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AN ANOMALY-FREE MODEL IN SIX DIMENSIONS *

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ABSTRACT

We show that a gauged supergravity theory based on $E_6 \times E_7 \times U(1)$ is free of gauge and gravitational anomalies in six dimensions. It compactifies to $(\text{Minkowski})^4 \times S^2$ by the standard monopole mechanism. With a monopole of strength n in E_6, the resulting four-dimensional theory exhibits chiral $SO(10) \times U(1)$ with $2|n|$ families (and no antifamilies). Supersymmetry is broken.

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I. INTRODUCTION

Green and Schwarz [1] have recently shown that 10-dimensional N = 1 supergravity coupled to O(32) or $E_8 \times E_8$ Yang-Mills fields is free of gauge and gravitational anomalies. Witten [2] has argued that the O(32) model might even be realistic in that it could compactify so as to yield a 4-dimensional effective chiral SU(5) gauge theory with any number of generations of quarks and leptons. Such a compactification however is conjectural [3]. If solutions can be found it would be interesting to know whether they correspond to Minkowskian or anti-de-Sitter space-time, and whether they are stable.

In this note we give criteria for constructing anomaly free gauge supergravities in six dimensions. These theories involve an antisymmetric tensor as part of the gravity supermultiplet and this field plays the same role in the structure of anomaly cancelling counter-terms as in the 10-dimensional model of Green and Schwarz. In six dimensions it is possible to introduce matter as well as gauge multiplets and, as we show, both are necessary in order to achieve cancellation of gauge and gravitational anomalies. We also exhibit explicit compactication on Minkowski $\times S^2$.

An important property of O(32), exploited by Green and Schwarz is the absence of an algebraically independent sixth-order Casimir invariant [4], i.e. $\mathrm{Tr}\, F^6 \sim \mathrm{Tr}\, F^2\, \mathrm{Tr}\, F^4$. For six dimensions the analogous property, which we use, is the absence of independent fourth-order invariants in the exceptional algebras [3]*). Apart from this, the structure of our model is heavily influenced by the purely gravitational anomalies. Specifically, the various contributions to the term $\mathrm{tr}\, R^4$ are required to cancel.

One of the aims in constructing the model was to find an explicit compactifying solution with the geometry of Minkowski space-time $\otimes$ internal 2-sphere. One way to achieve such compactification is through a non-vanishing magnetic monopole background configuration on the 2-sphere. It is therefore required that the model contain gauge fields. Among various anomaly free possibilities we found that the most elegant combination involved $E_6 \times E_7 \times U(1)$. The monopole background lies in the algebra of E_6 causing it to break spontaneously to SO(10) in the course of compactification. This compactification leads to a chiral SO(10) theory in four dimensions with the number of quark and lepton generations, $2|n|$, governed by the monopole strength, n. (Stability at the classical level requires $|n| = 1$.) Supersymmetry is broken by the E_6 background. The gravitino, along with the hypermatter is massive. The E_7 gauge symmetry remains unbroken but all light fermions are neutral with respect to it.

*) Note that SU(2) and SU(3) have the same property. A model based on $SU(3) \times SU(2) \times U(1)$ is being considered.

II. THE MODEL

In six dimensions the following $N = 2$ supermultiplets exist [5]:

gravity	$e_\mu^{\ m}$, $\psi_{\mu L}^{A}$, $B_{\mu\nu}^{+}$	$\mu, m = 0,1,\ldots,5;\ A = 1,2$
hypermatter	$\psi_R^{\ a}$, ϕ^α	$a = 1,\ldots,2n;\ \alpha = 1,\ldots,4n$
Yang-Mills	A_μ , $\lambda_L^{\ A}$	
tensor	$B_{\mu\nu}^{-}$, $\chi_R^{\ A}$, σ .	

The antisymmetric tensors $B_{\mu\nu}^{+}$ and $B_{\mu\nu}^{-}$ are constrained to have self-dual and anti-self dual field strengths, respectively. In the model to be treated here they appear only through the combination $B_{\mu\nu}^{+} + B_{\mu\nu}^{-}$, which is unconstrained. The index $A = 1,2$ indicates that $\psi_{\mu L}$, λ_L and χ_R are doublets with respect to Sp(1). They are also Weyl spinors ψ_L or ψ_R with respect to SO(1,5). They belong to <u>real</u> representations of SO(1,5) x Sp(1) and are subject to reality conditions. In our model the Sp(1) symmetry is explicitly broken to U(1), which is gauged. The hyperinos ψ_R^a are neutral with respect to U(1) although their partners ϕ^α are charged. In general, the hyperscalars ϕ^α – with 2n complex or 4n real components – must parametrize a non-compact quarternionic Kahler manifold [6].

In our model this turns out to be Sp(456,1)/Sp(456) x Sp(1). The hypermatter belongs to the pseudoreal $\underset{\sim}{912}$ of Sp(456) and the fermions are subject to a reality condition of the symplectic-Majorana type *). The dimensionality, $4n = 4 \times 456$, of the hyper-Kahler manifold is determined here by the requirement that the gravitational anomaly of the type $\mathrm{tr}\, R^4$ should be absent. It can be shown that this happens only if the number, n, of hyperinos is related to the number, m, of gauginos by **)

$$n = m + 244 \qquad (2.1)$$

We take $m = 78 + 133 + 1$ corresponding to the Yang-Mills group $E_6 \times E_7 \times U(1)$. With n = 456 it is then natural to assign the hyperinos to the 912 of $E_7 \subset Sp(456)$. What is then truly remarkable is the complete cancellation of all gauge and gravitational anomalies which results (see Sec.III).

Locally supersymmetric couplings of the above multiplets have been given in Ref.5. In particular, the antisymmetric tensor was included in order to overcome the difficulties associated with the action principle for constrained antisymmetric tensors. In Ref.5 the entire hypersymmetry Sp(n) was gauged. Here we shall gauge only the subgroup E_7 with respect to which the 912 of Sp(456) is irreducible.

The Lagrangian is given by

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{4\kappa^2}R - \frac{1}{4}(\partial_\mu\sigma)^2 - \frac{\kappa^2}{12}e^{2\kappa\sigma}G_{\mu\nu\rho}^{\ 2} - \frac{1}{4}e^{\kappa\sigma}\left(\frac{1}{g_6^2}F_{6\mu\nu}^{I'2} + \frac{1}{g_7^2}F_{7\mu\nu}^{I2} + \frac{1}{g_1^2}F_{1\mu\nu}^{\ 2}\right) - \frac{1}{2}g_{\alpha\beta}(\phi)D_\mu\phi^\alpha D^\mu\phi^\beta - \frac{1}{8\kappa^4}e^{-\kappa\sigma}\left(g_7^2 C_{7i}^{I2} + g_1^2 C_{1i}^{\ 2}\right) + \text{fermion terms}^{*)} , \qquad (2.2)$$

where $g_{\alpha\beta}(\phi)$ is the quarternionic-Kahler metric, and the usual Riemannian $g_{\mu\nu}(x)$ is tacit. The gauge field strengths and coupling constants of $E_6 \times E_7 \times U(1)$ are denoted, respectively, by F_6, F_7, F_1 and g_6, g_7, g_1. The hyperscalar potential is constructed from

$$C_{7i}^{\ I} = A_\alpha^i(\phi)\,(T^I\phi)^\alpha \quad , \qquad i = 1,2,3$$

$$C_{1i} = A_\alpha^i(\phi)\,(T^3\phi)^\alpha - \delta_{i3} \quad , \qquad (2.3)$$

where $A_\alpha^i(\phi)$ denotes the Sp(1) part of the canonical connection on the hyper-Kahler manifold. The generators of $E_7 \times Sp(1)$ are denoted by T^I and $T^i = \frac{-i\sigma^i}{2}$. The field strength tensor $G_{\mu\nu\rho}$ includes the curl of the antisymmetric tensor $B_{\mu\nu}$ and the Chern-Simons forms of the gauge group, $E_6 \times E_7 \times U(1)$. In the notation of differential forms,

$$G = dB + \frac{1}{g^2}\,\mathrm{tr}\left(AdA - \frac{2}{3}A^3\right) \quad . \qquad (2.4)$$

The presence of the non-negative hyperscalar potential in (2.2) is important for the Minkowskian compactification of the theory [8].

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*) Other non-compact quarternionic Kahler manifolds are the symmetric spaces SU(n,2)/SU(n) x U(1) x Sp(1), SO(n,4)/SO(n) x SO(3) x Sp(1), G_2/SO(3) x Sp(1), F_4/Sp(3) x Sp(1), E_6/SU(6) x Sp(1), E_7/SO(12) x Sp(1), E_8/E_7 x Sp(1). The associated hyperinos would be assigned, respectively, to the real representation $(n,1)_1 + (\bar{n},1)_{-1}$, (n,2,1), (4,1), (14,1), (20,1), (32,1) and (56,1) of the appropriate isotropy group. A model based on E_7/SO(12) x Sp(1) has been considered in Ref.7, but is anomalous.

**) This condition (2.1) changes to $n = m + 273 - 29k$, if k, the number of antisymmetric $B_{\mu\nu}^{-}$ fields, does not equal unity. Theories with $k \neq 1$ would not have a covariant Lagrangian formulation, only covariant field equations. In this note we concentrate on $k = 1$.

*) See appendix for fermion terms.

III. ANOMALY CANCELLATION

In six-dimensional space-time the various anomalies are associated with quadrangle graphs rather than triangles as in four dimensions. Another important difference is that the Weyl spinors of SO(1,5) are pseudoreal rather than complex. If anomalies are to be cancelled then fields of both chiral types must be involved. In the model discussed here the fermions and their classification with respect to $SO(1,5) \times E_6 \times E_7 \times U(1)$ are as follows: gravitino $\psi_{\mu L} \sim (4_+,1,1)_1$; hyperino $\psi_R \sim (4_-,1,912)_0$; gaugino $\lambda_L \sim (4_+,78,1)_1 + (4_+,1,133)_1 + (4_+,1,1)_1$; tensorino $\chi_R \sim (4_-,1,1)_1$, where the suffices L and R correspond to the Weyl spinors 4_+ and 4_- , respectively. Because of the U(1) charge (indicated by a suffix) all these fields are complex except for ψ_R which is real (because the representation 912 of E_7 is pseudoreal).

The quadrangle anomalies are usefully codified in the form of an invariant polynomial.[9] P(F,R), which is an 8-form made from the Yang-Mills and curvature 2-forms. The coefficients in this polynomial receive contributions from the fermions (listed above).According to the work of Alvarez-Gaume and Witten[10] the various contributions are given by:

$$P(\psi_\mu) = \frac{5}{24} F_1^4 - \frac{19}{96} F_1^2 \,\mathrm{tr}\, R^2 + \frac{245}{5760} \,\mathrm{tr}\, R^4 - \frac{5\times 43}{4\times 5760} (\mathrm{tr}\, R^2)^2$$

$$-2P(\psi_R) = \frac{1}{24} \mathrm{Tr}_{912}\, F_7^4 + \frac{1}{96}\, \mathrm{Tr}_{912}\, F_7^2 \,\mathrm{tr}\, R^2 + \frac{912}{5760} \left(\mathrm{tr}\, R^4 - \frac{5}{4} (\mathrm{tr}\, R^2)^2\right)$$

$$-P(\chi_R) = \frac{1}{24} F_1^4 + \frac{1}{96} F_1^2 \,\mathrm{Tr}\, R^2 + \frac{1}{5760} \left(\mathrm{tr}\, R^4 - \frac{5}{4} (\mathrm{tr}\, R^2)^2\right)$$

$$P(\lambda_L) = \frac{1}{24} \left\{\mathrm{Tr}_{78}\, F_6^4 + 6\, \mathrm{Tr}_{78}\, F_6^2\, F_1^2 + 78\, F_1^4\right\}$$
$$+ \frac{1}{24} \left\{\mathrm{Tr}_{133}\, F_7^4 + 6\, \mathrm{Tr}_{133}\, F_7^2\, F_1^2 + 133\, F_1^4\right\} + \frac{1}{24} F_1^4$$
$$+ \frac{1}{96} \left\{\mathrm{Tr}_{78} F_6^2 + \mathrm{Tr}_{133} F_7^2 + (78+133+1)\, F_1^2 \right\} \mathrm{tr}\, R^2 + \frac{78+133+1}{5760} \left\{\mathrm{tr}\, R^4 - \frac{5}{4}(\mathrm{tr} R^2)^2\right\} \quad (3.1)$$

where the field strength 2-forms corresponding to O(1,5), E_6, E_7 and U(1) are denoted by R, F_6, F_7 and F_1, respectively. The traces involving R are in the six-dimensional representation of O(1,5) while the other traces are specified by a subscript to indicate the representation. In the case of ψ_R an extra factor of 1/2 is needed to take account of the reality condition on ψ_R.

Because E_6 and E_7 have no independent quartic invariant it is possible to make some simplifications[3]:

$$\mathrm{Tr}_{78}\, F_6^4 = \frac{1}{2} (\mathrm{Tr}_{27}\, F_6^2)^2 \ , \quad \mathrm{Tr}_{78}\, F_6^2 = 4\, \mathrm{Tr}_{27}\, F_6^2$$

$$\mathrm{Tr}_{133}\, F_7^4 = \frac{1}{6} (\mathrm{Tr}_{56}\, F_7^2)^2 \ , \quad \mathrm{Tr}_{133}\, F_7^2 = 3\, \mathrm{Tr}_{56}\, F_7^2$$

$$\mathrm{Tr}_{912}\, F_7^4 = \frac{31}{12} (\mathrm{Tr}_{56}\, F_7^2)^2, \quad \mathrm{Tr}_{912}\, F_7^2 = 30\, \mathrm{Tr}_{56}\, F_7^2 \ .$$

The total P then reduces to

$$P = -\frac{1}{16} (\mathrm{tr}\, R^2)^2 + \frac{1}{24} \mathrm{tr}\, R^2\, \mathrm{Tr}_{27}\, F_6^2 - \frac{1}{16} \mathrm{tr}\, R^2\, \mathrm{Tr}_{56}\, F_7^2 + 2\, \mathrm{tr}\, R^2\, F_1^2$$
$$+ \frac{1}{48} (\mathrm{Tr}_{27}\, F_6^2)^2 + \mathrm{Tr}_{27}\, F_6^2\, F_1^2 - \frac{3}{64} (\mathrm{Tr}_{56}\, F_7^2)^2 + \frac{3}{4} \mathrm{Tr}_{56}\, F_7^2 F_1^2 + 9\, F_1^4 \ . \quad (3.2)$$

The tr R^4 contributions have cancelled. The four factors O(1,5), E_6, E_7 and U(1) are involved in a rather symmetrical fashion and it proves advantageous to express P in the form

$$P = \sum_{j,k} \beta_{jk}\, G_j\, G_k \qquad (3.3)$$

where

$$G_1 = \frac{1}{4} \mathrm{tr}\, R^2 \ , \quad G_2 = \frac{1}{4} \mathrm{Tr}_{27}\, F_6^2, \quad G_3 = \frac{3}{8} \mathrm{Tr}_{56}\, F_7^2 \ , \quad G_4 = 3\, F_1^2 \qquad (3.4)$$

and

$$\beta_{jk} = \frac{1}{3} \begin{pmatrix} -3 & 1 & -2 & 4 \\ 1 & 1 & 0 & 2 \\ -2 & 0 & -1 & 1 \\ 4 & 2 & 1 & 3 \end{pmatrix} \ . \qquad (3.5)$$

The 4-forms G_k are closed; they can be expressed locally in terms of Chern-Simons forms $G_k = d\omega_k^0$. Since G_k is invariant, it follows that the gauge[9] variation $\delta\omega_k^0$ is itself closed, i.e. $\delta\omega_k^0 = d\omega_k^1$. It can be shown that 6-forms $G_j\, \omega_k$ satisfy the Wess-Zumino consistency conditions and hence that the potential anomalies corresponding to (3.3) would take the form

$$\omega_6^1 = \sum_{jk} \beta_{jk}\, G_j\, \omega_k^1 \ . \qquad (3.6)$$

However, the anomaly is defined only up to terms of the form $\delta(\Delta\mathcal{L})$, the gauge variation of a counterterm. Our aim now is to show that the expression (3.6) can be completely eliminated by a counterterm.

What counterterms are available? Following the approach of Green and Schwarz we shall consider those of the form

$$\Delta\mathcal{L} = \sum_j \gamma_j\, G_j\, B + \frac{1}{2} \sum_{jk} \epsilon_{jk}\, \omega_j^0\, \omega_k^0 \ , \qquad (3.7)$$

where B is the 2-form specific to this type of supergravity. The parameters γ_j and $\varepsilon_{jk} = -\varepsilon_{jk}$ are at our disposal. We shall suppose that B transforms according to

$$\delta B = -\sum_j \alpha_j \omega_j^1 \quad , \qquad (3.8)$$

where, again, the parameters α_j are free. The gauge variation of (3.7) is

$$\delta(\Delta\mathcal{L}) = -\sum_{jk} (\gamma_j \alpha_k - \varepsilon_{jk}) G_j \omega_k^1 \quad . \qquad (3.9)$$

If this is to cancel the expression (3.6) we must have $\beta_{jk} = \gamma_j\alpha_k - \varepsilon_{jk}$. But β is symmetric while ε is antisymmetric. Therefore

$$\varepsilon_{jk} = \tfrac{1}{2}(\gamma_j\alpha_k - \gamma_k \alpha_j)$$
$$\beta_{jk} = \tfrac{1}{2}(\gamma_j\alpha_k + \gamma_k\alpha_j) \quad . \qquad (3.10)$$

In other words, the model is free of gauge and gravitational anomalies if the matrix β given by (3.5) can be expressed in the form (3.10) with real parameters γ and α. This highly non-trivial requirement is indeed satisfied. One easily finds

$$\alpha_j = (3, 1, 1, 1) \quad , \quad \gamma_j = (-1/3, 1/3, -1/3, 1) \quad . \qquad (3.11)$$

The complete counterterm is given by

$$\Delta\mathcal{L} = \sum_j \gamma_j G_j B + \frac{1}{2}\sum_{jk} \gamma_j \omega_j^0 \alpha_k \omega_k^0 + \mathcal{L}(G + \Delta G) - \mathcal{L}(G)$$
$$= \left(-\frac{1}{12} \operatorname{tr} R^2 + \frac{1}{12} \operatorname{tr} F_6^2 - \frac{1}{8} \operatorname{tr} F_7^2 + 3 F_1^2\right) B$$
$$+ \frac{1}{2}\left(-\frac{1}{3}\omega_1^0 + \frac{1}{3}\omega_2^0 - \frac{1}{3}\omega_3^0 + \omega_4^0\right)\left(3\omega_1^0 + \omega_2^0 + \omega_3^0 + \omega_4^0\right)$$
$$+ \mathcal{L}(G + \Delta G) - \mathcal{L}(G) \, . \qquad (3.12)$$

where $\Delta G = 3\omega_1^0 - 3\omega_2^0 - 5/3\,\omega_3^0 + 2/3\,\omega_4^0$ and the G-containing part of $\mathcal{L}$ is

$$\mathcal{L}(G) = -\frac{\kappa^2}{12} e^{2\kappa\sigma} G_{\mu\nu\rho}^2 +$$
$$+ \frac{1}{24}\kappa^2 e^{\kappa\sigma} G_{\mu\nu\rho}\Big(\bar\psi^\lambda \gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\psi^\tau + 2\bar\psi^\lambda\gamma^{\mu\nu\rho}\gamma_\lambda\chi +$$
$$- \bar\chi\gamma^{\mu\nu\rho}\chi + \bar\lambda^{\hat I}\gamma^{\mu\nu\rho}\lambda^{\hat I} + \bar\psi^a\gamma^{\mu\nu\rho}\psi_a\Big) \quad , \qquad (3.13)$$

where $\hat I$ labels the adjoint representation of $E_6 \times E_7 \times U(1)$.

IV. COMPACTIFICATION

At the classical level the equations of motion admit a vacuum solution in which the six-dimensional geometry factorizes into the product of four-dimensional Minkowskian space-time with an internal 2-sphere. This compactification is driven by one of the gauge fields which assumes a monopole configuration on the 2-sphere. All other fields either vanish or assume constant values. There are many ways to pick out the non-vanishing gauge field since we have a 212-dimensional local symmetry $E_6 \times E_7 \times U(1)$. For the purposes of this note we shall embed the monopole in the algebra of E_6, leaving $E_7 \times U(1)$ unbroken. The E_6 symmetry will itself break to SO(10). This means that the final vacuum symmetry in four dimensions will be Poincaré $\times SU(2)_{\text{Kaluza-Klein}} \times SO(10) \times E_7 \times U(1)$.

One of our tasks in the following is to isolate the zero mode sector and show that it is free of anomalies (in the usual four-dimensional sense).

First consider the relevant equations of motion derived from the bosonic part of (2.2). Since the only non-vanishing vacuum fields are the scalar σ, a U(1) gauge field from E_6 and the metric tensor, all equations are trivially satisfied except for

$$\frac{1}{\kappa^2} R_{AB} = \frac{2}{g_6^2} e^{\kappa\sigma} (F_{AC}^{I'} F_B^{I'C}) + D_A\sigma D_B\sigma - \frac{1}{2\kappa} g_{AB} \Box\sigma \quad , \qquad (4.1)$$

$$\frac{1}{\kappa}\Box\sigma = \frac{1}{2g_6^2} e^{\kappa\sigma} (F_{AB}^{I'} F_{I'}^{AB}) - \frac{g_1^2}{4\kappa^4} e^{-\kappa\sigma} \quad , \qquad (4.2)$$

$$D_A(\sqrt{-g}\, e^{\kappa\sigma} F^{AB}) = 0 \quad , \qquad (4.3)$$

where g_6 and g_1 denote the E_6 and $U(1)$ coupling constants, respectively, and $16\pi\kappa^2$ is the four-dimensional gravitation coupling.

We look for a solution of the form $M^4 \times S^2$ with constant σ and with the E_6 Yang-Mills field representing a monopole. The potential 1-form can then be written as

$$A_{\pm} = \nu Q(\cos\theta \mp 1)\, d\varphi \quad , \tag{4.4}$$

where θ and φ are the usual polar co-ordinates on the internal S^2, Q is a generator of E_6 and ν is a number. It is necessary to use two co-ordinate patches and two expressions for the potential, one for the northern hemisphere and one for the southern. On the equator these potentials must be connected by a single-valued gauge transformation, $A_- = S^{-1} A_+ S + i\, S^{-1} dS$. A suitable mapping is $S(\varphi) = \exp(2i\nu Q\varphi)$. The important condition of single valuedness, $S(2\pi) = S(0)$, implies that $2\nu Q$ must be an integer.

We now identify Q with the $U'(1)$ factor in the subgroup $SO(10) \times U'(1)$ of E_6. It can be normalized such that the 27 of E_6 branches according to

$$27 = 16_{1/3} + 10_{-2/3} + 1_{4/3}$$

while the adjoint representation gives

$$78 = 45_0 + 1_0 + 16_{-1} + \overline{16}_1 \quad .$$

If the system were to contain any 27's of E_6 then it is clear that 2ν could only be an integer multiple of 3. However, our system contains only the adjoint representation. In effect our group is E_6/Z_3 and we conclude that 2ν may be any integer, $\nu = \frac{n}{2}$. The field strength 2-form $F = dA_{\pm}$ is then given by

$$F = \frac{n}{2}\, Q \sin\theta\, d\theta \wedge d\varphi \quad . \tag{4.5}$$

In an orthonormal basis the Ricci tensor reduces to

$$R_{\alpha\beta} = \delta_{\alpha\beta}/a^2, \qquad \alpha,\beta = 4,5 \quad , \tag{4.6}$$

where a denotes the radius of S^2. In this basis the field strength (4.5) is represented by the component,

$$F_{45}^{\,Q} = n/2a^2 \quad . \tag{4.7}$$

On substituting (4.6) and (4.7) into (4.1) and (4.2) one obtains the algebraic equations

$$\frac{1}{\kappa^2 a^2} = \frac{n^2 e^{\kappa\sigma}}{2a^4 g_6^2} \quad \text{and} \quad 0 = \frac{n^2 e^{\kappa\sigma}}{4a^4 g_6^2} - \frac{g_1^2 e^{-\kappa\sigma}}{4\kappa^4} \quad . \tag{4.8}$$

These equations fix the radius of S^2, $a^2 = 2\kappa^2 e^{\kappa\sigma}/g_1^2$, but they also imply a rather delicate constraint on the values of g_6 and g_1,

$$g_6 = \pm \frac{n}{2}\, g_1 \quad , \tag{4.9}$$

Otherwise no solution of the desired type would be possible.

The fermionic zero modes are contained among the gauginos of E_6 since these are the only fields that couple to the monopole background. Indeed, according to (4.7) and $\nu = \frac{n}{2}$ the coupling is of strength $\nu Q = \pm n/2$ on the $\overline{16}_1$ and 16_{-1} contained in the 78. The four-dimensional massless chiral fermions therefore comprise $2|n|$ families of SO(10) and no antifamilies. They belong to the $|n|$-dimensional irreducible representation of the Kaluza-Klein SU(2) symmetry. These fermions are neutral with respect to E_7 but they carry the original U(1) charge, half the families are positive and the other half negative - so that no U(1) anomaly arises. The remaining symmetry, $SU(2) \times SO(10)$, is of course anomaly-free in four dimensions.

All other fermionic modes, including those coming from the gravitino are massive. Supersymmetry is broken in this compactification.

The $U'(1)$ associated with the monopole background is broken. The mechanism explained by Witten[2)] makes the gauge field massive but leaves the associated current conserved up to anomalies. Hence the four-dimensional theory should contain an axion.

In the boson sector the four-dimensional theory must contain the graviton and the gauge vectors corresponding to $SU(2) \times SO(10) \times E_7 \times U(1)$. The E_7 vectors do not seem to have any interesting physical role at present. They are "drones" pairing with the E_7 gauginos which were needed to cancel the gravitational anomaly. They should at least be confined, perhaps make glueballs, etc. The $SO(10) \times U(1)$ vectors couple to quarks and leptons but the SU(2) Kaluza-Klein vectors will couple to these fermions only if $|n| \geq 1$. Unfortunately, we may be limited to $|n| = 1$ (two families) for reasons of classical stability[11)] *).

The question of classical stability needs to be further examined. We observe only that the massless scalar contained in σ will presumably develop a mass at the 1-loop level. Along with this, the expectation value $\langle\sigma\rangle$, should be fixed, and hence the radius of the internal sphere.

*) Other but less elegant anomaly-free models are: $E_6 \times Sp(1)$ with $81 = 78+3$ gauginos and 325 hyperino singlets; and $E_7 \times U(1)$ with $134=133+1$ gauginos plus $355=2\times133+2\times56$ hyperinos, 22 gaugino and 22 hyperino singlets.

Other open questions are: is the safe $E_6 \times E_7 \times U(1)$ in D = 6 somehow descended from the safe $E_8 \times E_8$ in D = 10? Are the counter-terms which were needed to attain gauge and gravitational symmetry, sufficient to compensate any supercurrent anomalies which may arise, Or are further (gauge-invariant) counter-terms needed? Is the theory 1-loop finite? Like the Green-Schwarz theory which is supposed to represent a limit of the superstring in 10 dimensions, we may conjecture that our model relates to a string with internal symmetry or a membrane theory *) in 6 dimensions [12]. This may motivate finiteness.

*) We express appreciation to Dr. T. Jayaraman for pointing out Ref.(12) to us.

APPENDIX

The fermion terms are [5] *)

$$
\begin{aligned}
\mathcal{L}_\psi = {} & \frac{1}{2}\, \bar\psi_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho + \frac{1}{2} \bar\chi \gamma^\mu D_\mu \chi + \frac{1}{2} \bar\lambda^{\hat I} \gamma^\mu D_\mu \lambda^{\hat I} + \\
& + \frac{1}{2} \bar\psi^a \gamma^\mu D_\mu \psi_a + \frac{1}{2} \kappa \bar\psi_\mu \gamma^\nu \gamma^\mu \chi\, \partial_\nu \sigma - \kappa \bar\psi_\mu{}^A \gamma^\nu \gamma^\mu \psi^a (D_\nu \phi^\alpha) V_{\alpha a A} \\
& + \frac{\kappa^2}{24} e^{\kappa\sigma} G_{\mu\nu\rho} \Big\{ \bar\psi^\lambda \gamma_{[\lambda} \gamma^{\mu\nu\rho} \gamma_{\tau]} \psi^\tau + 2\bar\psi_\lambda \gamma^{\mu\nu\rho} \gamma^\lambda \chi - \bar\chi \gamma^{\mu\nu\rho} \chi \\
& \qquad + \bar\lambda^{\hat I} \gamma^{\mu\nu\rho} \lambda^{\hat I} + \bar\psi^a \gamma^{\mu\nu\rho} \psi_a \Big\} \\
& - \frac{\kappa}{2\sqrt{2}} e^{\kappa\sigma/2} \Big\{ \bar\psi_\lambda \gamma^{\mu\nu} \gamma^\lambda \lambda^{\hat I} F_{\mu\nu}{}^{\hat I} + \\
& \qquad + \bar\chi \gamma^{\mu\nu} \lambda^{\hat I} F_{\mu\nu}{}^{\hat I} \Big\} \\
& + \frac{\kappa}{\sqrt{2}} e^{-\kappa\sigma/2} \Big\{ g_7 \bar\psi_\mu \gamma^\mu T^i \lambda^I C_7{}^{iI} + g_1 \bar\psi_\mu \gamma^\mu T^i \lambda C_1{}^i \\
& \qquad - g_7 \bar\chi T^i \lambda^I C_7{}^{iI} - g_1 \bar\chi T^i \lambda C_1{}^i - 2g_7 \bar\psi^a \lambda^{IA} V_{\alpha a A} (T^I \phi)^\alpha \\
& \qquad - 2g_1 \bar\psi^a \lambda^A V_{\alpha a A} (T^3 \phi)^\alpha \Big\} \quad .
\end{aligned}
$$

where the hyperscalars ϕ^α constitute a complex 912-vector of $Sp(456) \times U(1)$ charged with respect to U(1). For inner products of Sp(1) symplectic spinors, the contractions with ϵ_{AB} are always understood, e.g. $\bar\chi \gamma^{\mu\nu} \lambda^I = \bar\chi^A \gamma^{\mu\nu} \lambda^I_A$,

*) We use the signature (- + + + + +) here.

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