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MATTER COUPLINGS, GAUGING AND MINKOWSKI COMPACTIFICATION

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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

d = 8 SUPERGRAVITY: MATTER COUPLINGS, GAUGING AND MINKOWSKI COMPACTIFICATION *

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ABSTRACT

We couple $d = 8$, $N = 1$ supergravity to n vector multiplets. The $2n$ scalars of the theory parametrize the Kahler manifold $SO(n,2)/SO(n) \times SO(2)$. The $n+2$ vector fields are used to gauge $\{SO(1,2) \times H\}$ subgroup of $SO(n,2)$ where $H \subset SO(n-1)$ and $\dim H = n-1$. It is shown that the theory compactifies to $(\mathrm{Minkowski})_6 \times S^2$ by a monopole configuration which is embedded in $SO(1,2)$. The field equations fix the monopole charge to be ± 1, which implies a stable, chiral $N = 2$ supergravity in $d = 6$.

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I. INTRODUCTION

It is well known that if quarks and leptons are elementary, the maximal supergravities, $d = 11$, $N = 1$ [1], or the chiral $d = 10$, $N = 2$ [2] or their direct descendents in lower dimensions, are inadequate as physical theories. To remedy this, one must consider additional matter couplings. Two examples are (1) $d = 6$, $N = 2$ chiral anomaly-free supergravity interacting with ($E_6 \times E_7 \times U(1)$) Yang-Mills plus hypermultiplets [3,4] and (2) $d = 10$, $N = 1$, chiral, anomaly-free supergravity interacting with ($SO(32)$ or $E_8 \times E_8$) Yang-Mills multiplets [5,6]. The first theory exhibits classical Minkowski compactification ($M^4 \times S^2$) to $d = 4$, and is the only theory known to do this [7]. It gives rise to two tachyon-free families of $SO(10)$ [3]. It is not a direct descendent of $d = 11$ or $d = 10$ theories mentioned above.

The purpose of this paper is to provide a third example of matter coupled even-dimensional supergravity. This is $N = 1$, $d = 8$ supergravity coupled to an arbitrary number of vector multiplets. (For $N = 2$, $d = 8$ supergravity, see Ref.[8].) Our results are as follows. The $2n$ scalars contained in the n vector multiplets parametrize the Kahler manifold $SO(n,2)/SO(n) \times SO(2)$. The n-vectors of the vector multiplets together with the two vectors of the supergravity multiplet form the $n+2$ dimensional representation of $SO(n,2)$ [9]. As in $d = 4$ analogs, it turns out that the theory has a Kahler invariance provided that shifts in the Kahler potential are accompanied by chiral $U(1)$ rotations of the fermions $^{*)}$[10]. We can gauge a subgroup of $SO(n,2)$, using all the $(n+2)$ vector fields of the theory. The gauge group is $SO(1,2) \times H$ where $H \subset SO(n-1)$ and $\dim H = (n-1)$. (The supersymmetry of the action is compatible with a gauge group which has the Cartan-Killing metric of the form $(--++\ldots+) = (--+)\ (++\ldots+)$.) Notwithstanding non-compactness, there are no ghosts [11].

The two non-compact generators of $SO(1,2)$ are realized non-linearly. Using the corresponding non-compact gauge transformations one can gauge away two scalars, thereby giving masses to the two vector fields of the supergravity multiplet [11]. In this way, $SO(1,2)$ breaks down to $U(1)$ which is the chiral $U(1)$ symmetry mentioned above. Its presence is important for it gives rise to a positive-definite potential which in turn triggers compactification to $(\text{Minkowski})_6 \times S^2$, leading to a chiral $N = 2$ supergravity. (This prompts the conjecture that our $d = 6$, $N = 2$ theory [7] may be descended from the gauged $d = 8$, $N = 1$ theory.)

*) In this note we formulate these transformations in a way which does not involve the Kahler potential explicitly. For this see Ref.[10] which discusses them in $d = 4$. Our approach is equivalent to that of Ref.[10].

Any anomalies in the theory arise due to the chiral $U(1)$ and its mixings with Lorentz and other gauge groups. However, since the theory has a two-form field, the Green-Schwarz mechanism [6] for cancelling such anomalies is available. This and the physical models which may be constructed will be discussed elsewhere.

II. FIELD CONTENT AND THE SCALAR MANIFOLD

The field content of $d = 8$, $N = 1$ supergravity is

$$\left(e_\mu^m ,\ \psi_\mu ,\ \chi ,\ B_{\mu\nu} ,\ A_\mu^i ,\ \sigma \right) , \quad i = 1,2 \tag{1}$$

where the gravitino ψ_μ, and χ are pseudo-Majorana spinors, that is to say, [12],

$$\chi = C \bar{\chi}^T , \quad C^T = C , \quad \gamma^{\mu\,T} = C \gamma^\mu C^{-1} \tag{2}$$

$B_{\mu\nu} = -B_{\nu\mu}$, A_μ^i and the scalar σ are real. Taking n vector multiplets of the form (λ, A_μ, ϕ^i) and combining them together we obtain

$$\left(e_\mu^m ,\ \psi_\mu ,\ \chi ,\ B_{\mu\nu} ,\ A_\mu^I ,\ \phi^\alpha ,\ \sigma \right) \tag{3}$$

where $I = 1,\ldots,n+2$, $a = 1,\ldots,n$ and $\alpha = 1,\ldots,2n$. Now from the known examples of matter couplings [9,13,14] we expect that the real $2n$ scalars ϕ^α must parametrize the Kahler manifold $SO(n,2)/SO(n) \times SO(2)$. In the ungauged theory, the vectors transform as the $n+2$ dimensional representation of the global $SO(n,2)$ while the gauge fermions transform as the n-dimensional representation of the local composite $SO(n)$. The $SO(2) \approx U(1)$ chiral transformations will be discussed below.

In order to construct an action for this system we first consider a typical representative of the coset space $SO(n,2)/SO(n) \times SO(2)$. A convenient one is [15]

$$L = \exp \begin{pmatrix} 0 & \phi \\ \phi^T & 0 \end{pmatrix} = \begin{bmatrix} \cosh\sqrt{\phi\phi^T} & \phi\,\dfrac{\sinh\sqrt{\phi^T\phi}}{\sqrt{\phi^T\phi}} \\ \phi^T\,\dfrac{\sinh\sqrt{\phi\phi^T}}{\sqrt{\phi\phi^T}} & \cosh\sqrt{\phi^T\phi} \end{bmatrix} \tag{4}$$

where ϕ is a $n \times 2$ rectangular matrix, describing the physical scalars of the theory. Denoting L by $L_I{}^A$, and its inverse by $L^I{}_A$, from (4) it follows that

$$L_I{}^A L^I{}_B = \delta^A_B \quad , \quad L_I{}^A L_J{}^B \eta_{AB} = \eta_{IJ} , \quad L^I{}_A = \eta^{IJ} \eta_{AB} L_J{}^B \tag{5}$$

where $I,A = 1,\dots,n+2$, and

$$\eta_{AB} = (- - + + + \cdots +) = \eta_{IJ} \tag{6}$$

Eqs.(5) and (6) are crucial for what follows. (Note especially that η_{IJ} is constant [13], and it can be used for raising and lowering the SO(n,2) indices.) It turns out that all the interactions of the scalar fields can be described in terms of $L_I{}^A$, and its derivatives which occur in the Maurer-Cartan form [16]

$$L^{-1}\partial_\mu L = \begin{bmatrix} Q_{\mu a}{}^b & P_{\mu a}{}^i \\ P_{\mu i}{}^a & Q_{\mu i}{}^j \end{bmatrix} \qquad \begin{matrix} a,b = 1,\dots,n \\ i,j = 1,2 \end{matrix} \tag{7}$$

$Q_{\mu ab} = -Q_{\mu ba} = Q_{\mu a}{}^c \delta_{cb}$ is the composite SO(n) connection, while $Q_{\mu ij} = -Q_{\mu ji} = Q_{\mu i}{}^k (-\delta_{kj})$ is the composite SO(2) connection. Often we shall use the notation $Q_{\mu 1}{}^2 \equiv Q_\mu$. The off-diagonal parts of the Maurer-Cartan form, which satisfy $P_{\mu a}{}^i = P_{\mu i}{}^a$, transform homogeneously under SO(n) x SO(2). The local supersymmetry of the action, which is constructed by the Noether procedure [17] and given in the next section, turns out to require the following covariant derivatives:

$$D_\mu \psi_\nu = \partial_\mu \psi_\nu + \tfrac{1}{4}\omega_{\mu rs}\gamma^{rs}\psi_\nu + \tfrac{i}{2}\gamma_9 Q_\mu \psi_\nu \tag{8a}$$

$$D_\mu \chi = \partial_\mu \chi + \tfrac{1}{4}\omega_{\mu rs}\gamma^{rs}\chi - \tfrac{i}{2}\gamma_9 Q_\mu \chi \tag{8b}$$

$$D_\mu \lambda_a = \partial_\mu \lambda_a + \tfrac{1}{4}\omega_{\mu rs}\gamma^{rs}\lambda_a + Q_{\mu a}{}^b \lambda_b + \tfrac{i}{2}\gamma_9 Q_\mu \lambda_a \tag{8c}$$

One then finds that the action is invariant under the following composite chiral U(1) transformations *)

$$\begin{aligned} \delta L_I{}^i &= \Lambda(\phi)\,\epsilon^{ij} L_I{}^j \quad , & \delta\psi_\mu &= -\tfrac{i}{2}\Lambda(\phi)\gamma_9\psi_\mu \\ \delta\chi &= \tfrac{i}{2}\Lambda(\phi)\gamma_9\chi \quad , & \delta\lambda_a &= -\tfrac{i}{2}\Lambda(\phi)\gamma_9\lambda_a \end{aligned} \tag{9}$$

From Eq.(7) it follows that

$$\delta Q_\mu = \partial_\mu \Lambda(\phi) \quad , \quad \delta P_{\mu a}{}^i = -\Lambda(\phi)\,\epsilon^{ij} P_{\mu a}{}^j \tag{10}$$

In the construction of the ungauged action also important are the following equations [17]:

$$D_\mu P_{\nu a}{}^i - \mu \leftrightarrow \nu = 0 \tag{11a}$$

$$P_{\mu 1}{}^a P_{\nu a}{}^2 - \mu \leftrightarrow \nu = -Q_{\mu\nu} \tag{11b}$$

where $Q_{\mu\nu} = \partial_\mu Q_\nu - \partial_\nu Q_\mu$.

III. THE ACTION AND THE TRANSFORMATION RULES

Using $SO(n,2)_{global} \times (SO(n) \times SO(2))_{local}$ invariance, and the objects $L_I{}^A$, $P_{\mu a}{}^i$, $Q_{\mu a}{}^b$ and Q_μ, one can now construct the coupling of n-vector multiplets to N = 1 d = 8 supergravity by the Noether method. The result (upto quartics) is ($\kappa = 1$) **)

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*) To make contact with the formulation of Ref.[10] note that $Q_\mu = \partial_\mu Q^\alpha A_\alpha$, $P_\mu{}^{ai} = \partial_\mu \phi^\alpha V_\alpha^{ai}$, $V_\alpha^{ai} V_{\beta ai} = g_{\alpha\beta}$ where A_α is the U(1) (Kahler) connection, V_α^{ai} is the 2n-bein and $g_{\alpha\beta}$ is the metric on SO(n,2)/SO(n) x SO(2), and that A_α and $g_{\alpha\beta}$ can be expressed in terms of the Kahler potential.

**) Our conventions are $\eta_{rs} = (-+++++++)$, $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} + \dots$, $\bar{\chi} = \chi^\dagger i\gamma_0$. Note that $\bar{\chi}\,\gamma^{\mu_1\dots\mu_n}\lambda = -\bar{\lambda}\,\gamma^{\mu_n\dots\mu_1}\chi$.

$$
\begin{aligned}
e^{-1}\mathcal{L}_0 = {} & \tfrac{1}{4}R + \tfrac{1}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \tfrac{1}{4}e^{\sigma} a_{IJ} F^{I}_{\mu\nu} F^{\mu\nu,J} - \tfrac{1}{12}e^{2\sigma} G_{\mu\nu\rho} G^{\mu\nu\rho} + \\
& + \tfrac{3}{4}\bar{\chi}\gamma^\mu D_\mu \chi + \tfrac{1}{2}\bar{\lambda}^a \gamma^\mu D_\mu \lambda^a + \tfrac{3}{8}\partial_\mu\sigma\,\partial^\mu\sigma + \tfrac{1}{4}P_\mu{}^{ai} P^{\mu ai} - \\
& - \tfrac{3}{4}i\,\bar{\chi}\gamma^\nu\gamma^\mu\psi_\nu\,\partial_\mu\sigma + \tfrac{i}{2}\bar{\lambda}^a\gamma^\nu\gamma^\mu \hat{P}_{\nu a}\psi_\mu + \tfrac{i}{4\sqrt{2}}e^{\sigma/2}F^{I}_{\mu\nu}\Big[\bar{\psi}^\lambda \gamma_{[\lambda}\gamma^{\mu\nu}\gamma_{\tau]}\hat{L}_I \psi^\tau + \\
& + i\bar{\chi}\gamma^\lambda\gamma^{\mu\nu}\hat{L}_I \psi_\lambda + \bar{\chi}\gamma^{\mu\nu}\hat{L}_I^{\dagger}\chi + 2i\bar{\lambda}^a\gamma^\lambda\gamma^{\mu\nu}\psi_\lambda L_I^a - 2\bar{\lambda}^a\gamma^{\mu\nu}\chi L_I^a - \bar{\lambda}^a\gamma^{\mu\nu}\hat{L}_I\lambda^a\Big] + \\
& + \tfrac{1}{24}e^{\sigma} G_{\mu\nu\rho}\Big[\bar{\psi}^\lambda \gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau - 2i\bar{\chi}\gamma^\lambda\gamma^{\mu\nu\rho}\psi_\lambda - \tfrac{1}{2}\bar{\chi}\gamma^{\mu\nu\rho}\chi - \bar{\lambda}^a\gamma^{\mu\nu\rho}\lambda^a\Big]
\end{aligned}
\tag{12}
$$

where

$$F^{I}_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu \tag{13a}$$

$$G_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + F^{I}_{\mu\nu} A^J_\rho \eta_{IJ} \quad + \;2\;\text{more} \tag{13b}$$

$$a_{IJ} = L_I{}^i L_J{}^i + L_I{}^a L_J{}^a \tag{13c}$$

$$\hat{L}_I = L_I{}^1 + i\gamma_9 L_I{}^2 \;, \quad \hat{P}_{\mu a} = P_{\mu a}{}^1 + i\gamma_9 P_{\mu a}{}^2 \tag{13d}$$

We recall that η_{IJ} is constant. (See Eq.(6)). The coefficient of the Chern-Simons form in $G_{\mu\nu\rho}$ is fixed by the requirement of supersymmetry in the Noether procedure. Note also that a_{IJ} is a positive definite function of scalars. Thus, although the global invariance group is non-compact (SO(n,2)) there are no ghosts [11].

Our action is invariant (upto cubics) under the following local supersymmetry transformations:

$$\delta_0 e_\mu{}^m = \bar{\varepsilon}\gamma^m\psi_\mu \;, \qquad \delta_0\sigma = -i\bar{\varepsilon}\chi$$

$$\delta_0 L_I{}^1 = -i\,\varepsilon\lambda_a L_I{}^a \;, \qquad \delta_0 L_I{}^2 = \bar{\varepsilon}\gamma_9\lambda_a L_I{}^a \;, \qquad \delta_0 L_I{}^a = -i\bar{\varepsilon}\hat{L}_I\lambda^a$$

$$\delta_0 A_\mu{}^I = \frac{-i}{\sqrt{2}}e^{-\sigma/2}\left(\bar{\varepsilon}\hat{L}^I\psi_\mu - \tfrac{i}{2}\bar{\varepsilon}\hat{L}^I\gamma_\mu\chi + i\bar{\varepsilon}\gamma_\mu\lambda^a L^I{}_a\right)$$

$$\delta_0 B_{\mu\nu} = e^{-\sigma}\left(\bar{\varepsilon}\gamma_{[\mu}\psi_{\nu]} - \tfrac{i}{2}\bar{\varepsilon}\gamma_{\mu\nu}\chi\right) - 2\eta_{IJ}A^I_{[\mu}\delta_0 A^J_{\nu]} \eta_{IJ}$$

$$\delta_0\psi_\mu = D_\mu\varepsilon + \frac{i}{12\sqrt{2}}F^I_{\rho\sigma}\left(\gamma_\mu{}^{\rho\sigma} - 10\delta_\mu^\rho\gamma^\sigma\right)\hat{L}_I\varepsilon - \frac{1}{36}e^{\sigma}G_{\rho\sigma\tau}\left(\gamma_\mu{}^{\rho\sigma\tau} - 6\delta_\mu^\rho\gamma^{\sigma\tau}\right)\varepsilon$$

$$\delta_0\chi = \frac{-i}{2}\partial_\mu\sigma\,\gamma^\mu\varepsilon + \frac{1}{6\sqrt{2}}e^{\sigma/2}F^I_{\mu\nu}\hat{L}_I\gamma^{\mu\nu}\varepsilon + \frac{i}{18}e^{\sigma}G_{\mu\nu\rho}\gamma^{\mu\nu\rho}\varepsilon$$

$$\delta_0\lambda_a = \frac{-i}{2}\gamma^\mu\hat{P}_{\mu a}\varepsilon + \frac{1}{2\sqrt{2}}e^{\sigma/2}F^I_{\mu\nu}L_I{}^a\gamma^{\mu\nu}\varepsilon \tag{14}$$

At this point since the vector fields are abelian, they are not obviously relevant for phenomenology. We will therefore consider the gauging of an appropriate non-abelian subgroup of SO(n,2) which also would use all the vector fields of the theory.

IV. GAUGING AN (n+2) PARAMETER SUBGROUP OF SO(n,2)

Consider an (n+2) dimensional Lie algebra, g_0, with structure constants $f_{IJ}{}^K$ [9]. (Restrictions on these will be discussed below.) Define

$$\mathcal{F}^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + f_{KL}{}^I A^K_\mu A^L_\nu \tag{15}$$

where the coupling constants of each simple factor in the Yang-Mills group can be different, and they are absorbed into $f_{IJ}{}^K$. Evidently, one must also non-abelianize the Chern-Simons form which appears in $G_{\mu\nu\rho}$ (see Eq.(13b))

$$\mathcal{G}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \mathcal{F}^I_{\mu\nu}A^J_\rho\eta_{IJ} - \tfrac{1}{3}f_{IJ}{}^L\eta_{LK}A^I_\mu A^J_\nu A^K_\rho + 2\;\text{more} \tag{16}$$

Next, we construct the gauged version of the Maurer-Cartan form as follows [18]:

$$L^I{}_A\left(\partial_\mu\delta_I^K + f_{IJ}{}^K A^J_\mu\right)L_K{}^B = \begin{bmatrix} Q_{\mu a}{}^b(A) & P_{\mu a}{}^i(A) \\ P_{\mu i}{}^a(A) & Q_{\mu i}{}^j(A) \end{bmatrix} \tag{17}$$

$Q_{\mu a}{}^b(A)$ $[Q_{\mu i}{}^j(A)]$ transforms as a gauge field of composite local SO(n) [SO(2) $\approx$ U(1)] induced by a local g_0 transformation (which generates the n+2 parameter group), provided that

$$f_{IJ}{}^{L}\,\eta_{LK} = \text{totally antisymmetric in } IJK\;. \tag{18}$$

This condition ensures that $Q_{\mu ab}(A) = -Q_{\mu ba}(A)$, and similarly for $Q_{\mu ij}(A)$. Moreover, the supersymmetry of the gauged action (checked by the Noether procedure) turns out to require the same condition as well. Now, Eq.(18) is satisfied provided that η_{IJ} is the Cartan-Killing metric of the gauge Lie algebra, g_0 [15]. This implies that g_0 has two non-compact generators (see Eq.(6)). Since only SO(1,2) can have two non-compact generators it follows that the gauge group, G_0, must be SO(1,2) x H where H is an (n-1) dimensional compact subgroup of SO(n,2).

Eq.(18) is not the only requirement to preserve supersymmetry. To describe the modifications to be made in the Lagrangian and transformation rules, it is convenient to define

$$C^{a} = f_{IJ}{}^{K}\, L^{I}{}_{1} L^{J}_{2}\, L_{K}{}^{a} \tag{19a}$$

$$C_{ab}{}^{i} = f_{IJ}{}^{K}\, L^{I}_{a} L^{J}_{b}\, L_{K}{}^{i} \tag{19b}$$

Note that $C_{ab}^{\ i} = -C_{ba}^{\ i}$ is charged, while C^{a} is neutral under U(1). Moreover from (18) and (19) it follows that

$$D_\mu C_a = C_{ab}{}^{1}\, P_\mu{}^{2b}(A) \; - \; 1 \leftrightarrow 2 \tag{20}$$

The functions C_a and $C_{ab}^{\ i}$ appear in the modification of Eq.(11),

$$D_\mu P_\nu{}^{ai} - \mu\leftrightarrow\nu = C^{a}\varepsilon^{ij} L_I{}^{j}\, \mathcal{F}^{I}_{\mu\nu} + C^{abi} L_I{}^{b}\, \mathcal{F}^{I}_{\mu\nu} \tag{21a}$$

$$P_{\mu 1}{}^{a}(A)\, P_{\nu a}{}^{2}(A) - \mu\leftrightarrow\nu = -\,Q_{\mu\nu}(A) + C^{a} L_I{}^{a}\, \mathcal{F}^{I}_{\mu\nu} \tag{21b}$$

where $Q_{\mu\nu}(A) = \partial_\mu Q_\nu(A) - \partial_\nu Q_\mu(A)$. These equations are needed in proving the supersymmetry of the action. In terms of the objects defined in (15), (16) and (19) the supersymmetric gauged action and the transformation rules can now be given as follows:

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_0\big(F\to\mathcal{F},\ G\to\mathcal{G},\ P\to P(A),\ Q\to Q(A)\big) + \mathcal{L}_g \tag{22}$$

where $\mathcal{L}_0$ is given in Eq.(12) and $\mathcal{L}_g$ is given by

$$e^{-1}\mathcal{L}_g = \frac{-i}{2\sqrt{2}}\, e^{-\sigma/2}\left[\bar{\psi}_\mu\gamma^\mu\gamma_9\lambda^a C^a - i\bar{\chi}\gamma_9\lambda^a C^a - \bar{\lambda}^a\hat{C}_{ab}\lambda^b\right] - \frac{1}{8}e^{-\sigma}C^aC^a \tag{23}$$

where $\hat{C}_{ab} = C_{ab}^{\ 1} + i\gamma_9 C_{ab}^{\ 2}$. $\mathcal{L}_g$ is invariant under

$$\delta_{\text{gauged}} = \delta_0\Big(F\to\mathcal{F},\ G\to\mathcal{G},\ P\to P(A),\ Q\to Q(A)\Big) + \delta_g \tag{24}$$

where δ_0 is given in Eq.(14), and δ_g is given by

$$\delta_g\lambda^a = \frac{i}{2\sqrt{2}}\, e^{-\sigma/2}\, C^a\gamma_9\,\varepsilon \tag{25}$$

These modifications are remarkably similar to those which arise in chiral N = 2, d = 6 supergravity [4]. The most interesting term in $\mathcal{L}_g$ is the potential which is positive definite. This fact turns out to be crucial for the Minkowski compactification of the theory, as in the N = 2 d = 6 theory [7]. This we discuss in the next section.

V. COMPACTIFICATION

The bosonic field equations of our theory are (M,N = 0,1,...,7)

$$R_{MN} = 2e^{\sigma}\mathcal{F}_{MP}{}^{I}\mathcal{F}_{N}{}^{P,J}\, a_{IJ} + e^{2\sigma}\mathcal{G}_{MPQ}\mathcal{G}_N{}^{PQ} + P_M{}^{ai}(A)\,P_N{}^{ai}(A) + \tfrac{3}{2}\partial_M\sigma\,\partial_N\sigma - \tfrac{1}{2}g_{MN}\Box\sigma \tag{26a}$$

$$\Box\sigma = \tfrac{1}{3}e^{\sigma}\mathcal{F}_{MN}{}^{I}\mathcal{F}^{MN,J}\, a_{IJ} + \tfrac{2}{9}e^{2\sigma}\mathcal{G}_{MNP}\mathcal{G}^{MNP} - \tfrac{1}{6}e^{-\sigma}C^aC^a \tag{26b}$$

$$D^M P_M{}^{ai}(A) = \tfrac{1}{2}\varepsilon^{ij}C_{ab}{}^{j}C^{b} - 2\left(\mathcal{F}^{I}_{MN}L_I{}^{a}\right)\left(\mathcal{F}^{MN,J}L_J{}^{i}\right) \tag{26c}$$

$$D_M\left(e^{2\sigma}G^{MNP}\right) = 0 \tag{26d}$$

$$D_M\left(e^{\sigma}a_{IJ}\mathcal{F}^{MN,J}\right) = \eta_{IJ}\, e^{2\sigma}\mathcal{G}^{NPQ}\mathcal{F}_{PQ}{}^{J} - \tfrac{1}{2}g^{MN}P_M^{ai}\left(C_{ab}{}^{i}L_I{}^{b} + \varepsilon^{ij}C_aL_I{}^{j}\right) \tag{26e}$$

Note that in the Einstein equation all the trace terms are absorbed into $g_{MN}\Box\sigma$, as in N = 2 d = 6 theory [7]. This fact facilitates Minkowski compactification. In this note we focus our attention on $(\mathrm{Minkowski})_6 \times S^2$ compactification and propose the following ansatz ($\mu,\nu = 0,1,\ldots,5$):*)

$$g_{MN}\, dx^M dx^N = \eta_{\mu\nu}\, dx^\mu dx^\nu + a^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$A_m^{(3)}\, dy^m = \frac{\nu}{g} (\cos\theta \mp 1)\, d\theta \ , \qquad \nu = \text{integer} \tag{27}$$

and all the other fields vanishing in the background. $A_m^{(3)}$ (m labels the co-ordinates of S^2) is the monopole field lying in the compact direction of the SO(1,2) algebra, and g is the coupling constant of SO(1,2). From (27) one finds

$$R_{mn} = \frac{1}{a^2} g_{mn} \ , \qquad F_{mn}^{(3)} = \frac{-\nu}{g a^2} \varepsilon_{mn} \tag{28}$$

Eqs.(26c,d,e) are trivially satisfied (note that since $L_I{}^A = \delta_I{}^A$ in our background, from (19) it follows that $C_{ab}{}^i = 0$, $C^a = 0$ if $a \neq 3$, and $C^3 = g$), while Eqs.(26a,b) require

$$g^2 a^2 = 2\nu^2 \qquad \text{(Einstein equation)} \tag{29a}$$

$$g^4 a^4 = 4\nu^2 \qquad \text{(scalar field equation)} \tag{29b}$$

from which it follows that [7]

$$\nu = \pm 1 \tag{30}$$

To see what this implies for the supersymmetry of the background, we first fix our conventions for the Γ-algebra as follows [19]:

$$[\Gamma_M, \Gamma_N] = 2\eta_{MN} \ , \quad \Gamma_\mu = \gamma_\mu \times \sigma^1 \ , \quad \Gamma_7 = \gamma_7 \times \sigma^1 \ , \quad \Gamma_8 = \mathbb{1} \times \sigma^2$$

$$\Gamma_9 = \Gamma_0 \Gamma_1 \cdots \Gamma_7 = \mathbb{1} \times \sigma^3 \ , \qquad \gamma_7^2 = \mathbb{1} \tag{31}$$

The supersymmetry of the background is ensured provided that

*) See Ref.[19] for a detailed explanation of such an ansatz.

$$\delta\psi_m = 0 \ : \quad \left(\partial_m + \tfrac{1}{2}\omega_{m78}\Gamma^{78} + \tfrac{1}{2} i \Gamma^9 Q_m(A)\right)\varepsilon = 0 \tag{32a}$$

$$\delta\lambda^{(3)} = 0 \ : \quad 2\nu\, \Gamma^{78} \varepsilon - i g^2 a^2 \Gamma^9 \varepsilon = 0 \tag{32b}$$

In the background

$$\omega_{m78} = e_m \ , \qquad Q_m(A) = g A_m^{(3)} = -\nu e_m \tag{33}$$

where $dy^m e_m = -d\phi(\cos\phi - 1)$ is the usual SU(2) invariant U(1) connection form on S^2 [19]. From (29), (31), (32) and (33) it now follows that

$$\left(\partial_m + \tfrac{1}{2}(\nu + \gamma_7)\, i\sigma^3 e_m\right)\varepsilon = 0 \tag{34a}$$

$$(\nu + \gamma_7)\, i\sigma^3 \varepsilon = 0 \tag{34b}$$

which is equivalent to (recalling (30))

$$\partial_m \varepsilon = 0 \ , \qquad (\nu + \gamma_7)\varepsilon = 0 \ , \qquad \nu = \pm 1 \tag{35}$$

For $\nu = +1$, ε is a single constant, Weyl spinor ($(1 + \gamma_7)\varepsilon = 0$), which is equivalent to two Majorana-Weyl spinors in d = 6. This shows that our compactification scheme yields an N = 2 chiral supergravity in d = 6.

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Note added in proof

After this paper was written we received a Cambridge preprint by M. Awada and P.K. Townsend, where ungauged couplings of n-vector multiplets to N = 1, d = 8 supergravity are exhibited.

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