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KALUZA-KLEIN PROPOSAL AND ELECTRO-NUCLEAR GRAVITY



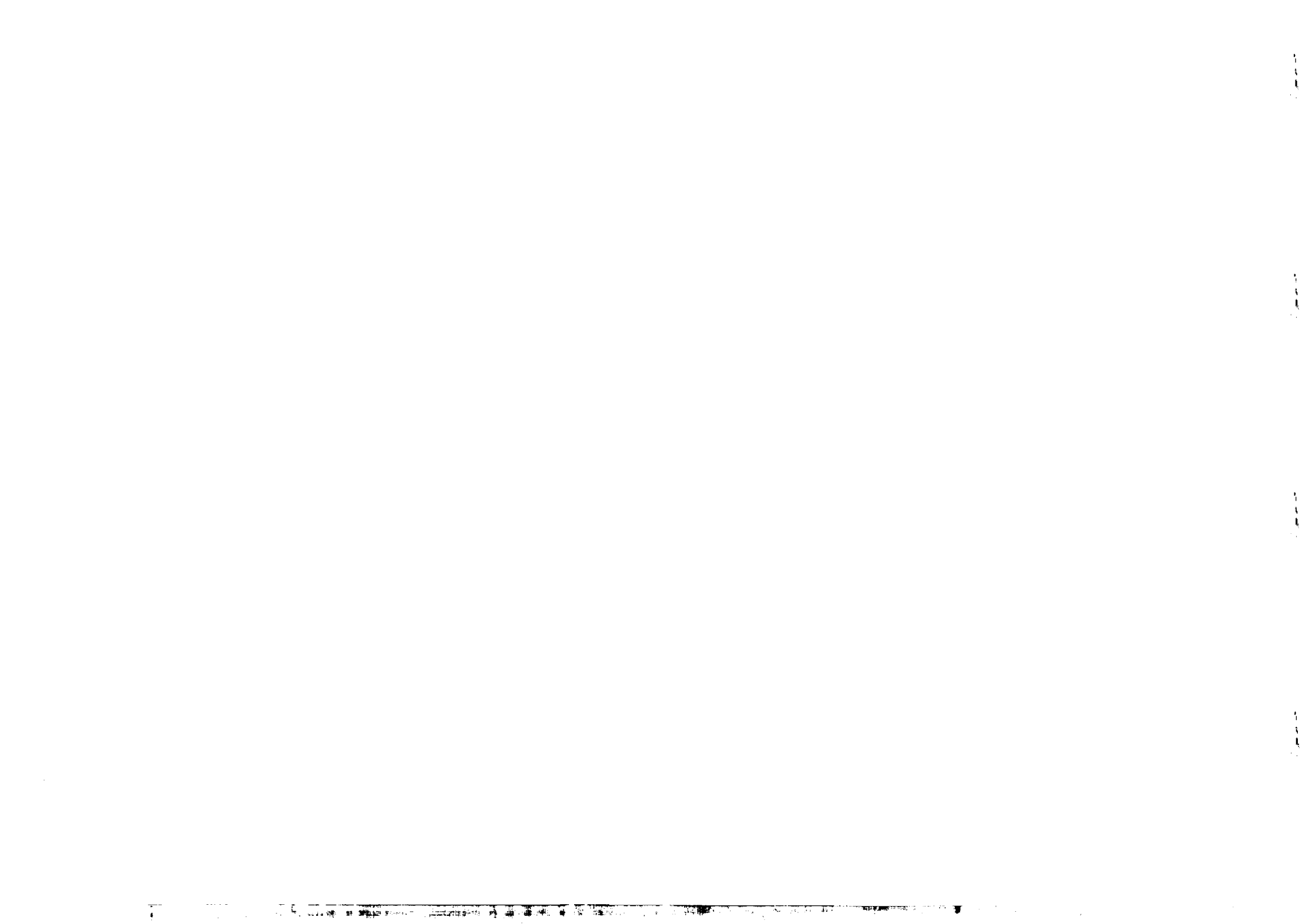
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ABSTRACT

A brief review of the present situation in higher dimensional Kaluza-Klein, Kaluza-Klein + Yang-Mills and Supersymmetrical Kaluza-Klein (SKK) theories is presented.

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I. INTRODUCTION

To my mind there is no question but that the revival of the Kaluza-Klein proposal (first made 65 years ago) represents the most exciting idea introduced into particle physics after supersymmetry and supergravity. Its importance lies, not so much in the electro-nuclear-gravity unification: it is rather that this proposal provides the only way in which the mysterious internal symmetries of particle physics can be given a physical meaning.

In this brief and condensed re-count, I discuss (1) The role of Kaluza-Klein cosmology as a window on extra dimensions; (2) Achieving of spontaneous compactification, with perturbative stability (no tachyons); (3) Realistic electro-nuclear-gravity models, either for quarks and leptons or for preons; (4) Chirality of fermions and (5) Prospects of Super-Kaluza-Klein (SKK).

II. NECESSITY OF INTRODUCING BOSONIC MATTER FOR SPONTANEOUS CLASSICAL COMPACTIFICATION AND SUPER-KALUZA-KLEIN (SSK) THEORIES

Minimal (and according to Weinberg, "conservative") K-K theory in d-dim. is given by:  $L = L_{\text{Einstein}} + L_{\text{fermions}}$ .

With this Lagrangian, since  $R_{MN} = T_{MN}(\psi) + g_{MN}(T+\lambda)/(2-d)$  and since  $\langle \psi \rangle = 0$ , compactification criteria of Eq.(A.1) of the Appendix ( $C_1 \leq 0$ ,  $C_2 > 0$ ) cannot be met (except for  $C_1 = 0 = C_2$ , i.e. Minkowski  $\times$  ( $S_1 \times S_1 \times S_1 \times \dots$ ) compactification with no non-Abelian symmetries). Thus we need EITHER

- (1) Dynamical compactification through Fermi condensates  $\langle \bar{\psi}\psi \rangle \neq 0$ ; OR
- (2) Radiatively induced compactification, i.e. computation of the effective action radiatively. Examples are, for odd d (dimensionally-regularized), finite one-loop computation of  $R_{MN}$  with internal fermion lines, by Candelas and Weinberg [1] and

with internal gravitational lines, by Sarmadi \*) [2a] and by Chodos \*) [3], OR

(3) Classical compactification, i.e. introduce extra bosonic matter fields, e.g.

(a) vector fields  $A_M$  [4] or (b) antisymmetric fields  $A_{MNP}$  [5,6] or (c) scalar fields, e.g. non-linear  $\sigma$ -model [7,8].

One method to reduce arbitrariness of introducing such basic matter fields - is to consider ungauged or gauged supergravity theories which from their very construction contain fields of the type  $A_M$ ,  $A_{MNP}$  or  $\sigma$ . The ideal K-K theory would, therefore, be Super-Kaluza-Klein - SKK - (i.e. extended supergravity for  $d \leq 11$ ), without extra matter multiplets. A second (in general non-super-symmetric) method is to think of 2 stages of a theory which starts with  $L_{\text{Einstein}} + L_{\text{Fermi}}$  in D dimensions. With the fermions present one may imagine that a radiative compactification [1] to d-dimensions  $M^d \times G/H$  has been achieved, where  $\dim G/H = D - d$ . At this stage a consistent effective Lagrangian may emerge in (the lower) d-dimensions, which contains massless fields only with  $L_{\text{Einstein}}$  (in d-dimensions) plus  $L_{\text{YM}}$  (based on G) plus minimally coupled fermions.

With either approach adopted, to illustrate the further problems of compactification, stability and chirality, in <sup>the</sup> sequel we consider effective theories with  $L = L_{\text{Einstein}} + L_{\text{YM}} + L_{\text{Fermi}}$  (i.e. minimally gauged spin 1/2 fermions) +  $\lambda$  which will be fine-tuned to give Minkowski compactification (in general one finds  $\lambda \sim G^{-1} a^{-2} \sim \alpha G^2$ ). But before this, consider the cosmological aspects of KK.

### III. KALUZA-KLEIN COSMOLOGY AS A WINDOW ON HIGHER DIMENSIONS

The best tests of K-K ideas (at present) appear to come from very early universe cosmology:

(1) Variation with time of the fine structure constant  $\alpha$  and the Newtonian constant  $G$

W. Marciano [9] has given the following empirical limits on variation of  $\alpha$  and G:

$$\begin{aligned} \dot{G}/G &< 1 \times 10^{-11} \text{ yrs}^{-1} && \text{(Astrophysics)} \\ \dot{\alpha}/\alpha &< 1 \times 10^{-17} \text{ yrs}^{-1} && \text{(Geochemical)} \\ \dot{\alpha}/\alpha &< 4 \times 10^{-12} \text{ yrs}^{-1} && \text{(Laboratory; Astrophysics)} \end{aligned}$$

\*) These off-shell computations of effective action are necessarily gauge dependent; even the minimum of the potential appears to be so. Randjbar-Daemi and Sarmadi [2b] working in the light-cone gauge, (with no manifest Lorentz-invariance) obtained no compactification up to  $d=21$ . Chodos [3] working in a relativistic gauge showed that the theory has tachyons. Ignoring their contribution he finds compactification ( $C_2 > 0$ ) for  $d=17$ . (Sarmadi has confirmed the results of Chodos [2a].) Clearly we do not yet know if this problem (of gauge dependence) is a fundamental problem or an avoidable technicality, (see in this connection Ref. [2b]).

In general, in KK theories  $\alpha(M_{\text{KK}}) = k G/a^2$  and  $G = \bar{c}/a^{d-4}$  (where  $\bar{c}$  is the constant which occurs in the Lagrangian for d-dimensions); thus  $\dot{\alpha}/\alpha(M_{\text{KK}}) = (2-d) \dot{a}/a$  if  $k = 0$ . If future experiments uphold this, the (near) constancy of G and  $\alpha$  (more precisely of  $\alpha(0)$ , which is related to  $\alpha(M_{\text{KK}})$  through renormalization group relations) may be traced to  $\dot{a} \approx 0$ .

(2) Near constancy of the internal radius ( $\dot{a} \approx 0$ )

Do cosmological solutions of KK equations exist - which after a certain epoch - or equivalently below a certain temperature - give a (near constant) internal radius  $a(t)$  with the external (4 space-time) radius  $R(t)$  of the standard Friedmann-Robertson-Walker type?

The answer appears to be, yes. Randjbar-Daemi, Salam and Strathdee [10] (hereafter referred to as RSS) find  $\dot{a} \approx 0$ ,  $R^2(t) = t(t_0 - t)$  as exact solutions of KK equations, for temperatures  $T < \frac{1}{a}$ , with a scalar-matter one-loop potential. The stability of this type of solution has been (positively) examined by Bailin, Love and Vayonakis [11]. Earlier, Ramond [12] had motivated a similar result for Zeldovich Cosmology ( $p = \rho$ ). Subsequently a number of configurations of matter density, (including SKK) have been shown to lead to  $\dot{a} \approx 0$  [13].

(3) Inflation from collapse of KK integral dimensions

A beautiful idea of motivating inflation in a KK context from the collapse of the internal dimensions has been suggested by several authors [14,11]. Three mechanisms have been considered. One idea is to assume that the total entropy in a co-moving volume remains constant; i.e.  $R \sim a^{(4-d)/3} T^{(1-d)/3}$ . Thus a modest decrease in "a" (the internal radius) and in temperature, T, may produce a large increase in R (by a factor of  $\sim 10^{28}$  or more) if d is large  $\approx 100$ . How exactly the decrease in "a" comes about is a dynamical question as is the reheating of the universe close to the original temperature. Shafi and Wetterich [14] (in a different approach) use non-minimal d-dimensional  $R^2$  terms; they show that a causal domain of the early universe gets trapped in a de-Sitter phase long enough for sufficient inflation to occur. Yoshimura [14], in a still different approach, considers the possibility, in KK context, that the universe may bounce in a closed model, and it may eventually, after some cycles of expansion and contraction, be driven into a nearly flat universe. In each cycle, presumably there are some kind of dissipative processes and entropy produced in each cycle may accumulate. Yoshimura shows that higher dimensional gravity quantum effects may provide the source mechanism, (see, however, Maeda, Bludman, Guth and Sher [15]).

(4) Problem of dark remnants of the KK spectrum

Kolb and Slansky[16] have raised the problem of dark remnants of the KK spectrum; the problem is particularly severe in some KK models, e.g. the unrealistic  $d=5$ , where the KK spectrum consists of charged massive particles ( $M_{KK} = n M_{\text{Planck}}$ ) which cannot decay into the lowest (zero mass) excitations (gravitons, photons, Brans-Dicke scalars). Thus if, at present,  $n_{KK} \approx n_\gamma$ , then  $M_{KK}$  must not exceed  $\approx 100$  eV, if the universe is not to be swamped by the KK density. Any stable KK particles must therefore annihilate with their antiparticles. Even so, Kolb and Slansky, with reasonable assumptions, estimate that  $M_{KK}$  must be less than  $10^6$  GeV. The problem is similar to the monopole abundance problem, only much worse. Kolb and Slansky suggest a number of ways out; e.g. (1) the internal radius  $a \sim \frac{1}{M_{KK}}$  may have been larger than  $10^6$  (GeV) $^{-1}$  at early times, (see Ref.[17]) shrinking later to its present value  $\sim \frac{1}{m_{\text{Planck}}}$ ; (2) Alternatively, the present entropy may have been created after compactification (see Koikawa and Yoshimura [14], who suggest a damped oscillation mechanism around compactified "a" of  $\approx 10^{40}$  sec $^{-1}$  which can produce energetic particles which later thermalize). In any case, this mystery (of the missing stable KK spectrum) will place important restrictions on the type of acceptable KK theory.

(5) Magnetic monopoles in higher dimensions

Finally, several authors [18] have considered the problem of magnetic monopoles in KK theory. For example, Ezawa and Iwasaki [18] show that for  $d=5$  the Hamiltonian is hermitean - suggesting that the Rubakov-Callan effect may not occur in  $d=5$ .

IV. RESTRICTIONS ON DIMENSIONALITY OF SPACE-TIME (d) AND ON MODELS OF ELECTRO-NUCLEAR-GRAVITY

Restriction from anomalies

- (1) Anomalies lead to a breakdown of gauge invariance, and thereby to breakdown of unitarity. They must be exorcised irrespective of the issue of renormalization. This has important bearings on the possible value of dimensionality parameter  $d$  and on possible models of electro-nuclear-gravity.
- (2) Since Weyl-fermions exist only for even  $d$ , odd  $d$  theories are anomaly free.
- (3) Three types of anomalies are known (a) Pure gauge (non-Abelian or U(1)); (b) Mixed gauge and gravitational and (c) Pure gravitational.

Types (a) and (b) are well known from  $d=4$  and exist for all even  $d$ . Type (c), pure gravitational anomalies were first discovered by Alvarez-Gaumé

and Witten [19]. They may be of Einstein or Lorentz varieties[20]; the equivalence of these two types has recently been shown by Bardeen and Zumino[21]. These anomalies exist for  $d = 2, 6, 10, \dots$ ; graphs with internal self-dual anti-symmetric tensor fields also being anomalous.

(4) All anomalies are proportional to  $\text{Sym. Trace} (\lambda^a \lambda^b \dots \lambda^p)$  ( $p = d/2 + 1, d/2 - 1, \dots$ ) where  $\lambda$ 's are the appropriate internal symmetry matrices. Right fermions anomalise oppositely to left fermions, likewise for self-dual and anti-self-dual tensors.

(5) Clearly for  $d = 2, 6, 10, \dots$  there is no hope of anomaly safe groups ( $p=d/2+1$  even). For  $d = 4, 8, \dots$  however, there do exist safe groups for non-Abelian gauge anomalies: e.g. SU(2);  $G_2$ ;  $F_4$ ;  $E_7$ ;  $E_8$ ; SO(2q+1),  $q \geq 2$ ; (Sp(q), SO(2q),  $q \geq 4$ ,  $q$  even) and (Sp(q), SO(2q),  $q$  odd, if  $d/2 \leq q - 3$ ).  $E_6$  is anomaly free for  $d=4$  (see Ref.[22]). (Note SO(10) is not safe for  $d=8$ .)

(6) Mixtures of left and right antisymmetric representations of spin-1/2 fermions for SU(N) groups have been examined by Frampton and Kephart[23] for possible anomaly cancellations. For each N, there are  $1/2 (N - d/2 - 1)$  independent combinations of anti-symmetric representations of SU(N), (e.g.  $6_L + \bar{6}_L + 15_R + 1\bar{5}_R + 20_L$  is anomaly-free for SU(6) for  $d=6$ ); these mixtures are most simply classified in terms of representations of super-algebras SU(N/M) with M satisfying  $N - d/2 - 1 \geq M$  [24a]. Since  $\frac{1}{2}(N - d/2 - 1) \geq 1$ , this places a lower limit,  $N \geq 6$  for  $d=6$  (this excludes SU(5));  $N \geq 7$  for  $d=8$  and  $N \geq 8$  for  $d=10$ , for this class of totally antisymmetric representations.

(7) For  $d=6$ , Alvarez-Gaumé and Witten [19] show that a mixture of one "spin 3/2", 21 "spin 1/2" and 8 self-dual tensors gives a gravitational anomaly-free theory. For  $d=10$ , there is one unique anomaly free theory, the  $N=2$ , chiral  $d=10$  Super-Kaluza-Klein theory. For  $d > 10$ , there is no possibility of cancelling the gravitational anomaly with any mixtures of 3/2, 1/2 and  $\hat{A}$  multiplets [24b]. In view of these anomalies restrictions, even considered as Yang-Mills + gravity theories, higher dimensions place strong restrictions on types of viable theories. We examine this in V.

V. COMPACTIFICATION, STABILITY AND CHIRALITY FROM MINIMAL GAUGING

Minimally coupled gauge vector fields in higher dimension (SKK) theories are favourable for compactification, stability and chirality, after descent to  $d=4$ .

(1) Percacci and Randjbar-Daemi [25] have shown that a theory with  $G_{\text{eff}} = H$  and  $L = L_{\text{Einstein}} + L_{\text{YM}} + \lambda$  (considered either as an approximation to SKK or arising as  $L_{\text{eff}}$  from a still higher-dimensional KK) compactifies on (Minkowski) $^4 \times G/H$  with

$\langle A \rangle = L^{-1}(y)dL(y)|_H$  where  $L(y)$  is a map from  $G/H$  into  $G$ ;  $L^{-1}dL|_H$  is the spin connection for symmetric spaces  $G/H$ . RSS [26] and Schellenkens [27] have shown that for  $G_{YM} \subseteq H$ , the compactified theory is perturbatively stable (tachyon-free). If, however,  $G_{YM} \supset H$ , stability depends on the type of embedding of  $H$  in  $G_{YM}$ .

(2) Even if we start in higher (even) dimensions with left (or right) Weyl fermions, descent to lower  $d$ , in general, would produce vector-like theories. However, with minimal gauging only, backgrounds of the PR type (on account of their topological properties) can give rise to chiral fermions in  $d=4$  (RSS, in preparation). In Tables I, II and III we examine this for  $d = 6, 8, 10$ .

(3) The final gauge-symmetry is  $G \times G'$  where  $G' \subseteq G_{YM}$ . One will obtain different types of models depending, for example, on whether it is  $G$  or  $G'$  which is identified with the electro-nuclear or with the family symmetry.

To summarize,  $d=8$  (with instanton compactification) is the safest dimension [33a, 33b, 34a, 34b]. A realistic model can be built (with three  $SU(5)$  families) or with families of  $SO(10)$  with an appropriate  $G_{YM}$  (see [33], [34], [36] and [37]). The gauge group  $G_{YM}$  is restricted by anomaly, stability and chirality considerations, but is otherwise arbitrary. In the next section we consider Super-Kaluza-Klein models for restricting this arbitrariness.

## VI. ELECTRO-NUCLEAR-GRAVITY MODELS FROM SUPER-KALUZA-KLEIN (SKK) THEORIES

The models considered fall into two classes:

(1) Quark-lepton models: here the emergence of at least 3 families of massless quarks and leptons after compactification to  $d=4$  and fermion chirality present two of the outstanding problems.

(2) Preonic models.

### Quark-lepton models

Of the two inequivalent pure supergravity maximal models, i.e.  $d=11$ ,  $N=1$  and  $d=10$ ,  $N=2$  chiral, the quark-lepton content of neither one appears to resolve the first problem. Quarks and leptons, if fundamental objects, must therefore be introduced in SKK as extra "matter". This, together with chirality considerations appear to limit the choices to the following cases:

(1)  $d=10$ ,  $N=1$ ; with quarks and leptons in (the adjoint representation of a Yang-Mills) vector multiplet. This model cannot be compactified to  $d=4$ , unless radiative corrections produce a desired compactifying potential [36].

The Yang-Mills group of choice is  $E_8$  - the only group with the remarkable properties that (1) its adjoint representation is also the fundamental. (2)  $E_8$  can descend to  $SU(5)$  through the chain  $E_8 \rightarrow E_7 \rightarrow E_6 \rightarrow SO(10) \rightarrow SU(5)$  [37].

(2)  $d=8$ ,  $N=1$  with quarks and leptons again in the  $N=1$  vector multiplet; an  $SU(2)$  gauged version of  $d=8$ ,  $N=2$  theory (descending from  $d=11$ ,  $N=1$ ) has recently been constructed [38].

The theory can be compactified to  $d=4$  in various ways (with or without instantons) All compactifications lead to  $(AdS)^4$ . Chiral fermions may emerge when composite gauging of a further  $SU(2) \times U(1)$  (present in the theory) is taken into account. Whether chirality will survive the compactification to  $d=4$ , still needs to be examined.

(3) Gauged chiral  $d=6$ ,  $N=2$  with monopole compactification [39, 40, 41]. Notwithstanding the presence of anomalies this offers the best model, at present, for quark and lepton matter. The model starts with chiral supergravity, plus  $N=2$  hyper-multiplets, (content: spin 1/2 and zero, only). Why hyper-multiplets are to be introduced in a pure SKK content is of course not clear. However, once they are introduced, they are not arbitrary. These hyper-multiplets must be non-linear (S/R) Kähler-quaternionic according to a result of Bagger and Witten [42]; that is S/R can only take the forms shown in Table IV with the fermionic content displayed according to the representation of  $R$ . Note all fermions are singlets of  $Sp(1)$ . Nishino and Sezgin [39] have shown that super-symmetric minimal gauging can be carried out for all the stability groups  $R$  (or any subgroups thereof). We are thus dealing in  $d=6$  with  $N=2$ , chiral supergravity, with chiral fermions of a specific content, minimally gauged.

The next task is of compactifying from  $d=6$  to  $d=4$ . Since we are dealing with a gauged Yang-Mills theory, a  $U(1)$  subgroup of  $Sp(1)$  or of the remainder of  $R$ , should give rise to chiral monopole gauging. Except for the case when  $R = SU(n) \times U(1) \times Sp(1)$ , where the monopole can be embedded in the  $U(1)$  factor of  $R$ , all other cases of embedding are unstable (Table I). To remedy this, one may either (A) concentrate on  $\frac{SU(N,2)}{SU(N) \times U(1) \times Sp(1)}$  or (B) gauge only a subgroup of  $R$  which contains an explicit  $U(1)$  factor. For type (A) theories, consider  $N=16$ ; the final symmetry group of the effective theory will be  $SU(16) \times U(1) \times Sp(1) \times SU_{KK}(2)$  where the last  $SU_{KK}(2)$  comes from the Kaluza-Klein side and can serve as the family group - the number of families equals the monopole charge  $|n| = 2q + 1$ , where  $q$  labels the  $SU_{KK}(2)$  representation. Each family is a 16-fold of  $SU(16)$ . An example of Type (B) theory is  $E_7/SO(12) \times Sp(1)$ , where only the  $SO(10) \times U(1)$  subgroup of  $SO(12)$  is gauged. Here the final gauge symmetry is  $SO(10) \times U(1) \times Sp(2) \times SU_{KK}(2)$ , where again the  $SU_{KK}(2)$  is the Kaluza-Klein family group, with  $|n| = 2q + 1$  families, each family containing a 16-fold of  $SO(10)$  [41]. A second example is the case of  $E_8/E_7 \times Sp(1)$ , where  $SO(10) \times U(1) \times U(1)$  subgroup of  $E_7$  is gauged. In this case the 56 Majorana-Weyl decomposes to the  $(1 + 1 + 10 + 10)$  Weyl representation of  $SO(10)$ .

### Anomalies

All these theories are gravitationally anomalous in  $d=6$ , except for the case of  $E_6/E_7 \times Sp(1)$ ; this is provided that the partial gauging is carried out only for the  $SU(3) \times SU(2) \times U(1) \times [U(1)]^4$  subgroup of  $E_7 \times Sp(1)$ . (This is to ensure that the gauge group has no more than 16 parameters<sup>\*</sup>, which is what is needed in the Alvarez-Gaumé-Witten anomaly cancellation condition [19], given that the hyperfermions are in the (Weyl)  $27$  of  $E_7$ .) None of the models is free of gauge anomalies (at least a  $U(1)$  anomaly survives even for  $d=4$ ). The anomaly problem is the most urgent to be solved for all such models.

To summarize  $d=6, N=2$  provides the best candidate SKK electro-nuclear-gravity for quarks and leptons. Its characterizations are:

- (1) In  $d=6$ , the model starts with chiral fermions; chirality is preserved for  $d=4$  after monopole compactification.
- (2) The theory compactifies on a Minkowski manifold; this is the only known theory with this property.
- (3) The theory possesses two length scales; one related to Planck length, the other related to the expectation value of a scalar field in a manifestly Lorentz-invariant formulation of the theory.
- (4) For the Maxwell-Einstein truncation of this theory, the monopole charge equals unity. For this theory  $N=2$  supersymmetry breaks down to  $N=1$  [40]. (Compare this with  $N=2$  supergravities in  $d=4$ , which have been shown not to break to  $N=1$  [43]).

### Preonic SKK

Following Cremmer and Julia [44], one may compactify  $d=11, N=1$  on  $(AdS)^4 \times (S^7)^1$ . This theory exhibits chiral  $SU(8)$  composite gauge symmetry. The  $SU(8)$  chiral symmetry led to the original preonic model of Ellis, Gaillard, Maiani and Zumino [45].

Although Witten made the seminal remark that  $\dim G/H = 7$ , for  $G/H = \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)}$ ; i.e.  $SU(3) \times SU(2) \times U(1)$  may arise naturally for  $d = 4+7 = 11$ , d'Auria, Castellani and Fre' and RSS have confirmed that quarks and leptons quantum numbers cannot be described within the compactification  $AdS \times G/H$  of  $d=11$  SKK. This is additional to the difficulty with chirality. Thus  $d=11$ , SKK can only be used for preons.

The recent compactification of  $d=11, N=1$  on  $(AdS)^4 \times S^7$  [6,46] gives rise to a gauged (vector-like)  $SO(8)$  symmetry ( $N=8$ ) in 4 dimensions; however

\*) Another gravitationally anomaly-free 16-parameter subgroup is  $SU(4) \times U(1)$  which may be relevant for preonic models, where the  $56$  (Majorana-Weyl of  $E_7$ ) goes to  $\underline{1} + \underline{6} + \underline{6}' + \underline{15}$  Weyl.

the fermions can still be classed in terms of a global, chiral  $SU(8)$ . Recently, Duff, Koh and Nilsson [47] have considered the squashed 7-sphere compactification which has  $SO(5) \times SO(3)$  ( $N=0$ ) symmetry [48]. They believe that  $SU(8)$  chiral now breaks to  $SU(5) \times SU(3) \times U(1)$ . This leads them to consider a new preonic model, where it is shown that 4 families of  $\underline{5}^* + \underline{10}$  (bound) massless fermions belonging to  $SU(5)$  may be motivated.

### VII. CONCLUDING REMARK

From this variety of attempts, the optimist may conclude that with gauged Super-Kaluza-Klein we may be at a stage when an acceptable electro-nuclear gravity model may emerge in not too distant a future. The pessimist - and the phenomenologist living in four dimensions - will of course point out that we are departing from the unadulterated KK ethic, if quarks and leptons are to be allowed in, as supplementary matter, however restricted. The lure of KK, at present, is aesthetic but this may change not too distantly.

Appendix I

NOTATION

Notation (A)  $d$  space-time dimensions  $(-+++,\dots)$ .

(Non)-Abelian gauge symmetries  $G$  emerge from spontaneous compactification  $M^4 \times B^{d-4}$ ;  $M^4$  is 4-space-time manifold (labels  $x_a$ ,  $a = 0,1,2,3$ ) and  $B^{d-4}$  = coset manifold  $G/H$  (labels  $y_\mu = 1,2,3,\dots$   $d-4 = \dim. G/H = \dim. G - \dim. H$ ).

(B) Spontaneously compactified background (vacuum) metric;

$$\langle g_{MN} \rangle = \langle g_{ab}(x), g_{\mu\nu}(y) \rangle$$

$$\langle R_{ab} \rangle = C_1 \langle g_{ab} \rangle, \quad C_1 = 0 \text{ for Ricci flat e.g. Minkowski; } C_1 < 0 \text{ for AdS} \quad (A.1)$$

$$\langle R_{\mu\nu} \rangle = C_2 \langle g_{\mu\nu} \rangle, \quad C_2 > 0, \quad C_2 = \text{constant}/a^2 \quad (\text{a is the "radius" associated with internal space}).$$

$M^4$  = Minkowski or AdS (rather than de Sitter) guarantees that energy  $E > 0$  and supertheories can be formulated;  $C_2 > 0$  guarantees compactness of  $B^{d-4}$ , i.e. discrete spectrum and a compact gauge group  $G$  (see Ref.[49]).

(C) Spectrum

Harmonically expand all fields on  $G/H$ ;  $\phi = \langle \phi \rangle + \phi^{\text{quantum}}$  [50],  $\phi(x,y) = \sum \phi_n(x) Y_n(y)$ ;  $\phi_n(x)$  are the physical fields in 4-dimension and  $M^2 Y_n = m_n^2 Y_n$  ( $M^2$  is a differential operator on  $G/H$ ).  $m_n$ 's are the masses in the KK spectrum (including zero masses) in  $d=4$ . Effective Lagrangian in  $d=4$  is obtained by integrating over  $y$ .

(D) Zero mass Yang-Mills fields corresponding to  $G$  have a coupling  $\alpha \approx G/a^2$ ,  $G$  = Newtonian constant. (The precise proportionality depends on mixing).

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TABLE CAPTIONS

- Table I : Monopole compactifications of  $d = 6$  Kaluza-Klein theories.
- Table II : Instanton induced compactification of  $d = 8$  Kaluza-Klein theories. Anomaly free representations of  $SO(14)$  will give a realistic one-family  $SO(10)$  electro-nuclear-gravity [30] or three families, e.g. take anomaly free  $SU(11)$  representation:  $[4]_L + [3]_R + [2]_R + [1]_R$  (or  $SU(9)$  representation =  $[3]_L + 9[1]_R$ ) where  $[k]$  is an anti-symmetric representation of order  $k$  [34a]. In the case of  $SO(5) \times SO(N-3)$  low energy symmetry, there exists a massless scalar  $(5, N-3)$  which may be used to break the symmetries using the Coleman-Weinberg mechanism.
- Table III:  $d = 10$  The Witten model [35]  $G_{YM} = O(16)$  with fermions in the anomaly-free representation:  $(+L, -R)$  where  $(L, R)$  specify tangent space  $O(1, q)$  chirality and  $+, -$  refer to Weyl decomposition of  $O(16)$  representations. This would be a fine model with no massless antifamilies of  $SO(10)$ . RSS show, however, that this model contains tachyons and is unstable [30].
- Table IV : Quaternionic Kalier manifolds.
- Table V : Status of SKK theories. Chiral theories are indicated by \*.  $M^4$  denotes 4-dimensional Minkowski space. In the fourth column gauging refers to vector fields of the supergravity multiplet. Whenever matter Yang-Mills multiplet exists (see first column) it can be coupled to the supergravity multiplet non-minimally (i.e. Pauli type coupling).

TABLE I

$d = 6$  Monopole compactification  $G/H = S^2 = \frac{SU(2)}{U(1)}$   $\langle A \rangle = L^{-1} dL|_H$

$G_{YM}$	Stability of embedding of $U(1)$ into $G_{YM}$	Low Energy Sym. of $L$ effective	# of chiral fermions	References
$U(1)$	Stable	$SU(2) \times U(1)$	$ n  = 2q+1$ $q$ rep. of $SU(2)$	[28,29]
$SO(N)$ $N > 2$	Stable only if under $SO(N) \rightarrow SO(N-2) \times O(2)$ we have $N \rightarrow (N-2)_{0, \pm 1/2} + 1_{-1/2}$	$SU(2) \times U(1) \times SO(N-2)$		[27,30,31]
$SU(N)/Z_N$ $N > 2$ e.g. $\frac{SU(3)}{Z_3}$	Stable  $A = 1/2d\phi(\cos\theta \mp 1)$ $\times \text{diag.}(1/3, 1/3, -2/3)$			[32,26]
e.g. $SU(N)$ , $Sp(N)$ , $Spin(N)$ or exceptional groups	perturbatively unstable			[27,26]

TABLE II

$d = 8$  Instanton compactification  $G/H = S^4 = \frac{SO(5)}{SU(2) \times SU(2)}$

$G_{YM}$	Embedding of instanton (instanton number $k=1$ )	Low energy symmetry	# chiral fermions	References
$SU(2)$	Stable	$SO(5)$ (note $SU_{YM}(2)$ is broken)	$(2,3)q(q+1)(2q+1)$	[33a,33b]
All groups	Stable			[30,34b]
$SO(N)$	$4 \times 4$ top corner of the $N \times N$ matrix for $SO(N)$	$SO(5) \times SU(2) \times SO(N-4)$	Starting with chiral rep. of $SO(1,7) \times SO(N)$ , massless fermion rep. is $(1,1,2^{N/(2-1)})$ of $SO(5) \times SU(2) \times SO(N-4)$ after compactification	[30]
$SO(N)$	$3 \times 3$ top corner	$SO(5) \times SO(N-3)$	massless rep. after compactification is $(1,2^{[N/2]})$	[30]

TABLE III

Embed. $\langle A \rangle$ = spin-connection	$G/H$	# chiral families of $SO(10)$	Remarks
$16 \times 16$ matrix	$S^6$	2 families	# family = Euler characteristic of $G/H$ (Witten)
$\begin{vmatrix} 0 & & \\ & & \\ & & 0(6) \end{vmatrix}$	$S^2 \times S^4$	4 families	
	$S^2 \times S^2 \times S^2$	8 families	
	$CP(3)$	4 families	
	$S^3 \times S^3$	0 families	
			No anti-families

TABLE IV

$S/R$	Fermionic content (R rep.)
$E_7/SO(12) \times Sp(1)$	$\psi^A: (32,1)$
$E_8/E_7 \times Sp(1)$	(56,1)
$E_6/SU(6) \times Sp(1)$	(20,1)
$F_4/Sp(3) \times Sp(1)$	(14,1)
$G_2/Sp(1) \times Sp(3)$	(4,1)
$SO(n,4)/SO(n) \times SO(3) \times Sp(1)$	(n,2,1)
$Sp(n,1)/Sp(n) \times Sp(1)$	(2n,1)
$SU(n,2)/SU(n) \times Sp(1) \times U(1)$	(n+n,1,q)

TABLE V

Dim.	N	Degrees of freedom	Gauging	Gauge group	Compactification	Anomaly free	References
d=11	N=1	128	No	-	AdS x (S <sup>7</sup> , J <sup>7</sup> , M <sup>pqr</sup> , ...)	Yes	[51]
d=10	N=2	128	No	-	AdS(CP <sup>3</sup> , CP <sup>2</sup> x S <sup>2</sup> , ...)	Yes	[52]
	*N=2	128	No	-	M <sup>8</sup> x Tear Drop, (AdS) <sub>5</sub> x K <sup>3</sup>	Yes	[53]
	*N=1+ matter	64	No	-	?	No	[54]
d=9	N=2	128	?	?	?		
	N=1+ matter	56	No	-	?		[55]
d=8	N=2	128	Yes	SU(2)	AdS x (S <sup>4</sup> , CP <sup>2</sup> , ...)	Yes	[38]
	N=1+ matter	48	No	-	?	Yes	
d=7	N=4	128	?	?	?	Yes	[56]
	N=4	128	Yes	SO(p,q) p+q=5	AdS x (S <sup>3</sup> , H <sup>3</sup> )		[57]
	N=2+ matter	40	Yes	SU(2)	AdS x H <sup>3</sup>		[57]
d=6	N=8	128	?	?	?	Yes	[58]
	*N=8	128	?	?	?	No	
	N=4+ matter	32	Yes	SU(2)	?	Yes	[59]
	*N=1+ matter	32	No	-	?	Yes	[60]
	*N=2+ matter	16	Yes	SU(2) (or U(1))	M <sup>4</sup> x S <sup>2</sup>	No	[39,40,41,61]