0 \qquad (5.20)$$

for the early universe. So, in the early universe, we are in the radiation dominated era and only later on (at $T \lesssim 1$ eV, $t \gtrsim 10^{13}$ sec), the universe enters the matter dominated era.

Another approximation, usually assumed, in the standard cosmology, is adiabatic expansion

$$\frac{d}{dt}(sR^3) = 0 \tag{5.21}$$

Provided $N(T)$ is constant, this implies

$$RT = \text{const.} \tag{5.22}$$

As we will see below, the assumption of adiabatic expansion is at the heart of the various cosmological problems of the standard cosmology and it is this assumption, which is abandoned for a brief period in the early history of the universe.

For the radiation dominated era, Eq.(5.9) yields

$$R(t) \propto t^{1/2} \tag{5.23}$$

Equivalently, if we write Eq.(5.9) as

$$\left(\frac{\dot{T}}{T}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \tag{5.24}$$

then, for the flat universe ($k = 0$), we can solve it explicitly using the initial condition $T(0) = \infty$;(*)

(*) $T(0) = \infty$ applies to an elementary particle picture for the early universe. On the contrary, in a composite particle picture, as it is suggested in some superunified theories, there exists a limiting temperature, of the order of the Planck temperature T_p [103a]. The same is true in superstring theories [103b]. Is really T_p an ultimate temperature of the universe or a critical temperature of some kind of a phase transition (like the confinement - deconfinement phase transition in QCD) ?

$$T^2(t) = \frac{3}{4\pi}\sqrt{\frac{5}{\pi G\, N(T)}}\;\frac{1}{t} \tag{5.25}$$

Numerically

$$T^2_{(GeV)} = 2.42 \times 10^{-6}\, N(T)^{-1/2}\, t^{-1}_{(sec)} \tag{5.26}$$

from which one can find the temperature T of the universe at some time t or vice-versa. E.g. for $T = 1$ MeV, with only photons, e^+e^- pairs and 3 left-handed neutrinos present, we find $t(1\text{ MeV}) \sim 1$ sec.

Later, during the matter dominated era, we have

$$\rho \propto T^3$$

$$R(t) \propto \begin{cases} t^{2/3} & \text{for flat universe } k=0 \\ t & \text{asymptotically for open universe } k=-1 \\ \text{cycloid} & \text{for closed universe } k=+1 \end{cases} \tag{5.27}$$

Finally, the so-called de Sitter phase of the universe is realized, for example, if in Eq.(5.11) $k = \Lambda = 0$ and $\rho = $ const. or alternatively if $\rho = k = 0$ and $\Lambda \neq 0$. The universe expands with a constant expansion rate H

$$R(t) \propto e^{Ht} \tag{5.28}$$

It is exactly after such an exponentially expanding (inflationary) period that the adiabaticity condition is violated, since at the end of it a large amount of entropy is produced, as we will see. After this brief background, let us now discuss the various cosmological problems, most of them inherent in the standard model.

1) Horizon problem

An important feature of standard cosmology is the existence of particle horizon [104], or causally connected distance at time t. The distance that a light signal could have propagated, since the big bang, is finite and given by

$$d_H(t) = R(t)\int_0^t \frac{dt'}{R(t')} = \frac{t}{1-n} \quad \text{for } R \propto t^n,\ n < 1 \tag{5.29}$$

In Friedman's cosmology

$$d_H(t) = \begin{cases} 2t & \text{in radiation dominated era} \\ 3t & \text{in matter dominated era} \end{cases} \tag{5.30}$$

The present visible universe, which is our present horizon distance, is

$$d_H(t_0) = R(t_0)\int_0^{t_0} \frac{dt'}{R(t')} \sim \frac{1}{H_0} \sim 10^{28}\ \text{cm} \tag{5.31}$$

The essence of the horizon problem lies in the fact that, whereas the "size" or the "scale factor" of the universe increases as $R \propto t^n$, with $n < 1$, in Friedman's cosmology, the horizon distance $d_H(t)$(Eq.(5.29)), increases linearly with time. Thus, when these two distances, which coincide today, are traced back in time, the horizon distance is always less than the "size" of the universe: For example, at decoupling time $t_{dec} \sim 10^{13}$ sec,

$$d_H(t_{dec}) = 2t_{dec} \sim 10^{23}\text{cm} \tag{5.32}$$

whereas the "size" of the universe at that time is (from the adiabaticity condition RT = const.)

$$\frac{R(t_{dec})}{R(t_0)} \times 10^{28}\text{cm} = \frac{T(t_0)}{T(t_{dec})} \times 10^{28}\text{cm} = \tag{5.33}$$

$$= \frac{3^0\text{k}}{3000^0\text{k}} \times 10^{28}\text{cm} = 10^{25}\text{cm}$$

which is two orders of magnitude bigger than (5.32). This means that the present universe would consist of $(10^2)^3 = 10^6$ causally disconnected regions in volume. At temperatures appropriate for grand unification, e.g. $T \sim 10^{16-17}$GeV, the ratio of the horizon distance to the "size" of the universe is $\sim O(10^{-28})$ and the problem becomes even more dramatic. We cannot understand how the present large scale homogeneity/isotropy has been emerged.

2) Flatness problem

The quantity

$$\Omega = \rho/\rho_c \tag{5.34}$$

measures the ratio of the energy density of the universe to the critical density

$$\rho_c \equiv \frac{3H^2}{8\pi G} \tag{5.35}$$

the largest density the universe can possess to expand for all future time. In general, Ω varies with time, but for $k = 0$ it is 1 for any time. There is a connection of Ω and another parameter, the deceleration parameter

$$q(t) = -\frac{\ddot{R}}{RH^2} \tag{5.36}$$

From Eqs.(5.7) and (5.9) we get

$$q = \frac{1}{2}\Omega\,(1 + 3\,\frac{p}{\rho}) \tag{5.37}$$

In the radiation dominated era, $p = \frac{\rho}{3}$ and $q = \Omega$, whereas in the present matter dominated epoch, $p \ll \rho$ and $q_0 = \Omega_0/2$. From $H_0 = 100\ h_0$ km sec^{-1} Mpc^{-1}, the present critical density is $\rho_{0cr} = 2 \times 10^{-29}\ h_0^2$ gr cm^{-3}, whereas the present matter density is written as $\rho_0 = 2 \times 10^{-29}\ \Omega_0 h_0^2$ gr cm^{-3}.

Although q_0 and Ω_0 are not known with great precision, we know that $0.05 < q_0 < 2$ and $0.1 < \Omega_0 < 4$.

From Eq. (5.9), we can write

$$\Omega(t) = \frac{1}{1-\varepsilon(t)}$$

$$\varepsilon(t) = \frac{k}{R^2}\frac{1}{8\pi G\rho/3} \tag{5.38}$$

For the epoch of nucleosynthesis, when $t_N = 10^{-2}$s and $T_N \simeq 10$MeV, $\varepsilon(t_N) = 10^{-16}$ and $\Omega(t_N) = 1 \pm O(10^{-16})$. At the GUT epoch, with $T \sim 10^{16-17}$GeV, $\varepsilon(t_{GUT}) = O(10^{-55})$ and $\Omega(t_{GUT}) = 1 \pm O(10^{-55})$. This means that, very early in the universe, the ratio of the curvature term k/R^2 to the density term $8\pi G\rho/3$ was extremely small, i.e. the expansion of the universe proceeded at the critical rate $H^2_{cr} = 8\pi G\rho/3$, to a very high degree of precision. So, our universe is today (and has been in the past) closely described by the $k = 0$ flat model ($\Omega = 1$ is a critical stable point). The smallness of the ratio ε requires an "initial condition" and constitutes the flatness problem [105, 106].

3) Small-scale inhomogeneity (or origin of density perturbations)

On small scales (< 100 Mpc), the Universe is very clumsy. Today, $\delta\rho/\rho \sim 10^5$ on the scale of a galaxy ($\simeq 10^{12}$ $M_\odot$). However, to create galaxies, it is necessary to assume density perturbations of amplitude $\delta\rho/\rho \sim 10^{-4}$-$10^{-5}$ or so, on the scale of a galaxy, in the early universe. On the other hand, the uniformity of the microwave background radiation on very small angular angles ($<<1^\circ$) indicates that the universe was smooth, even on galactic scales, in the early stages. In the standard cosmology, the initial seed for $\delta\rho/\rho$, was, in general, put in by hand.

4) Baryon asymmetry

The baryon asymmetry problem is to understand why, in the observable universe, the density of baryons is many orders greater than that of antibaryons and, also, why the density of baryons is much less than the density of photons $n_\beta/n_\gamma \sim 10^{-9}$. Again, in the standard cosmology, this value is put in by hand.

The above problems are problems associated with the standard cosmology. However, there exist some others, which are created when we put the unified theories in a cosmological context.

5) Magnetic monopole problem

When there occurs a phase transition, associated with the breaking of a group $G \rightarrow G' \times U(1)$, topologically stable magnetic monopoles are produced [107]. Their mass is

$$m_M \sim \frac{M_G}{\alpha_G} \tag{5.39}$$

and their number density is roughly

$$n_M \sim \frac{1}{\xi^3} \tag{5.40}$$

where ξ is the correlation length of the Higgs field. This cannot be greater than the causal horizon $d_H(t) = 2t$ (in a radiation dominated era). So, at the GUT time, it is

$$n_M \gtrsim \frac{1}{d_H^{\,3}} \sim \frac{1}{8t_G^{\,3}} \tag{5.41}$$

Since annihilation of such monopoles is rather negligible since t_G [108], at present their matter density is estimated

$$\rho_M = m_M n_M \sim 10^{15}\rho_c \; ! \tag{5.42}$$

where $\rho_c \sim 10^{-29}$ gr cm^{-3} is the critical density today. Under such circumstances the universe could not have survived until now. This primordial monopole problem is very acute, since it is present in practically all GUTS.

6) Domain wall problem

Domain walls, which are produced when discrete symmetries are broken, give rise to a somehow similar problem, since their surface density is so large that the observable part of the universe would be largely anisotropic [109]. For example, such domain wall problem arises in theories with the simplest SU(5) potential, in theories with spontaneous CP-violation, in axion models etc. and there must be a mechanism to avoid the undesirable effects.

7) The gravitino problem

In the early universe, gravitinos with mass $m_{3/2} \sim O(M_W)$ decay after the process of nucleosynthesis, which invalidates the successful prediction of light elements abundances [110]. Such undesirable effects from gravitino decays can be avoided only if their relative abundance was reduced to very small amounts [111a].

8) The Polonyi problem

The Polonyi problem [111b] is due to the fact that the Polonyi field is only coupled gravitationally to itself or to matter, so it takes long to reach its minimum and release the energy stored in it. As a result, the universe undergoes a late period of reheating and any baryon-to-photon ration is diluted by unacceptable amounts.

The interesting thing about the inflationary universe scenario, to be described in the next section, is that, in principle, it can solve all the above problems. To be true the solution to the baryon asymmetry problem was first suggested many years ago [112] and it has found its natural

realization inside GUTS[113]. The inflationary universe scenario simply makes the baryogenesis mechanism more effective [114]. Starting with a baryon symmetric universe, there must be three ingredients, necessary for baryosynthesis, all of which are present in a GUT: (i) B-nonconserving interactions (see Eq.(2.30)); (ii) violation of both C and CP, necessary to produce excess of matter over antimatter, and also present in GUTS; and (iii) departure from thermal equilibrium, something which naturally happens in our expanding universe. The standard scenario for baryogenesis is the out-of-equilibrium decays of superheavy (mainly Higgs) bosons (or even fermions), whose interactions violate B conservation.

To terminate this section we have to add two more cosmological problems which have not yet found a satisfactory answer, although some interesting speculations have been put forward for their solution. These are: (i) the singularity problem: the scale factor $R(t)$ vanishes at $t \to 0$ whereas the energy density becomes infinite [115] (interesting speculation: creation of universe from "nothing"[116]). (ii) the vacuum energy (cosmological constant) problem: Associated with every phase transition (GUT, Weinberg-Salam, QCD etc.), there is a reduction to the vacuum energy or, equivalently, to the cosmological constant Λ by many orders of magnitude ($\sim 10^{26}$ GeV^2, $\sim 10^{-30} GeV$, $\sim 10^{-40} GeV^2$ etc. respectively) and, today, we end up with $\Lambda < 10^{-120} M_p^2 \sim 10^{-82} GeV^2$. Such a series of cancellations, up to zero, with an accuracy of $10^{-82} GeV^2$, remains a big puzzle (there are many interesting speculations [117], but none really solves the problem. Some kind of symmetry must certainly be involved here [117a]).

VI. INFLATIONARY UNIVERSE SCENARIO

The inflationary model [118] is based on the idea that there was an epoch during which the vacuum energy density ρ_{vac}, associated with the GUT phase transition, dominated the total energy density ρ of the universe. Before and after this epoch, the radiation energy density $\rho_{rad} = \frac{\pi^2}{30} N(T)T^4 \propto R^{-4}$ is dominant, and the universe is in the Friedman phase. However, during this short epoch of the vacuum energy dominance, we have $\rho = \rho_{rad} + \rho_{vac}$, with $\rho_{rad} \ll \rho_{vac}$ = const., and $\rho \approx \rho_{vac}$ = const. Hence

$$H = \dot{R}/R = \sqrt{\frac{8\pi G}{3}\rho} \qquad (6.1)$$

and this yields the exponential expansion

$$R(t) \sim e^{Ht} \qquad (6.2)$$

During this period, the universe is in the de Sitter phase and, from Eq.(5.13), since $\dot{\rho} = 0$, we have

$$p = -\rho < 0 \qquad (6.3)$$

So, the pressure is negative and it is this which makes the universe expand exponentially.

As we have seen in the study of the GUT phase transitions, at very high temperatures $\phi = 0$ is the absolute minimum of the potential, whereas as temperature decreases, there appears a second local minimum $\phi = \sigma$. Until $T \gtrsim T_c$, the universe is trapped in the $\phi = 0$ vacuum and is in the Friedman phase. For $T < T_c$, the $\phi = \sigma$ minimum is lower than the $\phi = 0$ one, the non-zero vacuum energy ρ_{vac} appears and dominates over ρ_{rad}. This is the beginning of the inflationary period.

However, to assume that inflation takes place as the universe supercools in the false vacuum [106] does not work. The bubbles with the true vacuum inside do not collide fast enough to coalesce and make an homogeneous universe [119]. Simply speaking, one cannot inflate the false vacuum. The phase transition we want is not of the type with tunnelling from a false to true vacuum and bubble formation. Instead, one has to allow first the phase transition from the false to the true vacuum and then inflate the true vacuum [120, 121]. This becomes possible if the potential has a large flat region near $\phi = 0$. In this scenario, the phase transition is of the so-called "slow-rollover" type. During the "slow-rollover" transition, a single bubble (or fluctuation region) with the true vacuum inside, undergoes inflation and grows to a region which includes our visible universe. This is possible, since as the temperature decreases, the barrier between false and true vacuum becomes small enough and, then, due to quantum and/or thermal fluctuations, it is enough for one such bubble to be kicked out of the barrier to the "slow-rollover" region and be inflated there.

We distinguish three stages during the inflationary process:

1) The tunnelling

Due to thermal and/or quantum tunnelling, ϕ is taken across the barrier (remember that we need just one bubble with the true vacuum inside to explain our visible universe). The dynamics of the above process determine when and how the process occurs and, also, the value of ϕ after the barrier is penetrated. For definiteness, suppose that the barrier is overcome when the temperature is T_0 and the value of ϕ is $\phi_0 \ll \sigma$. From this point, the "slow-rollover" journey to the true vacuum $\phi = \sigma$ starts. This brings us to the second stage of the scenario, i.e.

2) Inflation or "slow-rollover" period

During this stage, the evolution of ϕ is adequately described by the semi-classical equation of motion. The expansion rate H is, in general, determined by the energy density of the universe

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \qquad (6.4)$$

with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_{rad} \qquad (6.5)$$

where ρ_{rad} represents the energy density in radiation, produced by the time variation of ϕ. For $T_0 << T_c$, the original thermal component makes a negligible contribution to ρ. Then, Eq.(5.13) can be split into two equations [122]: the semi-classical equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} = -\frac{\partial V(\phi)}{\partial\phi} \qquad (6.6)$$

and the evolution of ρ_{rad}

$$\dot{\rho}_{rad} + 4H\rho_{rad} = \Gamma\dot{\phi}^2 \qquad (6.7)$$

(ϕ is normalized so that its kinetic term is $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$). The $3H\dot{\phi}$ term acts like a frictional force and arises, because the expansion of the universe "red-shifts away" the kinetic energy of ϕ. The terms $\Gamma\dot{\phi}$ and $\Gamma\dot{\phi}^2$ account for particle creation, due to the time-variation of ϕ (they are not important during the "slow-rollover" period).

We assume ϕ to be spatially homogeneous. We take the size of the smooth small region (inside a bubble or fluctuation region) to be of the order of the "physical horizon" H^{-1} and follow the evolution of ϕ within it. For V

sufficiently flat between $\phi = \phi_0$ and $\phi = \sigma$, ϕ evolves very slowly in that region and the motion of ϕ is friction-dominated, so that Eq.(6.6) becomes

$$3H\dot{\phi} \simeq -\frac{\partial V}{\partial\phi} \qquad (6.8)$$

If one tries a solution $\phi \propto e^{\lambda t}$, ϕ takes off exponentially, after one e-fold. For V sufficiently flat, the "rollover" time t_r required for ϕ to transverse the flat region can be long compared to the expansion timescale H^{-1}. For definiteness, say $t_r \sim 60\,H^{-1}$.

During this period, $\rho = V(\phi) \simeq V(\phi = 0)$, since both $\frac{1}{2}\dot{\phi}^2$ and ρ_{rad} are $<<V(\phi)$. The expansion rate is then just

$$H \simeq \left(\frac{8\pi G}{3}V(0)\right)^{1/2} \simeq \frac{M_G^2}{M_P} \qquad (6.9)$$

where $V(0)$ is assumed to be of the order M_G^4. While H is constant, R grows exponentially

$$R(t) \sim e^{Ht} \qquad (6.10)$$

and for $t = t_r = 60\,H^{-1}$, R expands by a factor e^{60} during the "slow-rollover" period. As a result, the physical size of the smooth region increases to $e^{60}H^{-1} \sim 10^{28}H^{-1}$. This is sufficient to solve the horizon and flatness problems: Our observable universe today ($\sim 10^{28}$cm), at the grand unification temperature $T_G \sim 10^{16-17}$ GeV, had a physical size well within the physical size of a smooth region, which solves the horizon problem. On the other hand, the quantity $\varepsilon(t)$ of Eq.(5.38) becomes exponentially smaller after inflation $\varepsilon_{after} \simeq e^{-120}\varepsilon_{before}$. In fact, because of this exponential decrease of ε, the inflationary universe predicts that today Ω should be

$1(\pm\, O(10^{-\text{big number}}))$. Also, the magnetic monopole problem is obviously avoided, since Higgs fields are correlated and, therefore, approximately uniform, in a fluctuation region. For similar reasons, the domain wall problem is avoided.

3) Reheating

After inflation, ϕ will start to grow fast as the potential steepens to reach $\phi = \sigma$. Near $\phi = \sigma$, ϕ oscillates around the absolute minimum with frequency ω:

$$\omega^2 = V''(\sigma) = M_G^{\,2} >> H^2 = M_G^{\,4}/M_P^{\,2} \qquad (6.11)$$

As ϕ oscillates about $\phi = \sigma$, its motion is damped by, both, the expansion of the universe and particle creation. The Γ-dependent terms in Eqs.(6.6) and (6.7) are now important and account for particle creation, due to the time-variation of ϕ. $\Gamma(\phi)$ is determined by the particles with which ϕ is coupled and by the strength of that coupling. Near $\phi = \sigma$, Γ is the decay rate of the ϕ (Higgs) particle.

To be more precise [123], one needs a potential with a flat region, where $\ddot{\phi}$ term is negligible and the motion of ϕ is a friction dominated, i.e. $3H\dot{\phi} = -\frac{dV}{d\phi}$. This requires an interval, where

$$\frac{d^2V(\phi)}{d\phi^2} \lesssim 9H^2$$

$$\left| \frac{dV(\phi)}{d\phi} \Big/ V(\phi) \right| \lesssim (48\pi G)^{1/2} \qquad (6.12)$$

If $\phi_s \gtrsim \phi_0$, ϕ_e denote the starting and ending values of ϕ in this interval, we must have: $\phi_e - \phi_s \gtrsim O(10H)$, to insure that quantum fluctuations will not drive ϕ across

the flat region too quickly [123a]. This is because in de Sitter space, the scale of quantum fluctuations is set by H: $\Delta\phi \simeq H/2\pi$ ($H/2\pi$ is the Hawking temperature). Then, the time required for ϕ to transverse the flat region should be $\gtrsim 60H^{-1}$(to solve the horizon and flatness problems), that is we must have

$$\int H\, dt = -\int_{\phi_s}^{\phi_e} \frac{3H^2 d\phi}{dV/d\phi} \gtrsim 60 \qquad (6.13)$$

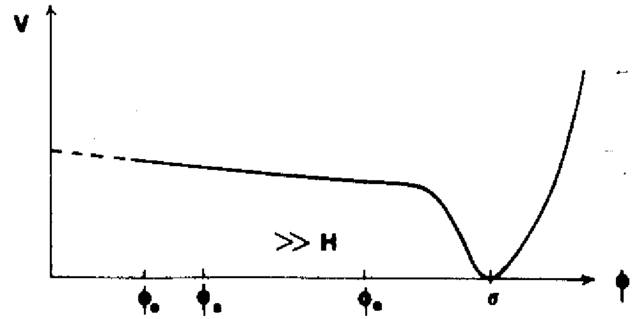

FIGURE The potential for a successful inflation

It is during this de Sitter phase that quantum fluctuations in ϕ produce density perturbations $\delta\rho/\rho$ necessary to explain the small-scale inhomogeneities. Quantum fluctuations in ϕ create density fluctuations $\delta\rho/\rho$, with different sizes (wavelengths) and amplitudes. Since this is a quantum effect, we expect that their sizes are much smaller than the horizon size (strictly speaking, the event horizon [104]) H^{-1}. As the universe expands, these also grow (their wavelength gets red-shifted). Since the event horison, within which we have causal microphysics (quantum fluctuations in ϕ) remains the same during inflation, the fluctuation size will reach H^{-1} and cross it. To an observer inside horizon, the fluctuation will seem to disappear. At that time, microphysics freezes out and, also, the fluctuation $\delta\rho/\rho$ freezes out, with an amplitude which can be calculated. After inflation, during the subsequent Friedman-Robertson-Walker (FRW) phase, the horizon distance grows faster than the fluctuation size and, eventually, there will come a time, when the fluctuation reenters the horizon. The observer will then rediscover the fluctuation, but now with perhaps a galactic size. This is how inflation provides the seeds for the formation of the large-scale structure in our universe. The theory for calculating the amplitude $\delta\rho/\rho$, when it freezes out, is rather involved (Refs. 80 and 124). The final result is that the amplitude of density perturbations, on a wavelength ℓ, is

$$\left.\frac{\delta\rho}{\rho}\right|_{\ell} \simeq \left(4 \text{ or } \frac{2}{5}\right) \frac{H\Delta\phi}{\dot\phi(t_f)} \qquad (6.14)$$

where $4(\frac{2}{5})$ applies if the scale in question ℓ reenters the horizon when the universe is radiation (matter) dominated; H is the value of the Hubble parameter during inflation; $\Delta\phi \simeq H/2\pi$ is the fluctuations in ϕ; and $\dot\phi(t_f)$ is the value of $\dot\phi$, evaluated at the time of freeze-out, when the fluctuation relevant for the scale ℓ leaves the horizon. The important thing is that this is not only an almost scale-invariant Harrison-Zeldovich spectrum [125], but, it also enables us to calculate the spectrum of density perturbations, from first principles, in terms of the parameters of an underlying field theory. To achieve an acceptable amplitude for density perturbations, we must have

$$\delta\rho/\rho \sim 10^{-4} - 10^{-5} \qquad (6.15)$$

Note that this limit also comes from the fact that, on angular scales $\gg 1^\circ$, the temperature fluctuations $\Delta T/T \simeq \frac{1}{2}\,\delta\rho/\rho$ must be $\leq 10^{-4} - 10^{-5}$ to be consistent with the observed isotropy of the background radiation [126]. Constraint (6.15) turned out to be the most stringent one on inflationary models [124].

The reheating scenario depends on whether $\Gamma > H$ or $\Gamma < H$. If $\Gamma > H$, the coherent field energy density $(\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi))$ is efficiently converted into radiation (good reheating). The reheating temperature is

$$T_{RH} \simeq \left[\frac{30\rho_\phi}{\pi^2 N(T)}\right]^{1/4} = \left[\frac{45}{4\pi^3 N(T)}\right]^{1/4} (H\,M_P)^{1/2} \sim M_G \qquad (6.16)$$

In this case, baryosynthesis occurs via the standard out-of-equilibrium decay mechanism [113, 114]. If, on the other hand, $\Gamma < H$, ρ_ϕ will not immediately convert into radiation, but ϕ, as it continues to oscillate, redshifts away with the expansion. The universe then consists of a very cold gas of ϕ particles. Eventually, when all ϕ particles decay, the reheating temperature T'_{RH} is smaller than previously, by a factor $(\Gamma/H)^{1/2}$:

$$T'_{RH} \simeq \left[\frac{30}{\pi^2 N(T)}\right]^{1/4} (\Gamma\, M_P)^{1/2} \simeq (\Gamma/H)^{1/2} M_G \qquad (6.17)$$

Baryosynthesis should occur now directly via very out-of-equilibrium decays of cold ϕ particles [127]. If the mass of the Higgs boson, responsible for generating the baryon asymmetry, is M_H, then the net baryon number produced is

$$n_B/n_\gamma \simeq \left(\frac{T'_{RH}}{M_H}\right) \Delta B \qquad (6.18)$$

where ΔB is the baryon number produced by the H, H^* decays.

Usually, there is another constraint, which must be also satisfied for a successful inflation, namely the thermal constraint [128]: the high temperature potential V_T must have a minimum at $\phi = 0$ or, more precisely, at a point where the zero temperature potential is flat (first and second derivatives of V vanish). This is not satisfied in minimal supergravity along with the other constraints [129], but it is possible to find non-minimal extensions, which satisfy all the constraints [130].

The last, but by no means least, requirement for a successful inflation must be the potential to come from a successful particle physics model. In fact, this must be a guide in the search of inflationary models. We will not give full details of the various models considered in turn, in the search for successful inflation. Initially, the Coleman-Weinberg potential was considered as a good candidate, but, above all, the magnitude of the density perturbations produced was uncomfortably large [124]. Supersymmetric models, based on the inverse hierarchy, also were unsuccessful [131]. However, supersymmetric models, in general, appeared to offer the best hopes to realize inflation [132]. Note also that, in order to avoid radiative corrections to spoil the flatness of the potential, it is almost essential that the field ϕ responsible for inflation, be a singlet field [133]. Since such gauge singlet fields are present in $N = 1$ supergravity theories, supergravity models have been considered extensively [134]. In particular, inflation has been implemented in the maximally symmetric $SU(N,1)$ supergravity models [135]. No model, however, seems to have managed to implement inflation in an unambiguous and natural way.

Moreover, the problem of initial conditions [136] seems still controversial. In particular, whether or not $\phi = 0$ is chosen as the initial condition has been questioned [137]. However, in "primordial" inflation models, where $\langle\phi\rangle \sim M_P$, proper initial conditions are obtained [138].

One scenario, which deserves particular attention, is the chaotic inflation [139], which seem to be realized in a wide class of models. The scenario is based on chaotic initial distribution of the scalar field ϕ. Domains of the universe with $|\phi| \gtrsim O(1)\ M_P$ can go through an inflationary period, as ϕ slowly rolls to the minimum of its potential $V(\phi)$. In this type of inflation, the expansion of the universe at the inflationary stage is quasi-exponential, since H slowly changes during inflation. In the models of chaotic inflation, the assumption is made that in the Lagrangian the kinetic energy terms do not dominate over the potential energy, which means that ϕ must be uniform on scales larger than the horizon, which is perhaps unreasonable. The chaotic inflationary scenario has been also considered under the combined action of a scalar field and gravitational vacuum polarization [140].

Before concluding this section, we recall that the inflationary universe predicts that today $\Omega_0 = 1$, to a high accuracy. However, luminous matter in galaxies accounts for only $\Omega_{0,lum} \sim 0.01$, whereas baryons seem to contribute $\Omega_{0,\ baryon} \lesssim 0.1$. There can be non-baryonic dark matter (e.g. white dwarfs, Jupiters, etc.). But certainly, to cover the gap between 0.1 and 1, as required by inflation, we need particles with only gravitational effects at present. This is the dark matter problem [141a]. There are many possibilities, like massive neutrinos, photinos, axions [141b] or quark nuggets [141c].

VII. KALUZA-KLEIN THEORIES AND COSMOLOGY

In recent years, there has been a considerable interest in the revival of the old Kaluza-Klein (K-K) proposal [142], namely that all fundamental interactions, today described by theories with local invariance under some given symmetries, could be (geometrically) unified in a higher dimensional theory of gravity, based on local coordinate and Lorentz invariance. Given that modern physics is based on two different concepts: local coordinate invariance, which leads to general relativity, and local gauge invariance, which leads to gauge theories, what the K-K approach actually does is a unification of these two basic principles into one: local coordinate invariance in more than four dimensions. Gauge invariance is then not an independent principle, but it can be deduced from the general covariance in higher dimensions. Through some compactification mechanism, the extra dimensions are curled up into a compact isometric space of presently unobservable dimensions, whereas, in the four dimensional space-time, there are left the usual gravitational and gauge interactions, as an effective low energy approximation of the fundamental theory [143].

The idea is certainly one of the most revolutionaries introduced into physics and there exists a widespread feeling among physicists that it must be relevant, at some stage, in Nature.

Still in the higher dimensional perspective, but in a different context, the recent emergence of superstring theories [32,33] has created a lot of excitement. The cancel-

lation [32] of all anomalies (including gravitational ones), as well as of infinities (up to one-loop level, this has been verified [32]), has led to hopes that one can have a consistent quantum theory of gravity, along with a phenomenologically acceptable particle physics. Various investigations [34,79] support these expectations (in particular, one can get the chiral quantum numbers of quarks and leptons, thus solving the chirality problem of conventional K-K theories). These theories are naturally formulated in ten dimensions and, upon compactification, one can get a manifold $M^4 \times N^6$, where M^4 is the flat Minkowski space-time and N^6 a compact six-dimensional space.

In this section, however, we will only discuss cosmology of a conventional K-K theory, in the case where the effective potential V_{eff} depends only on the scale b of the compactified internal space, i.e. $V_{eff} = V_{eff}(b)$. This includes a wide class of models, for example compactification by (one-loop) quantum fluctuations due to matter fields (Refs. 144, 145) (in analogy with the Casimir effect in QED) or by a vacuum expectation (vev) of a background matter field present in the theory [146, 147]. In a more general case, one would have a dependence of V_{eff} on another scalar field and/or higher derivative terms, like R^2, $R_{MN}R^{MN}$, $R_{MNPQ}R^{MNPQ}$, as in superstring theories. These more complicated cases remain to be fully investigated.

In K-K theories, the fundamental constants depend on the internal radius b. So, Newton's gravitational constant G_N and gauge coupling constant g are given by [148]

$$G_N \sim \frac{\bar{G}}{b^D} \quad , \quad g \sim \frac{G_N^{1/2}}{b} \qquad (7.1)$$

where $\overline{G}$ is the gravitational constant in the higher $N = (4 + D)$-dimensional Lagrangian. Any change in volume of the internal space will generate a change of the physical constants, e.g. G_N and α_{em}. There are a variety of empirical results [149], which suggest that G_N and α_{em} have remained unchanged over cosmological time

$$\dot{G}_N/G_N < 10^{-17}\ \mathrm{yr}^{-1}$$
$$\dot{\alpha}_{em}/\alpha_{em} < 10^{-11}\ \mathrm{yr}^{-1} \tag{7.2}$$

Something in the theory must keep the extra dimensions static. In the class of models we will discuss, this is achieved by an appropriate fine-tuning of b, which will also give zero four-dimensional cosmological constant $\overset{(4)}{\Lambda}$. If, however, before the time b had reached its equilibrium value b_0, it was ever away from that, there could exist an effective four-dimensional cosmological constant, which would drive an exponential (inflationary) period for the four-dimensional world [150]. In fact, this could be a rather general phenomenon. After the inflationary period, b will finally do a rapid oscillatory approach towards its equilibrium value b_0 [151,152]. This is a damped oscillation in which the damping rate of the oscillation amplitude is determined by the expansion of $a(t)$ ($a(t)$ is now the scale factor of the four-dimensional world) and the oscillation frequency is of order b_0^{-1} (presumably $\sim$ Planck energy). This rapid oscillation can then produce energetic particles, which in a realistic model will include X,Y bosons, W, Z, photons, gluons, quarks, leptons and/or their SUSY partners. This is the analogue of the last stage (reheating) in the inflationary universe scenario.

In applying K-K theories to the early universe, one must provide solutions of the type $F^4 \times N^D$, where F^4 is now the four-dimensional Friedman universe, and, moreover, give a positive answer to the stability of such solutions.

In discussing cosmology in higher dimensions, we choose a generalized FRW ground state metric

$$\overline{g}_{MN} = \mathrm{diag}\,(-1,\ a^2(t)\hat{g}_{ij}(x),\ b^2(t)\tilde{g}_{mn}(y)) \tag{7.3}$$

where $x(y)$ are $3(D)$ spatial coordinates (we assume a D dimensional compact Einstein space). Then, the components $\overline{R}_{MN}$ of the time dependent Ricci tensor for $4 + D$ dimensions are

$$\overline{R}_{00} = 3a^{-1}\,\ddot{a} + D\,b^{-1}\,\ddot{b}$$
$$\overline{R}_{ij} = -(2k + a\ddot{a} + 2\dot{a}^2 + Dab^{-1}\,\dot{a}\,\dot{b})\,\hat{g}_{ij}$$
$$\overline{R}_{mn} = -(2\tilde{k} + b\ddot{b} + (D-1)\,\dot{b}^2 + 3ba^{-1}\,\dot{b}\,\dot{a})\,\tilde{g}_{mn} \tag{7.4}$$

If we define the energy momentum tensor as

$$\overline{T}_{MN} = \begin{pmatrix} \rho & & \\ & p_3\hat{g}_{ij} & \\ & & p_D\tilde{g}_{mn} \end{pmatrix},\quad T \equiv \overline{T}_M{}^M = -\rho + 3p_3 + Dp_D \tag{7.5}$$

the $(4 + D)$-dimensional Einstein equations

$$\overline{R}_{MN} - \frac{1}{2}(\overline{R} + \overline{\Lambda})\,\overline{g}_{MN} = -8\pi\overline{G}\,T_{MN} \tag{7.6}$$

give the dynamical equations

$$3\,\frac{\ddot{a}}{a} + D\,\frac{\ddot{b}}{b} = \frac{\overline{\Lambda}}{D+2} + 8\pi\overline{G}\left[\frac{-T}{D+2} - \rho\right] \tag{7.7}$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + D\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) + \frac{2k}{a^2} = \frac{\overline{\Lambda}}{D+2} + 8\pi\overline{G}\left[\frac{-T}{D+2} + p_3\right] \tag{7.8}$$

$$\frac{\ddot{b}}{b} + (D-1)\left(\frac{\dot{b}}{b}\right)^2 + 3\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) + \frac{2\tilde{k}}{b^2} = \frac{\bar{\Lambda}}{D+2} + 8\pi\bar{G}\left[\frac{-T}{D+2} + p_D\right] \tag{7.9}$$

From these, one can get the first-order Friedman equation

$$\frac{6k}{a^2} + \frac{2\tilde{k}D}{b^2} + 6\left(\frac{\dot{a}}{a}\right)^2 + D(D-1)\left(\frac{\dot{b}}{b}\right)^2 + 6D\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) = \bar{\Lambda} + 16\pi\bar{G}\rho \tag{7.10}$$

One must supplement these equations with the conservation equation for ρ, p_3, p_D

$$\frac{d}{dt}(\Omega_3\Omega_D\rho) = -p_3\Omega_D\frac{d}{dt}\Omega_3 - p_D\Omega_3\frac{d}{dt}\Omega_D \tag{7.11}$$

where Ω_3 (Ω_D) are the three (D)-dimensional volumes:

$$\Omega_3 = \int d^3x\,\sqrt{g_3(x)}\ ,\quad \Omega_D = \int d^Dy\sqrt{g_D(y)} \tag{7.12}$$

For the three (D)-dimensional sphere, they are: $\Omega_3 = 2\pi^3R^3$, $\Omega_D = \frac{2\pi^{\frac{D+1}{2}}}{\Gamma(\frac{D+1}{2})}\,b^D$.

Now, one has to determine $\bar{T}_{MN}$, i.e. ρ, p_3, p_D. Given a specific model, we have to calculate the free energy F, from which we deduce ρ, p_3, p_D. From the free energy, we define the energy U and the entropy S by

$$S = -\left.\frac{\partial F}{\partial T}\right|_{a,b}$$

$$U = F + TS = F - T\frac{\partial F}{\partial T} \tag{7.13}$$

Then

$$\rho = \frac{U}{\Omega_3\Omega_D}\ ,\quad p_3 = -\frac{1}{3}\frac{1}{\Omega_3\Omega_D}\,a\left.\frac{\partial U}{\partial a}\right|_{b,S}\ ,\quad p_D = -\frac{1}{D}\frac{1}{\Omega_3\Omega_D}\,b\left.\frac{\partial U}{\partial b}\right|_{a,S} \tag{7.14}$$

In general, F, ρ, p_3, p_D will depend on a, b, $\dot{a}$, $\dot{b}$, as well as on temperature T. As we said at the beginning, we are going to consider here dependence of the effective potential on b only (and, of course, T). This is equivalent to say that we take the 3-dimensional space to be flat compared to the D-dimensional one (and also ignore $\dot{a}$, $\dot{b}$ dependence). We can then write

$$F = \Omega_3\, f(b, T) \tag{7.15}$$

Initially, the (4 + D) dimensional universe was very hot: $T > \frac{1}{a}, \frac{1}{b}$. In that case, the free energy is the (4 + D) dimensional radiation

$$F \propto -\Omega_3\Omega_D\,T^{4+D} \tag{7.16}$$

Then, the entropy is

$$S \propto \Omega_3\Omega_D\,T^{3+D} \tag{7.17}$$

As a result, if the entropy S in a (3 + D)-dimensional comoving volume is constant, we have that, in this initial radiation dominated stage,

$$T \propto (a^3b^D)^{-1/3+D} \tag{7.18}$$

Then, solutions of Eqs. (7.8)-(7.10) are of the form

$$a,\ b \propto t^{2/4+D} \tag{7.19}$$

Although the entropy per (3+D)-dim comoving volume remains constant, the entropy in the 3-dim comoving volume can increase. This is achieved at the time of collapse of the extra dimensions (through the compactification mechanism), when the 3 dimensions expand while the D dimensions contract, and the total mean volume decreases. Then, the temperature increases and a large entropy ($\sim 10^{88}$) can be obtained and squeezed into the horizon 3-volume (equivalently, a can, at the same time, increase largely, by a factor at least $\sim 10^{28}$). This approach to inflation, which does not require a cosmological constant, has been studied by various groups [153, 154]. It seems to need a large number of extra dimensions ($D > 40$) to work. Certainly, one can increase the entropy in the horizon 3-volume this way, but it is not known if there exist a consistent physical model for the above approach to be implemented.

Next, we come to a later stage characterized by a temperature T in the range $\frac{1}{a} < T < \frac{1}{b}$. At that stage, we usually have a free energy with contributions [155] coming from the 4-dimensional radiation, through a finite temperature effect from various kinds of particles contained in the theory, as well as from effective background matter fields, as for example quantum fluctuations of matter fields [144, 145] and/or the vev of matter fields present in the theory [146, 147]. Then, we can write

$$F = \Omega_3\left(\frac{A}{b^n} - BT^4\right) \qquad (7.20)$$

where the first term represents the background matter fields contribution (e.g. $n = 4$ for quantum fluctuations [144], $n = 7$ for 11-dim SUGRA [146] or $n = 2$ for 6-dim Einstein-Maxwell theory [147]), and the second one represents the 4-dim thermal effects. Then we have

$$\begin{aligned} S &= 4\Omega_3 BT^4 \\ \rho &= \frac{1}{\Omega_D}\left(\frac{A}{b^n} + 3BT^4\right) \equiv \rho_m + \rho_{rad} \\ p_3 &= \frac{1}{\Omega_D}\left(-\frac{A}{b^n} + BT^4\right) \equiv p_{3m} + p_{rad} \\ p_D &= \frac{1}{\Omega_D}\left(\frac{n}{D}\,\frac{A}{b^n}\right) \equiv p_{Dm} \end{aligned} \qquad (7.21)$$

These are now the expressions, which have to be substituted in Eqs.(7.8)-(7.10).

In order to see if we can recover the Friedman universe i.e. a solution $F^4 \times N^D$, we set b = const. Then, Eqs.(7.8) to (7.10) give

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2} - 8\pi G_N \overset{(4)}{p}_{rad} = \frac{\bar{\Lambda}}{D+2} + 8\pi\bar{G}\left(-\frac{T_m}{D+2} + p_{3m}\right) \qquad (7.22)$$

$$0 = -\frac{2\tilde{k}}{b^2} + \frac{\bar{\Lambda}}{D+2} + 8\pi\bar{G}\left(-\frac{T_m}{D+2} + p_{Dm}\right) \qquad (7.23)$$

$$\frac{6k}{a^2} + 6\left(\frac{\dot{a}}{a}\right)^2 - 16\pi G_N \overset{(4)}{\rho}_{rad} = -\frac{2\tilde{k}D}{b^2} + \bar{\Lambda} + 16\pi\bar{G}\rho_m \qquad (7.24)$$

where we have defined Newton's constant by $G_N = \frac{\bar{G}}{\Omega_D}$ and the 4-dim. radiation energy and pressure by $\overset{(4)}{\rho}_{rad} = \Omega_D \rho_{rad}$, $\overset{(4)}{p}_{rad} = \Omega_D p_{rad}$.

Now, we immediately recognize from Eq.(7.24)(compare with Eq.(5.11))that the effective four-dim. cosmological constant is

$$\overset{(4)}{\Lambda} = -\frac{\tilde{k}D}{b^2} + \frac{\bar{\Lambda}}{2} + 8\pi\bar{G}\rho_m \equiv 8\pi G_N V_{eff}(b) \qquad (7.25)$$

where $V_{eff}(b)$ is the total effective potential for the system of matter and gravitation, defined by (after assuming Poincaré invariance)

$$\begin{aligned} I_{total} &= -\int d^4x \; V_{eff} \\ V_{eff} &= \frac{1}{16\pi G}\int d^D y\,\sqrt{g_D(y)}\,\left(\bar{R}(y) + \bar{\Lambda}\right) + \int d^D y\sqrt{g_D(y)}\;\rho_m \equiv \\ &\equiv \frac{1}{16\pi G_N}\left(-\frac{2\tilde{k}D}{b^2} + \bar{\Lambda}\right) + \overset{(4)}{V} \end{aligned} \qquad (7.26)$$

where $\overset{(4)}{V} \equiv \rho_D \rho_m = \frac{A}{b^n}$ is the usual 4-dim energy density. V_{eff} is the minimum energy of any state in which the expectation value of the fields has the value indicated by the argument of the potential. Hence, a stable solution is one associated with a minimum of V_{eff}.

In fact, it is not difficult to see that the Friedman solution with $\overset{(4)}{\Lambda} = 0$ is the stable solution. Eqs.(7.22) to (7.24) give the Friedman solution in a radiation dominated universe if

$$\overset{(4)}{\Lambda} \equiv 8\pi G_N V_{eff} = 0$$
$$b \frac{dV_{eff}(b)}{db} = 0 \qquad (7.27)$$

which indicate that the Friedman universe corresponds to a stationary point of V_{eff}, which vanishes at that point. It has to be emphasized here that Eqs.(7.27) imply a fine-tuning of the (4 + D)-dim. cosmological constant $\bar{\Lambda}$, so that $\overset{(4)}{\Lambda} = 0$:

$$\bar{\Lambda} = \frac{2\tilde{k}D}{b_0^2} - 16\pi G_N \overset{(4)}{V} \qquad (7.28)$$

where b_0 is the constant solution obtained from (7.23)

$$\frac{2\tilde{k}}{b^2} = \frac{\bar{\Lambda}}{D+2} - \frac{8\pi\bar{G}}{D+2} T_m + 8\pi\bar{G}\, p_{Dm} \qquad (7.29)$$

To further check the stability of this solution, i.e. that the stationary point (7.27) of V_{eff} corresponds actually to a minimum, we consider [151,156] the small perturbations

$$a(t) = a_0(t) + \delta a(t)$$
$$b(t) = b_0 + \delta b(t) \qquad (7.30)$$

where $a_0(t) \propto t^{1/2}$ is the Friedman solution in the radiation dominated era (with k = 0). Then, Eq.(7.9) gives

$$\ddot{\delta b} + 3\frac{\dot{a}_0}{a_0}\dot{\delta b} + c\,\delta b = 0 \qquad (7.31)$$

where

$$c = \frac{16\pi G_N}{D(D+2)} \left[b^2 \frac{d^2 V_{eff}(b)}{db^2} \right]\Bigg|_{b=b_0} \qquad (7.32)$$

The general solution of Eq.(7.31) is

$$\delta b = t^{-1/4}\left| C_1 H^{(1)}_{1/4}(\sqrt{c}\, t) + C_2 H^{(2)}_{1/4}(\sqrt{c}\, t)\right| \qquad (7.33)$$

where $H^{(1)}_{1/4}$ and $H^{(2)}_{1/4}$ are the Hankel functions. So, with $c > 0$, i.e. with

$$\left[b^2 \frac{d^2 V_{eff}(b)}{db^2} \right]\Bigg|_{b=b_0} > 0 \qquad (7.34)$$

the solution is a damping oscillation with frequency

$$\omega = \sqrt{c} \propto b_0^{-1} \qquad (7.35)$$

and asymptotic behaviour

$$\delta b \underset{t\to\infty}{\sim} t^{-3/4} \qquad (7.36)$$

(at the same time, Eq.(7.10) gives that asymptotically $\delta a \underset{t\to\infty}{\sim} t^{-3/4}$, so δa is also damping with time). Thus, we see that the Friedman solution with $\overset{(4)}{\Lambda} = 8\pi G_N V_{eff} = 0$ is in fact a stable solution against small perturbations (the effective potential V_{eff} has a local minimum, with $V_{eff} = 0$, at that point).

However, in general, there may be more than one solutions to Eq.(7.29), with constant radius b. As we have just seen, the solution $b = b_0$ with $\overset{(4)}{\Lambda} = 0$ is the stable one. Let us denote by $b = b_1$ the (in general) unstable solution with $\overset{(4)}{\Lambda}(b_1) > 0$, for $b_0 < b_1$. Then, with the $\overset{(4)}{\Lambda}$ term dominating in Eq.(7.24), we will have

$$a \propto e^{Ht}, \quad H^2(b_1) = \frac{\overset{(4)}{\Lambda}(b_1)}{3} \tag{7.37}$$

and this corresponds to a de Sitter phase for the 3-dim universe. If it can last long enough, one can have the usual cosmological inflationary scenario. In fact, in the place of the (Higgs) scalar field in the usual inflation, we now consider the dynamical scalar field

$$\phi = \ln b \tag{7.38}$$

Then, Eq.(7.9) yields

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + D\dot{\phi}^2 = -\frac{dV(\phi)}{d\phi} \tag{7.39}$$

where

$$V(\phi) = -\int_{\phi_0}^{\phi} d\phi \left[-2\tilde{k}\, e^{-2\phi} + \frac{\bar{\Lambda}}{D+2} + 8\pi\bar{G}\left(-\frac{T}{D+2} + p_D\right)\right] =$$

$$= -\int_{\phi_0}^{\phi} d\phi\, \frac{8\pi G_N}{D+2}\left|2\, V_{eff} - \frac{2}{D}\frac{dV_{eff}}{d\phi}\right| \tag{7.40}$$

Note the existence of two potentials: V_{eff}, which determines the Hubble parameter (Eq.7.37) and $V(\phi)$, which determines the evolution of ϕ. It is easy to see that the solution $\phi = \phi_0$ is stable:

$$V(\phi_0) = 0$$

$$V'(\phi_0) = -\frac{8\pi G_N}{D+2}\left[2\, V_{eff} - \frac{2}{D}\frac{dV_{eff}}{d\phi}\right] = 0$$

$$V''(\phi_0) = \frac{16\pi G_N}{D(D+2)}\, V''_{eff}(\phi_0) > 0 \tag{7.41}$$

So, $\phi = \phi_0$ is a local minimum of $V(\phi)$, as $b = b_0$ is for V_{eff}. From Eqs.(7.28) and (7.29), the stable solution corresponds to $(\Omega_D = b^D v_D)$

$$b_0^{D+n-2} = \frac{8\pi\bar{G}\, A(n+D)}{2\tilde{k}Dv_D} \tag{7.42}$$

$$\bar{\Lambda} = \frac{2\tilde{k}D(D+n-2)}{b_0^2(n+D)} \tag{7.43}$$

which is the fine-tuning needed in order to get the Friedman universe $F^4 \times N^D$.

The potential $V(\phi)$ is

$$V(\phi) = -\tilde{k}\, e^{-2\phi} - \frac{\bar{\Lambda}}{D+2}\phi + \frac{16\pi\bar{G}(n+2D)A}{v_D D(D+2)(n+D)}\, e^{-(n+D)\phi} - (\phi \to \phi_0) \tag{7.44}$$

This potential has a local minimum $\phi = \phi_0$, whereas the point $\phi = \phi_1 = \ln b_1$ is unstable and the potential is unbounded from below at $\phi \to \infty$. It is obvious that successful inflation is not possible, since this potential is not flat enough to produce the required amount of inflation. Since the point ϕ_1 is unstable, thermal and/or quantum corrections will soon terminate the inflationary phase. Density perturbations and reheating due to the rapid oscillation of ϕ around ϕ_0, follow [150a, 157] the same lines, as discussed in the usual inflation, although detailed calculations remain to be done. Moreover, in the present model ϕ_0 is, in fact, a metastable state. Although, using the results of section 4.3, one can arrange to have a metastability lifetime, which is longer than the age of the universe [158a] (as in some supergravity models [158b]), it is not always possible to end up with the $\phi = \phi_0$ state. In other words, the $\phi = \phi_0$ solution, with Friedman universe $F^4 \times N^D$, is not always an attractor [156].

From Eq.(7.44), it is obvious that the potential is unbounded from below, because of the cosmological constant $\bar{\Lambda}$. What we apparently need is a potential, where the Friedman universe solution $F^4 \times N^D$ is the absolute minimum of the potential. Such models do exist. The first example is the pure gravity model in $4 + D$ dimensions, with higher derivative terms in the action [150a]. In fact, this was the first model, where inflation along these lines has been proposed. Another example is provided by the 6-dim $N = 2$ supergravity theory [159], which is also a strong attractor [160].

There are two other, potentially embarrassing [161], cosmological problems, associated with K-K theories. One is associated with the so-called pyrgons [162]: massive (with $m \simeq M_P$) excitations produced upon compactification, of the extra dimensions. If they are stable, they contribute $\sim 10^{26} \rho_c$! One then has to rid the universe of these massive stable particles through decay, or dilute their abundance through entropy creation. The other problem is associated with K-K magnetic monopoles [163]: massive (with $m \simeq M_P$) stable topological defects in the geometry of compactification. The study [164] of cosmological production of K-K monopoles has revealed an intrinsic problem in predicting the number of primordial monopoles, detection of which would be a potential source of information on the dynamics of dimensional reduction.

VIII. EPILOGUE

As we have already stressed, in the search of solution of the various cosmological problems (most notably through inflation), our guide must be a successful particle physics model. If superstring theories, under active scrutiny now, turn out to justify the present excitement as a finite anomaly-free quantum theory of all interactions, the problems, we have discussed here, have to be fully examined within these theories. In any case, the cosmology of a "hidden" world seems to play an interesting role in the evolution of the universe [165]. The so successful marriage between particle physics and cosmology continues, and many more fruitful results are expected in the future.

ACKNOWLEDGMENTS

This article is an updated form of a series of lectures given first, in 1983, at the University of Minnesota, USA, where I wish to thank S. Gasiorowicz and S. Rudaz, and then, in 1984, at the University of Sussex, England, where I wish to thank N. Dombey and D. Bailin. I am also indebted to Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for their kind hospitality at the International Centre for Theoretical Physics, Trieste, Italy, where this review has been written up.

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