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A MODEL FOR DIFFUSE AND GLOBAL IRRADIATION ON HORIZONTAL SURFACE*

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ABSTRACT

The intensity of the direct radiation and the diffuse radiation at any time on a horizontal surface are each expressed as fractions of the intensity of the extraterrestrial radiation. Using these and assuming a random distribution of the bright sunshine hours and not too wide variations in the values of the transmission coefficients, a number of relations for estimating the global and the diffuse irradiation are derived. Two of the relations derived, including the Angström correlation for estimating the global irradiation, are already known empirically while several new correlations have been derived. The relations derived in this paper are:

$$\frac{H_D}{H_0} = a_1 + b_1 \frac{S}{S_0} \quad \text{(i)}$$

$$\frac{H}{H_0} = a_2 + b_2 \frac{S}{S_0} \quad \text{(ii)}$$

$$\frac{H_D}{H_0} = a_3 + b_3 \frac{H}{H_0} \quad \text{(iii)}$$

$$\frac{H_D}{H} = a_4 + b_4 \frac{H_0}{H} \quad \text{(iv)}$$

$$\frac{H}{H - H_D} = a_5 + b_5 \frac{S_0}{S} \quad \text{(v)}$$

$$\frac{H_D}{H - H_D} = a_6 + b_6 \frac{S_0}{S} \quad \text{(vi)}$$

$$\frac{H}{H_D} = a_7 + b_7 \frac{S}{S_0} \quad \text{(vii)}$$

$$\frac{H}{H_D} = A_1 + A_2 \frac{S}{S_0} + A_3 \left(\frac{S}{S_0}\right)^2 \quad \text{(viii)}$$

The formulation lends more confidence in the use of the already empirically known relations (i) and (ii) by providing them a theoretical basis and affords more flexibility to the estimation techniques by supplying new equations (iii) to (viii). The study also enables to throw light into the nature of the constants appearing in the various equations. It identifies

three independent basic parameters and the constants appearing in the various equations as simple functions of these three basic parameters. This provides unification and interrelationships between the various constants. Experimental data for the diffuse irradiation, the global irradiation and the bright sunshine duration for Macerata (Italy), Salisbury and Bulawayo (Zimbabwe) is found to show good correlation for the linear eqs. (i) to (vii), and the nature and the interrelationships of the constants is found to be as predicted by the theory.

1. INTRODUCTION

A reasonably accurate knowledge of the amount of the global and the diffuse irradiation at any place is necessary for many solar energy applications. However the necessary equipment for their measurements are available only at a few places. For this reason there have been attempts at estimating them from theoretical models.

A number of approaches have been used for estimation purposes. In one of the approaches, use is made of the measurable atmospheric parameters like optical density, surface reflectivity, amount of precipitable water, etc. [1-3]. The other approach consists of using the empirical relations among the global irradiation, the diffuse irradiation, the extraterrestrial irradiation and the meteorological parameters such as the bright sunshine duration, relative humidity and temperature.

The first attempt at estimating the global irradiation is the famous Angström correlation between the global irradiation, the irradiation under clear sky conditions and the bright sunshine duration [4]. The modified version of the Angström's correlation used by Page [5] has been the most convenient and widely used correlation for estimating the global irradiation [6-16]. A host of other empirical relationships in terms of the sunshine duration, the relative humidity, and the temperature have been proposed to estimate the global irradiation. A brief account of them is given by Sayigh [17] .

For estimating the diffuse irradiation, the empirical correlations due to Liu and Jordan [18] and Page [19] are the most widely used. With the increasing awareness of solar energy applications a substantial amount of work has been done recently and several new empirical relations among the diffuse irradiation, the global irradiation, the extraterrestrial irradiation and the bright sunshine duration have been proposed and applied for the various locations [20-23]. The bright sunshine duration has emerged as the most dominating parameter for the estimation of the global as well as the diffuse irradiation.

The various equations proposed for estimating the irradiation are empirical in nature and there is no theoretical basis for them although

the oldest among them, the Angström linear regression, was proposed sixty years back. In the absence of a theoretical work the different empirical relations lack the existence of a unifying structure and stand completely apart from each other as if they did not bear any relationship with each other.

In this paper an attempt is made to develope a theory starting from some simple expressions. Two of the known empirical relations, including the Angström's modified relation, are then derived in a simple and natural way. Some new relations are also obtained which may be useful in the future work on the estimation of solar irradiation. The model also brings the important realization that there are only three independent basic parameters and all the constants in different equations are simple functions of these three basic parameters. The theory has enabled a deeper study and understanding of the nature of the constants and the interrelationships amongst them. Finally, from the data for the simultaneous measurement of the diffuse irradiation, the global irradiation and the sunshine duration for Macerata in Italy and Salisbury and Bulawayo in Zimbabwe, it is shown that the new equations derived in this paper do give a good correlation and that the relationaships among the different constants and their nature is indeed as predicted by the model.

2. FORMULATION AND DERIVATION OF EQUATIONS

The meaning of the symbols used is explained in the nomenclature at the end. At any time and place, the intensities of the diffuse radiation and the direct radiation on a horizontal surface can be expressed as fractions of the intensity of the extraterrestrial radiation on a horizontal surface, i.e.

$$(I_D)_{clear} = \alpha I_0 \qquad (1)$$

$$(I_D)_{cloud} = \beta I_0 \tag{2}$$

$$(I_d)_{cloud} = 0 \tag{3}$$

$$(I_d)_{clear} = \gamma I_0 \tag{4}$$

The values of the parameters α , β and γ , in general, will change from time to time as these depend on the altitude of the sun which determines the length of the air mass travelled by the radiation, and the radiation depleting factors such as the atmospheric water vapour content, dust content, ozone content and the amount of clouds.

We can write

$$H_o = \overline{\int_{day} I_o \, dt} \tag{5}$$

where the integration is carried over the whole day and the bar indicates the average over the month. The measurements of the diffuse and the global irradiation considered for the estimation purposes are generally over fairly long periods statistically covering the different parts of the days and the different days of the year. For such records the sunshine duration can be considered to be randomly distributed over the different times of the day. Then one can write

$$\overline{\int_{clear} I_o \, dt} = H_o \frac{S}{S_o} , \tag{6}$$

where $\int_{clear}$ indicates the integration is over clear sky hours of the day only and the bar indicates the average over the month. Using (1), we write

$$(H_D)_{Clear} = \overline{\int_{Clear} (I_D)_{Clear} \, dt} = \overline{\int_{Clear} \alpha I_o \, dt} . \tag{7}$$

Assuming that α does not vary too widely at one place one can take α out of the integration replacing by its average value. Then using (6)

$$(H_D)_{Clear} = \bar{\alpha} \overline{\int_{Clear} I_o \, dt} = \bar{\alpha} H_o \frac{S}{S_o} \tag{8}$$

Using eqs. (2), (3) and (4) and proceeding similarly one can obtain the following relations

$$(H_D)_{cloud} = \bar{\beta} H_0 (1 - \frac{S}{S_0}) \tag{9}$$

$$(H_d)_{clear} = \bar{\gamma} H_0 \frac{S}{S_0} \tag{10}$$

$$(H_d)_{cloud} = 0 \tag{11}$$

One may note from physical considerations that, in general

$$\bar{\alpha} < \bar{\beta} < \bar{\gamma} \tag{12}$$

This is so beacuse the diffuse irradiation on a cloudy hour, in general, is more than that on a clear hour and also the direct irradiation for clear sky is more than the diffuse irradiation. Moreover, all the three parameters necessarily have values between 0 and 1. The exact values depend on the atmospheric conditions and so may vary from place to place. However, to give a rough idea, the values of $\bar{\alpha}$ and $\bar{\beta}$ are expected to be below 0.5 and that of $\bar{\gamma}$ above 0.5. There may be seasonal changes in the values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ due to changes in the climatological parameters, but we assume the variations are small and take them as constants in the first approximation.

The average total diffuse irradiation can be written as

$$H_D = (H_D)_{Clear} + (H_D)_{Cloud} .$$

Using eqs. (8) and (9) this gives

$$H_D = \bar{\alpha} H_0 \frac{S}{S_0} + \bar{\beta} H_0 \left(1 - \frac{S}{S_0}\right)$$

or

$$H_D = \left[\bar{\beta} + (\bar{\alpha} - \bar{\beta}) \frac{S}{S_0}\right] H_0 \quad . \tag{13}$$

The average total amount of the direct irradiation can be written as

$$H_d = (H_d)_{clear} + (H_d)_{cloud} \quad .$$

Using eqs. (10) and (11) this gives

$$H_d = \bar{\gamma} \frac{S}{S_0} H_0 \tag{14}$$

The global irradiation is obtained by adding the total diffuse and the direct irradiation, i.e.

$$H = H_D + H_d \quad .$$

Using eqs. (13) and (14), we obtain

$$H = \left[\bar{\beta} + (\bar{\gamma} + \bar{\alpha} - \bar{\beta}) \frac{S}{S_0}\right] H_0 \quad . \tag{15}$$

Eliminating $\frac{S}{S_0}$ from eqs. (13) and (15), we get

$$\frac{1}{\bar{\alpha} - \bar{\beta}} \left(\frac{H_D}{H_0} - \bar{\beta}\right) = \left(\frac{H}{H_0} - \bar{\beta}\right) \frac{1}{\bar{\gamma} + \bar{\alpha} - \bar{\beta}} \quad ,$$

which, on simplification, gives

$$\frac{H_D}{H_0} = \frac{\bar{\beta}\bar{\gamma}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} + \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} \cdot \frac{H}{H_0} \tag{16}$$

Multiplying eq.(16) by $\frac{H_0}{H}$, we obtain

$$\frac{H_D}{H} = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} + \frac{\bar{\beta}\bar{\gamma}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} \cdot \frac{H_0}{H} \tag{17}$$

Dividing eq.(15) by (14), we obtain

$$\frac{H}{H_d} = \frac{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})}{\bar{\gamma}} + \frac{\bar{\beta}}{\bar{\gamma}} \frac{S_0}{S} \quad .$$

Since $H_d = H - H_D$, this gives

$$\frac{H}{H - H_D} = \frac{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})}{\bar{\gamma}} + \frac{\bar{\beta}}{\bar{\gamma}} \cdot \frac{S_0}{S} \quad . \tag{18}$$

Dividing eq.(13) by (14) we get

$$\frac{H_D}{H_d} = \frac{H_D}{H - H_D} = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma}} + \frac{\bar{\beta}}{\bar{\gamma}} \frac{S_0}{S_0} \quad . \tag{19}$$

We can write

$$\frac{H}{H_D} = \frac{H}{H_0} \cdot \frac{H_0}{H_D} \quad .$$

Using eqs. (15) and (13), we can rewrite the above equations as

$$\frac{H}{H_D} = \left[\bar{\beta} + (\bar{\gamma} + \bar{\alpha} - \bar{\beta}) \frac{S}{S_0}\right] \left[\bar{\beta} - (\bar{\beta} - \bar{\alpha}) \frac{S}{S_0}\right]^{-1}$$

$$= \left(1 + \frac{\bar{\gamma} + \bar{\alpha} - \bar{\beta}}{\bar{\beta}} \cdot \frac{S}{S_0}\right) \left[1 - \frac{\bar{\beta} - \bar{\alpha}}{\bar{\beta}} \frac{S}{S_0} - \left(\frac{\bar{\beta} - \bar{\alpha}}{\bar{\beta}}\right)^2 \left(\frac{S}{S_0}\right)^2 + \cdots\right]$$

Since $\frac{\bar{\beta} - \bar{\alpha}}{\bar{\beta}} < 1$ and also $\frac{S}{S_0} < 1$, the series in the second term is convergent and only a few terms of the series can be retained. Retaining the term only up to $(S/S_0)^2$ in the overall product, we have

$$\frac{H}{H_D} = 1 + \frac{\bar{\gamma} + 2(\bar{\alpha} - \bar{\beta})}{\bar{\beta}} \cdot \frac{S}{S_0} - \frac{\bar{\gamma}(\bar{\beta} - \bar{\alpha})}{\bar{\beta}^2} \left(\frac{S}{S_0}\right)^2 . \tag{20}$$

3. DISCUSSION OF THE EQUATIONS

In the previous section, we obtained eqs. (13) to (15) as a natural consequence of eqs. (8) to (11). Eqs. (13) to (15) were then used to obtain eqs.(16) to (20). Out of the equations (13) to (20), some can be identified as the already known empirical equations used for estimation of solar irradiation while the others are new relations. All these equations are derived on logical reasoning. The formulation therefore puts the already known empirical equations on a firm theoretical footing. By furnishing the new equations it has provided flexibility by adding substantially to the existing wealth of known relations for the estimation of solar irradiation. The other important feature revealed by the formulation is that there are only three basic constants $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$; all the constants occurring in the various equations are simple functions of these three constants. This formulation should therefore enable us to establish interrelationships between the various constants and throw light into the nature of these constants. We now discuss each of the equations separately.

Eq. (13) is of the form

$$\frac{H_D}{H_o} = a_1 + b_1 \frac{S}{S_o} , \qquad (21)$$

$$\text{where} \quad a_1 = \bar{\beta} \quad \text{and} \quad b_1 = \bar{\alpha} - \bar{\beta} \qquad (21a)$$

Eq. (21) connects the diffuse irradiation and the sunshine duration. This equation is useful as it does not require the knowledge of the global irradiation for estimating the diffuse irradiation. This is not a completely new relationship, but it has been tried very sparingly. It appears that only Barbaro et al. [20] have tried it along with several other equations. The simplicity and convenience of this equation coupled with the theoretical basis provided here should encourage more use of this equation. From eqs. (21a) it is redaily seen that a_1 has to be a definitely positive constant and due to the inequality (12), the constant b_1 , in general, should be negative. Further, the magnitude of b_1 is less

than that of a_1 and the magnitudes of a_1 and b_1 both are expected to be below 0.5. All these predictions are seen to be correct by the analysis of the three stations carried out in the next section and summarized in Table 2.

Eq.(15) is of the form

$$\frac{H}{H_o} = a_2 + b_2 \frac{S}{S_o} , \qquad (22)$$

$$\text{where} \quad a_2 = \bar{\beta} \quad \text{and} \quad b_2 = [\bar{\gamma} - (\bar{\beta} - \bar{\alpha})] . \qquad (22a)$$

Eq. (22) is nothing but the well-known Angström's regression equation. It is again easily seen that in the light of the inequality (9) the coefficients a_2 and b_2 are both positive. The value of the coefficient a_2 is likely to be below 0.5. Also b_2 has to be definitely less than unity. Angström's equation has been widely used to estimate the global irradiation and these facts are found to hold true. However, it appears difficult to predict with reasonable certainty about the relative magnitudes of a_2 and b_2 . Their magnitude depends on the magnitude of the parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$.

Eqs. (21a) and (22a) further show that the constant terms in eqs. (21) and (22) are equal because $a_1 = \bar{\beta} = a_2$.

Eq.(16) is of the form

$$\frac{H_D}{H_o} = a_3 + b_3 \frac{H}{H_o} , \qquad (23)$$

$$\text{where} \quad a_3 = \frac{\bar{\beta}\,\bar{\gamma}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} \quad \text{and} \quad b_3 = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} . \qquad (23a)$$

Eq. (23) is a new correlation. As is seen from eqs.(23a) the constant a_3 is positive and b_3 is negative. Again it is difficult to say anything regarding the relative magnitudes of a_3 and b_3 as these depend on the values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, which may vary from one place to another due to varying atmospheric conditions.

Eq. (17) is of the form

$$\frac{H_D}{H} = a_4 + b_4 \frac{H_0}{H} , \qquad (24)$$

where $a_4 = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} = b_3$, and $b_4 = \frac{\bar{\beta}\,\bar{\gamma}}{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})} = a_3$. (24a)

Eq. (24) is a new equation which suggests a correlation between H_D/H and H_0/H. The equation is linear and therefore simple to apply. The validity of this equation is tested for three locations in the next section. It may be interesting to test this correlation for more locations in future. Though eq.(17) is obtained from eq.(16) by a mere multiplication of H_0/H, the two equations nevertheless represent different correlations and can give rise to different values of the correlation coefficients when applied to the same set of data. This is seen in Table 2 for the Italian location Macerata. The constants in eq.(24) are the same as those in eq. (21) with interchange of positions only.

Eq.(18) is of the form

$$\frac{H}{H - H_D} = a_5 + b_5 \frac{S_0}{S} , \qquad (25)$$

where $a_5 = \frac{\bar{\gamma} - (\bar{\beta} - \bar{\alpha})}{\bar{\gamma}}$ and $b_5 = \frac{\bar{\beta}}{\bar{\gamma}}$ (25a)

Eq. (25) is again a new correlation between $H/(H-H_D)$ and S_0/S. The correlation is again linear and simple to apply. This correlation requires the values of the global irradiation, the diffuse irradiation and the sunshine duration, but is independent of the extraterrestrial irradiation. The coefficients a_5 and b_5 are both expected to be positive and less than unity. The validity of the equation is again seen in the next section for three locations and it would be interesting to try this equation for more locations. The nature of the coefficients is also found to be as predicted here.

Eq. (19) is of the form

$$\frac{H_D}{H - H_D} = a_6 + b_6 \frac{S_0}{S} \qquad (26)$$

where $a_6 = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\gamma}} = a_5 - 1$ and $b_6 = \frac{\bar{\beta}}{\bar{\gamma}} = b_5$. (26a)

Eq. (26) is also a new correlation similar to eq.(25). The coefficient a_6 is negative and its magnitude is less than unity. The coefficient b_6 is the same as b_5. These predictions are seen to be true in the next section.

Eq. (20) is of the form

$$\frac{H}{H_D} = A_1 + A_2 \frac{S}{S_0} + A_3 \left(\frac{S}{S_0}\right)^2 \qquad (27)$$

where $A_1 = 1$, $A_2 = \frac{\bar{\gamma} + 2(\bar{\alpha} - \bar{\beta})}{\bar{\beta}}$ and $A_3 = \frac{\bar{\gamma}(\bar{\alpha} - \bar{\beta})}{\beta^2}$. (27a)

Eq. (27) is another new equation . This is the only non-linear correlation derived in this paper. The number of terms to be included in this equation depends on the accuracy desired and the values of the parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$, and the fraction S/S_0. It is expected that eq.(27) would represent a sufficiently accurate correlation if the terms beyond $A_3(\frac{S}{S_0})^2$ are neglected. Or more conveniently, one can even try a linear relationaship of the form

$$\frac{H}{H_D} = a_7 + b_7 \frac{S}{S_0} , \qquad (28)$$

where a_7 and b_7 are constants to be determined by regression analysis. It may be cautioned that the constants in eqs.(27) and (28) will not, in general, be given by eq. (27a). Both eqs.(27) and (28) have been obtained after truncating the series and are therefore approximate equations. The values of the coefficients will therefore be given by actual regression analysis of the experimental data.

4. APPLICATION AND VERIFICATION OF THE MODEL

It would be interesting to see the validity of the correlations derived in the previous section and also the predicted interrelationships between the various constants. The equations require the experimental values of the global irradiation, the diffuse irradiation and the sunshine duration. We surveyed the literature and tried to find the places for which these experimental values are provided for the same duration. We found the two Zimbabwean locations, Salisbury and Harare, in Lewis' paper [23] and one Italian location, Macerata, in Barabaro et al's paper [22] for which the needed data is provided. In Barbaro et al.'s paper, we did not consider the other two locations, Genova and Palermo, as the radiation data given therein is for a short period which is different from that of the sunshine duration. It is necessary that the two records should be for the same duration particularly when one or both of them are only for a few years. In the following we discuss and analyze the data for these locations in relation to the equations derived earlier.

Table 1 lists the given data for the three locations and the values of S_0 and H_0 calculated using the standard expression given in Duffie and Beckmann's book [24] and the values of the day of year as recently recommended by Jain [25] . A computer programme was prepared for linear regression analysis using the least square method and the data for the three locations was analyzed. The linear equations (21) to (26) and (28) were all fitted to the data of each of the three locations and the values of the regression constants and the correlation coefficients were computed. The results of the analysis are summarized in Table 2.

It is readily seen from Table 2 that a good correlation is found to exist in most of the cases with values of the correlation coefficient above 0.9 or close to it except for Macerata for eqs.(21) and (23) for which the correlation is not good. Except eq.(23) for the case of Macerata only, the five new equations (23) to (26) and (28) have all given good correlations for all the three locations. This lends justification to the formulation presented in this paper and should encourage the use of the new equations in future work on the estimation of the diffuse irradiation.

The results in Table 2 also show that all the values of the various coefficients are indeed as predicted in the previous section. The signs of all the coefficients are fully in accordance with the predictions. Also the qualitative predictions regarding their magnitudes are all found to hold true. It may be noted that the coefficients of eq. (28) are completely different than those in eq.(27). As pointed out in the previous section it was expected to be so. Unlike eqs.(21) to (26), eq.(28) is an approximate equation only and so its coefficients cannot be predicted. They should be solely determined from the analysis of the experimental data.

Let us investigate the interrelationships of the various constants in our equations more deeply and qualitatively. For any place the values of the twelve constants in eqs.(21) to (26)-six a's and b's - are not expected to be independent according to the formulation presented in this paper. There are only three independent parameters, $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. The relationships in the previous section should enable one to calculate, within reasonable limits of errors, the values of all the twelve constants for one place from an appropriate single set of the values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ characteristic of that place. To verify this assertion we made a computer programm to find an appropriate set of the values for each of the three places. We call the values of the constants as determined by linear regression analysis (Table 2) the "estimated values" of the constants and those calculated from a given set of the values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ and using the relationship of the previous section as the "calculated values". Then the most appropriate values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ are choosen to be those which minimize the sum, S, of the square of the relative deviations of the estimated values of the constants, i.e.

$$\text{Min } S = \sum_{i=1}^{6} \left[\left(\frac{(a_i)_{est} - (a_i)_{cal}}{(a_i)_{est}} \right)^2 + \left(\frac{(b_i)_{est} - (b_i)_{cal}}{(b_i)_{est}} \right)^2 \right].$$

The computed values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ are presented in Table 3 for the three places. The estimate of the error was done by calculating the root mean square error, i.e., $\sqrt{\frac{S}{12}}$.

As is seen in Table 3 the estimated error is about 4%, 13% and 9% for Macerata, Salisbury and Bulawayo, respectively. These errors are well within reasonable limits. The values of the constants were then calculated using the set of values of $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ and the relationships in the previous section. The calculated values of the constants are presented in Table 4. For convenience in comparison the values of the constants as estimated from the linear regression are also presented in parenthesis. The agreement is very close in the case of Macerata and fairly close for Salisbury and Bulawayo. In fact, we regard the agreement as excellent for all the three places as we should not have expected a closer agreement between the theory and the data which is very often prone to certain errors. Moreover the computing of the regression coefficients using the least square fit approximation is simply a suitable criterion and not an absolute measure of the values of the coefficients. If some other criterion is used, substantial deviations in the values of the constants may be obtained. A logical snag in the formulation could have led to wide differences in the values.

In the end it may be remarked that a knowledge of the parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ would enable one to predict the global as well as the diffuse irradiation. It may be possible to determine these parameters experimentally by simply measuring the instantaneous values of the parameters α , β and γ for a sufficiently large number of times distributed randomly over different times of the day and days of the year and taking their averages. Such measurements may be less laborious and accomplishable even at places where facilities for continuous measurements do not exist, and could thus enable a fair assessment of the global and the diffuse irradiation.

5. SUMMARY AND CONCLUSION

The intensities of the diffuse and the direct radiation both under clear sky conditions as well as cloudy sky conditions have been expressed as

fractions of the extraterrestrial radiation intensity. Using these and the assumptions that (i) for long-range records the bright sunshine hours may be considered to be randomly distributed over the different times of the day, and (ii) that there are not wide variations in the transmission coefficients for different times of the day and different days of the year, the expressions for the total diffuse irradiation and total direct irradiation are obtained which are then used to obtain an expression for the global irradiation. This has resulted in the well-known modified Angström linear equation for estimating the global irradiation and another already known empirical relation for estimating the diffuse irradiation. Some algebraic manipulation of these equations lead to 5 more equations which are new correlations.

This study provides a theoretical basis to some of the known empirical relations and therefore lends more confidence in their use. The study has also established five new correlations for estimating the diffuse irradiation, thus providing more flexibility and novelty to the future attempts at estimating the diffuse irradiation.

The other important finding of the study is that there are only three basic parameters; all the constants appearing in different equations are simple functions of these three basic parameters. This has enabled to understand and predict qualitative nature of the constants in different equations and their interrelationships. The experimental data for the irradiation and the sunshine duration for three places - Macerata in Italy and Salisbury and Bulawayo in Zimbabwe - is used to show that the new equations provide a good correlation and that the interrelationships of the constants is in agreement with those predicted by the model.

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NOMENCLATURE

$(I_D)_{clear}$	Instantaneous diffuse radiation intensity on horizontal surface for clear sky conditions
$(I_D)_{cloud}$	Instantaneous diffuse radiation intensity on horizontal surface for cloudy sky conditions.
$(I_d)_{cloud}$	Instantaneous direct radiation intensity on a horizontal surface for cloudy sky conditions.
$(I_d)_{clear}$	Instantaneous direct radiation intensity on a horizontal surface for clear sky conditions.
I_0	Extraterrestrial radiation intensity on a horizontal surface
α	Instantaneous transmission coefficient for diffuse radiation on horizontal surface for clear sky conditions.
β	Instantaneous transmission coefficient for diffuse radiation on horizontal surface for cloudy sky conditions
γ	Instantaneous transmission coefficient for direct radiation for clear sky conditions.
$(H_D)_{cloud}$	Monthly average daily diffuse irradiation on a horizonatal surface during cloudy hours only
$(H_D)_{clear}$	Monthly average daily diffuse irradiation on a horizontal surface during clear sky hours only
$(H_d)_{cloud}$	Monthly average daily direct irradiation on a horizontal surface during cloudy hours only
$(H_d)_{clear}$	Monthly average daily direct irradiation on a horizontal surface during clear hours only.
H_0	Monthly average daily extraterrestrial irradiation
H_D	Total monthly average daily diffuse irradiation
H_d	Total monthly average daily direct irradiation

$\bar{\alpha}$	Average of α for an assumed random distribution of the cloudy hours over the different times and days of the years and for varying atmospheric conditions.
$\bar{\beta}$	Average of β for an assumed random distribution of the cloudy hours over the different times and days of the years and for varying atmospheric conditions.
$\bar{\gamma}$	Average of γ for varying atmospheric conditions.
S	Monthly average daily number of sunshine hours
S_0	Monthly average daily maximum possible number of sunshine hours.
a's, b's, A's	constants

Table 1 (a) Measured monthly mean daily irradiation and S/S_0 values (1958-73)(from Barbaro et al. [22]) and the calculated values of H_0 for Macerata (43°18'N; altitude 338 m).

Month	Global Irradiation (MJ/m^2 day)	Diffuse Irradiation (MJ/m^2 day)	S/S_0	H_0
Jan	6.4	3.3	0.34	13.1
Feb	10.0	5.0	0.41	18.2
Mar	13.9	5.6	0.39	25.6
Apr	18.8	7.1	0.45	33.4
May	24.8	8.0	0.57	39.0
Jun	26.9	8.6	0.57	41.3
Jul	27.0	8.3	0.67	40.1
Aug	24.2	7.3	0.65	35.4
Sep	20.4	6.1	0.56	28.2
Oct	12.9	4.9	0.50	20.4
Nov	7.7	3.6	0.39	14.2
Dec	5.9	2.7	0.33	11.6

Table 1(b) Measured monthly mean daily irradiation and sunshine data (1968-78) (from Lewis [23]) and the calculated values of S_0 and H_0 for Salisbury (17°50'S; 31°01'E; altitude 1471m)

Month	Global irradiation (MJ/m^2 day)	Diffuse irradiation (MJ/m^2 day)	Sunshine hours S	S_0	H_0
Jan	22.3	10.2	6.2	12.9	41.0
Feb	21.5	9.8	6.4	12.6	39.6
Mar	21.3	8.4	7.4	12.1	36.5
Apr	20.0	6.1	8.0	11.6	31.8
May	18.6	4.3	8.5	11.2	27.3
Jun	17.2	3.8	8.6	10.9	25.1
Jul	18.4	3.9	9.2	11.0	26.0
Aug	21.1	4.1	9.6	11.4	29.7
Sep	23.9	5.6	9.7	11.9	34.6
Oct	24.9	7.1	9.1	12.4	38.4
Nov	23.5	8.1	6.8	12.9	40.5
Dec	21.8	10.2	6.0	13.1	41.2