VACUUM TENSION EFFECTS ON THE EVOLUTION OF DOMAIN WALLS
IN THE EARLY UNIVERSE

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ABSTRACT

The "vacuum pressure" mechanism of the hadronic bag model is taken as a guide to formulate the dynamics of closed domain walls in the cosmological case. The effective action functional suggested by this analogy is a straightforward generalization of the Einstein-Maxwell action: it involves a 3-index antisymmetric potential whose coupling to matter generates two effective cosmological constants, one inside and one outside the domain wall. It is suggested that this mechanism, which is alternative to the introduction of a Higgs potential, is the source of the bubble nucleation process envisaged in the New Inflationary Cosmology. The dynamics of a spherical domain in a de Sitter phase is analyzed and is consistent with the geometrical formulation of shell dynamics proposed long ago by Israel.

T. INTRODUCTION

The interplay between particle physics and cosmology has led to profound changes in our perception of the physics of the early universe. As opposed to the picture offered by the hot big-bang model, the ground state of the early universe resembles an emulsion of different phases. The phase transitions proceed via the creation of vacuum bubbles in one phase and their subsequent evolution in the background of a different phase. Such a characteristic domain structure is common to other physical systems; ferromagnets and superconductors are some of such systems whose properties have undoubtedly inspired some of the current cosmological scenarios. However, what comes to our mind as a particularly fitting analogy with the case of the early universe is the hadronic vacuum pictured in the quark-bag model with surface tension. Our objective is to illustrate the precise sense of this analogy and to explore some of its physical consequences in cosmology and astrophysics.

The hadronic vacuum envisaged in the bag model is also regarded as a medium consisting of two phases separated by a closed domain wall. The interior of the "bag" represents the hadronic phase while the exterior region is regarded as a different phase of the vacuum inaccessible to the hadronic constituents. A key ingredient of the bag model is a sort of "cosmological constant" confined to the interior of the bag. This is tantamount to a non-vanishing vacuum energy density associated with the hadronic phase. Hopefully, this phenomenological scenario of the ground state of strong interactions will be substantiated by a deeper understanding of the hadronization phase in non-perturbative QCD. By the same reasoning, since at present the quantum properties of the cosmological ground state are poorly understood, it seems desirable to formulate an effective approach to the bubble nucleation process in the early universe. We suggest that the process of formation of vacuum bubbles in the cosmological case be interpreted in terms of a vacuum energy density which is completely analogous to that of the hadronic phase in the physics of strong interactions. Once formed, the later evolution of the domain boundaries is governed by their surface
tension and their interaction with matter.

The problem of "bubble dynamics" is of interest in its own right and was considered recently by Berezin, Kuzmin and Tkachev in connection with the vacuum state of the early universe and also by Maeda, Samaki and H. Sato in connection with a computer simulated evolution of intergalactic "voids" in an expanding universe. The relevant mathematical formalism for this type of problem in General Relativity was developed some time ago by Israel and later by Chase. In particle physics, on the other hand, Dirac used his formalism of constrained hamiltonian systems to develop an electrodynamic model of a conducting bubble in an attempt to resolve the electron-muon puzzle. Later, with the recognition that strongly interacting particles possess a finite spatial extension, the study of the evolution of extended structure in spacetime became a major line of research in particle physics.

Our approach to bubble dynamics is based on a general theory of relativistic extended objects formulated previously in ref. 7. In order to discuss the cosmological and the astrophysical case we propose an effective action modelled on the hadronic bag. The resulting equation governing the dynamics of one vacuum in the medium of a different vacuum can be obtained either as a junction equation à la Israel, or from the generally covariant definition of "the total mass energy of the system". Particular solutions to our equation of motion fit into two classes: we find "oscillating bubbles", which expand from zero to a maximum radius and then recollapse, and de Sitter type bubbles which contract from infinity up to a minimum radius and then reexpand indefinitely. A special case of this type of solution was advocated by Vilenkin who pictures the birth of the universe as a tunnelling effect from an initial "void" with total energy equal to zero into a de Sitter space.

When the surface tension is zero we also find an isolated static solution representing a spherical vacuum domain of de Sitter type. This kind of bubble admits a limiting configuration where its radius, the Schwarzschild and the de Sitter radii are coincident. This solution was proposed some time ago as a model of hadronic matter in the framework of "strong" gravity. The astrophysical meaning of such a solution, if any, remains an open question. Finally, when the action is formulated in "flat" Minkowski space, the equations of motion reproduce the confinement mechanism postulated in the quark-bag model.

This paper is organized as follows: in Sec. II we introduce and discuss the coupling of the membrane and its generalized gauge field in curved space. In Sec. III the existence of vacuum domains is shown as a solution of the generalized Maxwell equations. In Sec. IV we find a spherical bubble solution of the Einstein equation and discuss its evolution in some particular cases.

II. THE ACTION

In the current literature the vacuum state of the early universe is often modelled on the classical de Sitter solution to Einstein's equation with a cosmological term. This new type of 'de Sitter vacuum' has been successfully exploited to get around some of the well-known problems of the standard big-bang model. A similar artifact (the bag constant) is the key to the success of the quark-bag model in matching hadronic spectroscopy. Our approach is based on the recognition that the cosmological constant as well as the bag constant can be formulated, in a gauge invariant way, in terms of a 3-index antisymmetric potential to which we associate a totally antisymmetric field strength tensor . In geometric notation the field is a 4-form which trivially satisfies the Bianchi identities and obeys the Maxwell-like equations . The source will be specified shortly. For the moment we note that the field F remains invariant under the gauge transformation where the gauge function A is a differential 2-form. The field F is non-propagating and, in the absence of a source term, it represents a background field constant over the entire universe; indeed, since F, the
Hodge dual field strength is a zero-form, the equation \( d^*F = 0 \) implies \( ^*F = c \), with \( c \) constant everywhere. This constant can be interpreted as the cosmological constant in General Relativity. As a simple calculation (cf. eq. (2.4)), the energy momentum tensor of the F-field is given by

\[ T_{\mu\nu} = \Lambda g_{\mu\nu} \]

with \( \Lambda = 4\pi c^2 \). Therefore the coupling of the F-field to the Einstein tensor \( G \) can be described by the equivalent sets of equations,

\[ G = 8\pi T \quad d^*F = 0 \implies \quad G - \Lambda g = 0 - \quad (2.1) \]

This is the basic property of the F-field; recently it has generated a lot of interesting applications not only in connection with the long standing question of the actual size of the cosmological constant \(^{13}\) but also in connection with the problem of the compactification of extra dimensions in Kaluza-Klein supergravity. \(^{14}\)

From our viewpoint the usefulness of the gauge invariant formulation of the cosmological constant is seen in the possibility of coupling the F-field to matter whereas the standard cosmological term is frozen in the action of General Relativity. The gauge invariant coupling of the F-field to matter is easily understood by analogy with electrodynamics: as the standard electromagnetic field \( A_{\mu} \) mediates the interaction between the line elements along the world line of point particles, so the 3-index potential \( A_{\mu\nu\rho} \) mediates the interaction between the hypersurface elements along the world-tube swept by the surface of a bubble in space-time. With this analogy in mind, our effective action is given by

\[ S = \int \frac{1}{16\pi} \sqrt{-g} \, F_{\mu\nu}^2 - \frac{1}{2} \int \sqrt{-g} \, \tilde{g} F_{\mu\nu}^2 \]

\[ - \frac{1}{2} \int \sqrt{-g} \, d^*F_{\mu\nu\rho}(x) \Rightarrow \int \sqrt{-g} \, d^*F_{\mu\nu\rho}(x) \]

\[ \cdot \left[ F_{\mu\nu}(x) F_{\rho\sigma}(x) \right] \quad (2.2) \]

where we have used \( G_{\mu\nu} = 1 \).

Here the first term is the usual Hilbert-Einstein action and the fourth is a straightforward generalization of Maxwell's. The second (kinetic) term represents the invariant "area" of the world-track swept by a bubble in spacetime. In terms of local coordinates \((t^1, t^2, t^3)\) on the bubble, the symbol \( \| \hat{y} \|^3 = \frac{1}{3!} \hat{y}^{\mu\nu\rho} \hat{y}_{\mu\nu\rho} \) represents the norm of the time-like tangent element \( \hat{y}^{\mu\nu\rho} \) to the world-tube of the bubble. The surface tension \( \rho \) is assumed positive and has dimension \( L^{-2} \). Finally, the interaction term involves the 3-vector density

\[ J_{\mu}(x) = \int \hat{y}^{\mu\nu\rho} \delta([x - \delta(t)]] d^3t \]

which acts as a source of the F-field. Therefore the coupling constant \( C \) has dimension \( L^{-1} \). As noted before, the current \( J_{\mu\nu\rho}(x) \) is an obvious generalization of the current associated with point-charge particles and represents the conserved Noether current corresponding to the gauge invariance of the F-field quoted previously. The action (2.2) is also manifestly invariant under general coordinate transformations on the spacetime manifold as well as on the embedded submanifold representing the world-track of the bubble. These invariance properties are reflected by the field equations:

\[ \delta S = 0 \quad \text{leads to} \quad \text{Einstein's equation} \]

\[ G = 8\pi (T_{\text{Bubble}} + T^F) \]

while \( \delta S = 0 \) leads to "Maxwell's equations" for the F-field

\[ d^*F = 0 \quad \quad d^*F = - C^*J \quad (2.5) \]

where \( \ast J \) represents the 1-form current dual to \( J^{\mu\nu\rho}(x) \)

\[ J_{\mu}(x) = \frac{1}{3!} \left[ F_{\mu\nu\rho}(x) \right] J^{\nu\rho}(x) \quad (2.6) \]
where $[\mu^\nu \rho^\sigma] = 1$ is the fully antisymmetric symbol in 4 dimensions,

$\{0\ 1\ 2\ 3\} \cdot 1$.

Finally, $\Sigma S = 0$ leads to the "Lorentz force" equation governing the evolution of the bubble

$$\square x^\alpha + (\Gamma^\alpha)_{\beta\gamma} x^\beta \partial x^\gamma \partial x^\gamma = \frac{1}{3!} \frac{\xi}{\beta} F^\alpha \partial x^\alpha \cdot \chi_{\alpha \beta \gamma} \cdot \pi_{\alpha \beta \gamma}$$ (2.7)

where $\Gamma^\alpha_{\beta\gamma}$ is the connection of the metric $g$ on the spacetime manifold and

$\square \chi$ is the Laplace-Beltrami operator with the metric $\chi_{\alpha \beta} = \partial_{\alpha \beta} \partial_{\alpha \beta} \partial_{\alpha \beta}$ (a,b = 1,2,3) induced by $g$ on the world-track of the bubble. Just as in electrodynamics, the field equations imply the conservation of the Noether current (2.5b) as well as the conservation of the symmetric energy-momentum tensor

$$\mathcal{T} = \mathcal{T}_{\text{bubble}} + \mathcal{T}_{\text{F}}$$ (2.8)

where

$$\mathcal{T}_{\text{bubble}} = \frac{\beta}{3!} \int \frac{x^\alpha \delta}{\delta_{\alpha}} \frac{x^\beta \delta}{\delta_{\beta}} \delta(x - T(t)) \, d^3x$$ (2.9)

and

$$\mathcal{T}_{\text{F}} = \frac{1}{3!} F_{\alpha \beta \gamma} F_{\alpha \beta \gamma} = \frac{1}{2} \frac{\beta}{3!} \int \frac{x^\alpha \delta}{\delta_{\alpha}} \frac{x^\beta \delta}{\delta_{\beta}} \delta(x - T(t)) \, d^3x$$ (2.10)

III. THE ORIGIN OF VACUUM DOMAINS

In principle, the field equations completely determine the motion of the bubble as well as the spacetime geometry. The general solution of Maxwell equations (2.7) in an arbitrary spacetime was derived in ref. (7)

$$F = -\xi \Theta \cdot \alpha'$$ (3.1)

where $\Theta$ is the characteristic function of the open subset $\mathcal{U}$ of the spacetime manifold associated with the interior of the bubble. The constant $\alpha'$ is a solution of the homogeneous equation representing an additional cosmological background. Indeed, when the physical parameters $\xi$ and $\alpha'$ are zero, the field equations reduce to Einstein's equation with a cosmological term as indicated previously (cf. eq. (2.1))

The general solution (3.1) represents the origin of the bubble nucleation process in the cosmic vacuum as well as in the hadronic vacuum. To appreciate the connection with the bag model it may help to gain a qualitative understanding of the solution in Minkowski space. In the absence of curvature, our action (2.2) represents the embedding in Minkowski space of a closed surface endowed with the energy momentum tensor (2.9) and coupled, in a gauge invariant way, to the generalized Maxwell field $F$. By construction, the dual current $J$ is directed along the spacelike normal $n_{\alpha}$ to the world-track of the bubble. In the instantaneous rest frame of a point on the bubble $n_0 = 0$, $n_1$ is the ordinary unit space normal and $S_{\alpha}(x)$ possesses a surface $\delta$-type singularity. Thus, at each point on the surface of the bubble, the Maxwell equation becomes

$$\frac{\partial}{\partial x} S_{\alpha}(x) = -\xi \frac{\partial}{\partial x} S_{\alpha}(x)$$ (3.2)

where $S_{\alpha}(x)$ is the surface $\delta$-function with the property

$$\int d^3x S_{\alpha}(x) f(x) = \int (d^3x)_{2} f(x)$$ (3.3)

and the integral on the r.h.s. is restricted to the volume enclosed by the membrane. A special solution of eq. (3.2) is therefore, $\xi = 0$.

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\[ \varepsilon F = - C \Theta_u(x) \]  

(3.4)

where the volume step function \( \Theta_u(x) = 1 \) inside the bubble and \( \Theta_u(x) = 0 \) outside the bubble. We define the value of the field strength on the surface \( \varepsilon F/\text{Bubble} = - C/2 \) as the arithmetic mean of the values inside and outside. The \( F \)-field evidently acts as an order parameter in spacetime. Equation (3.2) with the solution (3.4) represents the basic mechanism of confinement postulated in the bag model

\[ \varepsilon F = - \frac{1}{2} C^2 \Theta_u(x) \]  

(3.5)

which follows immediately from the solution (3.4) combined with eq. (2.10). Thus the net result of the coupling of the \( F \)-field to the bubble's degree of freedom is a static effect: in the bubble's interior there exists a non-vanishing volume energy density which, in a Minkowskian background, is precisely the "volume tension" advocated in the bag model in order to confine quarks and gluons. From our viewpoint the vacuum energy density of the bag is simply the self-energy density of the \( F \)-field. Note that, unlike electrodynamics, the total self-energy of the \( F \)-field is finite and equal to \( \frac{1}{2} C^2 V \) (when \( \alpha' = 0 \)) where \( V \) is the volume enclosed by the membrane at a given time. Note also that, unlike the original formulation of the MIT bag model, the bubble's surface is an independent dynamical system evolving according to the "Lorentz force" (2.6). When the metric is Minkowskian, and the bubble is spherical, the Lorentz force equation can be solved analytically. The full details of this solution were given in Ref. (16) and will not be reproduced here. The results which concern us here are as follows: the dynamical behaviour of the spherical solution depends on the combination \( \lambda = \alpha C / f \) of the physical parameters (with \( \alpha = - \frac{1}{2} C' \)) and is dynamically unstable for all possible values of \( \lambda \). In particular, when \( \alpha' = 0 \) then \( \lambda < 0 \) and there exist no static configurations whatever the value of the initial radius. Under the combined effect of surface tension and volume tension, the bag collapses to the central singularity unless an extra agent provides the necessary balancing pressure. In the quark-bag model it is the quark-gluon system which stabilizes the bag into a physical hadron. It would cost however an infinite amount of energy to isolate the hadronic constituents as free, non-interacting particles.

IV. COSMIC VOIDS

In the presence of a gravitational field the constant \( \alpha' \) cannot be safely taken equal to zero because a constant \( F \)-field provides a non-trivial energy-momentum tensor coupled to gravity through the Einstein equations. Even in this case the problem admits a "spherical bubble" solution which is represented in a locally static coordinate system, by an interior de Sitter metric

\[ ds^2_u = (1 - \frac{\Lambda_1}{3} x^2) dt^2 + \left( 1 + \frac{\Lambda_1'}{3} x^2 \right)^{-1} dx^2 + \pi^2 d\Omega^2 \]  

(4.1)

and an exterior Schwarzschild-de Sitter geometry

\[ ds^2_o = (1 - \frac{2G M}{\alpha' x^2}) dt^2 + \left( 1 - \frac{2G M}{\alpha' x^2} - \frac{\Lambda_2}{3} x^2 \right)^{-1} dx^2 + \pi' d\Omega^2 \]  

(4.2)

where

\[ \Lambda_1 = \frac{4 \pi (C - \alpha')}{\alpha' x^2}, \quad \Lambda_2 = \frac{4 \pi \alpha'}{\alpha' x^2} \]

and \( \mathcal{E} \) is the total mass-energy of the system. Due to the spherical symmetry of the solution we have on the bubble surface the metric of a two sphere

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\[ ds_{(b)}^2 = -d\tau^2 + R(\tau)^2 d\Omega^2 \]  

(4.3)

with a time dependent radius \( R(\tau) \). Equations (4.1)(4.2), (4.3) completely fix the geometry of the problem. The dynamical equation for the bubble, i.e. the evolution equation for \( R(\tau) \), can be obtained either through the purely geometrical method of Israel (5), (6) or using the integral expression for the total mass energy (8)

\[ E = \int_T \kappa \cdot d\Sigma \nu \]  

(4.4)

where \( \kappa \) is the unit-norm, timelike Killing vector. Both methods lead to the following equation for \( R(\tau) \):

\[ \left( \frac{dR}{d\tau} \right)^2 = -1 + \frac{H^2 R^2}{E} \left( \frac{\Lambda \gamma - \Lambda_0}{4\pi G R^2} - I \right) + \frac{E^2}{16\pi G R^4} \]  

(4.5)

where \( H = (24\pi G)^{-1} \left( \Lambda_1 + \Lambda_0 + \frac{48\pi^2 f^2}{2} k^2 - 4\Lambda_1 \Lambda_0 \right)^{1/2} \).

Except for the last term, (4.5) is formally equivalent to the dynamical equation for a "matter dominated", spatially closed, Friedman universe

\[ \left( \frac{dR}{d\tau} \right)^2 = -1 + \frac{\Lambda}{3} R^2 + \left( \frac{8\pi G}{3} \rho \right)^{1/2} \]  

(4.6)

with a positive cosmological constant \( \Lambda = 3H^2 \) and an "exotic" matter density \( \rho = 3H(\Lambda_1 - \Lambda_0) - 1/8\pi R^2 \). Therefore we can treat the bubble evolution like a standard cosmological problem. To our knowledge there is no general way to solve eq. (4.5), other than by numerical methods. Presently we shall only consider some suitable approximation in order to obtain fully analytical solutions.

A) In the case \( H^2 > 0, \ E^2 > 16\pi G \rho \) equation (4.5) reads

\[ \left( \frac{dR}{d\tau} \right)^2 = -1 + \frac{E}{R} \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) \]  

(4.7)

and can be viewed as the classical equation describing the one-dimensional motion

\[ \left( \frac{dR}{d\tau} \right)^2 = \frac{1}{2} \left( \frac{dR}{d\tau} \right)^2 = E - V(R) \]  

of a unit-mass point particle, moving with constant total energy \( E = -1/2 \) in the potential

\[ V(R) = -\frac{E}{2R} \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) \]  

(4.8)

This problem admits a physically meaningful solution only if \( \Lambda_1 - \Lambda_0 > 48\pi^2 f^2 \) (in the opposite case the particle always stays in a classically forbidden region), and the motion is confined to the interval \( \Omega R \ll E \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) \).

A parametric solution of equation (4.7) can be easily obtained in terms of the conformal time \( \tau = R(\eta) d\eta \) where

\[ R(\eta) = E \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right)^{1/2} \]  

(4.9a)

and therefore

\[ \tau = \frac{1}{2} \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) E \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) \]  

(4.9b)

Solution (4.9) represents an "oscillating bubble" which expands from a vanishing initial radius up to \( R \), and then recollapses. The whole cycle occurs in the time \( \Delta \tau = \pi E \left( \frac{\Lambda_1 - \Lambda_0}{4\pi G R^2} - I \right) \).

Near the origin the dominant contribution to (4.5) comes from the \( 1/R^4 \) term we have discarded, so we expect solution (4.9) to become unreliable as \( \tau \to 0 \).

B) The case \( H^2 = 0, \ \Lambda = \Lambda_1 + 48\pi^2 f^2 \) gives

\[ \left( \frac{dR}{d\tau} \right)^2 = -1 + \left( \frac{E}{4\pi G} \right)^{1/2} \]  

(4.10)

The classical meaning of (4.10) is qualitatively the same as in the case A)
and we expect again a motion confined to the interval \( 0 \leq R \leq (E/4M^2)^{1/2} \).

Equations (4.9a) and (4.9b) are replaced by

\[
R(\tau) = \left(\frac{E}{4M^2}\right)^{1/2} \sin^2 \frac{\pi}{2} \sin \frac{\pi}{2} \eta
\]  

(4.11a)

\[
\tau = \frac{R}{2} \left[ F(d, 1/\eta^2) - 2 E(d, 1/\eta^2) \right]
\]  

(4.11b)

where

\[
\alpha = \arcsin \left( \sqrt{2} \sin \left( \frac{\eta}{2} - \frac{\eta}{4} \right) \right)
\]  

(4.11c)

\( F(d, 1/\eta^2) \) and \( E(d, 1/\eta^2) \) are the elliptic integrals of the first and second kind respectively \( \eta \). An expansion-recontraction cycle requires a time

\[
\Delta \tau = \left( \frac{E}{4M^2} \right)^{1/2} \left[ \frac{(F(\eta))}{4M^2} \right] - \frac{2\pi \sqrt{2}}{2 \sqrt{\eta^2}}
\]  

(4.12)

We notice that (4.11) represent a good approximation for a general solution to (4.5) near the origin, where the \( 1/R^2 \) term dominates over the others.

C) From \( E = 0 \), it follows that

\[
\left( \frac{dR}{d\tau} \right)^2 = -1 + H^2 R^2
\]  

(4.13)

This is the case of "vacuum bubbles" expanding according to the De Sitter law

\[
R(\tau) = H^{-1} \cos H \tau
\]  

(4.14)

This kind of solution acquires a particular relevance in the framework of the New Inflationary Cosmology because it provides the exponentially expanding phase needed to solve the inconsistency of the Standard Cosmological Model \( \text{[11,18]} \). Moreover it has been suggested by Tryon that a closed universe with total vanishing energy could emerge as a spontaneous vacuum fluctuation \( \text{[19]} \). This approach to "cosmogenesis" was recently reconsidered by Vilenkin who interpreted the solution (4.14) as describing a quantum tunnelling from "nothing" to De Sitter space \( \text{[9]} \).

We have already argued elsewhere that in our classical approach "nothing" is rather replaced by the exterior de Sitter phase corresponding to a vanishing total energy for the system \( \text{[8]} \).

D) \( \frac{\rho}{\sigma} = 0 \) and for sake of simplicity \( A_0 = 0 \) given

\[
E = \frac{\Lambda}{6} R^3
\]  

(4.15)

In this case the vanishing surface tension \( \sigma \) turns a dynamical problem into a static one: the membrane does no longer have a physical meaning, it is simply the geometrical boundary of the bubble. The dynamical equation (4.5) becomes the condition (4.15) and the total energy \( E \) is completely saturated by the volume energy. One can read equation (4.15) also from a geometrical point of view: once the Schwarzschild and De-Sitter radii, \( R_s = 2E \) and \( R_A = (3/\Lambda)^{1/2} \) respectively, are fixed, the radius of the bubble is \( R = (R_s/R_A)^{1/2} \). As an extreme case we can choose \( R = R_s \).

In view of the peculiar equation of state \( P = -\rho \) associated with the De Sitter vacuum, it is somewhat surprising that

if one takes for \( \Lambda_0 \) the typical energy density of the nuclear matter i.e. \( \rho \approx 10^{14} \text{ gr/cm}^3 \), then the resulting value of \( R \) is close to 10 km, the correct size of a neutron star. The physical meaning, if any, of such a solution is presently unclear. Similar objects, but with \( R \approx 10^{-13} \text{ cm} \), have also been obtained in the framework of Strong Gravity, where they provide a geometrical model for the hadron structure and dynamics \( \text{[10]} \).

On the basis of the former particular cases, and by analogy with the motion of a classical, unit mass particle, with total energy \( E = \frac{1}{2} \), in the potential

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we can deduce some features about the general solutions of (4.5). We expect
all the bubbles for $E > 0$, to exhibit either an oscillating or an
asymptotically de Sitter like behaviour.

In the first case $R \to 0$, and (4.5) can be approximated, in this
region, by

$$\left( \frac{dR}{d\tau} \right)^2 = \left( \frac{E}{4\pi f} \right)^2 \frac{1}{R^4}$$

so that near the origin we have the following asymptotic behaviour:

$$R(\tau) \sim \left( \frac{3E}{4\pi f} \right)^{1/3} \tau^{1/3}$$

It is easy to check that (4.18) agrees with the asymptotic form of (4.11)
in the same limit.

In the second case $R \to \infty$ so in the remote past and in the
distant future (4.5) approaches (4.13). As a consequence, also for $E > 0$,
there exist solutions whose asymptotic form is (4.14).

Both kind of bubbles have a rest point (the particle's turning
point) corresponding to the maximum expansion or the minimum contraction
radius. The mass of the bubble at the rest point is given by

$$M = \frac{1}{24\pi} \left( \frac{2}{\hbar} \right)^{1/2} \left( \Lambda - \Lambda_0 - 4\pi f \right)$$

The function $y(x)$ exhibits a single maximum in the interval

$$x = \left[ \frac{4 + 3a^2 - a \left( 3a^2 + 8 \right)^{1/2}}{6(1+a^2)} \right]$$

when $a > 0$ (4.21)

or

$$x = \left[ \frac{4 + 3a^2 - a \left( 3a^2 + 8 \right)^{1/2}}{6(1+a^2)} \right]$$

when $a < 0$ (4.22)

Finally we notice that for $a > 0$ the curve (4.9) intersects the x-
axis. The intercept represent the minimum radius of a zero total energy
bubble: $R_0 = H^{-1}$ in agreement with the exact solution (4.14).

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