REFERENCE

IC/84/184

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
6121/84
27 NOV 1984
* LIBRARY *

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

d = 8 SUPERGRAVITY

Abdus Salam

and

E. Sezgin

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1984 MIRAMARE-TRIESTE

IC/84/184

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

d = 8 SUPERGRAVITY *

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy
and
Imperial College, London, England

and

E. Sezgin
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

SU(2) gauged N = 2 supergravity in d = 8 is constructed by generalized dimensional reduction of d = 11 supergravity on SU(2) group manifold. The relation between the field equations of the d = 8 and those of d = 11 supergravities is established. As a byproduct of this, it is shown that certain compactifications of d = 11 supergravity give rise to anti-de Sitter space-time $(AdS) \otimes S^4$ or $AdS \otimes CP^2$ (with or without SU(2) instanton) or $AdS \otimes S^2 \otimes S^2$ compactifications of d = 8 supergravity. The latter two solutions have no supersymmetry, while $AdS \times S^4$ has N = 0 or N = 1 supersymmetry.

MIRAMARE - TRIESTE
October 1984

* To be submitted for publication.

I. INTRODUCTION

In conventional super-Kaluza Klein theories [1] one way of obtaining chiral fermions in four dimensions is to start from a Weyl theory in even dimensions (d = 6,8,10) and coupling vector fields (fundamental or composite) to the fermions. For monopole or instanton solutions in the background, chiral fermions emerge after compactification to d = 4 [2,3].

In d = 6, an N = 2 chiral supergravity theory and its couplings to matter supermultiplets have already been constructed [3,4]. This theory has several unusual and welcome properties such as chiral fermions [3,5], Minkowski compactification and a mass scale which is not necessarily Planckian. Unfortunately, however, the theory is plagued with anomalies *). In d = 10, there are two chiral supergravities. The N = 1 chiral theory [7] is anomalous **) [8] moreover it has no compactifications. The N = 2 chiral theory [10], however, most remarkably, is anomaly free [8]. However, it is not known whether it can non-trivially compactify to d = 4 ***) [11].

For d = 8 not much is known. In this paper we wish to close this gap. At the beginning of our search we were especially motivated by two

*) There exists an N = 4 chiral theory (at the linearized level) in d = 6 which is anomaly free [6]. However, it has no manifestly Lorentz invariant action, and it is not known whether it compactifies to d = 4.

**) In this connection, see a recent paper of Green and Schwarz [9] who obtained the remarkable result that the type I superstring theory corresponding to supersymmetric ten-dimensional Yang-Mills theory coupled to N = 1 d = 10 supergravity may be anomaly free but only for SO(32) or $E_8 \times E_8$ Yang-Mills group.

***) The known compactifications are: $(AdS)_5 \otimes K^3$ [11], $(\text{Minkowski})_8 \times$ Tear drop [12] (i.e. a two dimensional topologically non-compact surface with finite area and U(1) isometry group).

facts. (a) The anomaly situation is far better in $d = 8$, compared to $d = 6, 10$. (The gravitational anomalies are absent [8] , gauge anomalies can be cured with relative ease [13].) (b) A chiral $d = 8$ Kaluza-Klein theory coupled to *minimal* $SU(2)$ Yang-Mills, compactifying on $(\text{Minkowski})_4 \times S^4$ with an $SU(2)$ instanton configuration on S^4 would certainly lead to chiral fermions in $d = 4$ [14]. Thus it was natural to ask if $d = 8$ supergravity theory could be constructed with an $SU(2)$ Yang-Mills sector. In this paper we answer this question in affirmative. Furthermore, we show that $SU(2)$ instanton induced compactification to $d = 4$ is possible. On the negative side, however, we do not find Minkowski compactification, nor do we find chiral fermions. These aspects may not persist in the future. After presenting our model, we shall have more to say on this in Sec. V.

In a sense, the problems which arise in $d = 8$ are no worse than those which arise in $d = 11$ [15] . One can regard an $d = 8$ theory as a non-trivial truncation of the $d = 11$ theory, which provides a $d = 8$ language for studying some of the $d = 11$ problems (such as compactification, or chirality) more conveniently.

Our construction and results can be summarized as follows. We begin by considering the field content of the $d = 8$, $N = 2$ theory which can be easily deduced from $d = 11$ and is given by [16]

$$(g_{\mu\nu},\ 2\psi_\mu\ ,\ 6\chi\ ,\ A_{\mu\nu\rho}\ ,\ 3A_{\mu\nu}\ ,\ 6A_\mu\ ,\ 7\phi\ )\ .$$

The spinors are pseudo-Majorana [17] and the bosons are real. The automorphism group of the superalgebra is $SU(2)$. With respect to this $SU(2)$, ψ^a_μ $(a = 1,2)$ is a 2-spinor, χ^a_i $(i = 1,2,3)$ is a vector-spinor, $A^i_{\mu\nu}$ is a triplet, and the vectors split into two triplets. Anticipating an $SL(3,R)/SO(3)$ coset structure for the scalars, [16] they must then transform as $5 + 1 + 1$ under $SU(2)$.

It is a formidable problem to construct the most general SU(2) gauged consistent interactions of the fields listed above , directly in $d = 8$. The reason is that it is not known, *in general*, how to couple

charged vector fields, or antisymmetric tensor fields to Yang-Mills gauge fields [18,19]. (One quickly runs into inconsistencies in the field equations.) Thus, we cannot proceed by ignoring the $d = 11$ supergravity, especially in view of the fact that a consistent and appropriate truncation of it may solve this problem. Now, no compactification of the $d = 11$ theory to $d = 8$ is known. (It may very well not exist.) However, according to a much neglected but remarkable result of Scherk and Schwarz [20] , this is not necessary for consistency, at least in the case when the internal space is a group manifold. This is so essentially because, according to what they called generalized dimensional reduction, one can introduce a special dependence on the internal coordinates for the fields so that the symmetries of the higher dimensional theory carry over to symmetries of the lower dimensional one, and the latter are sufficient to decouple ghosts and tachyons from the theory.

Starting in $d = 11$, by using Scherk-Schwarz generalized dimensional reduction scheme we in fact obtain an $SU(2)$ gauged $N = 2$ consistent supergravity in $d = 8$. Remarkably, the so called T-tensor [21,22,19] which is a certain function of the scalars, naturally arises in $d = 8$, and we find a potential which has exactly the same form as was found in $d = 7$ gauged supergravities [22,19] . This observation should be useful in uncovering the relation between $d = 11$ supergravity, and gauged $SO(8)$ supergravity of de Wit- Nicolai in four dimensions [21], where the T-tensor occurs.

Having obtained the $SU(2)$ gauged $d = 8$ theory, we then examine the field equations, and we establish a relation with the corresponding field equations in $d = 11$. [23] Using the relation from the compactifications of the $d = 11$ theory which involves a 3-dimensional internal space either as a factor in a product space or as a fibre-space in a seven dimensional fibre bundle [24] , we deduce various compactifications of the $d = 8$ theory to $d = 4$. ($AdS \times CP^2$ or $AdS \times S^4$ with/without $SU(2)$ instanton, and $AdS \times S^2 \times S^2$). This generalizes the results of Pope and Nilsson [25] , who established connections between the compactifications of the $d = 11$ supergravity and those which are obtained from it by torus compactification.

This paper is organized as follows. In Sec. II the $d = 11$ supergravity action is reduced on an $SU(2)$ group manifold to $d = 8$. In Sec. III, the generalized dimensional reduction of the supersymmetry transformation rules is carried out. In Sec.IV the field equations, and connection with $d = 11$ is discussed, various spontaneous compactification schemes are found. In Sec. V we remark further on our results, and open problems. The results for the $SU(2)$ gauged $N = 2$, $d = 8$ action and transformation rules are collected in the Appendix.

II. DIMENSIONAL REDUCTION OF $d = 11$ SUPERGRAVITY TO $d = 8$

(A) Bosonic Sector: The key point which simplifies dramatically the computation of dimensional reduction is to work in the tangent space [20]. The bosonic part of $d = 11$ supergravity Lagrangian [15] in the tangent space reads

$$\mathcal{L}_{\text{Bosonic}} = \frac{V}{4\kappa^2} R - \frac{V}{48} F_{ABCD} F^{ABCD} + \frac{2\kappa}{(144)^2} \varepsilon^{A_1 \cdots A_{11}} F_{A_1 \ldots} F_{\ldots} V_{\ldots A_{11}} \tag{1}$$

where $V = \det V_M^{\ A}$, and

$$F_{ABCD} = 4\partial_{[A} V_{BCD]} + 12\, \omega_{[AB}{}^{E} V_{CD]E} \tag{2}$$

Here $\partial_A = V_A^M \partial_M$, $M, N, \ldots = 0,1,\ldots 10$ are the curved indices, $A, B \ldots = 0,1,\ldots 10$ are the flat tangent space indices, and V_{ABC} is the torsion free spin connection defined below. The flat Lorentz metric is chosen to be $\eta_{AB} = (- + + \ldots +)$. Since this choice differs from that of Ref. [20], the definitions of various geometrical objects will be given here for the reader's convenience. The Riemann tensor and the curvature scalar are

$$R^A{}_{BMN} = \partial_M \omega_N{}^A{}_B + \omega_M{}^A{}_E\, \omega_N{}^E{}_B - M \leftrightarrow N \tag{3.a}$$

$$R = e^M_A e^N_C R^A{}_{BMN}\, \eta^{BC} \tag{3.b}$$

The spin connection is defined by

$$\omega_M{}^A{}_B = e_M{}^C \left(\eta^{AE} \eta_{BF}\, \Omega_{CE}{}^F - \eta^{AE} \eta_{CF}\, \Omega_{EB}{}^F + \Omega_{BC}{}^E \right), \tag{4}$$

where the Ricci relation coefficients are

$$\Omega_{AB}{}^C = \tfrac{1}{2} \left(e^M_A e^N_B - e^M_B e^N_A \right) \partial_M e_N^C \tag{5}$$

After some manipulations, the curvature scalar can now be written as [20]

$$R = \omega_{ABC}\, \omega^{CAB} + \omega_A \omega^A - 2 V^{-1} \partial_M \left(V V^M_A \omega^A \right), \tag{6}$$

where $\omega_A \equiv \eta^{BC} \omega_{BCA}$. Thus, the dimensional reduction of the curvature scalar amounts to dimensional reduction of the spin connection coefficients which are defined in terms of the vielbein and its inverse. At this point we adopt the ansatz of Scherk-Schwarz [20] for the vielbein given by

$$V_M{}^A = \begin{pmatrix} e^{-\kappa\phi/3} e_\mu^a & 0 \\ 2 e^{2\kappa\phi/3} A_\mu^\alpha L_\alpha^i & e^{2\kappa\phi/3} L_\alpha^i \end{pmatrix} \tag{7}$$

where $\mu, a = 0,1,\ldots,7$, $\alpha, i = 8,9,10$, and L_α^i is a unimodular matrix

$$\det L_\alpha^i = 1 \tag{8}$$

representing five scalars of the $d = 8$ theory. ϕ is the sixth scalar which arises in the gravity sector, and the exponentials in (7) are chosen in such a way that the $d = 8$ Hilbert - Einstein action has the canonical form. Most importantly, the objects defined in (7) have the following coordinate dependences [20];

$$e_\mu^a(x, y) = e_\mu^a(x) \tag{9.a}$$

$$A_\mu^\alpha(x,y) = \mathcal{U}^{-1\,\alpha}{}_\beta(y)\, A_\mu^\beta(x) \quad , \tag{9.b}$$

$$L_\alpha^i(x,y) = \mathcal{U}_\alpha{}^\beta(y)\, L_\beta^i(x) \quad ; \tag{9.c}$$

where x labels $d = 8$ space-time, while y refers to x_8, x_9 and x_{10} of $d = 11$ space-time. The matrix $U_\alpha^\beta(y)$ is an 3×3 invertible matrix, which is taken to be an SU(2) group element in this paper. Hence, [20]

$$f^\gamma{}_{\alpha\beta} = \mathcal{U}_\alpha^{-1\,\alpha'} \mathcal{U}_\beta^{-1\,\beta'} \left(\partial_{\beta'} \mathcal{U}_{\alpha'}{}^\gamma - \partial_{\alpha'} \mathcal{U}_{\beta'}{}^\gamma \right) \tag{10}$$

where $f^\gamma_{\alpha\beta}$ are proportional to the structure constants of SU(2),

$$f^\gamma{}_{\alpha\beta} = \frac{-1}{2\kappa}\, g\, \varepsilon_{\alpha\beta\gamma} \tag{11}$$

Here $\varepsilon_{\alpha\beta\gamma}$ is the usual invariant tensor of SU(2), g is a dimensionless arbitrary gauge coupling constant, and -2κ is introduced for later convenience. Note that although U is y-dependent, $f^\gamma_{\alpha\beta}$ is not. The proof of this can be found in standard textbooks on group theory (see for example Ref. [26]). The motivation for the ansatz described in (9) has been discussed extensively by Scherk and Schwarz [20] , and will not be repeated here. It suffices to recall that, the ansatz (a) preserves the number of physical modes, (b) allows the factorization of the y-dependence in the transformation laws, (c) ensures the absence of ghost particles in the reduced theory. The authors of Ref. [20] also require a vanishing cosmological term in the reduced theory, something which cannot be achieved if $f^\gamma_{\alpha\beta}$ represents the structure constants of a semi-simple Lie group .This leads them to consider the so called flat groups [20] , which in the present case is nothing but the 2-dimensional Poincaré group i.e. rotation in the $y_1 - y_2$ plane, and the two translations in the y_1 and y_2 directions. [27] . In this paper, we do not require a vanishing

cosmological constant, since the reduced theory is not in $d = 4$, but in $d = 8$ *). We now proceed to calculate the $d = 8$ theory with the local SU(2) symmetry. First, it is useful to know the inverse vielbein given by

$$V^M{}_A = \begin{pmatrix} e^{\kappa\phi/3} e^\mu{}_a & -2\kappa e^{\phi/3} e^\mu_a A_\mu^\alpha \\ 0 & e^{-2\kappa\phi/3} L^\alpha{}_i \end{pmatrix} \tag{12}$$

where

$$L_\alpha^i L_j^\alpha = \delta_j^i \quad , \quad L_\alpha^i L_\beta^j \delta_{ij} = g_{\alpha\beta} \; , \quad L_\alpha^i L_\beta^j g^{\alpha\beta} = \delta^{ij} \tag{13}$$

In fact, L_α^i can be considered as a representative of the SL(3,R)/SO(3) coset. Using Eqs. (7)-(12) and the definition of ω_{CAD}, it is straight-forward to compute the components of ω_{CAB}, as projected to $d = 8$. The result is [20] ,

$$\omega_{cab} = e^{\kappa\phi/3} \left(\omega'_{cab} - \frac{\kappa}{3} \eta_{ca} \partial_b \phi + \frac{\kappa}{3} \eta_{cb} \partial_a \phi \right) \tag{14.a}$$

$$\omega_{abi} = \kappa e^{4\kappa\phi/3} F_{abi} \tag{14.b}$$

*) Since, one looks for the spontaneous compactification of the $d = 8$ theory down to $d = 4$, it may even be necessary to start with a non-vanishing cosmological constant in $d = 8$. Moreover, even if the resulting $d = 4$ theory does have a non-vanishing cosmological constant of AdS variety, a stable field theory can still be set up in such backgrounds as has been shown by Breitenlohner and Freedman [28]. Of course, a large cosmological constant would still be unwanted, unless one deals with preon theory, or invents a mechanism which would flatten the AdS space to Minkowskian one.

$$\omega_{aij} = e^{\kappa\phi/3} Q_{aij} \tag{14.c}$$

$$\omega_{iab} = -\kappa e^{4\kappa\phi/3} F_{abi} \tag{14.d}$$

$$\omega_{ija} = e^{\kappa\phi/3} \left(P_{aij} + \tfrac{2}{3}\kappa\, \delta_{ij}\, \partial_a \phi \right) \tag{14.e}$$

$$\omega_{kij} = \frac{-1}{4\kappa} g\, e^{-2\kappa\phi/3} \left(-\varepsilon_{kin} T_j{}^n + \varepsilon_{ijn} T_k{}^n - \varepsilon_{jkn} T_i{}^n \right) \tag{14.f}$$

where ω°_{cab} is the torsion free spin connection in $d = 8$, and $F^i_{ab} = e^\mu_a e^\nu_b F^i_{ab}$ and

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g\, \varepsilon_{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu \tag{15}$$

Furthermore, $P_{aij} = e^\mu_a P_{\mu ij}$ and $Q_{aij} = e^\mu_a Q_{\mu ij}$ are defined as follows:

$$L^\alpha_i \left(\partial_\mu \delta^\beta_\alpha + g\, \varepsilon_{\alpha\beta\gamma} A^\gamma_\mu \right) L_{\beta j} = P_{\mu ij} + Q_{\mu ij} \, , \tag{16}$$

where $P_{\mu ij}$ is the symmetric and traceless, while $Q_{\mu ij}$ is the antisymmetric part of the left-hand side of Eq.(16), which can be considered to describe the Cartan decomposition of the SL(3,R)/SO(3) coset. P_μ and Q_μ, of course, depend on the scalars parametrizing this coset, as well as the SU(2) gauge fields A^α_μ. A further definition is that [22,19]

$$T^{ij} = L^i_\alpha L^j_\beta\, \delta^{\alpha\beta} \tag{17}$$

Note that the contraction is with the Kronecker $\delta^{\alpha\beta}$. Substitution of the Latin connections given in (14) into the curvature scalar given in (6)*) now yields the results:

*) $I_B = \int d^8x \int \frac{d^3y}{\rho} \mathcal{L}_B$ where $\rho = \int d^3y U(y)$. See Ref. [20] for details on this point.

$$\int d^{11}x\, R = \int d^8x\, e \Big[R - \kappa^2 e^{2\kappa\phi} F^\alpha_{\mu\nu} F^{\mu\nu\beta} \delta_{\alpha\beta} - \kappa^2 P_{\mu ij} P^{\mu ij}$$
$$- 7\kappa^2 (\partial_\mu \phi)^2 - \tfrac{1}{4} \kappa^2 g^2 e^{-2\kappa\phi} \left(T_{ij} T^{ij} - \tfrac{1}{2} T^2 \right) \Big] \, , \tag{18}$$

where $T = T^{ij}\delta_{ij}$, $e = \det e^a_\mu$, and $R = R^{\ \lambda}_{\mu\lambda\nu} g^{\mu\nu}$. It is remarkable that the potential has the exact form that was found in Refs. [22,19] in the case of SO(4) gauged $d = 7$ supergravity theory. The suggests a close relation between the $d = 7$ theory of Ref. [22], and the $d = 11$ supergravity [15].

Turning now to the second term in Eq.(1), firstly it is convenient to make the following field redefinitions*)

$$B_{\mu\nu\rho} = e^{-\kappa\phi} e^a_\mu e^b_\nu e^c_\rho V_{abc} \tag{19.a}$$

$$B_{\mu\nu\alpha} = e^a_\mu e^b_\nu L^i_\alpha V_{abi} \tag{19.b}$$

$$B_{\mu\alpha\beta} = e^{\kappa\phi} e^a_\mu L^i_\alpha L^j_\beta V_{aij} \equiv \varepsilon_{\alpha\beta\gamma} B_{\mu\gamma} \tag{19.c}$$

$$B_{\alpha\beta\gamma} = e^{\kappa\phi} L^i_\alpha L^j_\beta L^k_\gamma V_{ijk} \equiv \varepsilon_{\alpha\beta\gamma} B \tag{19.d}$$

Similarly, the field stregths in the $d = 8$ curved space can be defined as

$$G_{\mu\nu\rho\sigma} = e^{-4\kappa\phi/3} e^a_\mu e^b_\nu e^c_\rho e^d_\sigma F_{abcd} \tag{20.a}$$

$$G_{\mu\nu\rho\alpha} = e^{-\kappa\phi/3} e^a_\mu e^b_\nu e^c_\rho L^i_\alpha F_{abci} \tag{20.b}$$

$$G_{\mu\nu\alpha\beta} = e^{2\kappa\phi/3} e^a_\mu e^b_\nu L^i_\alpha L^j_\beta F_{abij} \equiv \varepsilon_{\alpha\beta\gamma} G_{\mu\nu\gamma} \tag{20.c}$$

$$G_{\mu\alpha\beta\gamma} = e^{5\kappa\phi/3} e^a_\mu L^i_\alpha L^j_\beta L^k_\gamma F_{aijk} \equiv \varepsilon_{\alpha\beta\gamma} \partial_\mu B \tag{20.d}$$

*) Note that with each $e^a_\mu (L^i_\alpha)$ a factor of $e^{-\kappa\phi/3} (e^{2\kappa\phi/3})$ is associated.

Using (19) and (20) in the definition of F_{ABCD} given in (2), one finds

$$G_{\mu\nu\rho\sigma} = (\partial_\mu B_{\nu\rho\sigma} + 3 \text{ perms}) + (2\kappa F^{\alpha}_{\mu\nu} B_{\rho\sigma\alpha} + 5 \text{ perms}) \tag{21.a}$$

$$G_{\mu\nu\rho\alpha} = \mathcal{D}_\mu B_{\nu\rho\alpha} + 2\kappa \varepsilon_{\alpha\beta\gamma} F^{\beta}_{\mu\nu} B_{\rho\gamma} + 2 \text{ perms} \tag{21.b}$$

$$G_{\mu\nu\alpha} = \mathcal{D}_\mu B_{\nu\alpha} - \mathcal{D}_\nu B_{\mu\alpha} + 2\kappa F^{\alpha}_{\mu\nu} B + \tfrac{1}{\kappa} g B_{\mu\nu\alpha} \tag{21.c}$$

An example of the covariant derivative is

$$\mathcal{D}_\mu B_{\nu\rho\alpha} = \partial_\mu B_{\nu\rho\alpha} - 2\kappa f^{\gamma}{}_{\alpha\beta} A^{\beta}_{\mu} B_{\nu\rho\gamma} \tag{22}$$

Note that the "F B" terms in (21), i.e. the analogues of Chern Simons forms, arise from the connection term in (2). Using Eqs. (20.c), (20.d) and (21), it is straightforward to dimensionally reduce the second and third term of the bosonic action in Eq.(1). Together with the expression for R given in Eq.(18), the total bosonic Lagrangian of the $SU(2)$ gauged $d = 8$ supergravity is then obtained. The result is exhibited in the Appendix.

It is important to realize that the action is invariant under the ξ-transformations given by [20]

$$\delta e^{a}_{\mu}(x) = 0 \tag{22.a}$$

$$\delta A^{\alpha}_{\mu}(x) = \frac{1}{2\kappa}\partial_\mu \xi^{\alpha}(x) + f^{\alpha}{}_{\beta\gamma}\xi^{\beta}(x) A^{\gamma}_{\mu}(x) \equiv \frac{1}{2\kappa}\mathcal{D}_\mu \xi^{\alpha}(x) \tag{22.b}$$

$$\delta L^{i}_{\alpha}(x) = f^{\gamma}{}_{\alpha\beta}\xi^{\beta}(x) L^{i}_{\gamma}(x) \tag{22.c}$$

$$\delta L^{\alpha}{}_{i}(x) = f^{\alpha}{}_{\beta\gamma}\xi^{\beta}(x) L^{\gamma}{}_{i}(x) \tag{22.d}$$

These laws are obtained from the general coordinate transformation of V^{A}_{M} in $d = 11$, by using (7) and (9), and by assuming that $\xi^{\alpha}(x,y) = U^{-1\alpha}{}_{\beta}(y)\,\xi^{\beta}(x)$ $\xi^{\mu}(x,y) = \xi^{\mu}(x)$. Note that the y-dependence has

indeed factored out in the transformation laws, as promised.

The action is also invariant under the antisymmetric gauge transformations originating from the $d = 11$ law ; $\delta V_{MNP} = 3\partial_{[M}\Lambda_{NP]}$. Expressing this law in tangent space, and performing field (and parameter) redefinition as in (19), these transformations in $d = 8$ become [20]

$$\delta B_{\mu\nu\rho} = \partial_\mu \Lambda_{\nu\rho} - 2\kappa F^{\alpha}_{\mu\nu}\Lambda_{\rho\alpha} + 2 \text{ perms} \tag{23.a}$$

$$\delta B_{\mu\nu\alpha} = \mathcal{D}_\mu \Lambda_{\nu\alpha} - \mathcal{D}_\nu \Lambda_{\mu\alpha} - 2\kappa \varepsilon_{\alpha\beta\gamma} F^{\beta}_{\mu\nu}\Lambda_{\gamma} \tag{23.b}$$

$$\delta B_{\mu\alpha} = \mathcal{D}_\mu \Lambda_\alpha - \tfrac{1}{\kappa} g \Lambda_{\mu\alpha} \tag{23.c}$$

where all the fields and parameters depend only on x . Again the y-dependence has factorized.

In closing this section, we note that the field strengths defined in (21) are invariant under ξ-transformations, and covariant under Λ-transformations [20]; moreover they satisfy the following Bianchi identities:

$$\mathcal{D}_{[\lambda} G_{\mu\nu\rho\sigma]} = 4\kappa F^{\alpha}_{[\lambda\mu} G_{\nu\rho\sigma]\alpha} \tag{24.a}$$

$$\mathcal{D}_{[\lambda} G_{\mu\nu\rho]\alpha} = 3\kappa \varepsilon_{\alpha\beta\gamma} F^{\beta}_{[\lambda\mu} G_{\nu\rho]\gamma} \tag{24.b}$$

$$\mathcal{D}_{[\lambda} G_{\mu\nu]\alpha} = 2\kappa F^{\alpha}_{[\lambda\mu}\partial_{\nu]} B - \tfrac{1}{3\kappa} g G_{\lambda\mu\nu\alpha} \tag{24.c}$$

These equations will be useful in the derivation of the field equations, in Sec. IV.

(B) <u>Fermionic Sector</u>: The complete fermionic part of the $d = 11$ supergravity Lagrangian [15] is given by

$$\mathcal{L}_{Fermionic} = \tfrac{1}{2} V \bar{\eta}_C \Gamma^{CAB} D_A \eta_B + \frac{\kappa V}{96}\left(\bar{\eta}_E \Gamma^{EFABCD}\eta_F + 12\,\bar{\eta}^{A}\Gamma^{BC}\eta^{D}\right) F_{ABCD} +$$

$$+ \frac{\kappa^2}{16}\, \bar\eta^A \Gamma^B \eta^C \left(\bar\eta_A \Gamma_B \eta_C - 2 \bar\eta_B \Gamma_C \eta_A - 4 \eta_{AB}\, \bar\eta_C \Gamma_E \eta^E + \frac{3}{2} \bar\eta_E \Gamma^{EFABC} \eta_F \right) -$$

$$- \frac{\kappa^2}{32}\, \bar\eta^A \Gamma^{CD} \eta^B \left(\bar\eta^A \Gamma^{CD} \eta^B + 4 \bar\eta^A \Gamma^{BC} \eta^D - \bar\eta^C \Gamma^{AB} \eta^D + \frac{1}{2} \bar\eta_E \Gamma^{EFABCD} \eta_F \right) -$$

$$- 2 V^{-1} \partial_M \left(V V^M{}_A\, \bar\eta_E \Gamma^E \eta^A \right) \qquad (25)$$

where η_A denotes the gravitino field, and <u>all</u> the quartic fermion terms have been explicitly exhibited. The covariant derivative in (25) is [20]

$$D_A \eta_B = \partial_A \eta_B + \frac{1}{4} \omega_{AEF} \Gamma^{EF} \eta_B + \omega_{AB}{}^E \eta_E \qquad (26)$$

with torsion free spin connection ω_{ABC} as defined in (4). The virtue of this covariant derivative is that it commutes with the Γ-matrices: $[D_A, \Gamma_B] = 0$. The field strength F_{ABCD} is defined in (2). The Γ-matrices obey the algebra

$$\{ \Gamma_A, \Gamma_B \} = 2 \eta_{AB} = 2\, \mathrm{diag}\,(-+++++++) \qquad (27)$$

A convenient representation for these Γ-matrices is

$$\Gamma^a = \gamma^a \times \mathbb{1} \qquad (28.a)$$

$$\Gamma^i = \gamma_9 \times \sigma^i \;, \quad \gamma_9 = i\gamma^0\gamma^1 \ldots \gamma^7 \;, \quad \gamma_9^2 = 1 \qquad (28.b)$$

Starting with the gravitino kinetic term, one soon discovers that there is a mixing between ψ_a and $\Gamma^i \psi_i$. [29]. To bring the fermion kinetic terms into canonical form the following field redefinition is required

$$\eta_a = e^{\kappa\phi/6} \left(\psi_a - \frac{1}{6} \Gamma_a \Gamma^i \chi_i \right)$$
$$\eta_i = e^{\kappa\phi/6} \chi_i \qquad (29)$$

In general, the dimensional reduction of $d = 11$ supergravity to a dimension D would involve the field redefinition

$$\eta_a = e^{\kappa\phi/(D-2)} \left(\psi_a - \frac{1}{(D-2)} \Gamma_a \Gamma^i \chi_i \right)$$
$$\eta_i = e^{\kappa\phi/(D-2)} \chi_i \qquad (30)$$

The field redefinitions given in Eq. (29), and the components of the spin connection given in Eq.(14), upon substitution into Eq.(26) yield the "field strengths"

$$D_{[a} \eta_{b]} = e^{\kappa\phi/2} \Big[D_a \psi_b - \frac{1}{2} e^{\kappa\phi} F_{aci} \Gamma^i \Gamma^c \psi_b + \frac{1}{6} \Gamma_a \Gamma^i D_b \chi_i$$
$$+ \frac{1}{12} e^{\kappa\phi} F_{aci} \left(\Gamma^i \Gamma^j \Gamma^c \Gamma_b + 12\, \delta^{ij} \delta^c_b \right) \chi_j - \frac{\kappa}{6} (\partial_c \phi) \Gamma^a \Gamma^c \left(\psi_b - \frac{1}{6} \Gamma_b \Gamma^i \chi_i \right) \Big] \qquad (31.a)$$

$$D_{[i} \eta_{j]} = e^{\kappa\phi/2} \Big[-\frac{\kappa}{4} e^{\kappa\phi} F_{abi} \Gamma^{ab} \chi_j + \frac{1}{2} P_{ai\ell} \Gamma^\ell \Gamma^a \chi_j + \qquad (31.b)$$
$$+ \frac{\kappa}{3} (\partial_a \phi) \Gamma_i \Gamma^a \chi_j - \frac{1}{16\kappa} g e^{-\kappa\phi} \left(2 \varepsilon_{ikn} T_\ell{}^n + \varepsilon_{k\ell n} T_i{}^n \right) \left(\Gamma^{k\ell} \delta^{jm} + 4 \delta^{kj} \delta^{\ell m} \right) \chi_m \Big]$$

$$D_a \eta_i - D_i \eta_a = e^{\kappa\phi/2} \Big[D_a \chi_i + \frac{\kappa}{4} e^{\kappa\phi} F_{cdi} \Gamma^{cd} \psi_a + \frac{1}{3} (\partial_c \phi) \Gamma^c \Gamma^i \psi_a - \qquad (31.c)$$
$$- \frac{\kappa}{24} e^{\kappa\phi} F_{cdj} \left(\Gamma^{cd} \Gamma_a \delta^j_i \Gamma^k + 12\, \Gamma^c \delta^d_a \delta^k_i \Gamma^j \right) \chi_k - \frac{1}{2} P_{cij} \Gamma^j \Gamma^c \psi_a +$$
$$+ \frac{1}{12} P_{cij} \left(\Gamma^j \Gamma^k \Gamma^c \Gamma_a + 12\, \delta^{jk} \delta^c_a \right) \chi_k + \frac{1}{16\kappa} g e^{-\kappa\phi} \left(-2 \varepsilon_{ijn} T_k{}^n + \varepsilon_{jkn} T_i{}^n \right) \Gamma^{jk} \times$$
$$\times \left(\psi_a - \frac{1}{6} \Gamma_a \Gamma^\ell \chi_\ell \right) + \frac{1}{18} \kappa (\partial_c \phi) \left(\Gamma^i \Gamma^j \Gamma^c \Gamma_a + 15\, \delta^{ij} \delta^c_a + 3 \Gamma^{ca} \delta^{ij} \right) \chi_j \Big]$$

Antisymmetrization in ab and ij on the right-hand sides of (31.a) and (31.b), respectively is understood. The Lorentz $\otimes$ SU(2) covariant derivatives are

$$D_a \psi_b = \partial_a \psi_b + \frac{1}{4} \omega_{acd} \Gamma^{cd} \psi_b + \omega_{ab}{}^{c} \psi_c + \frac{1}{4} Q_{aij} \Gamma^{ij} \psi_b$$

$$D_a \chi_i = \partial_a \chi_i + \frac{1}{4} \omega_{acd} \Gamma^{cd} \chi_i + Q_{ai}{}^{j} \chi_j + \frac{1}{4} Q_{akl} \Gamma^{kl} \chi_i \tag{32}$$

$P_{\mu ij}$ and $Q_{\mu ij}$ are defined in (16). Defining $\psi_\mu = e_\mu^a \psi_a$, and using (31) and (20), from the first three terms in (25), after considerable algebra one obtains the complete fermionic Lagrangian in $d = 8$, modulo the quartic fermion terms. The result is exhibited in the Appendix. To obtain the quartic fermion terms one must insert (29) into the $d = 11$ quartics, all of which are given explicitly in (25). This is a straightforward but extremely tedious process, and as it will not be needed in what follows we shall skip it.

III. DIMENSIONAL REDUCTION OF SUPERSYMMETRY TRANSFORMATION LAWS

In this section, in addition to the supersymmetry transformation laws, it also is important to consider the Lorentz transformations. The reason is that in Eq.(8), a triangular gauge, $V_\alpha^a = 0$, has been chosen which breaks the $SO(1,10)$ Lorentz group to $SO(1,7) \times SO(3)$. In order to preserve this gauge the off-diagonal part of the Lorentz group has to be fixed [30]. This will affect the supersymmetry transformation laws of all those fields which transform under the Lorentz group.

To begin, the combined supersymmetry and Lorentz transformation rules of the $d = 11$ theory are [15]

$$e^M_B\, \delta e_{MA} = -\kappa \bar{\xi} \Gamma_A \eta_B + \lambda_{AB} \tag{33.a}$$

$$\delta V_{ABC} = -\frac{3}{2} \bar{\xi} \Gamma_{[AB} \eta_{C]} - 3 V_{E[AB} e^M_{C]} \delta e_M^E \tag{33.b}$$

$$\begin{aligned}\delta \eta_A = {} & \frac{1}{\kappa} D_A \xi - \frac{1}{144} \left(\Gamma^{CDEF}{}_A - 8 \Gamma^{CDE} \delta_A^F \right) F_{CDEF}\, \xi + \\ & + \frac{\kappa}{8} \left(\bar{\eta}_E \Gamma_A \eta_F - 2 \bar{\eta}_A \Gamma_F \eta_E \right) \Gamma^{EF} \xi + \frac{\kappa}{48} \bar{\eta}_{[C} \Gamma_{DE} \eta_{F]} \left(\Gamma^{CDEF}{}_A - 8 \Gamma^{CDE} \delta_A^F \right) \xi + \\ & + e^M_A (\delta e_M^B) \eta_B + \frac{1}{4} \lambda_{CD} \Gamma^{CD} \eta_A \end{aligned} \tag{33.c}$$

where ξ is the supersymmetry parameter, $D_A \xi$ is the torsion free covariant derivative, and $\lambda_{AB} = -\lambda_{BA}$ is the $SO(1,10)$ Lorentz group parameter. To put the gravitino transformation law into canonical form, ξ must be redifined,

$$\xi = e^{\kappa \phi / 6} \varepsilon \tag{34}$$

The ab component of (33.a) now reads

$$e^\mu_b\, \delta e_{\mu a} = -\bar{\varepsilon} \gamma_a \psi_b + \lambda'_{ab} \quad , \tag{35}$$

where Eqs.(7), (12), (29) and (34) have been used, and $\lambda'_{ab} = -\lambda'_{ba}$ is the redefined local $SO(1,7)$ Lorentz transformation parameter [29,31,32] ,

$$\lambda'_{ab} = \lambda_{ab} - \frac{i}{6} \bar{\varepsilon} \gamma_{ab} \sigma^i \chi_i \tag{36}$$

The ia projection of (33.a) tells us that in order to preserve the triangular gauge $V_\alpha^a = 0$, the Lorentz parameter λ_{ia} must be fixed as

$$\lambda_{ia} = -\bar{\varepsilon} \gamma_a \chi_i \tag{37}$$

The ai projection of (33.a) gives $L_\alpha^i \delta A_\mu^\alpha$ which is exhibited in the Appendix. Considering the ij projection of (33.a) and tracing with δ^{ij} one finds $\delta\phi$ which is given in the Appendix. Finally the part of (33.a) which is traceless in ij reads

$$L^\alpha_i\, \delta L_{\alpha j} = \frac{i}{2} \bar{\varepsilon} \left(\sigma_i \delta_j^k + \sigma_j \delta_i^k - \frac{2}{3} \sigma^k \delta_{ij} \right) \chi_k + \lambda'_{ij} \tag{38}$$

where the first part is symmetric and traceless in ij, thus it lies in the $SL(3,R)/SO(3)$ coset, while $\lambda'_{ij} = -\lambda'_{ji}$ is the redifined composite local $SO(3)$ parameter,

$$\lambda'_{ij} = \lambda_{ij} - \frac{i}{2}\left(\bar{\varepsilon}\sigma_i \chi_j - \bar{\varepsilon}\sigma_j \chi_i\right) \tag{39}$$

Turning to (33.b) and recalling the field redefinitions given in (23), (29) and (34), a straightforward algebra yields the supersymmetry transformation rules for B, $B_{\mu\alpha}$, $B_{\mu\nu\alpha}$ and $B_{\mu\nu\rho}$. The results are given in Appendix. Note that the first terms on the right-hand side of $\delta B_{\mu\alpha}$, $\delta B_{\mu\nu\alpha}$ and $\delta B_{\mu\nu\alpha}$ have originated from the $e^M_a\,\delta e^i_M$ terms in (33.b).

As to the gravitino transformation law, again using Eqs.(23), (29) and (34), after some algebra, one finds the result for $\delta\psi_\mu$ and $\delta\chi_i$ which is given in the Appendix. The terms which are trilinear in the fermionic fields have not been given, and all of them can be derived from (i) the use of (29) and (34) in the explicitly cubic fermion terms in (33.c) (ii) the use of (7),(12),(33.a),(33.b) and (34) in the $e^M_A\,(\delta e^B_M)\,\eta_B$ term in (33.c) (iii) the use of (34), (36), (37) and (39) in the Lorentz transformation $\frac{1}{4}\lambda_{CD}\Gamma^{CD}\eta_A$, in (33.c).

IV. SPONTANEOUS COMPACTIFICATION OF D = 8 SUPERGRAVITY

We shall first give the field equations of the $d = 8$ theory, and then comment on their derivation. The bosonic field equations of the gauged $N = 2$ $d = 8$ supergravity are

$$\begin{aligned}\kappa^{-1} R_{\mu\nu} = {} & P^{ij}_\mu P^{ij}_\nu + 2\,\partial_\mu\phi\,\partial_\nu\phi + 2\,e^{2\kappa\phi} F^i_{\mu\lambda} F_\nu{}^{\lambda,i} - \frac{\kappa^{-1}}{3} g_{\mu\nu}\Box\phi \\ & + \frac{1}{3} e^{2\kappa\phi}\left(G_{\mu\lambda\tau\sigma} G_\nu{}^{\lambda\tau\sigma} - \frac{1}{12} g_{\mu\nu} G_{\lambda\tau\rho\sigma} G^{\lambda\tau\rho\sigma}\right) \\ & + \left(G^i_{\mu\lambda\tau} G_\nu{}^{\lambda\tau,i} - \frac{1}{9} g_{\mu\nu} G^i_{\lambda\tau} G^{\lambda\tau,i}\right) \\ & + 2\,e^{-2\kappa\phi}\left(G^i_{\mu\lambda} G_\nu{}^{\lambda,i} - \frac{1}{6} g_{\mu\nu} G^i_{\lambda\tau} G^{\lambda\tau,i}\right) \\ & + 2\,e^{-4\kappa\phi}\left(\partial_\mu B\,\partial_\nu B - \frac{1}{3} g_{\mu\nu} (\partial_\lambda B)^2\right)\end{aligned} \tag{40.a}$$

$$\begin{aligned}\kappa^{-1} \mathcal{D}_\mu P^{\mu\,ij} = {} & \frac{-2}{3}\kappa\,\Box\phi\,\delta^{ij} + e^{2\kappa\phi} F^i_{\mu\nu} F^{\mu\nu,j} + \frac{1}{2\kappa^4} g^2 e^{-2\kappa\phi}\Big[T^i{}_n T^j{}_n - \frac{1}{2} T\,T^{ij} - \\ & - \frac{1}{2}\delta_{ij}\left(T_{mn}T^{mn} - \frac{1}{2}T^2\right)\Big] - \frac{1}{3}\left(G_{\mu\nu\rho}{}^i G^{\mu\nu\rho,j} - \frac{1}{3}\delta^{ij} G_{\mu\nu\rho,k} G^{\mu\nu\rho,k}\right) + \\ & + e^{-2\kappa\phi}\left(G^i_{\mu\nu} G^{\mu\nu,j} - \frac{2}{3}\delta^{ij} G^k_{\mu\nu} G^{\mu\nu,k}\right) + \\ & - \frac{4}{3} e^{-4\kappa\phi} (\partial_\mu B)^2 \delta^{ij} + \frac{1}{36} e^{2\kappa\phi} G_{\mu\nu\rho\sigma} G^{\mu\nu\rho\sigma}\delta^{ij} \quad .\end{aligned} \tag{40.b}$$

$$\begin{aligned}\kappa\,\mathcal{D}_\mu\left(e^{2\kappa\phi} F^{\mu\nu,i}\right) = {} & -\kappa\,e^{2\kappa\phi} P^{ij}_\mu F^{\mu\nu,j} - g\kappa^{-1} g^{\mu\nu}\varepsilon^{ijk} P^{jl}_\nu T_{kl} + \\ & + \frac{1}{3} e^{2\kappa\phi} G^{\nu\rho\sigma\tau} G_{\rho\sigma\tau}{}^i - \varepsilon^{ijk} G^{\nu\rho\sigma}{}_j G_{\rho\sigma,k} + 2\,e^{-2\kappa\phi} G^{\nu\rho,i}\partial_\rho B\end{aligned} \tag{40.c}$$

$$\begin{aligned}\mathcal{D}_\mu\left(e^{2\kappa\phi} G^{\mu\nu\rho\sigma}\right) = \frac{-1}{12}\varepsilon^{\nu\rho\sigma\nu_1\cdots\nu_5}\Big(& G_{\nu_1\cdots\nu_4}\partial_{\nu_5} B + \\ & + 2\,G^i_{\nu_1\nu_2\nu_3} G^i_{\nu_4\nu_5}\Big)\end{aligned} \tag{40.d}$$

$$\begin{aligned}L^i_\alpha\,\mathcal{D}_\mu G^{\mu\nu\rho\alpha} = \frac{-1}{72}\varepsilon^{\nu\rho\nu_1\cdots\nu_6}\Big(& 3\,G_{\nu_1\cdots\nu_4} G^i_{\nu_5\nu_6} - 2\,\varepsilon^{ijk} G_{\nu_1\nu_2\nu_3,j} G_{\nu_4\nu_5\nu_6,k}\Big) + \\ & + e^{2\kappa\phi} G^{\nu\rho\lambda\tau} F^i_{\lambda\tau} - \frac{1}{2} g\kappa^{-1} e^{-2\kappa\phi} T^{ij} G^{\nu\rho,j}\Big)\end{aligned} \tag{40.e}$$

$$L^i_\alpha D_\mu (e^{-2\kappa\phi} G^{\mu\nu\lambda}) = \frac{1}{4!}\varepsilon^{\nu_1\cdots\nu_7} G_{\nu_1\cdots\nu_4} G^i_{\nu_5\nu_6\nu_7} +$$

$$+ \varepsilon^{ijk} G^{\nu\rho\sigma}{}_j F^k_{\rho\sigma} \tag{40.f}$$

$$D^\mu (e^{-4\kappa\phi} D_\mu B) = \frac{-1}{2\cdot 4!^2}\varepsilon^{\nu_1\cdots\nu_8} G_{\nu_1\cdots} G_{\cdots\nu_8} + \tag{40.g}$$

$$+ \frac{1}{3} e^{2\kappa\phi} G^{\mu\nu,i} F^i_{\mu\nu} .$$

The field equation for ϕ has been incorporated into that of L^i_α equation. Accordingly, the trace of (40.b) yields the ϕ-field equation. Eq.(40.a) is obtained by varying the $d = 8$ action with respect to $g^{\mu\nu}$ and eliminating the potential term by using the ϕ-field equation. The variation $L^\alpha_i \, \delta L_{\alpha j}$ which gives Eq.(40.b) is straightforward. The A^α_μ equation, (40.c), involves extremely cumbersome calculations, which have been simplified by using gauge invariance arguments. In deriving Eqs. (40.d) - (40.g), the Bianchi identities, Eqs.(24) have been used. The "GF" and "GT" terms in these equations arise from the variation of the Chern-Simon forms.

The form of the field equations suggests a close relation with those of $d = 11$ theory. In fact, a closer inspection of Eqs. (40.a)-(40.g) shows that they can be obtained directly from the field equations of $d = 11$ supergravity given by

$$R_{AB} = \frac{1}{3}\left(F_{ACDE} F_B{}^{CDE} - \frac{1}{12}\gamma_{AB} F_{CDEF} F^{CDEF}\right) \tag{41.a}$$

$$\partial_A F^{ABCD} - \omega_A F^{ABCD} + 3\omega_{AE}{}^{[B} F^{CD]AE} = \frac{-1}{(24)^2}\varepsilon^{BCDE_1\cdots E_8} F_{E_1\cdots} F_{\cdots E_8} \tag{41.b}$$

where $R_{AB} = V^M_A V^N_B R_{MN}$, $\partial_A = V^M_A \partial_M$. Projecting the free indices of Eq.(41) to $d = 8$, and using the definitions of y-independent fields given in Eqs. (19) and (20), the Latin spin connections, (14), one finds precisely the $d = 8$ field equations, (40.a)-(40.g). The GF and GT terms of Eqs.(40.a)-(40.g) are seen to arise from the spin connection terms of (41.b). Especially, it should be noted that the terms which do not contain the field strengths of the B-fields (i.e. the G-terms) in Eqs. (40.a), (40.b) and

(40.c) are precisely $R_{MN} V^M_a V^N_b e^a_\mu e^b_\nu e^{-2k\phi/3}$, $R_{MN} V^M_i V^N_j e^{-2k\phi/3}$ and $R_{MN} V^M_i V^N_a e^{\nu a} e^{-k\phi/3}$, respectively. One way to understand these connections is to relate the Euler-Lagrange variation of $d = 8$ action to that of $d = 11$. Denoting the latter by $\hat{I}$, the chain rule gives

$$V^\mu_a V^\nu_b \frac{\delta \hat{I}}{\delta g^{\mu\nu}} = V^\mu_a V^\nu_b \frac{\delta \hat{I}}{\delta g^{MN}} \frac{\delta g^{MN}}{\delta g^{\mu\nu}} = \left[\frac{\delta \hat{I}}{\delta \hat{g}^{\mu\nu}} e^{2\kappa\phi/3} - \right.$$

$$\left. -4\kappa\left(\frac{\delta \hat{I}}{\delta \hat{g}^{\mu\alpha}} A^\alpha_\nu + \mu \leftrightarrow \nu\right) + 4\kappa^2 e^{2\kappa\phi/3} \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha\beta}} A^\alpha_\mu A^\beta_\nu \right] V^\mu_a V^\nu_b$$

$$= e^{2\phi/3} V^M_a V^N_b \left(\frac{\delta \hat{I}}{\delta g^{MN}}\right) . \tag{42}$$

It should be emphasized that $\hat{I}^{d=11}(\phi(x,y)) = I^{d=8}(\phi(x))$ due to the special y-dependence the fields, generically denoted by $\phi(x,y)$, have assumed. Hence, Eq.(42) gives the relation between the $d = 8$ field equation $\delta\hat{I}/\delta g^{\mu\nu}$, and eleven dimensional one $\delta\hat{I}/\delta g^{MN}$. This argument can also be applied to the other field equations.

The connection with the $d = 11$ theory discussed above, also suggests that those compactifying solutions of the $d = 11$ supergravity which contain a three-dimensional manifold as a part of the seven-dimensional product space, or as a fibre space in a seven-dimensional fibre bundle, will also form a compactifying solution to the $d = 8$ gauged theory. For this to happen it is important that the SU(2) coupling constant g is non-vanishing. If $g = 0$, then the $d = 8$ theory is just the $d = 11$ supergravity dimensionally reduced on $S^1 \otimes S^1 \otimes S^1$. Pope and Nilsson [25] have argued that such a theory cannot compactify non-trivially to $d = 4$, for there is no $U(1) \otimes U(1) \otimes U(1)$ bundle over B^4 (a 4-dimensional internal manifold) for which all the eigenvalues of the Ricci tensor of B^4 are positive. In the $g = 0$ case, indeed we have not succeeded in solving the background field equations, in agreement with the result of Ref. [25]. Therefore, in the following g will always be taken to be non-vanishing.

The seven-dimensional homogeneous product spaces which compactify the $d = 11$ theory, à la Freund-Rubin [33], and which contain a three-dimensional factor, are $S^4 \otimes S^3$, $CP^2 \otimes S^3$ and $S^2 \otimes S^2 \otimes S^3$ [24]. These solutions have no supersymmetries, and they are unstable [34]. The seven dimensional fibre bundles with $SU(2)$ (or $SO(3)$) fibres which compactify the $d = 11$ theory à la Freund-Rubin are: (a) Round S^7 [33,35] which can be regarded as $SU(2)$ bundle over S^4, (b) Squashed S^7 [36] which is $SU(2)$ bundle over S^4 with squashed metric. (c) N^{pqr} spaces, which can be considered as $SO(3)$ bundle over CP^2 [37]. (d) Squashed N^{pqr} [37,38] which is again $SO(3)$ bundle over CP^2, but with squashed metric.

Thus the candidate compactifying solutions to $d = 8$ supergravity are: S^4, CP^2 or $S^2 \otimes S^2$ with no instantons; and S^4 or CP^2 with $SU(2)$ instanton. Although the compactifications without instantons would be descending from unstable solutions of the $d = 11$ theory, it is not ruled out that the unstable modes are truncated in the effective $d = 8$ theory. Therefore, compactifications with or without instantons will be considered below, though the emphasis will be on the first, in the light of chiral fermion problem, and the fact that all the $d = 11$ compactifications they are related to are stable [38]. We now proceed to examine those cases separately.

(A) S^4 Compactification:

It is a simple matter to check that the following ansatz is a solution of the background field equations, (40)*):

$$\mathrm{AdS}: \qquad R_{\mu\nu} = -12\, m^2 g_{\mu\nu} \tag{43.a}$$

$$S^4: \qquad R_{\bar\mu\bar\nu} = 6\, m^2 g_{\bar\mu\bar\nu} \tag{43.b}$$

*) In the rest of this section; $\mu = 0,\dots,3$, $\bar\mu = 4,\dots,7$, $i = 8,9,10$, $\Gamma^\mu = \gamma^\mu \otimes \mathbb{1}_4 \otimes \mathbb{1}_2$, $\Gamma^{\bar\mu} = \gamma_5 \otimes \gamma^{\bar\mu} \otimes \mathbb{1}$, $\Gamma^i = \gamma_5 \times \bar\gamma_5 \otimes \sigma^i$, $\gamma_5 = i\gamma^0\gamma^1\dots\gamma^3$, $\bar\gamma_5 = \gamma^4\dots\gamma^7$, $\gamma_5^2 = 1$, $\gamma_5^2 = 1$.

$$\kappa\phi = c \tag{43.c}$$

$$G_{\mu\nu\rho\sigma} = 3m\, e^{-c}\, \varepsilon_{\mu\nu\rho\sigma} \tag{43.d}$$

$$g = 4\sqrt{3}\, \kappa m\, e^{c} \tag{43.e}$$

where m and c are constants, and the unmentioned fields vanish Eqs.(43.a) and (43.b) follow from Einstein equation, while (43.e) is implied by the scalar field equation. Substitution of this ansatz into the supersymmetric variation of χ_i, given in the Appendix, yields the result

$$\delta\chi_i = \frac{3\sqrt{3}}{2}\, i m\, \gamma_5 \otimes \bar\gamma_5\, \varepsilon - \frac{1}{2}\, i m\, \gamma_5 \otimes \mathbb{1}\, \varepsilon \tag{44}$$

which cannot be vanishing for non-trivial ε. Thus, S^4 compactification is non-supersymmetric, as expected. The bosonic symmetry of this solution is $SO(5) \otimes SO(3)$, where $SO(5)$ is the isometry group of S^4, and $SO(3)$ is the Yang-Mills group of the $d = 8$ theory.

(B) CP^2 compactification:

The ansatz is exactly the same as in (43) with the exception that the Ricci tensor $R_{\mu\nu}$ now is that of the CP^2 metric.

$$CP^2: \qquad R_{\bar\mu\bar\nu} = 6\, m^2 g_{\bar\mu\bar\nu} \tag{45}$$

Although the Ricci tensors of S^4 and CP^2 have the same eigenvalues, the Riemann tensors do not. While S^4 metric implies, $R_{\bar\mu\bar\nu\bar\rho\bar\sigma}(S^4) = 2m^2 (g_{\bar\mu\bar\rho} g_{\bar\nu\bar\sigma} - g_{\bar\mu\bar\sigma} g_{\bar\nu\bar\rho})$ the CP^2 metric gives [39],

$$R_{\bar\mu\bar\nu\bar\rho\bar\sigma}(CP^2) = m^2 \left(g_{\bar\mu\bar\rho} g_{\bar\nu\bar\sigma} - g_{\bar\mu\bar\sigma} g_{\bar\nu\bar\rho} + J_{\bar\mu\bar\rho} J_{\bar\nu\bar\sigma} - J_{\bar\mu\bar\sigma} J_{\bar\nu\bar\rho} + 2 J_{\bar\mu\bar\nu} J_{\bar\rho\bar\sigma} \right) \tag{46}$$

where $J_{\mu\nu}$ is the complex structure on CP^2. Eq.(44) continues to hold. Thus, there are no supersymmetries. The bosonic symmetry is $SU(3) \otimes SU(2)$.

(C) $S^2 \otimes S^2$ Compactification:

Eqs. (43.a,c,d,e) remain the same, while (43. b) is replaced by

$$S^2 \times S^2: \quad R_{\mu_1\nu_1} = 6m^2 g_{\mu_1\nu_1} \quad , \quad R_{\mu_2\nu_2} = 6m^2 g_{\mu_2\nu_2} \tag{47}$$

where $\mu_1 \nu_1$ labels the first S^2, and $\mu_2\ \nu_2$ labels the second S^2. Since (44) still holds, $S^2 \times S^2$ is non-supersymmetric. The bosonic symmetry now is $SU(2) \otimes SU(2) \otimes SU(2)$.

(D) S^4 plus on $SU(2)$ instanton compactification:

Let g and $F_{\mu\nu\rho\sigma}$ be non-vanishing*), and moreover let the only other nonvanishing background field be $F^{\alpha}_{\bar\mu\bar\nu}$ and taking the value of an $SU(2)$ self-dual instanton with unit charge, $k = 1$, on S^4 [14,40,41]. The ansatz can then be stated as follows (henceforth $\kappa = 1$):

$$\text{AdS}: \quad R_{\mu\nu} = -12 m^2 g_{\mu\nu} \tag{48.a}$$

$$S^4: \quad R_{\bar\mu\bar\nu} = \frac{3}{a^2} g_{\bar\mu\bar\nu} \tag{48.b}$$

$$F^i_{\bar\mu\bar\nu} = \frac{1}{g a^2} J^i_{\bar\mu\bar\nu} \quad , \quad A_{\bar\mu} = \frac{1}{2g} J^i_{ab}\, \omega_{\bar\mu}{}^{ab} \tag{48.c}$$

$$G_{\mu\nu\rho\sigma} = 3m\, \varepsilon_{\mu\nu\rho\sigma} \tag{48.d}$$

where a is the radius of S^4, $J^{\alpha}_{\bar\mu\bar\nu}$ is a triplet of ($\alpha = 1,2,3$) covariantly constant complex structures obeying the quaternion algebra [41,42]

$$J^i_{\bar\mu\bar\nu} J^{\bar\nu}{}_{\bar\rho}{}^j = -\delta^{ij} g_{\bar\mu\bar\rho} + \varepsilon^{ijk} J^k_{\bar\mu\bar\rho} \tag{49}$$

$$J_4{}^i{}_j = -\delta^i_j \quad , \quad J^i_{jk} = -\varepsilon_{ijk} \quad , \quad \varepsilon^{ijk} J^j_{\bar\mu\bar\nu} J^k_{\bar\rho\bar\sigma} = g_{\bar\nu\bar\rho} J^i_{\bar\mu\bar\sigma} + 3 \text{ more}.$$

*) $\langle\phi\rangle = 0$, henceforth, without loss of generality[25].

In fact, $J^i_{\bar\mu\bar\nu}$ is nothing but the 't Hooft symbol [41] usually denoted by $\eta^i_{\bar\mu\bar\nu}$. It further satisfies the relations:

$$\tfrac{1}{2} \varepsilon_{\bar\mu\bar\nu\bar\rho\bar\sigma} J^{\bar\rho\bar\sigma,i} = J^i_{\bar\mu\bar\nu} \quad , \quad J^i_{[\bar\mu\bar\nu} J^j_{\bar\rho\bar\sigma]} = \varepsilon_{\bar\mu\bar\nu\bar\rho\bar\sigma} \quad . \tag{50}$$

The fields unstated in (48) have vanishing background values. Inserting the ansatz,(48), into the field equations, we see that Einstein's equation yields

$$3a^{-2} + 6g^{-2}a^{-4} = 6m^2 \tag{51}$$

while the scalar field equation gives

$$4 g^{-2} a^{-4} - \tfrac{1}{8} g^2 = 6m^2 \tag{52}$$

Eqs(51) and (52) give two solutions.

$$\text{(A)} \quad g^2 a^2 = 4 \qquad\qquad \text{(B)} \quad g^2 a^2 = 20 \tag{53}$$

Therefore g and a are related to m as follows:

$$\text{(A)} \quad g = \pm 4m \quad , \quad a^{-2} = 4m^2 \tag{54}$$

$$\text{(B)} \quad g = \pm \frac{20}{\sqrt{3}} m \quad , \quad a^{-2} = \frac{20}{3} m^2 \tag{55}$$

From Eqs. (51) and (52) it appears that these two solutions are related to the round S^7 and squashed S^7 solutions of $d = 11$ supergravity, since the left-hand side of (51) and (52) can be interpreted as the values which the Ricci tensors on S^4 and S^3 assume, and ga is the scale of the $SU(2)$ instanton. As is well known [36,43], on S^7 there are two values of these scale which give Einstein's metrics (i.e. L.H.S. of (51) and (52) are equal). In one case (Eq.(53.a)) one has the S^7, in the other case (Eq.(53.b)) the squashed S^7.

The bosonic symmetry of our solutions is $SO(5)$. To find the supersymmetry of these solutions, consider first the condition $\delta\chi_i = 0$

$$\delta\chi_i = \frac{-1}{4} g^{-1} a^{-2} J^i_{\bar\mu\bar\nu} \Gamma^{\bar\mu\bar\nu} \varepsilon + \frac{i}{8} g \gamma_5 \otimes \bar\gamma_5 \Gamma_i \varepsilon - \frac{im}{2} \Gamma_i \bar\gamma_5 \varepsilon \tag{56}$$

where only Eqs.[48] have been used. Defining [25]

$$Q \equiv i J^i_{\bar\mu\bar\nu} \gamma_5 \Gamma^i \Gamma^{\bar\mu\bar\nu} \tag{57}$$

one finds

$$Q \varepsilon = -Q \bar\gamma_5 \varepsilon \tag{58}$$

which implies

$$(1 + \bar\gamma_5) \varepsilon = 0 \tag{59}$$

Moreover from the definition of Q, and using the properties of $J^i_{\bar\mu\bar\nu}$ given in Eqs.(49) and (50) one finds:

$$(Q^2 - 8Q - 48) \varepsilon = 0 \tag{60}$$

Thus Q has the eigenvalues $(-4, -4, -4, 12)$. Eq.(56) can now be written as

$$\Gamma^i \delta\chi_i = \frac{i}{4} g^{-1} a^{-2} \gamma_5 Q \varepsilon - \frac{3i}{2} g \gamma_5 \varepsilon - \frac{7i}{2} m \gamma_5 \varepsilon \tag{61}$$

In the case of Eq.(54), this equation yields

$$\Gamma^i \delta\chi_i = \frac{im}{4} \gamma_5 \left(\pm Q \mp 6 - 6 \right) \varepsilon \tag{62}$$

where $\pm$ refers to the sign of g in Eq. (54). Thus, for $g = +4\kappa m$, the eigenvalue 12 of Q solves Eq.(62), while the eigenvalue -4 of Q

is clearly not a solution. If one inserts the "squashed" solution, (55) into (61) one obtains

$$\Gamma^i \delta\chi_i = \frac{1}{\sqrt{3}} i \gamma_5 \left(\pm Q \mp \frac{5}{2} - \frac{3\sqrt{3}}{2} \right) \varepsilon \tag{63}$$

which cannot be satisfied for either eigenvalue of Q. In summary, among the four solutions given in (54) and (55), only the one for which $g = +4m$, $4m^2a^2 = 1$ possesses the $N = 1$ supersymmetry, while the others have no supersymmetry. For the $N = 1$ supersymmetric case we have checked that no further conditions on the parameters arise from the $\delta\psi_\mu = 0$ and $\delta\psi_{\bar\mu} = 0$ equations. In fact, in all the cases studied so far [25,44], once $\delta\chi = 0$ holds, the equations $\delta\psi_\mu = 0 = \delta\psi_{\bar\mu}$ hold as well. It would be interesting to prove that a relation exists among these equations, in general. (In fact, such a relation was found in $d = 4$ $N = 2$ supergravity [45].)

(E) <u>CP^2 plus an SU(2) instanton compactification:</u>

In this case the only non-vanishing structures are

$$\text{AdS}: \quad R_{\mu\nu} = -12 m^2 g_{\mu\nu} \tag{64.a}$$

$$CP^2: \quad R_{\bar\mu\bar\nu\bar\rho\bar\sigma} = a^{-2} \left(g_{\bar\mu\bar\rho} g_{\bar\nu\bar\sigma} + J_{\bar\mu\bar\rho} J_{\bar\nu\bar\sigma} + J_{\bar\mu\bar\nu} J_{\bar\rho\bar\sigma} - \bar\mu \leftrightarrow \bar\nu \right) \tag{64.b}$$

$$F^i_{\bar\mu\bar\nu} = 2 g^{-1} a^{-2} \bar J^i_{\bar\mu\bar\nu} \quad , \quad A^i_{\bar\mu} = \frac{1}{2} g^{-1} \bar J^i_{ab} \omega_{\bar\mu}{}^{ab} \tag{64.c}$$

$$G_{\mu\nu\rho\sigma} = 3m \, \varepsilon_{\mu\nu\rho\sigma} \tag{64.d}$$

Here $\bar J^i_{\bar\mu\bar\nu}$ is anti-selfdual 't Hooft tensor [41] : $\frac{1}{2} \varepsilon_{\bar\mu\bar\nu\bar\rho\bar\sigma} \bar\eta^i_{\bar\rho\bar\sigma} = -\bar\eta^i_{\bar\mu\bar\nu}$. This tensor obeys the same relations that $\eta^i_{\bar\mu\bar\nu}$ obey. The ansatz (64.c) can be motivated as follows. Firstly, the factor $\frac{1}{2g}$ in the expression for A^i_μ is chosen so that $F^i_{\bar\mu\bar\nu}$ is related to the Riemann tensor on CP^2:

$$F^{i}_{\mu\bar\nu} = \frac{1}{2g} \bar J^{i}_{ab} R_{\mu\bar\nu ab} \qquad (65)$$

To simplify this expression, one notes that in terms of the <u>self-dual</u> Weyl tensor on CP^2 [39]

$$W_{\mu\bar\nu\rho\bar\sigma} = a^{-2}\left(-g_{\mu\bar\rho}\, g_{\bar\nu\sigma} + J_{\mu\bar\rho} J_{\bar\nu\sigma} + J_{\mu\bar\nu} J_{\rho\bar\sigma} - \bar\mu \leftrightarrow \bar\nu\right) \qquad (66)$$

the Riemann tensor reads

$$R_{\mu\bar\nu\rho\bar\sigma} = W_{\mu\bar\nu\rho\bar\sigma} + 2a^{-2}\left(g_{\mu\bar\rho}\, g_{\bar\nu\sigma} - \bar\mu \leftrightarrow \bar\nu\right) \qquad (67)$$

Substituting this into (65) it is seen the Weyl term drops out due to the fact that we have chosen the anti-self dual connection on CP^2. Thus one obtains the expression for $F^{i}_{\bar\mu\bar\nu}$ as given in (64.c). With the ansatz stated in (64), the Einstein equation yields

$$6a^{-2} - 24\, g^{-2} a^{-4} = 6m^2 \qquad (68)$$

while the scalar field equation yields

$$\frac{1}{8} g^2 + 16\, g^{-2} a^{-4} = 6m^2 \qquad (69)$$

Eqs. (68) and (69) have two solutions:

$$(A) \quad g^2 a^2 = 8 \qquad\qquad (B) \quad g^2 a^2 = 40 \qquad (70)$$

Thus, g and a are related to m as follows:

$$(A) \quad g = \pm 8m, \qquad a^{-2} = 2m^2 \qquad (71)$$

$$(B) \quad g = \pm \frac{4}{3}\sqrt{5}\, m, \qquad a^{-2} = \frac{10}{9} m^2 \qquad (72)$$

From the results of Ref. [37,38] one finds out that Case B is related to the N^{010}, while Case A is associated with the squashed N^{010} compactifications [38] of d = 11 supergravity. The bosonic symmetry of our solutions is SU(3). To decide the supersymmetry of these solutions it is simplest to first consider the condition $\delta\chi^{i} = 0$ which reads

$$\delta\chi^{i} = -\frac{1}{2} g^{-1} a^{-2} \bar J^{i}_{\mu\bar\nu} \Gamma^{\mu\bar\nu} \varepsilon + \frac{i}{8} g\, \gamma_5 \otimes \bar\gamma_5 \Gamma^{i} \varepsilon - \frac{im}{2} \Gamma^{i} \gamma_5 \varepsilon = 0 \qquad (73)$$

where only Eqs.(64) have been used. Defining

$$\tilde Q \equiv i \bar J^{i}_{\mu\bar\nu} \gamma_5 \Gamma^{i} \Gamma^{\mu\bar\nu} \qquad (74)$$

it follows that

$$\tilde Q \varepsilon = + \tilde Q \bar\gamma_5 \varepsilon \qquad (75)$$

Thus

$$(1 - \bar\gamma_5)\, \varepsilon = 0 \qquad (76)$$

On the other hand, from (75) and (76) it follows that

$$(\tilde Q^2 + 8\tilde Q - 48)\, \varepsilon = 0 \qquad (77)$$

which implies that Q has eigenvalues (4, 4, 4, - 12). Noting that (73) leads to

$$\Gamma^{i} \delta\chi_i = \frac{i}{2} g^{-1} a^{-2} \tilde Q \varepsilon + \frac{3i}{8} g\, \gamma_5 \varepsilon - \frac{3im}{2} \gamma_5 \varepsilon \qquad (78)$$

for the $g^2 a^2 = 8$ case one finds

$$(\tilde Q + 6 \mp 3)\, \varepsilon = 0 \qquad (79)$$

where the $\pm$ corresponds to the two possible values of the g given in (71). Since (79) cannot be satisfied for either value of g and Q, one concludes that the background for Case A has no supersymmetry. A similar result is easily established for the Case B, given in (72).

V. COMMENTS

Can chirality arise in a preonic interpretation of the d = 8 theory? In this context, we recall that in d = 4 N = 8 supergravity the scalars are described by the coset $E_7/SU(8)$, where SU(8) is a chiral composite local symmetry acting on Majorana spinors. (Gravitino is in $\underset{\sim}{8}$, and the spinors in $\underset{\sim}{56}$ of SU(8)). This chiral SU(8) symmetry has been considered in preonic context in Ref.46. *) Now starting with d = 8 a similar situation exists here also. As was conjectured by Cremmer and independently by Julia [16], in addition to the SL(3,R)/SO(3) coset structure, the two scalars of the theory, B and ϕ, must belong to an SU(1,1)/U(1) coset in d = 8 and thereby exhibit a global hidden SU(1,1) symmetry acting on the bosons and composite local U(1) acting on the fermions. This U(1) coupling will be chiral in the sense that the SU(1,1) algebra is realized by $i\tau_2$ and $\tau_1\Gamma_9, \tau_3\Gamma_9$. If the scalar fields B, ϕ could be used for "tear-drop" compactification, we may even expect this chirality to carry through to d = 6, in analogy with Ref.12. We do not know if this chirality may carry further to d = 4.

Besides chirality, another major problem in Kaluza-Klein theories is how to break the local Yang-Mills symmetries whcih arise from the symmetry of the internal space. What aggravates the situation is the lack of charged massless scalars (or appropriate couplings of them) in the effective theory to trigger a Higgs mechanism. Witten [2] has given an interesting example for d = 8 Kaluza-Klein theory with fundamental SU(2) gauge fields, where charged massless scalars in d = 4 do arise if a component of SU(2) gauge fields takes the value of a monopole configuration over $S^2 \times S^2$, with equal monopole charge on each of the 2-spheres. It would be interesting to see whether such a scenario can exist in the d = 8 supergravity constructed here, though we expect difficulties in solving the scalar field equations in doing this.

*) More recently, the SU(5) $\otimes$ SU(3) $\otimes$ U(1) subgroup of SU(8) has been conjectured to arise as a composite symmetry from squashed seven-sphere compactification of d = 11 supergravity, and this symmetry has been used to construct preonic models in d = 4 [47]

Another open problem of interest is to find out whether the S^4 or CP^2 compactifications without SU(2)instantons are stable or not. If stability can be found here, in view of the fact that the $S^4 \otimes S^3$ compactification of d = 11 supergravity is unstable, it would signify a very remarkable truncation phemomenon, which would be of utility in the study of alternative Kaluza-Klein supergravity theories.

APPENDIX

The Lagrangian*)

$$e^{-1}\mathcal{L} = \frac{1}{4\kappa^2}R - \frac{1}{4}e^{2\kappa\phi}F^{\alpha}_{\mu\nu}F^{\mu\nu,\beta}g_{\alpha\beta} - \frac{1}{4}P_{\mu ij}P^{\mu ij} - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{4}e^{-4\kappa\phi}(\partial_\mu B)^2$$

$$- \frac{1}{16\kappa^4}g^2 e^{-2\kappa\phi}\left(T_{ij}T^{ij} - \frac{1}{2}T^2\right) - \frac{1}{48}e^{2\kappa\phi}G_{\mu\nu\rho\sigma}G^{\mu\nu\rho\sigma} - \frac{1}{12}G^{i}_{\mu\nu\rho}G^{\mu\nu\rho,i}$$

$$- \frac{1}{4}e^{-2\kappa\phi}G^{i}_{\mu\nu}G^{\mu\nu,i} - \frac{e^{-1}}{24\times 72}\varepsilon^{\nu_1\cdots\nu_8}\Big(8\, G_{\nu_1\ldots}G_{\ldots\nu_8} -$$

$$- 8\, G_{\nu_1\ldots}G^{i}_{\ldots\nu_7}B^{i}_{\nu_8} - 8\,\varepsilon_{ijk}G^{i}_{\nu_1\nu_2\nu_3}G^{j}_{\nu_4\nu_5\nu_6}B^{k}_{\nu_7\nu_8} + 12\, G_{\nu_1\ldots}G^{i}_{\nu_5\nu_6}B^{i}_{\nu_7\nu_8} -$$

$$- 12\, G^{i}_{\nu_1\nu_2\nu_3}G^{i}_{\nu_4\nu_5}B_{\nu_6\nu_7\nu_8} + 8\, G_{\nu_1\ldots\nu_4}B_{\nu_5\nu_6\nu_7}\partial_{\nu_8}B\Big) + \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\mathcal{D}_\nu\psi_\rho +$$

$$+ \frac{1}{12}\bar{\chi}_i(\sigma^i\sigma^j + 6\,\delta^{ij})\gamma^\mu\mathcal{D}_\mu\chi_j + \frac{i}{2}(P_\mu^{ij} + 3\,\delta^{ij}\partial_\mu\phi)\bar{\psi}_\nu\gamma^\mu\gamma^\nu\sigma^i\chi^j -$$

$$- \frac{i}{8}e^{\kappa\phi}F^{i}_{\mu\nu}\Big[\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu}\gamma_{\tau]}\sigma^i\psi^\tau + \frac{1}{3}\bar{\psi}_\lambda\gamma^{\mu\nu}\gamma^\lambda(\sigma^i\sigma^j + 6\,\delta^{ij})\chi_j +$$

$$+ \frac{1}{9}\bar{\chi}_k\gamma^{\mu\nu}\left(-\sigma^k\sigma^i\sigma^l + 6\,\sigma^k\delta^{il} + 9\,\sigma^i\delta^{kl}\right)\chi_l\Big] +$$

$$+ \frac{1}{96}e^{\kappa\phi}G_{\mu\nu\rho\sigma}\Big[\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho\sigma}\gamma_{\tau]}\psi^\tau - i\,\bar{\psi}_\lambda\gamma^{\mu\nu\rho\sigma}\sigma^i\chi_i +$$

$$+ \frac{1}{3}\bar{\chi}_i\gamma^{\mu\nu\rho\sigma}(\sigma^i\sigma^j - 3\,\delta^{ij})\chi_j\Big] - \frac{i}{24}G^{i}_{\mu\nu\rho}\Big[\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu\rho}\gamma_{\tau]}\sigma^i\psi^\tau -$$

$$- \frac{2i}{3}\bar{\psi}_\lambda\gamma^{\mu\nu\rho}\gamma^\tau(\sigma^i\sigma^j - 3\,\delta^{ij})\chi_j + \frac{1}{18}\bar{\chi}_k\gamma^{\mu\nu\rho}\left(\sigma^i\sigma^k\sigma^l - 24\,\sigma^k\delta^{il} + 18\,\sigma^i\delta^{kl}\right)\Big] +$$

*) Eq.(28.b) has been used, and the γ_q's have been eliminated by field redefinitions.

$$+ \frac{i}{8}e^{-\kappa\phi}G^{i}_{\mu\nu}\Big[\bar{\psi}^\lambda\gamma_{[\lambda}\gamma^{\mu\nu}\gamma_{\tau]}\sigma^i\psi^\tau + 2i\,\bar{\psi}_\lambda\gamma^{\mu\nu}\gamma^\lambda\left(\delta^{ij} - \frac{5}{6}\sigma^i\sigma^j\right)\chi_j +$$

$$- \frac{1}{9}\bar{\chi}_k\gamma^{\mu\nu}\left(11\,\sigma^k\sigma^i\sigma^l - 30\,\sigma^k\delta^{li} + 9\,\sigma^i\delta^{kl}\right)\Big] - \frac{1}{4}e^{-\kappa\phi}\partial_\mu B\Big[\bar{\psi}_\lambda\gamma^{\lambda\mu\tau}\psi_\tau -$$

$$+ 2i\,\bar{\psi}_\lambda\gamma^\mu\gamma^\lambda\sigma^i\chi_i + \frac{1}{6}\bar{\chi}_i\left(-6\,\delta^{ij} + 11\,\sigma^i\sigma^j\right)\chi_j\Big] + \frac{1}{16}g\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu T +$$

$$+ \frac{i}{48}g\,T^{ij}\bar{\psi}_\mu\gamma^\mu\left(-13\,\sigma^k\delta^{ij} + 30\,\delta^{ki}\sigma^j\right)\chi_k + \frac{1}{48}g\,T^{ij}\bar{\chi}_m\left(13\,\sigma^{mn}\delta^i_j + 4\,\delta^m_n\delta^i_j + 30\,\delta^m_i\delta^n_j - 48\,\sigma^m\sigma^i\delta^n_j\right)\chi_n$$

$$+ \text{QUARTICS}.$$

The Supersymemtry Transformations Laws

$$\delta e^a_\mu = -\bar{\varepsilon}\gamma^a\psi_\mu, \qquad \delta\phi = \frac{i}{2}\bar{\varepsilon}\sigma^i\chi_i, \qquad \delta B = -i\,e^{2\kappa\phi}\bar{\varepsilon}\sigma^i\chi_i$$

$$L^\alpha_i\,\delta L_{\alpha j} = \frac{i}{2}\bar{\varepsilon}\left(\sigma_i\delta^k_j + \sigma_j\delta^k_i - \frac{2}{3}\sigma^k\delta_{ij}\right)\chi^k$$

$$L^i_\alpha\,\delta A^\alpha_\mu = e^{-\kappa\phi}\left[\frac{-i}{2}\bar{\varepsilon}\sigma^i\psi_\mu - \frac{1}{12}\bar{\varepsilon}\gamma_\mu(\sigma^i\sigma^j + 6\,\delta^{ij})\chi_j\right]$$

$$\delta B_{\mu\nu\rho} = -6\kappa\, e^{\kappa\phi}(\delta A^\alpha_\mu)B_{\nu\rho\alpha} + e^{-\kappa\phi}\left(-\frac{3}{2}\bar{\varepsilon}\gamma_{[\mu\nu}\psi_{\rho]} - \frac{i}{4}\bar{\varepsilon}\gamma_{\mu\nu\rho}\sigma^i\chi_i\right)$$

$$L^\alpha_i\,\delta B_{\mu\nu\alpha} = -4\kappa(\delta A^\beta_\mu)B_{\nu\alpha\beta}L^\beta_i - i\,\bar{\varepsilon}\sigma^i\gamma_{[\mu}\psi_{\nu]} + \frac{1}{6}\bar{\varepsilon}\gamma_{\mu\nu}(\sigma^i\sigma^j - 3\,\delta^{ij})\chi_j$$

$$L^\alpha_i\,\delta B_{\mu\alpha} = -2\kappa(\delta A^\gamma_\mu)L_{\gamma i}B - \frac{i}{2}\bar{\varepsilon}\sigma_i\psi_\mu + \frac{1}{2}\bar{\varepsilon}\gamma_\mu\left(\delta_{ij} - \frac{5}{6}\sigma_i\sigma_j\right)\chi^j$$

$$\delta\psi_\mu = \frac{1}{\kappa}\mathcal{D}_\mu\varepsilon + \frac{i}{24}e^{\kappa\phi}F^{i}_{\rho\sigma}\sigma^i\left(\gamma_\mu{}^{\rho\sigma} - 10\,\delta^\rho_\mu\gamma^\sigma\right) + \frac{1}{48}g\,e^{-\kappa\phi}\gamma_\mu T\varepsilon -$$

$$- \frac{1}{96}G_{\nu_1\ldots\nu_4}\left(\gamma_\mu{}^{\nu_1\cdots\nu_4} - 4\,\delta^{\nu_1}_\mu\gamma^{\nu_2\nu_3\nu_4}\right)\varepsilon - \frac{i}{36}G^{i}_{\nu_1\nu_2\nu_3}\sigma^i\left(\gamma_\mu{}^{\nu_1\nu_2\nu_3} - 6\,\delta^{\nu_1}_\mu\gamma^{\nu_2\nu_3}\right)\varepsilon -$$

$$- \frac{i}{24}G^{i}_{\nu_1\nu_2}\sigma^i\left(\gamma_\mu{}^{\nu_1\nu_2} - 10\,\delta^{\nu_1}_\mu\gamma^{\nu_2}\right)\varepsilon - \frac{1}{2}(\partial_\mu B)\varepsilon + \text{CUBICS}.$$

$$\delta\chi^i = \frac{i}{2}\left(P^{ij}_\mu + \frac{2}{3}\delta^{ij}\partial_\mu\phi\right)\sigma^j\gamma^\mu\varepsilon - \frac{e^{\kappa\phi}}{4}F^{i}_{\mu\nu}\gamma^{\mu\nu}\varepsilon - \frac{1}{4}g\,e^{-\kappa\phi}\left(T_{ij} - \frac{1}{2}\delta_{ij}T\right)\sigma^j\varepsilon - \frac{ie^{-\kappa\phi}}{3}(\partial_\mu B)\sigma^i\gamma^\mu\varepsilon -$$

$$- \frac{ie^{\kappa\phi}}{144}G_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\sigma^i\varepsilon + \frac{1}{36}G^{j}_{\mu\nu\rho}(\sigma^{ij} - 2\,\delta^{ij})\gamma^{\mu\nu\rho}\varepsilon - \frac{1}{24}G^{j}_{\mu\nu}(\sigma^i\sigma^j - 3\,\delta^{ij})\gamma^{\mu\nu}\varepsilon + \text{CUBICS}.$$

REFERENCES

[1] P. v. ieuwenhuizen, Lectures presented at Cargèse (1983) (to be published); M.J. uff, B.E.W. Nilsson and C.N. Pope, to appear in Physics Reports, and references therein.

[2] L. Palla, Tokyo Conf. Report, p. 629 (1978);
S. Randjbar-Daemi, Abdus Salam and J.Strathdee Nucl.Phys. B214, 491 (1983);
E. Witten, Princeton Univ., preprint (1983).

[3] Abdus Salam and E. Sezgin, ICTP, Trieste, preprint IC/84/57, to appear in Phys. Lett.B (1984).

[4] H. Nishino and E. Sezgin, Phys.Lett. 144B, 187 (1984).

[5] I.G. Koh and H. Nishino, ICTP, Trieste, preprint IC/84 129 (1984).

[6] P.K. Townsend, Univ. of Texas, preprint, UTT 6-11-83 (1983).

[7] E. Bergshoeff, M. De Roo, B. de Wit and P. van Nieuwenhuizen, Nucl.Phys. B195, 97 (1982);
G.F. Chapline and N.S. Manton, Phys.Lett. 120B, 105 (1983).

[8] L. Alvarez-Gaume and E. Witten, Nucl.Phys. B234, 269 (1984).

[9] M.B. Green and J.H. Schwarz, CALTECH, preprint, CALT-68-1182 (1984).

[10] M.B. Green and J.H. Schwarz, Phys.Lett. 104B, 444 (1982).
J.H. Schwarz and P.C. West, Phys.Lett. 126B, 301 (1983).
P.S. Howe and P.C. West, Nucl.Phys. B238, 181 (1984).

[11] J.H. Schwarz, CALTECH, preprint, CALT-68-1049 (1982).
J.H. Schwarz, Nucl.Phys. B226, 269 (1983).

[12] M. Gell-Mann and B. Zweibach, Phys.Lett. 141B, 333 (1984).

[13] P.M. Frampton and K. Yamamoto, Univ. of North Carolina, preprint, IFT-229-UNC (1984).

[14] S. Randjbar-Daemi, Abdus Salam and J. Strathdee, Phys.Lett. 132B, 56 (1983); Nucl.Phys. B242, 447 (1984).

[15] E. Cremmer, B. Julia and J. Scherk, Phys.Lett. 76B, 409 (1978).

[16] E. Cremmer, in "Proceedings of the Nuffield Workshop on Superspace and Supergravity" (Cambridge Univ.Press., 1981)p. 267;
B. Julia, ibid, p.331.

[17] T. Kugo and P.K. Townsend, Nucl.Phys. B221, 357 (1983).

[18] S. Deser and E. Witten, Nucl.Phys. B178, 491 (1981).
L. Baulieu and J. Thierry-Mieg, Univ. of Paris, preprint (1983).

[19] M. Pernici, K. Pilch and P. van Nieuwenhuizen, Phys.Lett. 143B, 103 (1984).

[20] J. Scherk and J.H. Schwarz, Nucl.Phys. B153, 61 (1979).

[21] B. de Wit and H. Nicolai, Nucl.Phys. B208, 323 (1982).

[22] Abdus Salam and E. Sezgin, Phys.Lett. 126B, 295 (1983).

[23] E. Cremmer and S. Ferrara, Phys.Lett. 91B, 61 (1980).

[24] L. Castellani, L.J. Romans and N.P. Warner, CALTECH, preprint, CALT-68-1055 (1983).

[25] B.E.W. Nilsson and C.N. Pope, Imperial College, preprint, TP/83-84/41 (1984).

[26] R. Gilmore, Lie Groups, Lie Algebras and Some of Their Applications, (John Wiley & Sons, New York 1974).

[27] Abdus Salam and J. Strathdee, Ann. Phys. 141, 316 (1982).

[28] P. Breitenlohner and D.Z. Freedman, Ann.Phys. 144, 249 (1982).

[29] E. Cremmer and B. Julia, Nucl.Phys. B159, 141 (1979).

[30] J. Scherk and J.H. Schwarz, Phys.Lett. 82B, 60 (1979).

[31] E. Sezgin and P. van Nieuwenhuizen, Nucl.Phys. B195, 325 (1982).

[32] M. Huq and M.A. Namazie, ICTP, Trieste, preprint IC/83/210 (1983).

[33] P.G.O. Freund and M.A. Rubin, Phys.Lett. 97B, 223 (1980).

[34] M.J. Duff , B.E.W. Nilsson and C.N. Pope, to appear in Phys.Lett.B(1984).

[35] M.J.Duff and C.N. Pope, in Supersymmetry and Supergravity 82, Eds. S. Ferrara, J.G. Taylor and P. van Nieuwenhuizen (World Scientific Singapore, 1983).

[36] M. Awada, M.J. Duff and C.N. Pope, Phys.Rev.Lett. 50, 294 (1983).

[37] L. Castellani and L.J. Romans, CALTECH, preprint, CALT-68-1053 (1983).

[38] D.N. Page and C.N. Pope, Pennsylvania State Univ., preprint (1984).

[39] T. Eguchi, P.B. Gilkey and A.J. Hanson, Phys.Report 66 No.8 (1980) 213-393;
K. Yano, Differential Geometry on Complex and Almost Complex Spaces, (Pergamon Press, 1965).

[40] J.M. Charap and M.J. Duff, Phys.Lett. 69B, 445 (1977).

[41] G.'t Hooft, Phys.Rev. D14, 3432 (1976).

[42] F. Gürsey and C. Tse, Ann.Phys. 128, 29 (1980).

[43] G. Jensen, J. Diff.Geo. 8, 599 (1973).

[44] L. Mezincescu, P.K. Townsend and P. van Nieuwenhuizen, Texas Univ., preprint, UTT-G-6-84 (1984).

[45] S. Cecotti, L. Girardello and M. Porrati, Milan Univ., preprint (1984).

[46] J. Ellis, M. Gaillard, L. Maiani and B. Zumino in Unification of the fundamental interactions, Eds. J. Ellis, S. Ferrara and P. van Nieuwenhuizen (Plenum Press, 1980).

[47] M. J. Duff, I.G. Koh and B.E.W. Nilsson, Imperial College, preprint, 83-84/50 (1984).