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TOWARDS A SELF-CONSISTENT COMPUTATION OF VACUUM ENERGY IN 11-DIMENSIONAL SUPERGRAVITY *

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ABSTRACT

An attempt is made to balance the negative vacuum energy associated with the Freund-Rubin compactification of the 11-dimensional supergravity theory against the contribution from vacuum fluctuations. We do this in order to obtain a ground state geometry which has four physical (flat) dimensions and is of the form, $(\mathrm{Minkowski})^4 \times B^7$ where B^7 is one of the 7-dimensional manifolds: S^7, $S^5 \times S^2$, $S^4 \times S^3$, $\mathbb{C}P^2 \times S^3$, $S^3 \times S^2 \times S^2$ or a 5-parameter family of $SU(3) \times SU(2) \times U(1)$ invariant spaces M^{pqrst}. We find that all of these solutions are unstable. As a side-issue the facility for computation of the particle spectra, which results from the use of lightcone gauge, is emphasized.

1. INTRODUCTION

A major problem with supergravity models in more than four dimensions is the large vacuum energy which always arises in the spontaneously compactified phase. For example, consider the Freund-Rubin class of compactifying solutions to eleven-dimensional supergravity [1]. These solutions to the classical field equations take the form of a product geometry, $AdS^4 \times B^7$ where B^7 denotes a compact internal, 7-dimensional Einstein space whose Ricci curvature is of comparable magnitude to that of the 4-dimensional anti-de Sitter spacetime, AdS^4. This curvature is generated by vacuum stress energy associated with a non-vanishing Cremmer-Julia-Scherk (CJS) field strength [2].

To overcome the vacuum energy problem it is necessary to find a solution in which the non-vanishing components of the stress energy tensor are essentially confined to "internal" directions so that the 4-dimensional geometry becomes Ricci flat. This effect can be achieved in non-supersymmetric models by including an adjustable cosmological parameter in the higher dimensional Lagrangian [3]. Supergravities however do not allow such an adjustable parameter and this probably means that there is no chance of solving the problem at the strictly classical level.

Some thought has already been given to the contribution of non-classical mechanisms. In particular, the appearance of a fermionic vacuum condensate may drastically modify the stress energy and it has been argued that the 4-dimensional vacuum geometry could become Ricci flat [4]. Such mechanisms are of course not perturbative and hence somewhat conjectural. In this paper we attempt to formulate a simple kind of self-consistent computation of the vacuum energy in which at least some non-perturbative effects are allowed for.

Our approach is to assume a radiatively induced cosmological parameter which is large enough to effectively flatten the 4-dimensional geometry. More specifically, we add a bare cosmological term, $\lambda_0(-g)^{-1/2}$ to the 11-dimensional Lagrangian and compute the 1-loop correction, $\lambda = \lambda_0 + \delta\lambda$, where

$$\delta\lambda \sim \Omega_7^{-1} \Sigma M^4(\lambda) \ln M^2(\lambda) \quad , \tag{1.1}$$

Ω_7 is the volume of B_7, and $M(\lambda)$ denotes a mass in the (flat) effective 4-dimensional theory. The spectrum of masses, $M(\lambda)$, is computed in the usual way except that a cosmological term $\lambda(-g)^{1/2}$ is present and is assumed to take a value appropriate to the background geometry $M^4 \times B^7$, where M^4 denotes Minkowski spacetime. Since the original supergravity theory includes no bare cosmological term, we must have $\lambda_C = 0$, i.e.

$$\delta\lambda = \lambda \qquad (1.2)$$

... defines a programme for the self-consistent computation of λ. This computation certainly envisages the dynamical breakdown of supersymmetry since the sum in (1.1) is assumed not to vanish.

Unfortunately, there are some difficulties which may be mentioned before we state our results. The rather symbolic expression (1.1) does not do justice to the realities of the 1-loop contribution to the effective action functional

$$\Gamma_1(g,A) = \frac{\hbar}{2i} \ \ln \mathrm{Det}\ G(g,A) \qquad (1.3)$$

where $G(g,A)$ denotes the classical propagator on a background configuration specified by the classical fields g, A. The determinant in (1.3) can be evaluated for the more symmetrical configurations which are presumably appropriate to the ground state,

$$\Gamma_1 = -\int d^4x \ V_1$$
$$= -\int d^4x \ \frac{\hbar}{64\pi^2} \ \Sigma\, M^4 \ \ln M^2 \ . \qquad (1.4)$$

The "potential" V_1 depends on the parameters of the background geometry (scales of B^7). To go beyond the 1-loop approximation to a self-consistent one, it is necessary to allow for some influence of V_1 on the propagator G used in (1.3). Our proposal is to make the identification,

$$\langle V_1 \rangle = \Omega_7 \lambda \quad , \qquad (1.5)$$

where $\langle V_1 \rangle$ denotes the ground state value of V_1. The parameter λ is used in a cosmological term, $-\lambda(-g)^{1/2}$ to be adjoined to the classical Lagrangian and hence carried into the expressions for G, M and, ultimately, V_1 itself.

But leaving these difficulties aside, in eleven dimensions the ultraviolet cut-off cancels from the sum (1.4) and the self-consistent computation of λ envisaged in (1.1) and (1.2) could be feasible. However in all the cases we have considered, we find that the solution appears to be unstable. That is, the propagator $G(\lambda)$ includes tachyonic states - at the classical level. These might be corrected by one-loop self-energy effects. We have not tried to discover whether this happens since such a 1-loop computation would be vexed and perhaps unpersuasive.

The tachyonic difficulty may originate in an oversimplification such as (1.5) or it may be reflecting the familiar resistance of supersymmetries to spontaneous breakdown - in this case to a breakdown of anti-de Sitter geometry in four dimensions to a Ricci flat one. For some of the cases studied in this paper, the correponding anti-de Sitter x B^7 do not give rise to instability though whenever B^7 is a product space, instability sets in.[5] Alternatively, the appearance of tachyons could be an indication that stable Minkowskian compactification must be motivated through a mechanism for which a non-zero topological number associated with external Yang-Mills fields exists. In this sense this paper is inconclusive. In writing it, however, one of our aims has been to make known some of the virtues of working with the light-cone gauge. Although not Lorentz invariant, this gauge is valuable in that unphysical modes and other gauge artifacts such as Faddeev-Popov ghosts do not arise. The extraction of particle spectra from the Lagrangian is more direct than in other gauges.

2A. THE LIGHT CONE GAUGE

To establish notation and remind the reader of some of the properties of the light cone gauge, we consider briefly the coupling of a U(1) gauge field, A_μ, to a complex scalar, ϕ. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{\ 2} - |\nabla_\mu\phi|^2 - V(\phi) + C\, n_\mu A_\mu \qquad (2.1)$$

where C is a Lagrange multiplier whose role is to give the light cone gauge condition

$$n_\mu A_\mu = 0 \quad , \qquad (2.2)$$

where n is lightlike. In (2.1) $V(\phi)$ is a potential and $\nabla_\mu\phi = (\partial_\mu - ieA_\mu)\phi$.

Suppose that the background scalar field, φ, is a non-vanishing and real constant. Write

$$\phi = \frac{1}{\sqrt{2}}(\varphi + \phi_1 + i\phi_2) \quad , \qquad (2.3)$$

where the real components ϕ_1 and ϕ_2 are to be treated, along with A_μ as small fluctuations. The bilinear terms in $\mathcal{L}$ are then given by

$$\mathcal{L}_2 = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}(\partial_\mu \phi_1)^2 - \frac{1}{2}(\partial_\mu \phi_2 - e\varphi A_\mu)^2$$
$$- \frac{m_1^2}{2}\phi_1^2 - \frac{m_2^2}{2}\phi_2^2 + C\, n_\mu A_\mu \quad , \tag{2.4}$$

where $m_1^2 = \partial^2 V/\partial\phi_1^2$ at $\phi_1 = \phi_2 = 0$, etc. These bilinears determine the "masses" as functions of the background field φ. The physical masses are obtained by setting φ equal to the value which minimizes V.

Subsequent analysis is greatly facilitated by the use of light cone co-ordinates. In these co-ordinates the scalar product takes the form

$$A\cdot B = A_+B_- + A_-B_+ + A_i\, B_i \quad , \tag{2.5}$$

where $i = 1,2$. We shall conveniently regard X^+ as "time" so that equations which involve the derivative ∂_+ are dynamical, while those which do not are mere constraints. Such constraints are to be used freely in simplifying the Lagrangian. The vector n_μ which determines the light cone gauge is now chosen such that (2.2) reduces to

$$A_- = 0 \quad . \tag{2.6}$$

Use of (2.6) in the expression (2.4) causes the bilinears to take the form

$$\mathcal{L}_2 = -\frac{1}{2}(\partial_\mu A_i)^2 + \frac{1}{2}(\partial_- A_+ + \partial_i A_i)^2 - \frac{1}{2}(\partial_\mu \phi_1)^2 -$$
$$- (\partial_+\phi_2 - e\varphi A_+)\partial_-\phi_2 - \frac{1}{2}(\partial_i\phi_2 - e\varphi A_i)^2 - \frac{m_1^2}{2}\phi_1^2 - \frac{m_2^2}{2}\phi_2^2 \quad . \tag{2.7}$$

Observe that the equation of motion for A_+,

$$0 = -\partial_-(\partial_- A_+ + \partial_i A_i) + e\varphi\partial_-\phi_2$$

is in fact a constraint. It can be solved for A_+,

$$A_+ = \frac{1}{\partial_-}\left[e\varphi\phi_2 - \partial_i A_i\right] \quad , \tag{2.8}$$

where $1/\partial_-$ represents a non-local operator, viz.

$$\frac{1}{\partial_-}\delta(x^- - y^-) = \frac{1}{2}\varepsilon(x^- - y^-) \;(= \pm 1) \quad . \tag{2.9}$$

If (2.8) is used to eliminate A_+ from (2.7) then the bilinear term simplifies to read

$$\mathcal{L}_2 = -\frac{1}{2}(\partial_\mu A_i)^2 - \frac{1}{2}e^2\varphi^2 A_i^2$$
$$- \frac{1}{2}(\partial_\mu\phi_2)^2 - \frac{1}{2}(m_2^2 + e^2\varphi^2)\,\phi_2^2$$
$$- \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}m_1^2\phi_1^2 \quad . \tag{2.10}$$

Now m_1^2 and m_2^2 are in general dependent on φ. The value of φ which minimizes V, also causes m_2 to vanish (Goldstone theorem) and it is at this point that ϕ_2 is seen to represent the helicity 0 component of a massive vector. The helicity ± 1 components are represented by A_i. (Of course, if the minimum occurs at $\varphi = 0$ then Goldstone's theorem does not apply and one finds $m_2 = m_1$.) Values of φ which do not minimize the potential will yield masses m_1 and m_2 which are not compatible with Poincaré invariance. This merely confirms the non-covariance of off-shell, gauge dependent quantities in the light cone gauge. Notwithstanding this, it should be noted, that the 1-loop effective potential

$$V_1(\varphi) = \frac{\hbar}{64\pi^2}\Big[2(e^2\varphi^2)^2 \ln e^2\varphi^2 + m_1^4 \ln m_1^2 +$$
$$+ (m_2^2 + e^2\varphi^2)^2 \ln (m_2^2 + e^2\varphi^2)\Big]$$

is manifestly Lorentz invariant for arbitrary φ. Each term in this sum represents the zero-point energy of a field in the light cone Lagrangian (2.10). No Faddeev-Popov contribution would be needed in the non-Abelian case.

It can certainly be argued that our use of the light cone gauge in the above example has yielded simplifications which are, at best, only marginal. But, as we shall show, in the case of 11-dimensional supergravity, the advantages are significant. We shall consider the bosonic sector of the Cremmer-Julia-Scherk (CJS) Lagrangian, with the significant modification of an added cosmological term.

B. THE ELEVEN DIMENSIONAL SUPERGRAVITY IN THE LIGHT CONE GAUGE

The fluctuation field, h_{AB} and V_{ABC} of the 11-bein and CJS fields are defined by

$$E_M{}^A = \overset{o}{E}_M{}^B \, (\delta^A_B + h^A_B)$$
$$A_{ABC} = \overset{o}{A}_{ABC} + V_{ABC} \quad , \qquad (2.11)$$

where the background basis, $\overset{o}{E}_M{}^A$, is orthogonal and the fluctuation field h_{AB} is symmetric, $h_{AB} = h_{BA}$. The bilinears are

$$\begin{aligned}
\mathcal{L}_2 = \det \overset{o}{E} \Big[& -\tfrac{1}{4} \nabla_A h_{BC} \nabla_A h_{BC} + \tfrac{1}{8} \nabla_A h \nabla_A h \\
& + \tfrac{1}{2} \left(\nabla_A h_{AB} - \tfrac{1}{2}\nabla_B h\right)\left(\nabla_C h_{CB} - \tfrac{1}{2}\nabla_B h\right) \\
& + \tfrac{1}{4}\left(\overset{o}{R} + \tfrac{\kappa^2}{48}\overset{o}{F}{}^2 + \kappa^2\lambda\right)\left(h_{AB} h_{AB} - \tfrac{1}{2} h^2\right) \\
& + \tfrac{1}{2}\left(-h_{AC} h_{BC} \overset{o}{R}_{AB} + h\, h_{AB} \overset{o}{R}_{AB} + h_{AB} h_{CD} \overset{o}{R}_{ACBD}\right) \\
& - \tfrac{\kappa^2}{12} \overset{o}{F}_{AA_1A_2A_3} \overset{o}{F}_{BA_1A_2A_3} \left(h_{AC} h_{BC} - \tfrac{1}{2} h\, h_{AB}\right) \\
& - \tfrac{\kappa^2}{8} \overset{o}{F}_{ACA_1A_2} \overset{o}{F}_{BDA_1A_2} h_{AB} h_{CD} \\
& + \tfrac{2}{3}\kappa \overset{o}{F}_{ABCD} h_{AE} \nabla_{[E} V_{BCD]} - \tfrac{\kappa}{12} \overset{o}{F}_{ABCD} h \nabla_A V_{BCD} \\
& - \tfrac{1}{3} \nabla_{[A} V_{BCD]} \nabla_{[A} V_{BCD]} + \tfrac{\kappa}{(12)^3} \varepsilon_{A_1 \cdots A_{11}} \nabla_{A_1} V_{A_2A_3A_4} \overset{o}{F}_{A_5 \cdots A_8} V_{A_9A_{10}A_{11}} \Big]
\end{aligned} \qquad (2.12)$$

where

$$h = h_{AA} = 2h_{+-} + h_{11} + \cdots + h_{10,10}$$

$$F_{ABCD} = 4\nabla_{[A}A_{BCD]} = \nabla_A A_{BCD} + 3 \text{ terms} \qquad (2.13)$$

Covariant derivatives are with respect to the Riemannian connection of the background *). We shall be concerned with background geometries which are the product of 4-dimensional Minkowski spacetime with compact 7-dimensional Einstein space. The field strength, $\overset{o}{F}_{ABCD}$, has non-vanishing components only in the Minkowskian directions, and is independent of position,

$$\overset{o}{F}_{12+-} = f \quad . \qquad (2.14)$$

The Riemann tensor $\overset{o}{R}_{ABCD}$ has non-vanishing components in the compact directions and the Ricci tensor satisfies

$$\overset{o}{R}_{\alpha\beta} = \Lambda\, \delta_{\alpha\beta} \quad . \qquad (2.15)$$

If the background field equations are satisfied then the constants f and Λ are given by

$$\kappa^2 f^2 = -2\Lambda \quad \text{and} \quad \kappa^2\lambda = -6\Lambda \quad . \qquad (2.16)$$

The expression (2.12) becomes somewhat unwieldy when the background fields (2.14) and (2.15) are substituted. However, if the light cone gauge conditions

$$h_{A-} = 0 \quad \text{and} \quad V_{AB-} = 0 \qquad (2.17)$$

are used, then a number of constraints emerge,

$$\begin{aligned}
\partial_-^2 h &= 0 \\
-\partial_- h_{i+} &= \partial_j h_{ij} + \nabla_\beta h_{i\beta} - \tfrac{1}{2}\partial_i h \\
-\partial_- h_{\alpha+} &= \partial_j h_{\alpha j} + \nabla_\beta h_{\alpha\beta} - \tfrac{1}{2}\nabla_\alpha h - \kappa f V_{\alpha 12} \\
-\partial_- V_{\beta\gamma+} &= \partial_j V_{j\beta\gamma} + \nabla_\alpha V_{\alpha\beta\gamma} \\
-\partial_- V_{\beta j+} &= \partial_i V_{i\beta j} + \nabla_\alpha V_{\alpha\beta j} - \epsilon_{jk}\, \kappa f h_{\beta k} \qquad \epsilon_{12} = +1 = -\epsilon_{21}, \;\; \epsilon_{11} = \epsilon_{22} = 0 \\
-\partial_- V_{12+} &= \nabla_\alpha V_{\alpha 12} + \kappa f h_{ii} \; .
\end{aligned} \qquad (2.18)$$

---

*) Our conventions for the Riemann and Ricci tensors are

$$R^M{}_{LPQ} = \partial_P \Gamma^M_{QL} - \partial_Q \Gamma^M_{PL} + \Gamma^M_{PK}\Gamma^K_{QL} - \Gamma^M_{QK}\Gamma^K_{PL} \qquad R_{MN} = R^P{}_{MNP}$$

sign $g_{MN} = (-, +, \ldots, +)$.

If these are used as well, then the simplifications are very substantial. The Lagrangian $\mathcal{L}_2$ separates into sectors according to the transverse O(2) quantum number of the fields,

$$\mathcal{L}^0 = \frac{9}{56} h_{ii} \left(\partial^2 + \nabla^2 + \frac{55}{9}\Lambda + \kappa^2\lambda - \frac{23}{18}\kappa^2 f^2 \right) h_{jj}$$
$$+ \frac{1}{4} \tilde{h}_{\alpha\beta} \left(\partial^2 + \nabla^2 + 5\Lambda + \kappa^2\lambda - \frac{\kappa^2 f^2}{2} \right) \tilde{h}_{\alpha\beta} + \frac{1}{2} \tilde{h}_{\alpha\beta} \mathring{R}_{\alpha\gamma\beta\delta} \tilde{h}_{\gamma\delta}$$
$$+ \frac{1}{2} V_{\alpha 12} \left(\partial^2 + \nabla^2 + \Lambda - \kappa^2 f^2 \right) V_{\alpha 12}$$
$$+ \kappa f \, V_{\alpha 12} \left(\nabla_\beta \tilde{h}_{\alpha\beta} + \frac{6}{7} \nabla_\alpha h_{ii} \right)$$
$$+ \frac{1}{12} V_{\alpha\beta\gamma} \left(\partial^2 + \nabla^2 + 3\Lambda \right) V_{\alpha\beta\gamma} + \frac{1}{2} V_{\alpha\beta\varepsilon} \mathring{R}_{\alpha\gamma\beta\delta} V_{\gamma\delta\varepsilon}$$
$$- \frac{\kappa f}{72} \varepsilon_{\alpha_1 \cdots \alpha_7} \nabla_{\alpha_1} V_{\alpha_2\alpha_3\alpha_4} V_{\alpha_5\alpha_6\alpha_7} \tag{2.19}$$

$$\mathcal{L}^{\pm 1} = \frac{1}{2} h_{\alpha i} \left(\partial^2 + \nabla^2 + 5\Lambda + \kappa^2\lambda - \frac{\kappa^2 f^2}{2} \right) h_{\alpha i}$$
$$+ \frac{1}{4} V_{\alpha\beta i} \left(\partial^2 + \nabla^2 + 2\Lambda \right) V_{\alpha\beta i} + \frac{1}{2} V_{\alpha\beta i} \mathring{R}_{\alpha\gamma\beta\delta} V_{\gamma\delta i}$$
$$- \kappa f \, V_{\alpha\beta i} \, \varepsilon_{ij} \nabla_\alpha h_{\beta j} \tag{2.20}$$

$$\mathcal{L}^{\pm 2} = \frac{1}{4} \tilde{h}_{ij} \left(\partial^2 + \nabla^2 + 7\Lambda + \kappa^2\lambda + \frac{\kappa^2 f^2}{2} \right) \tilde{h}_{ij} \, , \tag{2.21}$$

where $\partial^2 = 2\partial_+\partial_- + \partial_1^2 + \partial_2^2$ is the Minkowski part of the Laplacian and $\nabla^2 = \nabla_\alpha \nabla_\alpha$ is the compact part. The traceless parts of h_{ij} and $h_{\alpha\beta}$ are denoted by $\tilde{h}_{ij}$ and $\tilde{h}_{\alpha\beta}$ respectively.

The spectrum of the internal Laplacian can be found by algebraic methods if the internal space is a coset space [6]. In particular, if G/H is symmetric then the eigenvalues take the form

$$-\nabla^2 \sim \Lambda \left| C_2(G) - C_2(H) \right| \, ,$$

where $C_2(G)$ and $C_2(H)$ are second order Casimir operators.

If the spectrum of ∇^2 is known then it is possible to compute the 1-loop potential from (2.19)-(2.21) as a function of the parameters Λ and f. It will be manifestly Lorentz invariant. No Faddeev-Popov contribution is needed. On the other hand, to see that the spectrum of states is Poincaré invariant it is necessary to use the on-shell values (2.16) for f^2 and λ. When these values are substituted, the expressions reduce to

$$\mathcal{L}^0 = \frac{9}{56} h_{ii} \left(\partial^2 + \nabla^2 + \frac{8}{3}\Lambda \right) h_{jj}$$
$$+ \frac{1}{4} \tilde{h}_{\alpha\beta} \left(\partial^2 + \nabla^2 \right) \tilde{h}_{\alpha\beta} + \frac{1}{2} \tilde{h}_{\alpha\beta} \mathring{R}_{\alpha\gamma\beta\delta} \tilde{h}_{\gamma\delta}$$
$$+ \frac{1}{2} V_{\alpha 12} \left(\partial^2 + \nabla^2 + 3\Lambda \right) V_{\alpha 12}$$
$$+ \frac{1}{12} V_{\alpha\beta\gamma} \left(\partial^2 + \nabla^2 + 3\Lambda \right) V_{\alpha\beta\gamma} + \frac{1}{2} V_{\alpha\beta\varepsilon} \mathring{R}_{\alpha\gamma\beta\delta} V_{\gamma\delta\varepsilon}$$
$$- \frac{\sqrt{-2\Lambda}}{72} \varepsilon_{\alpha_1 \cdots \alpha_7} \nabla_{\alpha_1} V_{\alpha_2\alpha_3\alpha_4} V_{\alpha_5\alpha_6\alpha_7}$$
$$+ \sqrt{-2\Lambda} \, V_{\alpha 12} \left(\nabla_\beta \tilde{h}_{\alpha\beta} + \frac{6}{7} \nabla_\alpha h_{ii} \right) \tag{2.19'}$$

$$\mathcal{L}^{\pm 1} = \frac{1}{2} h_{\alpha i} \left(\partial^2 + \nabla^2 + \Lambda \right) h_{\alpha i}$$
$$+ \frac{1}{4} V_{\alpha\beta i} \left(\partial^2 + \nabla^2 + 2\Lambda \right) V_{\alpha\beta i} + \frac{1}{2} V_{\alpha\beta i} \mathring{R}_{\alpha\gamma\beta\delta} V_{\gamma\delta i}$$
$$- \sqrt{-2\Lambda} \, V_{\alpha\beta i} \, \varepsilon_{ij} \nabla_\alpha h_{\beta j} \tag{2.20'}$$

$$\mathcal{L}^{\pm 2} = \frac{1}{4} \tilde{h}_{ij} \left(\partial^2 + \nabla^2 \right) \tilde{h}_{ij} \tag{2.21'}$$

The massless graviton states evidently must correspond to the zero mode of ∇^2 acting on $\tilde{h}_{ij}$. The non-zero eigenvalues must give the helicity ± 2 states of massive spin 2 multiplets. The helicity ± 1 and 0 partners of these states will have to be found in the systems $\mathcal{L}^{\pm 1}$ and $\mathcal{L}^0$. To see in a general way how the components must arrange themselves, it is useful to resolve the various fields into transverse and longitudinal parts as follows:

$$
\begin{aligned}
\tilde{h}_{\alpha\beta} &= h^{tt}_{\alpha\beta} + (\nabla_\alpha H^t_\beta + \nabla_\beta H^t_\alpha) + (\nabla_\alpha \nabla_\beta - \tfrac{1}{7}\delta_{\alpha\beta}\nabla^2) H \\
h_{\alpha i} &= h^t_{\alpha i} + \nabla_\alpha H_i \\
V_{\alpha 12} &= V^t_{\alpha 12} + \nabla_\alpha v_{12} \\
V_{\alpha\beta i} &= V^t_{\alpha\beta i} + (\nabla_\alpha v^t_{\beta i} - \nabla_\beta v^t_{\alpha i}) \\
V_{\alpha\beta\gamma} &= V^t_{\alpha\beta\gamma} + (\nabla_\alpha v^t_{\beta\gamma} + \nabla_\beta v^t_{\gamma\alpha} + \nabla_\gamma v^t_{\alpha\beta})
\end{aligned}
\tag{2.22}
$$

where $h^{tt}_{\alpha\alpha} = 0$, $\nabla_\alpha h^{tt}_{\alpha\beta} = 0$, $\nabla_\alpha H^t_\alpha = 0$, etc. If these expressions are substituted into (2.19')-(2.21') the various bilinears will be found to comprise the following J^{PC} sectors *):

1) $\tilde{h}_{ij}$, H_i, h_{ii}, H, v_{12} $= 2^{++}, 0^{++}, 0^{++}$
2) $h^t_{\alpha i}$, $v^t_{\alpha i}$, H^t_α, $V^t_{\alpha 12}$ $= 1^{--}, 1^{--}$
3) $h^{tt}_{\alpha\beta} = 0^{++}$
4) $v^t_{\alpha\beta i}$, $v^t_{\alpha\beta}$ $= 1^{-+}$
5) $v^t_{\alpha\beta\gamma} = 0^{+-}, 0^{+-}$. (2.23)

We have confirmed this in detail for an arbitrary internal Einstein space in sector 1) and for the O(8) invariant 7-sphere in all sectors.

Because of the 3x3 mixing between h_{ii}, H and v_{12}, sector 1) is the most complicated, and it is here that a 0^{++} tachyon seems always to arise. In fact, a lengthy computation is needed to get these bilinears into the form

*) We are associating C-parity with reflections in the tangent space of B^7 .

$$
(h_{ii} \quad \xi \quad \eta)
\begin{pmatrix}
\frac{9}{56}(\partial^2+\nabla^2+\frac{8}{3}\Lambda) & -\frac{3}{7}\sqrt{-2\Lambda} & 0 \\
-\frac{3}{7}\sqrt{-2\Lambda} & -\frac{\partial^2+\nabla^2+2\Lambda}{2\nabla^2} & -\sqrt{-2\Lambda} \\
0 & -\sqrt{-2\Lambda} & \frac{\partial^2+\nabla^2-2\Lambda}{\frac{6}{7}\nabla^2-\Lambda}
\end{pmatrix}
\begin{pmatrix} h_{ii} \\ \xi \\ \eta \end{pmatrix}
\tag{2.24}
$$

where

$$
\xi = \nabla^2 v_{12} \quad \text{and} \quad \eta = (\tfrac{6}{7}\nabla^2 - \Lambda) H \tag{2.25}
$$

The spectrum of ∂^2 is given by the zeros of the determinant of (2.24), i.e. by the roots of a cubic equation. The roots are

$$
\partial^2 = -\nabla^2 \tag{2.26a}
$$

$$
\partial^2 = -\nabla^2 - \frac{4}{3}\Lambda\left[1 + \sqrt{1 + \frac{9}{2}\frac{\nabla^2}{\Lambda}}\right] \tag{2.26b}
$$

$$
\partial^2 = -\nabla^2 - \frac{4}{3}\Lambda\left[1 - \sqrt{1 + \frac{9}{2}\frac{\nabla^2}{\Lambda}}\right] , \tag{2.26c}
$$

where ∇^2 is the internal space Laplacian, acting on scalars. The root (2.26a) had to be present since it corresponds to the helicity 0 component of the $J^{PC} = 2^{++}$ multiplet, whose other components are carried by $\tilde{h}_{ij}$ and H_i . The root (2.26b) is clearly not tachyonic; it describes a series of massive 0^{++} states. The root (2.26c) is potentially dangerous. In order for the spectrum to be non-tachyonic we must have

$$
-\nabla^2 - \frac{4}{3}\Lambda\left[1 - \sqrt{1 + \frac{9}{2}\frac{\nabla^2}{\Lambda}}\right] \geq 0
$$

i.e.

$$
\nabla^2 = 0 \quad \text{or} \quad -\nabla^2 \geq \frac{16}{3}(-\Lambda) \tag{2.27}
$$

In all cases where we have evaluated the spectrum of ∇^2 we find values which violate (2.27). Here we shall give the example of $B^7 = S^7$ in some detail and then exhibit the tachyonic modes for other cases [7]).

For the case in which B^7 is a sphere of radius a we have $\Lambda = -6/a^2$ and it is relatively easy to exhibit the full complement of bilinears, sector by sector. The sectors are now distinguished by O(8) quantum numbers and we find it convenient to follow the usage of Gel'fand and Tseytlin . The results are

$$\begin{aligned}
\mathcal{L}^{n000} = {} & \tfrac{1}{4}\, \bar{\tilde{h}}_{ij} \left(\partial^2 - \frac{n(n+6)}{a^2} \right) \tilde{h}_{ij} \\
& + \tfrac{9}{56}\, \bar{h}_{ii} \left(\partial^2 - \frac{n(n+6)+16}{a^2} \right) h_{ii} \\
& + \tfrac{1}{2}\, \bar{H} \left(\partial^2 - \frac{n(n+6)-12}{a^2} \right) H \\
& + \tfrac{1}{2}\, \bar{v}_{12} \left(\partial^2 - \frac{n(n+6)+12}{a^2} \right) v_{12} \\
& - \frac{\sqrt{12}}{a^2}\, \bar{v}_{12} \left(\sqrt{\tfrac{12}{7}(n-1)(n+7)}\; H + \tfrac{6}{7} \sqrt{n(n+6)}\; h_{ii} \right) \\
& + \tfrac{1}{2}\, \bar{H}_i \left(\partial^2 - \frac{n(n+6)}{a^2} \right) H_i
\end{aligned} \qquad (2.28a)$$

where H_i, v_{12} and H are absent for n = 0 and H is also absent for n = 1.

$$\begin{aligned}
\mathcal{L}^{n100} = {} & \tfrac{1}{2}\, \bar{h}^t_i \left(\partial^2 - \frac{n(n+6)+5}{a^2} \right) h^t_i \\
& - \frac{1}{a^2} \sqrt{12(n+1)(n+5)} \left(\bar{v}^t_1 h^t_2 - \bar{v}^t_2 h^t_1 \right) \\
& + \tfrac{1}{2}\, \bar{v}^t_i \left(\partial^2 - \frac{n(n+6)+5}{a^2} \right) v^t_i \\
& + \tfrac{1}{2}\, \bar{H}^t \left(\partial^2 - \frac{n(n+6)-7}{a^2} \right) H^t \\
& - \frac{1}{a^2} \sqrt{12(n-1)(n+7)}\; \bar{V}^t_{12} H^t \\
& + \tfrac{1}{2}\, \bar{V}^t_{12} \left(\partial^2 - \frac{n(n+6)+17}{a^2} \right) V^t_{12}
\end{aligned} \qquad (2.28b)$$

$$\mathcal{L}^{n200} = \tfrac{1}{2}\, \bar{h}^{tt} \left(\partial^2 - \frac{n(n+6)}{a^2} \right) h^{tt} \qquad (2.28c)$$

$$\begin{aligned}
\mathcal{L}^{n110} = {} & \tfrac{1}{2}\, \bar{V}^t_i \left(\partial^2 - \frac{n(n+6)+8}{a^2} \right) V^t_i \\
& + \tfrac{1}{2}\, \bar{v}^t \left(\partial^2 - \frac{n(n+6)+8}{a^2} \right) v^t
\end{aligned} \qquad (2.28d)$$

$$\mathcal{L}^{n11\pm} = \tfrac{1}{2}\, \bar{V}^{t\pm} \left(\partial^2 - \frac{n(n+6)+9 \mp \sqrt{12}\,(n+3)}{a^2} \right) V^{t\pm} \qquad (2.28e)$$

The Lagrangians (2.28a–e) correspond, respectively, to the sectors 1)–5) indicated in (2.23) but now broken down into O(8)-irreducible pieces. The tachyons referred to above are found in the (n000) sectors with n = 1,2,3.

We shall not go into details for cases in which B^7 is a product space; the tachyonic modes are listed in Table I

B^7	∇^2	Tachyonic modes
$S^5 \times S^2$	$-\frac{\Lambda}{4}\,[n(n+4)+4j(j+1)]\; n \geq 0, j \geq 0$	(n=0,j=1), (n=1,j=0), (n=j=1)
$S^4 \times S^3$	$-\frac{\Lambda}{6}\,[2n(n+3)+3j(j+1)]\; n \geq 0, j \geq 0$	(n=0,j=1), (n=1,j=0), (n=j=1)
$S^3 \times \mathbb{C}P^2$	$-\frac{\Lambda}{2}\,[n(n+2)+\frac{4}{3}\, j(j+2)]\; n \geq 0, j \geq 0$	(n=0,j=1), (n=1,j=0), (n=j=1)
$S^2 \times S^2 \times S^2$	$-\frac{\Lambda}{2}\,[n(n+2)+2j(j+1)+2\ell(\ell+1)]$	(n=1,j=ℓ=0), (j=n=0,ℓ=1), (ℓ= 0=n,j=1)

Table I

Finally we consider the general case of $B^7 = M^{pqrst}(\alpha,\beta,\gamma)$ where p,q,r,s,t are five parameters characterizing the family of SU(3) x SU(2) x U(1) invariant 7-dimensional manifolds and α^2, β^2, γ^2 are three metrical scales. Elsewhere [8)] we have given a detailed exposition of these spaces and indicated the technique of Harmonic expansions on them. Here we only need the formula for ∇^2 acting on a scalar function on $M^{pqrst}(\alpha,\beta,\gamma)$. This is given by

$$\nabla^2 = -\frac{2}{\alpha^2}\left[\frac{1}{6}(n_1-n_2+3)^2+\frac{1}{6}(n_2-n_3+3)^2+\frac{1}{6}(n_1-n_3)^2-3-\frac{3}{4}s^2y^2\right]$$
$$-\frac{2}{\beta^2}\,[T(T+1)-t^2y^2]$$
$$-\frac{1}{\gamma^2}\,(r-ps-qt)^2\,y^2 \quad .$$

Here n_1, n_2, n_3 (subject to $n_1+n_2+n_3 = 0,1$, or 2) are integers characterizing the irreducible representations of SU(3). T and Y are SU(2) x U(1) quantum numbers.

We now show that the mode $n_1 = -n_3 = 1$, $n_2 = 0$ and $T = Y = 0$ is tachyonic. To see this we remark that for this mode $\nabla^2 = -6/\alpha^2$. If we impose the requirement on the SU(3) x SU(2) x U(1) invariant metric to be Einstein, then α^2 is related to the cosmological constant Λ by

$$-\frac{1}{\alpha^2} = \frac{2}{3}\,\frac{\Lambda}{1+\sqrt{1-\frac{1}{2}\left[\frac{s}{r-ps-qt}\right]^2 Z}} \quad .$$

Here Z is a complicated function of p,q,r,s, and t whose explicit form is not needed. Clearly,

$$\left|\frac{1}{\alpha^2}\right| \leq \left|\frac{2}{3}\Lambda\right| \quad .$$

For a choice, like for example, $-\frac{1}{\alpha^2} = \frac{2}{3}\Lambda$, we would get $\nabla^2 = 4\Lambda$ which would violate the bound imposed by Eq.(2.27) signalling instability.

We had chosen the case of flat M^4 x SU(3) x SU(2) x U(1)invariant 7-dimensional manifold, since this appeared to us to be nearest to the physical situation. However, we must remark that the breaking of this symmetry to the final physical symmetry M^4 x SU(3)xU(1) would in any case have induced a non-zero cosmological constant. We have found that $B^7 = \frac{SU(3)\ x\ U(1)}{U(1)\ x\ U(1)}$ suffers from the shortcoming similar to the one pointed out in Ref.[8], in that one cannot find a representation for coloured quarks or uncoloured leptons with appropriate electric charges among the allowed spinor representations. We have however not yet checked the particle spectrum for tachyons, whether here as well as in our approximation, the 11-dimensional supergravity theory does not wish to break from AdS to flat Minkowski M^4 .

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