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CHIRAL COMPACTIFICATION ON MINKOWSKI x S^2 OF N = 2 EINSTEIN-MAXWELL SUPERGRAVITY IN SIX DIMENSIONS *

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ABSTRACT

We show that the U(1) gauged Einstein-Maxwell supergravity in six dimensions, spontaneously compactifies on Minkowski x S^2, with a monopole-valued Maxwell field on S^2. The bosonic symmetry of the background is SU(2) x U(1). The field equations fix the monopole charge to be ±1. The consequence of this is that the N = 2 supersymmetry breaks down to N = 1, and chiral fermions emerge.

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I. INTRODUCTION

In Ref.1 it was shown that the spontaneous compactification of the six-dimensional Einstein-Maxwell-Dirac system on Minkowski x S^2, with the Maxwell field assuming the value of a background monopole solution with charge $n = \pm 1, \pm 2, \ldots$ on S^2 [2], leads to chiral fermions in the four-dimensional effective theory. For Minkowski compactification, it was essential that the six-dimensional theory had a cosmological constant of magnitude $-\frac{1}{2} g^2/n^2\kappa^4$, g being the electric charge, n the S^2-monopole charge and κ the gravitational coupling constant.

In this note, we consider the N = 2 Einstein-Maxwell supergravity in six dimensions which is a special case of the theory constructed in Ref.3, and show that Minkowski x S^2 compactification with U(1) monopole on S^2 occurs also here, and that the chirality mechanism of Ref.1 is operative as well.

There are new features in our compactification. Firstly, the cosmological term is not a constant: instead it is a function of the single real scalar field of the d = 6 theory. Furthermore, it is not introduced by hand, nor is its sign and magnitude arbitrary. Rather, local supersymmetry requires a very specific cosmological potential term, and it does so because the Maxwell field is minimally coupled to the gravitino in d = 6. In fact, the cosmological potential term is given by $-\frac{1}{2} g^2\kappa^{-4}e^{-\kappa\sigma}$, where g is the U(1) coupling constant, and σ is the single real scalar field of the d = 6 theory. This cosmological term has the right sign, and since it does not involve the S^2-monopole charge n, it signals the possibility of fixing that charge. For compactification, the background solution necessitates $\langle\sigma\rangle \equiv \sigma_0$ = constant. Thus we find that the field equations fix the S^2-monopole charge n to be ±1 . This means that S^2 plus the monopole system is really the Hopf fibration of the 3-sphere [4]. More importantly, substituting this background into supersymmetry transformation laws, we see that, consistency requires the vanishing of either the right-handed supersymmetry parameter ϵ_R <u>or</u> the left-handed one ϵ_L, depending on whether $n = +1$ or -1. Therefore, N = 2 supersymmetry is broken down to N = 1. Accordingly, recalling that all the fermions of the d = 6 theory are chiral, and using the mechanism of Ref.1, we find that either the right-handed gravitino $\psi_{\mu R}$ <u>or</u> the left-handed one $\psi_{\mu L}$ is massless, hence the chirality in d = 4 effective theory.

It should be pointed out that the theory presented here does not have a realistic spectrum, nor is it anomaly free [5]. It does illustrate, however certain important phenomena such as Minkowski compactification, chirality and breakdown in supersymmetry to $N = 1$. Conceivably, inclusion of a hypermultiplet in $d = 6$, in a suitable way, may lead to a realistic $N = 2$ $SU(2) \times U(1)$ theory. We now turn to a more detailed description of the theory.

II. EINSTEIN-MAXWELL SUPERGRAVITY IN SIX DIMENSIONS

Supergravity in $d = 6$ contains the following set of fields [3,6]:

$$\left(e_M{}^A ,\ \psi_M ,\ B^+_{MN} \right) , \tag{1}$$

where e_m^A is the vielbein $(M,A = 0,1,\ldots,5)$, ψ_M is the single, complex Weyl spinor satisfying

$$\Gamma_7 \psi_M = \psi_M$$

and B^+_{MN} is a real antisymmetric tensor field with self-dual field strength. Note that this multiplet unlike its $d = 4, N = 2$ counterpart does not contain a vector field. As is well known, fields with self-dual field strengths do not admit a manifestly Lorentz invariant action formulation, in dimensions 2 mod 4 [6]. This problem in our case can be circumvented by coupling a single antisymmetric tensor multiplet [6]

$$\left(B^-_{MN} ,\ \chi ,\ \sigma \right) \tag{2}$$

to supergravity, where B^-_{MN} has anti-self dual field strength, χ is a single complex Weyl spinor obeying the relation

$$\Gamma_7 \chi = - \chi \tag{3}$$

and σ is a real scalar. B^+_{MN} and B^-_{MN} can now be treated as a single entity, B_{MN}, with no (anti) self duality conditions.

In order to describe the Einstein-Maxwell system, we finally couple a single vector multiplet

$$\left(A_M \quad \lambda \right) . \tag{4}$$

where A_M is the real Maxwell field, and λ is a Weyl spinor with the same handedness as the gravitino

$$\Gamma_7 \lambda = \lambda . \tag{5}$$

A shortcut to build the model is to make use of the results of Ref.3, where Yang-Mills multiplet, with local gauge group $Sp(n) \times Sp(1)$ and hypermultiplets containing scalars which parametrise the coset $Sp(n,1)/Sp(n) \times Sp(1)$, were coupled to supergravity in $d = 6$. One can consider the $U(1)$ subgroup of $Sp(1)$, and truncate away the hypermultiplet to obtain the Einstein-Maxwell system. One more technical point to be taken care of is to switch from a pair of symplectic Majorana spinors to a single complex Weyl spinor. This is a matter of convenience. It turns out that the complex spinors are more suitable for studying the mass spectrum of the theory.

After carrying out the procedure briefly outlined above, we obtain the following Lagrangian *) for the Einstein-Maxwell system in $d = 6$:

$$\begin{aligned}
e^{-1} \mathcal{L} = {} & \frac{1}{4\kappa^2} R - \frac{1}{4} (\partial_M \sigma)^2 - \frac{1}{12} e^{2\kappa\sigma} G_{MNP} G^{MNP} - \frac{1}{4} e^{\kappa\sigma} F_{MN} F^{MN} - \frac{1}{2} g^2 \kappa^{-4} e^{-\kappa\sigma} \\
& + \bar{\psi}_M \Gamma^{MNP} \mathcal{D}_N \psi_P + \bar{\chi} \Gamma^M \mathcal{D}_M \chi + \bar{\lambda} \Gamma^M \mathcal{D}_M \lambda + \frac{\kappa}{2} (\partial_M \sigma) (\bar{\chi} \Gamma^N \Gamma^M \psi_N + \\
& + \bar{\psi}_N \Gamma^M \Gamma^N \chi) + \frac{\kappa}{12} e^{\kappa\sigma} G_{MNP} (\bar{\psi}^R \Gamma_{[R} \Gamma^{MNP} \Gamma_{S]} \psi^S + \bar{\psi}_R \Gamma^{MNP} \Gamma^R \chi - \\
& - \bar{\chi} \Gamma^R \Gamma^{MNP} \psi_R - \bar{\chi} \Gamma^{MNP} \chi + \bar{\lambda} \Gamma^{MNP} \lambda) \\
& - \frac{\kappa}{2\sqrt{2}} e^{\kappa\sigma/2} F_{MN} (\bar{\psi}_Q \Gamma^{MN} \Gamma^Q \lambda + \bar{\lambda} \Gamma^Q \Gamma^{MN} \psi_Q + \bar{\chi} \Gamma^{MN} \lambda - \bar{\lambda} \Gamma^{MN} \chi) \\
& + \frac{i}{\sqrt{2}} g \kappa^{-1} e^{-\kappa\sigma/2} (\bar{\psi}_M \Gamma^M \lambda + \bar{\lambda} \Gamma^M \psi_M - \bar{\chi} \lambda + \bar{\lambda} \chi)
\end{aligned} \tag{6}$$

*) We use the signature (-+++++) and $R_{MN} = R^Q_{MQN} = \partial_Q \Gamma^Q_{MN} + \ldots$, $\bar{\chi} = \chi^\dagger i \Gamma_0$.

The action of this Lagrangian is invariant under the following local supersymmetry transformations (modulo the trilinear fermion terms):

$$\delta e_M^A = -\kappa \bar{\varepsilon} \Gamma^A \psi_M + \kappa \bar{\psi}_M \Gamma^A \varepsilon$$

$$\delta \sigma = \bar{\varepsilon} \chi + \bar{\chi} \varepsilon$$

$$\delta B_{MN} = 2\kappa A_{[M} \delta A_{N]} + \tfrac{1}{2} e^{-\kappa\sigma} \Big(\bar{\varepsilon} \Gamma_M \psi_N - \bar{\psi}_N \Gamma_M \varepsilon - \bar{\varepsilon} \Gamma_N \psi_M + \bar{\psi}_M \Gamma_N \varepsilon + $$

$$+ \bar{\varepsilon} \Gamma_{MN} \chi - \bar{\chi} \Gamma_{MN} \varepsilon \Big)$$

$$\delta \chi = -\tfrac{1}{2} (\partial_M \sigma) \Gamma^M \varepsilon + \tfrac{1}{12} e^{\kappa\sigma} G_{MNP} \Gamma^{MNP} \varepsilon$$

$$\delta \psi_M = \tfrac{1}{\kappa} \mathcal{D}_M \varepsilon + \tfrac{1}{24} e^{\kappa\sigma} G_{PQR} \Gamma^{PQR} \Gamma_M \varepsilon$$

$$\delta A_M = \frac{1}{\sqrt{2}} e^{-\kappa\sigma/2} \left(\bar{\varepsilon} \Gamma_M \lambda - \bar{\lambda} \Gamma_M \varepsilon \right)$$

$$\delta \lambda = \frac{1}{2\sqrt{2}} e^{\kappa\sigma/2} F_{MN} \Gamma^{MN} \varepsilon - \frac{i}{\sqrt{2}} g e^{-\kappa\sigma/2} \varepsilon \tag{7}$$

The field strength G_{MNP} is defined as follows:

$$G_{MNP} = 3\, \partial_{[M} B_{NP]} + 3\kappa F_{[MN} A_{P]} \tag{8}$$

All the spinors carry the same charge, g. The covariant derivative of the gravitino, for example, is

$$\mathcal{D}_M \psi_N = \left(\partial_M + \tfrac{1}{4} \omega_{MAB} \Gamma^{AB} - i g A_M \right) \psi_N \,. \tag{9}$$

In addition to the local U(1), the action also has the antisymmetric gauge invariance

$$\delta B_{MN} = 2\, \partial_{[M} \Lambda_{N]} - \kappa \Lambda F_{MN} \,, \tag{10}$$

where Λ is a U(1) parameter, as well as a global scale invariance given by

$$\sigma \to \sigma + c$$

$$A_M \to e^{-c/2} A_M$$

$$B_{MN} \to e^{-c} B_{MN}$$

$$g \to e^{c/2} g \,, \tag{11}$$

where c is an arbitrary constant.

A noteworthy aspect of this Lagrangian is that it has a positive definite potential. As we shall see in the next section, this positivity is crucial for the spontaneous compactification to four dimensions.

III. SPONTANEOUS COMPACTIFICATION ON $(\text{MINKOWSKI})_4 \times S^2$

Assuming that the background fermion fields are vanishing, it suffices to consider the bosonic field equations for compactification. From (6), they are

$$\kappa^{-2} R_{MN} = \partial_M \sigma \partial_N \sigma + e^{2\kappa\sigma} G_{MPQ} G_N{}^{PQ} + 2 e^{\kappa\sigma} F_{MQ} F_N{}^Q - \tfrac{1}{2} \kappa^{-1} g_{MN} \Box \sigma \tag{12a}$$

$$\tfrac{1}{2} \kappa^{-1} \Box \sigma = \tfrac{1}{4} e^{\kappa\sigma} F_{MN} F^{MN} + \tfrac{1}{6} e^{2\kappa\sigma} G_{MNP} G^{MNP} - \tfrac{1}{2} g^2 \kappa^{-4} e^{-\kappa\sigma} \tag{12b}$$

$$\partial_M \left(e\, e^{2\sigma} G^{MNP} \right) = 0 \tag{12c}$$

$$\partial_M \left(e\, e^{\kappa\sigma} F^{MN} \right) = \kappa\, e^{2\kappa\sigma} G^{NPQ} F_{PQ} \,. \tag{12d}$$

It is remarkable that the trace terms in the energy momentum tensor of all the fields have been absorbed to the $-g_{MN} \Box \sigma$ term in (12a). Assuming that none of the background fields depend on the four dimensional space time (so as to preserve a maximally symmetric space time), and assuming that the scalar field is a constant, Eq.(12a) forces on us (no fine tuning) a Ricci flat (e.g. Minkowski) space time. Furthermore, in order to have a compact internal space, from (12a), it is clear that the background Maxwell field must be non-trivial. Thus, it is natural to consider $(\text{Minkowski})_4 \times S^2$ with an S^2-monopole field of charge $n = \pm 1, \pm 2, \ldots$ [1,2]. Accordingly we characterize the background as follows:

$$g_{MN}\, dx^M dx^N = \eta_{\mu\nu}\, dx^\mu dx^\nu + a^2 (d\theta^2 + \sin^2\theta\, d\phi^2) \tag{13a}$$

$$A_m\, dy^m = \frac{n}{2g} (\cos\theta \mp 1)\, d\phi \tag{13b}$$

$$\sigma = \sigma_0 = \text{Constant}, \qquad B_{MN} = 0, \tag{13c}$$

where the index m labels the extra two dimensions; θ, ϕ are the usual spherical co-ordinates on S^2; a is the radius of S^2, and the sign in (13b) refers to the two patches of S^2, on which A_m is defined. From (13) it follows that

$$R_{mn} = \frac{1}{a^2} g_{mn} \tag{14a}$$

$$F_{mn} = \frac{-n}{2ga^2} \epsilon_{mn} \quad , \tag{14b}$$

where R_{mn} is the Ricci tensor on S^2, $F_{mn} = \partial_m A_n - \partial_n A_m$, and ϵ_{mn} is the antisymmetric SO(2) invariant tensor, $\epsilon_{12} = \epsilon^{12} = 1$. Field equations (12c) and (12d) are trivially satisfied by this background, while Eq.(12a) yields the single relation

$$g^2 = n^2 \frac{\kappa^2 e^{\kappa\sigma_0}}{2a^2} \; . \tag{15}$$

Substituting (13) and (14) into Eq.(12b), we find

$$g^4 = n^2 \frac{\kappa^4 e^{2\kappa\sigma_0}}{4a^4} \tag{16}$$

Comparing (15) and (16), we discover that the monopole charge is fixed to be

$$n = \pm 1 \quad . \tag{17}$$

It is remarkable that the field equations fix the S^2-monopole charge to be ± 1 *). This will have interesting consequences for the residual supersymmetry as well as the chirality of the fermions, as we shall see in the next section. Moreover, S^2 with a singly charged monopole field defined over it, is

*) In general, if there are a number of U(1) fields $A_M{}^i$, each coupling with constant g_i and each carrying background S^2-monopole charges n_i, we would obtain $\sum_i n_i{}^2/g_i{}^4 = 4(\kappa/a)^4\, e^{-2\kappa\sigma_0} = (\sum_i n_i{}^2/g_i{}^2)^2$.

nothing but S^3 in disguise *). One wonders: Does our theory have a seven dimensional (space time) origin which admits S^3 compactification?

IV. SUPERSYMMETRY OF THE BACKGROUND

In this section we show that our solution preserves N = 1 supersymmetry. In the next section we will come back to the relevance of this to the chirality problem.

The condition for the supersymmetry of the background is the vanishing of the supersymmetric variations of the fermionic fields, since the background fermions were assumed to be vanishing themselves. From Eq.(7), we see that $\delta\chi = 0$ automatically, while in view of Eq.(14), $\delta\lambda = 0$ gives

$$\Gamma_{56}\, \varepsilon + 2i \frac{a^2}{n\kappa^2} g^2 e^{-\kappa\sigma_0} \varepsilon = 0 \quad . \tag{18}$$

Using (15), as well as $n^2 = 1$, this implies

$$(\Gamma_{56} + in)\, \varepsilon = 0 \quad . \tag{19}$$

In conventions of Ref.1 ,

$$\Gamma_\mu = \gamma_\mu \times \sigma^1 \;, \quad \Gamma_5 = \gamma_5 \times \sigma^1 \;, \quad \Gamma_6 = 1 \times \sigma^2$$
$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN} \;, \qquad \gamma_5^2 = 1 . \tag{20}$$

*) This is the celebrated example of Hopf fibering. See for example, Ref.4.

and thus

$$\Gamma_{56} = \gamma_5 \times i\sigma^{-3} , \quad \Gamma_7 = \Gamma_0\Gamma_1 \cdots \Gamma_6 = 1 \times \sigma^{3} \tag{21}$$

The Weyl condition $\Gamma_7\epsilon = \epsilon$ implies

$$\sigma^3 \mathcal{E} = \mathcal{E} \tag{22}$$

Using Eqs.(21) and (22) in (19) we find that

$$(n + \gamma_5)\, \mathcal{E} = 0 \quad . \tag{23}$$

Since n = ±1, this equation clearly shows that the right or the left handed ϵ will be vanishing, but not both *). This signals, but not yet proves N = 1 supersymmetry. The crucial check now is to examine the gravitino transformation law. From (7) we have

$$\mathcal{D}_m \mathcal{E} = 0 = \left(\partial_m + \tfrac{1}{2}\,\omega_{m56}\,\Gamma^{56} - ig\,A_m\right)\mathcal{E} \quad . \tag{24}$$

The background values of ω_{m56} and A_m are given by [1]

$$\omega_{m56} = e_m \,, \qquad A_m = \frac{-n}{2g}\, e_m \tag{25}$$

where e_m is the canonical SU(2) invariant U(1) connection obtainable from the Maurer-Cartan form and is given by [1]

$$dy^m\, e_m = -\, d\phi\,(\cos\phi - 1) \quad . \tag{26}$$

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*) If n ≠ ±1, this equation also shows that both left and the right-handed $\mathcal{E}$ would have to vanish, in which case the background could not have been supersymmetric.

Substituting (25) into (24) we find

$$\left[\partial_m + \tfrac{i}{2}\,(n - i\Gamma_{56})\, e_m\right]\mathcal{E} = 0 \tag{27}$$

Again, employing Eqs.(21) and (22) yield

$$\left[\partial_m + \tfrac{i}{2}\,(n + \gamma_5)\, e_m\right]\mathcal{E} = 0 \quad . \tag{28}$$

Consistency with (23) implies that

$$\partial_m \mathcal{E} = 0 \tag{29}$$

Obviously ϵ = constant is a solution of this equation. This corresponds to a singlet of the isometry group SU(2). Thus, we have shown that our background indeed has N = 1 supersymmetry. If n = 1, then $\frac{1}{2}(1 + \gamma_5)\epsilon = 0$ while $\frac{1}{2}(1 - \gamma_5)\,\epsilon \neq 0$, (and if n = -1, then $\frac{1}{2}(1 - \gamma_5)\epsilon = 0$ while $\frac{1}{2}(1 + \gamma_5)\epsilon \neq 0$). In both cases the surviving supersymmetry parameter is a constant.

V. CHIRAL FERMIONS

The fermion spectrum splits into two parts. The spin $\frac{3}{2}$ sector which involves the gravitini $\psi_{\mu R,L}$ has no mixing problem. In the spin $\frac{1}{2}$ sector there are mixings among five right and left handed fields, which in a suitable gauge, can be chosen to be $(\chi, \lambda, \partial_\mu\psi_\mu, \frac{1}{\sqrt{2}}(\psi_5 \pm i\psi_6)_{R,L}$. Here we will consider only the spin $\frac{3}{2}$ sector, which will be sufficient to illustrate the emergence of N = 1 supersymmetry and chirality.

The mass term for the gravitino is

$$\bar{\Psi}_\mu\, \gamma^{\mu\nu}\, \Gamma^m \left(\partial_m + \tfrac{1}{2}\,\omega_{m56}\,\Gamma^{56} - ig\,A_m\right)\Psi_\nu \quad . \tag{30}$$

As before, substituting (25) into this equation and using (21), and recalling that $\sigma^3\psi_\mu = \psi_\mu$, we obtain

$$\bar{\Psi}_\mu\, \gamma^{\mu\nu}\, \Gamma^m \left[\partial_m + \tfrac{i}{2}\,(n + \gamma_5)\, e_m\right]\Psi_\nu \quad . \tag{31}$$

For n = +1, Eq.(31) can be written as

$$\bar{\Psi}_{\mu L}\, \gamma^{\mu\nu}\, \Gamma^m \partial_m \Psi_{\nu R} + \bar{\Psi}_{\mu R}\, \gamma^{\mu\nu}\, \Gamma^m (\partial_m + i e_m)\Psi_{\nu L} \quad . \tag{32}$$

We see that, $\psi_{\mu R}$ has a vanishing effective charge [1], while $\psi_{\mu L}$ has an effective charge -1. Therefore, the harmonic expansion of $\psi_{\mu R}$ involve the $(2\ell+1)$ dimensional representations of SO(3) with $\ell = 0,1,2,\ldots$ while for the expansion of $\psi_{\mu L}$, $\ell = 1,2,3,\ldots$ Hence for $\ell = 0$, we have an SO(3) singlet, $\psi_{\mu R}$, which is evidently massless (see Eq.(32)). For $\ell > 0$, Eq.(32) yields the mass term

$$\bar{\psi}_{\mu R}\gamma^{\mu\nu}\sigma^1\nabla_+\psi_{\nu L} = \frac{1}{a}\sqrt{\ell(\ell+1)}\;\bar{\psi}_{\mu R}\gamma^{\mu\nu}\sigma^1\psi_{\nu L}\,, \quad \ell \geq 1\,. \tag{33}$$

This is in accord with $N = 1$ supersymmetry. In the massless sector in addition to the graviton and one spin 3/2, we expect four gauge vector mesons of $SU(2) \times U(1)$ and their four spin 1/2 companions. For the massive sector, in addition to the massive spin 3/2 tower exhibited in Eq. (33), we have also computed the bosonic spectrum. The spin 1/2 sector then follows from $N = 1$ supersymmetry. The final result is summarized in the following table. (The label ℓ is integer and it labels the $(2\ell+1)$ dimensional representations of SU(2). The entries in the table are the degeneracies of states with a given mass and spin. Masses are in units of $2\,g e^{-\kappa\sigma_0/2} M^2_{\text{Planck}}$.)

Spin / Mass^2	2	3/2	1	1/2	0
$\ell(\ell+1)$	1	2	1		
$\ell(\ell+1)$			1	2	1
$(\ell+1)(\ell+2)$			1	2	1
$\ell(\ell-1)$			1	2	1

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