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ANOMALY FREEDOM IN CHIRAL SUPERGRAVITIES

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ANOMALY FREEDOM IN CHIRAL SUPERGRAVITIES *

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In honour of Stig Lundqvist's 60th birthday

ABSTRACT

We give criteria for anomaly cancellations in chiral Yang-Mills supergravities (including dual formulations of the theories) in 6, 8 and 10 dimensions.

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1. Yang-Mills coupled supergravities in higher dimensions are becoming increasingly attractive [1] because (i) they have large enough symmetries for physics, (ii) some of them admit chiral compactification, (iii) the requirement of anomaly freedom restricts the possible Yang-Mills groups [2] and finally (iv) at least for $d = 10$ they are intimately related to strings [3]. In this note we consider the issue of anomaly freedom in $d = 6,8,10$ chiral supergravities, and also in their dual formulations.

Recently, Green and Schwarz [2] discovered that in $d = 10$, $N = 1$ chiral supergravity [4] all anomalies (gauge, gravitational and mixed) cancel for only two Yang-Mills groups; $SO(32)$ and $E_8 \times E_8$. Since we will use their mechanism for anomaly cancellations in $d = 6,8$, it is useful to recall its central features. In the following, we shall use the notations of Ref.[2] (e.g. the Yang-Mills curvature is $F = dA + A^2$). The anomalies in $d = 10$ are derivable from a 12-form, Ω_{12}, made of Riemann and Yang-Mills curvature 2-forms, R and F, respectively. Given the field content of the theory, Ω_{12} can be easily determined from the work of Alvarez-Gaumé and Witten [17]. Since the theory contains a 2-form field B, the necessary and sufficient conditions for anomaly cancellation is found to be the factorization of Ω_{12} in the form $\Omega_{12} = X_4 X_8$, where X_4 and X_8 are closed 4 and 8 forms made of R and F [5]. Only for $SO(32)$ and $E_8 \times E_8$ such factorization can occur [2] F1). Since X_4 and X_8 are closed, one can write $X_4 = dX_3^0$, $X_8 = dX_7^0$. These equations define X_3^0 and X_7^0. Moreover since X_4 is gauge invariant it follows that $\delta X_3^0 = dX_2^1$ and $\delta X_7^0 = dX_6^1$. These equations define X_2^1 and X_6^1. (The superscripts refer to the ghost number.)

One can now deduce the anomaly, G, from Ω_{12} as follows

F1) For the factorization to occur it is necessary (but not sufficient) that the coefficient of $\mathrm{tr}R^6$ vanish, and that $\mathrm{Tr}F^6$ factorizes as $\mathrm{tr}F^4\,\mathrm{tr}F^2$ or $(\mathrm{tr}F^2)^3$. In fact, both requirements are satisfied for $SU(N)$ $(N < 5)$ Yang-Mills plus $496 - N^2 + 1$ supersymmetric Maxwell multiplets - each multiplet containing a gaugino and a non-minimally coupled Maxwell field - or $SO(5)$ Yang-Mills plus $496 - 10$ supersymmetric Maxwell multiples. However, we have checked that the factorization $\Omega_{12} = X_4 X_8$ does not occur for these groups.

$$G = (1-\alpha) X_2^1 X_8 + \alpha X_4 X_6^1 \quad (1a)$$

$$= X_2^1 X_8 - \alpha\, \delta(X_3^0 X_7^0) \quad . \quad (1b)$$

Here $0 \leqslant \alpha \leqslant 1$ is a constant arising from the Bose symmetrization of the anomaly amplitude [2]. From (1 b) we see that the value of α does not matter for the anomaly issue - one can always add a local counter-term $X_3^0 X_7^0$ to cancel the α-dependent term. Therefore it is convenient to take $\alpha = 0$ or 1 and thus deduce from $\Omega_{12} = X_4 X_8$ the anomaly to be $X_2^1 X_8$ or $X_4 X_6^1$, respectively. Taking $\alpha = 0$, we have to cancel the potential anomaly $X_2^1 X_8$. This is achieved [2] by adding to the classical Lagrangian the counter-term $\Delta\mathcal{L} = B X_8$ and modifying the gauge transformation $\delta B = -\omega_{2Y}^1$, (where ω_{2Y}^1 is defined by the variation of the Chern-Simons form $\delta\omega_{3Y}^0 = d\omega_{2Y}^1$) to be $\delta B = -X_2^1$. Accordingly the field strength $H = dB + \omega_{3Y}^0$ is modified to $H = dB + X_3^0$ so that it is invariant under the modified gauge transformation [2].

At this point we wish to make the following observations. If one considers the alternative formulation [6] of the $d = 10$, $N = 1$ theory which uses a 6-form Chamseddine field C, instead of the 2-form field B, the Green-Schwarz mechanism is guaranteed to work, and it works as follows.

We first observe that Ω_{12} is precisely as before, i.e. $\Omega_{12} = X_4 X_8$. Next, we take $\alpha = 1$ in Eq.(1a). We emphasize that the difference between $X_4 X_6^1$ and $X_2^1 X_8$ is a gauge variation of a local object, thus immaterial for the anomaly. Now, to cancel the potential anomaly $X_4 X_6^1$, we add the counter-term $\Delta\mathcal{L} = C X_4$, and modify the C-gauge transformation $\delta C = 0$ to be $\delta C = -X_6^1$, and the field strength $G = dC$ to be $G = dC + X_7^0$. Thus, the conclusion is that the anomalies in the Chamseddine formulation again cancel, and they do so for $SO(32)$ and $E_8 \times E_8$ only. It is worth noting that the Witten condition [7], (namely the fact that the integral of dH over a closed 4 manifold, Q, vanishes: $\int_Q dH = \int_Q X^4 = 0$), now changes into the statement, $\int_\eta dG = \int_\eta X^8 = 0$, where η is a closed 8-dimensional manifold. In the 2-form formulation, the Witten condition puts a restriction on the candidate backgrounds for compactification to $d = 4$ [8,9]. In the 6-form Chamseddine formulation, X_8 must contain space-time as well as internal indices. Since for Minkowski compactification, both R as well as F must vanish in space-time directions, it is clear that X_8 will vanish automatically [F2]. This summarizes the anomaly issue in $d = 10$.

2. In $d = 6$, the chiral $N = 2$ supergravity and its couplings to Yang-Mills and hypermultiples were constructed in Ref.[10]. The question of anomaly freedom in this theory was considered in Ref.[11]. In this case, anomalies are derived from an 8-form, Ω_8, which is a function of R and F and can easily be extracted from Ref.[17]. The anomaly cancellation occurs if Ω_8 factorizes as $\Omega_8 = X_4 Y_4$, where X_4 and Y_4 are closed 4-forms which are functions of R and F. For this factorization to occur the coefficient of $\mathrm{tr}R^4$ must vanish and $\mathrm{tr}F^4$ must factorize as $(\mathrm{tr}F^2)^2$. The first condition is satisfied if $m-n-29k+273 = 0$, where m is the number of gaugino, n the number of hyperino and k the number of tensorino multiplets. (Only in the case of $k = 1$ a manifestly Lorentz invariant Lagrangian formulations exists so far [12].) As shown by Okubo [13], the condition $\mathrm{tr}F^4 \sim (\mathrm{tr}F^2)^2$ is satisfied for all the representations of all the exceptional Lie groups and of $SU(3)$, $SU(2)$, $U(1)$, for $\underset{\sim}{28}$ of $Sp(4)$, $\underset{\sim}{28}$ of $SU(8)$ and all irreps of $SO(2n)$ with highest weight $(f_1, f_2, f_1+f_2, 0, 0, \ldots, 0)$ [F3].

Supposing that the two necessary requirements are satisfied (i.e. no $\mathrm{tr}F^4$ and $\mathrm{tr}F^4$ terms in Ω_8), then in the case of a group $G_1 \times G_2 \times \ldots \times G_r$ (<u>including Lorentz</u>), all the anomalies of the theories can be characterized by a symmetric $r \times r$ matrix β_{ij}

$$\Omega_8 = \sum_{i=1}^{r} \beta_{ij} F^{2(i)} F^{2(j)} \quad , \quad (2)$$

F2) One may entertain the possibility of compactifying the $d = 10$, $N = 1$ supergravity to $d = 2$ and to use the resulting two-dimensional field theory as a candidate for a $d = 10$ string theory.

F3) Of course, we can take a special combination of irreps. for a group which is not included in the above list, and arrange that $\Sigma \mathrm{tr}F^4 \sim \Sigma(\mathrm{tr}F^2)^2$. This line of search is currently under study [14].

where $F^{2(1)}$ for Lorentz group is R^2. The sufficiency condition for anomaly cancellation then is the requirement that β_{ij} has the eigenvalues $\lambda_1 = a$, $\lambda_2 = b$, $\lambda_3 = \lambda_4 = \ldots = \lambda_r = 0$, where a,b are constants satisfying $ab \leq 0$. Three non-trivial solutions were found in Ref.[11]. (i) $E_6 \times E_7 \times U(1)$; (ii) $E_7 \times U(1)$ with 22 gaugino and 22 hyperino singlets; (iii) $E_6 \times Sp(1)$ with 155 gaugino and 480 hyperino singlets [F4]. All these singlets are non-minimally coupled. Since then we have also found the following case. Take $k = 9$ and the Yang-Mills group to be <u>any</u> (a) $G_1 \times G_2 \times \ldots \times G_r \times Sp(1)$ or (b) $G_1 \times G_2 \times \ldots \times G_r \times U(1)$. If one now takes the hyperinos to be in the adjoint representation, then the condition on the β-matrix mentioned above is satisfied, and all anomalies cancel, provided there are (a) 15 or (b) 13 non-minimally coupled hyperino singlets. This is due to the fact that, in this case the β-matrix has the form

$$\beta_{ij} = \begin{array}{c} \begin{array}{cccccccc} G_1 & G_2 & . & . & . & G_n & L & U(1)/Sp(1) \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & . & . & . & 0 & 0 & c_1 \\ 0 & 0 & . & . & . & 0 & 0 & c_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & . & c_n \\ 0 & 0 & . & . & . & . & 0 & c_{n+1} \\ c_1 & c_2 & . & . & . & . & c_{n+1} & c_{n+2} \end{array}\right] \end{array} , \qquad (3)$$

where $c_1,\ldots,c_{n+2}$ are numbers which are irrelevant for the desired property of the β-matrix. What is crucial for this property is the vanishing of the $\beta_{n+1,n+1}$ entry, which is ensured by the choice of $k = 9$ (i.e. 9 tensor multiplets). This theory is <u>not</u> vectorlike, since the gauginos are (a) Sp(1) or (b) U(1) non-inert, while the hyperinos are (a) Sp(1) or (b) U(1) inert. (Thus, a $\text{Minkowski}_4 \times S_2$ compactification of the theory with a monopole lying in appropriate U(1) will yield a chiral theory in $d = 4$.) It should be noted that, in a recent paper other anomaly free theories in $d = 6$ were found by Green-Schwarz and West [9], by considering the compactification of the $d = 10$, $N = 1$ anomaly-free supergravity on K_3. Interestingly enough, these theories do not seem to satisfy our criteria of anomaly freedom [14]. We believe that this is due to the fact that the anomaly freedom of those theories requires the presence of infinitely many massive Kaluza-Klein modes which would arise in the K_3 compactification, unlike for the theories which we have considered which start life in $d = 6$. There are no such massive modes in our theories. In fact, Green and Schwarz [2] had pointed out earlier that infinitely many massive modes, contrary to the common belief do effect the anomaly situation in the context of strings.

F4) Note the misprint in Ref. [11] regarding the number of singlets in case (iii).

3. Consider now the $d = 8$ case. The Yang-Mills coupled $d = 8$ supergravity has recently been constructed [15]. The field content of the theory is

$$(e_\mu^m,\ \psi_\mu,\ \chi,\ \lambda^a,\ A_\mu^I,\ \phi^\alpha,\ B_{\mu\nu},\ \sigma) , \qquad (4)$$

where $a = 1,\ldots,n$, $I = 1,\ldots,n+2$, $\alpha = 1,\ldots,2n$. (In the dual formulation $B_{\mu\nu}$ is replaced by $B_{\mu\nu\rho\sigma}$ [16].) The scalars ϕ^α are described by the coset $G/K = SO(n,2)/SO(n) \times SO(2)$. K is the composite local gauge group. The fundamental gauge group is $SO(2,1) \times H \subset SO(n,2)$ with $\dim H = (n-1)$. $SO(2,1)$ is non-linearly realized and therefore naturally breaks down to U(1). <u>The theory is chiral with respect to this U(1).</u> More explicitly the $U(1) \times H$ content of the fermions is

$$\begin{array}{cccc} \psi_{\mu L} & (1)_1 & \psi_{\mu R} & (1)_{-1} \\ \chi_L & (1)_{-1} & \chi_R & (1)_1 \\ \lambda_L^{\underline{a}} & (n-1)_1 & \lambda_R^{\underline{a}} & (n-1)_{-1} \\ \lambda_L^0 & (1)_1 & \lambda_R^0 & (1)_{-1} \end{array} , \qquad (5)$$

where λ_a has been decomposed to the adjoint of H, $\lambda^{\underline{a}}$ ($\underline{a} = 1,\ldots,n-1$), and of U(1), λ^0. If one does not gauge the chiral U(1), then with respect to the fundamental Yang-Mills symmetry the theory is vectorlike. Therefore no anomalies arise in that case. Note, however, even without U(1) minimal gauging, the theory is still chiral with respect to the composite local U(1) (i.e. where the U(1) gauge field is a function of scalars, and not fundamental). If the chiral U(1) is gauged, however, all pentagon diagrams with one, three, or five U(1) external gauge line, will be anomalous [F5]. These anomalies are derivable from a 10-form invariant polynomial of the form

$$P = f[\beta_{11}(\mathrm{Tr}F^2)^2 + \beta_{12}(\mathrm{Tr}F^2)f^2 + \beta_{13}\,\mathrm{Tr}F^2\,\mathrm{tr}R^2 + \beta_{22}(f^2)^2 + \beta_{23}\,f^2\,\mathrm{tr}R^2 + \beta_{33}(\mathrm{tr}R^2)^2 + a\,\mathrm{Tr}F^4 + b\,\mathrm{tr}R^4] \quad , \qquad (6)$$

where f is the U(1) curvature, and $\mathrm{Tr}F^2$ is the trace of Yang-Mills curvature squared in the adjoint representation of H, while $\mathrm{tr}R^2$ is the trace of the Riemann curvature squared in the fundamental representation of SO(7,1). The coefficients β_{ij} $(i,j = 1,\ldots,3)$, a and b can be easily obtained from the work of Alvarez-Gaumé [17]. All the terms except the last two, have essentially the same structure as for the anomalies of the chiral d = 6 theory. For the Green-Schwarz mechanism to work, P must factorize as $P = f\,X_4\,Y_4$, where X_4 and Y_4 are closed 4 forms which are made of f, F and R. In P, we see that only $a\,\mathrm{tr}F^4$ and $b\,\mathrm{tr}R^4$ do not allow this factorization. If one chooses the Yang Mills group to contain only E_8, E_7, E_6, F_4, G_2, SU(3), SU(2), U(1) or SO(8), then according to Okubo, $\mathrm{Tr}F^4 \sim (\mathrm{Tr}F^2)^2$. However, there remains the term $\mathrm{tr}R^4$. Although, R takes values in SO(7,1) (which is the non-compact form of SO(8)), $\mathrm{tr}R^4$ unfortunately does not factorize because R is in the fundamental representation of SO(1,7) and not in the adjoint [F6]. On the other hand, the coefficient of $f\,\mathrm{tr}R^4$ is given by (1/5760)(247 - 1 + (n-1) + 1). Thus unlike in d = 10, where the contribution

F5) Note that even if U(1) is not minimally gauged, anomalies of the form discussed in the text will appear where U(1) curvature is to be replaced by composite U(1) curvature [18]. These anomalies do not lead to breakdown of unitary, however, and are therefore innocuous [19].

F6) As mentioned earlier, in the case of SO(8) the factorization occurs only for those irreps. with highest weight $(f_1, f_2, f_1-f_2, 0)$. These contain the adjoint representation, (1,1,0,0), but not the fundamental representation (1,0,0,0).

$\psi_{\mu L}$ has opposite sign to that of λ_L, and which make the cancellation of $\mathrm{tr}R^6$ terms possible, here for d = 8, the contribution (247) of ψ_L has the same sign as that of (n-1), the contribution of λ_L and no cancellation is possible. Therefore we conclude that anomalies in d = 8 supergravities with minimally gauged chiral U(1) cannot be cancelled. As remarked earlier, however, the d = 8 supergravities with minimally ungauged U(1), are chiral with respect to the global U(1), are anomaly free [20]. Finally we note that in the context of strings, Nahm [21] has speculated that string models should exist in 6 and 8 dimensions.

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