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SO(4) GAUGING OF N = 2 SUPERGRAVITY IN SEVEN DIMENSIONS *

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ABSTRACT

We obtain N = 2 supergravity theory coupled to N = 2 matter by truncation of the N = 4 theory in seven dimensions. The truncated theory possesses global GL(4,R) ⊗ local composite SO(4) invariance. We then gauge the global SO(4) subgroup of GL(4,R), and preserve the composite SO(4) symmetry, following the method of de Wit and Nicolai in d = 4. The resulting action has a potential which contains all ten scalars of the theory. The single second rank antisymmetric tensor field in the theory has a generalized field strength, which contains the Chern-Simons 3-form.

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1. In a previous paper [1)] we constructed the maximal supergravity theory in seven dimensions (i.e. N = 4, with 128 + 128 modes). The theory possesses global SL(5,R) symmetry which acts on the bosons, and a "local" USp(4) ~ SO(5) symmetry which acts on the fermions. We conjectured that the global SO(5) subgroup of the SL(5,R) could be gauged, leaving the composite local SO(5) invariance intact. This is similar to the gauging of the global SO(8) subgroup of E_7 symmetry of d = 4, N = 8 supergravity, in which the composite SU(8) group is left intact [2)]. The fact that the vector fields of the d = 7, N = 4 theory are in the adjoint representation of the global SO(5) group, makes the similarity to the d = 4, N = 8 case even closer. However, there is an obstacle in carrying out the gauging of SO(5) in d = 7, due to the presence of the antisymmetric tensor fields, which are not singlets of SO(5), but are in the 5-fold representation of the global SO(5). There is no known technique of constructing covariantly transforming field strengths for such objects if they belong to a non-singlet representation of the internal symmetry group. Presumably a new type of differential algebra [3)] is needed to solve this problem.

In this note we consider a truncation of the N = 4 theory which yields N = 2 supergravity (containing 40 + 40 modes) coupled to a three fold of N = 2 matter (which comprises a total of 24 + 24 modes). This theory by-passes the problem mentioned above since it contains only a singlet antisymmetric tensor field. The theory possesses global GL(4,R) ⊗ local composite SO(4) symmetry. The vectors form an SO(4) anti-self dual triplet in the supergravity sector, as well as an SO(4)-self dual triplet in the matter sector. Our main result is the gauging of the $SO(3)_I \otimes SO(3)_{II} \approx SO(4)$ global subgroup of the GL(4,R), in which $SO(3)_I$ and $SO(3)_{II}$ are associated with the anti-self dual and self dual triplets of the vectors, respectively, which leaves the composite SO(4) invariance intact. (The gauging has been carried out with the same coupling constant, both for $SO_I(3)$ and $SO_{II}(3)$.) Our method follows closely that of de Wit and Nicolai, and our results are similar to those obtained by them for the gauged N = 8. However their results are for four dimensions, whereas ours are for seven dimensions.

In Secs.2 and 3 we rewrite the N = 4 theory in a notation appropriate for the truncation to N = 2, which is carried out in Sec.4, while the gauging of the N = 2 theory is carried out in Secs.5 and 6. Our index conventions are: for the N = 4 theory, M,N = 1,...,5 are curved SL(5,R), A,B = 1,...,5 are flat SO(5), α,β = 1,...,4 are curved USp(4) and a,b = 1,...,4 are flat USp(4) indices. For the N = 2 theory, α,β = 1,...,4 are curved GL(4,R) and i,j = 1,...,4 are flat SO(4) indices. The SO(4) spinorial indices a,b = 1,...,4 will often be suppressed. We use the signature (+ - - - - - -).

2. In order to perform the truncation of the $N = 4$ theory down to $N = 2$, it is best to rewrite the $N = 4$ theory [1)] in a manifestly $SL(5,R) \otimes SO(5)$ invariant form. To this effect, we make use of the 4×4 matrices which obey the SO(5) Clifford algebra [4)]

$$\{\gamma_A, \gamma_B\} = 2\eta_{AB} \quad , \quad \eta_{AB} = \mathrm{diag}(-,-,-,-,-) \quad , \tag{1}$$

$$\Omega\gamma_A\Omega^{-1} = \gamma_A^T \; ; \qquad \Omega\gamma_A\Omega^{-1} = -\gamma_A^* \quad , \quad \Omega^T = -\Omega \; ; \quad \Omega^2 = -1 \quad . \tag{2}$$

Our convention for the suppressed indices are such that $(\gamma_A\Omega)_{ab} = (\gamma_A)_a{}^c\Omega_{cb}$, $(\Omega\gamma_A)^{ab} = \Omega^{ac}(\gamma_A)_c{}^b$. Note that $\gamma_A\Omega$ are antisymmetric, whereas $\gamma_{AB}\Omega$ are symmetric *). Using these matrices we redefine the fields of the $N = 4$ theory as follows [1)]:

$$A_\mu^{\alpha\beta} = (\gamma_{MN})^{\alpha\beta} A_\mu^{MN} \qquad B_{\mu\nu,\alpha\beta} = \frac{1}{2}(\gamma^M)_{\alpha\beta} B_{\mu\nu M}$$

$$\mathcal{V}_{\alpha\beta}^{ab} = \frac{1}{4}(\gamma^M)_{\alpha\beta}(\gamma_A)^{ab}\mathcal{V}_M^A \qquad \chi_{abc} = i(\gamma_A)_{ab}\,\chi_c^A \quad , \tag{3}$$

$A_\mu^{MN} = -A_\mu^{MN}$ are the ten vectors of the theory. $B_{\mu\nu M}$ are the antisymmetric tensor fields which belong to the 5 of SL(5,R). $\mathcal{V}_M{}^A$ are the 5-beins associated with the SL(5,R)/SO(5) coset, where $\mathcal{V}_M{}^A\mathcal{V}_B{}^M = \delta_B^A$, $\det \mathcal{V}_M{}^A = 1$. They are parametrized by the 14 scalars of the $N = 4$ theory. χ_a^A is the 16 dimensional vector-spinor of SO(5), which obeys the symplectic Majorana condition [4)] and the SO(5) irreducibility condition given by

$$(\chi_A)^* = -\Gamma_0\Omega\chi_A \quad , \quad \gamma^A\chi_A = 0 \quad . \tag{4}$$

All boson fields are real.

We recall that the raising and lowering of the SL(5,R) indices are with the metric $g_{MN} = \mathcal{V}_M{}^A\mathcal{V}_N{}^B\eta_{AB}$, and those of SO(5) with η_{AB}.

3. Using the transcription given in Eq.(3) in the $N = 4$ theory [1)] we can rewrite the Lagrangian (Eq.(12) of Ref.1) as follows ($\kappa = 1$):

*) Our antisymmetrizations are always with unit strength, e.g. $\gamma_{AB} = \frac{1}{2}[\gamma_A, \gamma_B]$.

$$\begin{aligned}
V^{-1}\mathcal{L} = & -\frac{1}{4}R - \frac{1}{4}g_{MK}g_{NL}F_{\mu\nu}^{MN}F^{\mu\nu KL} - \frac{1}{12}g^{MN}G_{\mu\nu\rho M}G_N^{\mu\nu\rho} \\
& + \frac{1}{4}P_\mu^{AB}P_{AB}^\mu - \frac{i}{2}\bar\psi_\mu\Gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{i}{2}\bar\chi^A\Gamma^\mu D_\mu\chi_A \\
& + \frac{iV^{-1}}{90\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda\tau\kappa}\left(G_{\mu\nu\rho M}G_{\sigma\lambda\tau N}A_\kappa^{MN} - 6\,G_{\mu\nu\rho M}B_{\lambda\tau N}F_{\sigma\kappa}^{MN}\right) \\
& + \frac{1}{8\sqrt{2}}\mathcal{V}_M^A\mathcal{V}_N^B F_{\mu\nu}^{MN}\left(\bar\psi^\lambda\Gamma_{[\lambda}\Gamma^{\mu\nu}\Gamma_{\tau]}\gamma_{AB}\psi^\tau - 4\bar\psi_\lambda\Gamma^{\mu\nu}\Gamma^\lambda\gamma_A\chi_B + \frac{1}{2}\bar\chi_C\gamma^D\gamma_{AB}\gamma^C\Gamma^{\mu\nu}\chi_D\right) \\
& - \frac{1}{24}\mathcal{V}_A^M G_{\mu\nu\rho M}\left(\bar\psi^\lambda\Gamma_{[\lambda}\Gamma^{\mu\nu\rho}\Gamma_{\tau]}\gamma^A\psi^\tau - 2\bar\psi_\lambda\Gamma^{\mu\nu\rho}\Gamma^\lambda\chi^A - \bar\chi^C\Gamma^{\mu\nu\rho}\gamma^A\chi_C\right) ,
\end{aligned} \tag{5}$$

where

$$G_{\mu\nu\rho M} = \partial_\mu B_{\nu\rho M} + \frac{1}{2}\epsilon_{MPQRS}F_{\mu\nu}^{PQ}A_\rho^{RS} + \ 2 \text{ perms.} \tag{6a}$$

A typical covariant derivative is:

$$D_\mu\chi_A = \left(\partial_\mu + \frac{1}{4}Q_{\mu CD}\gamma^{CD}\right)\chi_A + Q_{\mu A}{}^B\chi_B \ . \tag{6b}$$

The composite SO(5) gauge fields $Q_{\mu AB}$ and the covariant derivative of the scalars are defined by

$$\mathcal{V}^M{}_A\,\partial_\mu\mathcal{V}_{MB} = P_{\mu(AB)} + Q_{\mu[AB]} \quad ; \quad \eta^{AB}P_{\mu AB} = 0 \ . \tag{6c}$$

The supersymmetry transformation laws (Eq.(13) of Ref.1) transcribe as:

$$\begin{aligned}
\delta V_\mu^r &= -i\,\bar\epsilon\Gamma^r\psi_\mu \qquad\qquad \mathcal{V}_A^M\,\delta\mathcal{V}_{MB} = \frac{i}{2}\left(\bar\epsilon\gamma_A\chi_B + \bar\epsilon\gamma_B\chi_A\right) \\
\mathcal{V}_M^A\mathcal{V}_N^B\,\delta A_\mu^{MN} &= \frac{1}{2\sqrt{2}}\left(\bar\epsilon\gamma^{AB}\psi_\mu + \bar\epsilon\gamma^A\chi^B - \bar\epsilon\gamma^B\chi^A\right) \\
\mathcal{V}_A^M\,\delta B_{\mu\nu M} &= \mathcal{V}_A^M\epsilon_{MPQRS}A_\mu^{PQ}\delta A_\nu^{RS} - \frac{1}{2}\left(\bar\epsilon\gamma_A\Gamma_\mu\psi_\nu - \bar\epsilon\gamma_A\Gamma_\nu\psi_\mu - \bar\epsilon\Gamma_{\mu\nu}\chi_A\right) \\
\delta\psi_\mu &= D_\mu\epsilon + \frac{i}{20\sqrt{2}}F_{\rho\sigma}^{AB}\gamma_{AB}\left(\Gamma^{\rho\sigma}{}_\mu + 8\Gamma^\rho\delta_\mu^\sigma\right)\epsilon - \frac{i}{30}G_{\rho\sigma\tau A}\gamma^A\left(\Gamma^{\rho\sigma\tau}{}_\mu + \frac{9}{2}\Gamma^{\rho\sigma}\delta_\mu^\tau\right)\epsilon \\
\delta\chi_A &= -\frac{1}{2}P_{\mu AB}\gamma^B\epsilon + \frac{i}{10\sqrt{2}}F_{\mu\nu}^{CD}\Gamma^{\mu\nu}\left(\gamma_{ACD} - 3\eta_{AC}\gamma_D\right)\epsilon - \frac{i}{60}G_{\mu\nu\rho}^B\Gamma^{\mu\nu\rho}\left(\gamma_{AB} - 4\eta_{AB}\right)\epsilon
\end{aligned} \tag{7}$$

As a check, we have re-established the invariance of (5) (up to the total derivative and quartic fermion terms) under (7).

4. In order to obtain the N = 2 theory, we require the consistent truncation of the supersymmetry transformation laws (i.e. the variations of the vanishing fields must also vanish) in such a way that the SL(5,R) $\otimes$ SO(5) conditions on the fields are taken into account. We find that this leads to a truncation scheme which retains

$$\frac{1-i\gamma_5}{\sqrt{2}}\,\psi_\mu \equiv \psi_\mu\,, \qquad \frac{1+i\gamma_5}{\sqrt{2}}\,\chi_k \equiv \chi_k\,, \qquad \chi_5 = -i\gamma^k\chi_k$$

$$g_{\mu\nu},\ B_{\mu\nu 5} \equiv B_{\mu\nu} \qquad A^{ij}_\mu\,,\quad \mathcal{V}^i_\alpha\,,\quad \mathcal{V}^5_5 = (\det \mathcal{V}^i_\alpha)^{-1} \equiv \mathcal{V}^{-1} \tag{8}$$

This set contains 64 + 64 modes. The multiplet they form is not irreducible under supersymmetry. It splits into a pure N = 2 supergravity multiplet which consists of 40 + 40 modes

$$g_{\mu\nu},\ \psi_\mu,\ \chi,\ B_{\mu\nu},\ \mathcal{V},\ A^{ij-}_\mu = \tfrac{1}{2}\,(A^{ij}_\mu - \tfrac{1}{2}\epsilon^{ijk\ell}A^{k\ell}_\mu) \tag{9}$$

and the N = 2 matter multiplet consisting of 24 + 24 modes

$$(\chi_k - \tfrac{i}{4}\gamma_k\chi) \equiv \lambda_k\,,\quad \mathcal{V}^i_\alpha\mathcal{V}^{-1/4} \equiv \mathcal{E}^i_\alpha,\quad A^{ij+}_\mu = \tfrac{1}{2}\,(A^{ij}_\mu + \tfrac{1}{2}\epsilon^{ijk\ell}A^{k\ell}_\mu)\,, \tag{10}$$

where $\gamma^k\lambda_k = 0$ and $\det \mathcal{E}^i_\alpha = 1$. Note that the vector fields contained in the supergravity multiplet (and the matter multiplets) are anti-self dual (and self dual).

We can now obtain from (4) the N = 2 Lagrangian in terms of the fields given by (9) and (10). If we wished we could have dropped the matter multiplet at this stage and gauged $SO(3)_I$ associated with A^{ij-}_μ only. However, in this note we shall take the advantage of the global GL(4,R) (and hence SO(4)) invariance of the total system and gauge $SO(3)_I \otimes SO(3)_{II} \sim SO(4)$ with equal coupling constants *) . It is therefore more appropriate to work with the set of fields given by (8); namely: $g_{\mu\nu}$, ψ_μ, $B_{\mu\nu}$, A^{ij}_μ, χ_k and $\mathcal{V}^i_\alpha$. In this way, one treats the gravity and matter fields, in particular the six vector fields, on an equal footing. Note also that $\mathcal{V}^i_\alpha$ now is associated with the GL(4,R)/SO(4) coset. In terms of these fields, from (5), we obtain the ungauged N = 2 Lagrangian with 64 + 64 modes as follows:

*) We have not investigated if $SO_{II}(3)$ could be gauged with a different coupling constant from $SO_I(3)$, though we believe this should be possible.

$$\begin{aligned}
V^{-1}\mathcal{L}_0 = &-\tfrac{1}{4}R - \tfrac{1}{4}(\mathcal{V}^i_\alpha\mathcal{V}^j_\beta F^{\alpha\beta}_{\mu\nu})^2 + \tfrac{1}{12}(\mathcal{V}G_{\mu\nu\rho})^2 + \tfrac{1}{4}P^{ij}_\mu(\eta_{ik}\eta_{j\ell}+\eta_{ij}\eta_{k\ell})P^{k\ell}_\mu \\
&-\tfrac{i}{2}\bar\psi_\mu\Gamma^{\mu\nu\rho}D_\nu\psi_\rho - \tfrac{i}{2}\bar\chi_i(\eta^{ij}+\gamma^i\gamma^j)\Gamma^\mu D_\mu\chi_j - \tfrac{i}{2}(P^{ij}_\mu+\eta^{ij}P_\mu)\bar\psi_\nu\Gamma^\mu\Gamma^\nu\gamma_i\chi_j \\
&+\tfrac{1}{8\sqrt{2}}\mathcal{V}^i_\alpha\mathcal{V}^j_\beta F^{\alpha\beta}_{\mu\nu}\Big(\bar\psi^\lambda\gamma_{ij}\Gamma_{[\lambda}\Gamma^{\mu\nu}\Gamma_{\tau]}\psi^\tau - 4\bar\psi_\lambda\Gamma^{\mu\nu}\Gamma^\lambda\gamma_i\chi_j + \bar\chi_k(\delta^k_\ell\gamma_{ij}+\gamma^k\gamma_{ij}\gamma^\ell - 4\delta^k_i\delta^\ell_j)\Gamma^{\mu\nu}\chi_\ell\Big) \\
&+\tfrac{i}{24}\mathcal{V}G_{\mu\nu\rho}\Big[\bar\psi^\lambda\Gamma_{[\lambda}\Gamma^{\mu\nu\rho}\Gamma_{\tau]}\psi^\tau + 2\bar\psi_\lambda\Gamma^{\mu\nu\rho}\Gamma^\lambda\gamma^i\chi_i - \bar\chi_i\gamma^{ij}\Gamma^{\mu\nu}\chi_j\Big]
\end{aligned} \tag{11}$$

where $P_\mu \equiv \eta^{ij}P_{\mu ij}$. We recall that $\alpha,\beta = 1,\ldots,4$ are the global GL(4,R) and $i,j = 1,\ldots,4$ are the composite local SO(4) indices. $P_{\mu ij}$ and $Q_{\mu ij}$ are obtained simply by the restriction of the corresponding SO(5) objects, P^{AB}_μ and Q^{AB}_μ . The field strength for $B_{\mu\nu}$ is $G_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}_{\mu\nu}A^{\gamma\delta}_\rho$ +2perms. Note that $\epsilon_{\alpha\beta\gamma\delta}$ is a constant tensor of SO(4), and the FA term contains the matter as well as gauge vectors, in the form $(F^+A^+ - F^-A^-)$. The supersymmetry transformation laws are to be obtained from (7) and read:

$$\begin{aligned}
&\delta_0 V^r_\mu = -i\bar\epsilon\,\Gamma^r\psi_\mu \qquad \mathcal{V}^\alpha_i\,\delta_0\mathcal{V}_{\alpha j} = \tfrac{i}{2}\,(\bar\epsilon\,\gamma_i\chi_j + \bar\epsilon\,\gamma_j\chi_i) \\
&\mathcal{V}^i_\alpha\,\mathcal{V}^j_\beta\,\delta_0 A^{\alpha\beta}_\mu = \frac{1}{2\sqrt{2}}\,(\bar\epsilon\,\gamma^{ij}\psi_\mu + \bar\epsilon\,\Gamma^\mu\gamma^i\chi^j - \bar\epsilon\,\Gamma^\mu\gamma^j\chi^i) \\
&\delta_0 B_{\mu\nu} = \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}_\mu\,\delta_0 A^{\gamma\delta}_\nu - \tfrac{i}{2}\mathcal{V}^{-1}\,(\bar\epsilon\,\Gamma_\mu\psi_\nu - \bar\epsilon\Gamma_\nu\psi_\mu + \bar\epsilon\Gamma_{\mu\nu}\gamma^i\chi_i) \\
&\delta_0\psi_\mu = D_\mu\epsilon + \frac{i}{20\sqrt{2}}F^{ij}_{\rho\sigma}\gamma_{ij}\,(\Gamma^{\rho\sigma}{}_\mu + 8\Gamma^\rho\delta^\sigma_\mu)\epsilon - \frac{\mathcal{V}}{30}\,G_{\rho\sigma\tau}\,(\Gamma^{\rho\sigma\tau}{}_\mu + \tfrac{9}{2}\Gamma^{\rho\sigma}\delta^\tau_\mu)\epsilon \\
&\delta_0\chi_k = -\tfrac{1}{2}P_{\mu k\ell}\,\Gamma^\mu\gamma^\ell\epsilon + \frac{i}{10\sqrt{2}}F^{ij}_{\mu\nu}\,\Gamma^{\mu\nu}(\gamma_{ij}\gamma_k + \gamma_i\eta_{jk})\epsilon - \frac{\mathcal{V}}{60}\,G_{\mu\nu\rho}\,\Gamma^{\mu\nu\rho}\,\gamma_k\epsilon\,,
\end{aligned} \tag{12}$$

where $F^{ij}_{\mu\nu} = \mathcal{V}^i_\alpha\mathcal{V}^j_\beta F^{\alpha\beta}_{\mu\nu}$.

5. As has been emphasized by de Wit and Nicolai [2], in gauging the global symmetries of supergravity theory with scalars, it is important to preserve the composite local symmetries of the theory. In this way, the construction of the gauged action with non-polynomial scalars becomes dramatically simplified, as we shall see below. Thus, in accordance with the procedure of Ref.2 we shall be gauging the global SO(4) subgroup of GL(4,R). We begin by covariantizing the field strength $F_{\mu\nu}$ in the standard fashion

$$\mathcal{F}^{\alpha\beta}_{\mu\nu} = \partial_\mu A^{\alpha\beta}_\nu - \partial_\nu A^{\alpha\beta}_\mu + g\,A^{\alpha\epsilon}_\mu A^{\epsilon\beta}_\nu - g\,A^{\alpha\epsilon}_\nu A^{\epsilon\beta}_\mu \tag{13}$$

The contraction in the AA term is with $\eta_{\alpha\beta}$; thus, evidently, we are giving up the full global $GL(4,R)$ invariance, while maintaining the global $SO(4)$ invariance.

The next step is to covariantize $G_{\mu\nu\rho}$. In order to preserve as much as possible the supersymmetry of the ungauged action, it is desirable that the antisymmetric curl of the new $G_{\mu\nu\rho}$ be of the form $\mathcal{F}^2$ and thus gauge invariant. This situation is not new and has been encountered before, by Chapline and Manton [5] in their construction of the $N = 1$ supergravity coupling to a Yang-Mills multiplet in $d = 10$, who made use of the Chern-Simons 3-form to secure gauge invariance. Adopting a similar approach we find the appropriate modification for our theory to be:

$$\mathcal{G}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \frac{1}{2}\epsilon_{\alpha\beta\delta\gamma}\left(\mathcal{F}^{\alpha\beta}_{\mu\nu} A^{\gamma\delta}_{\rho} - \frac{2}{3} A^{\alpha\epsilon}_{\mu} A^{\epsilon\beta}_{\nu} A^{\gamma\delta}_{\rho}\right) + 2 \text{ perms.} \tag{14}$$

This satisfies

$$\partial_{[\sigma}\mathcal{G}_{\mu\nu\rho]} = \frac{3}{4}\epsilon_{\alpha\beta\gamma\delta}\mathcal{F}^{\alpha\beta}_{[\mu\nu}\mathcal{F}^{\gamma\delta}_{\rho\sigma]} . \tag{15}$$

The next step in the gauging process is to covariantize the derivative which acts on the scalars. Consequently,

$$\mathcal{V}^{\alpha}{}_{i}\,(\partial_\mu \delta^{\beta}_{\alpha} + g A^{\alpha\beta}_{\mu})\,\mathcal{V}_{\beta j} = \tilde{P}_{\mu(ij)} + \tilde{Q}_{\mu[ij]} , \tag{16}$$

where $\mathcal{V}^{\alpha}{}_{i} A^{\alpha\beta}_{\mu} = \mathcal{V}^{\gamma}{}_{i} A^{\delta\beta}_{\mu}\eta_{\gamma\delta}$. Eq.(16) defines $\tilde{P}_\mu$ and $\tilde{Q}_\mu$. Clearly, these depend on both the vector fields and the scalars. From (16) one can define the local $SO(4) \otimes$ composite local $SO(4)$ covariant derivative as follows:

$$\mathcal{D}_\mu \mathcal{V}_{\alpha i} = \left(\partial_\mu \delta^{\beta}_{\alpha}\delta^{j}_{i} + g A^{\alpha\beta}_{\mu}\delta^{j}_{i} + \tilde{Q}_{\mu i}{}^{j}\,\delta^{\beta}_{\alpha}\right)\mathcal{V}_{\beta j} . \tag{17}$$

From (16) and (17) one finds:

$$(\mathcal{D}_\mu\tilde{P}_\nu - \mathcal{D}_\nu\tilde{P}_\mu)_{ij} = \frac{1}{2} g\,(T_{ik}\,\mathcal{F}_{\mu\nu}{}^{k}{}_{j} + i \leftrightarrow j) , \tag{18a}$$

$$[\tilde{P}_\mu,\tilde{P}_\nu]_{ij} = -\tilde{Q}_{\mu\nu ij} + \frac{1}{2} g\,(T_{ik}\mathcal{F}_{\mu\nu}{}^{k}{}_{j} - i \leftrightarrow j) , \tag{18b}$$

where $\tilde{Q}_{\mu\nu} = \partial_\mu\tilde{Q}_\nu - \partial_\nu\tilde{Q}_\mu + [\tilde{Q}_\mu,\tilde{Q}_\nu]$ and

$$T_{ij} \equiv \mathcal{V}^{\alpha}{}_{i}\,\mathcal{V}^{\beta}{}_{j}\,\eta_{\alpha\beta} . \tag{19}$$

This is the analogue of the T-tensor that was encountered in the gauging of $SO(8)$ supergravity in $d = 4$ in Ref.2.

6. For the gauging of $SO(4)$, in the Lagrangian given by (11) and in the transformation laws given by (12), we now replace $F^{\alpha\beta}_{\mu\nu}$ by $\mathcal{F}^{\alpha\beta}_{\mu\nu}$, $G_{\mu\nu\rho}$ by $\mathcal{G}_{\mu\nu\rho}$, $P_{\mu ij}$ by $\tilde{P}_{\mu ij}$ and $Q_{\mu ij}$ (which appears in the covariant derivative of the fermions) by $\tilde{Q}_{\mu ij}$. This procedure violates supersymmetry for the following reasons. Although, all the previously vanishing variations of the Lagrangian will simply covariantize, and thus still vanish, new, non-vanishing variations which are proportional to g will be generated, due to the fact that the fundamental $SO(4)$ gauge fields now appear in the covariant derivative of the scalars (Eq.(16)) and furthermore there are g-dependent terms in Eq.(18). The terms we encounter are the following:

i) From the variation of $A^{\alpha\beta}_{\mu}$ in the scalar kinetic term (i.e. $\tilde{P}^2_\mu$) we obtain

$$\frac{1}{2} g\,T_{ij}\,\tilde{P}^{ik}_{\mu}\,(\bar{\epsilon}\,\gamma^{j}{}_{k}\,\psi_\mu + \bar{\epsilon}\,\Gamma^\mu\gamma^j\chi_k - \bar{\epsilon}\,\Gamma^\mu\gamma_k\chi^j) . \tag{20}$$

ii) The variation of χ_i in the Yukawa term (i.e. in $\tilde{P}\psi\chi$) yields

$$-\frac{i}{8} g\,T_{ij}\,\mathcal{F}_{\mu\rho k}{}^{j}\,\bar{\psi}_\nu\,\Gamma^{\mu\nu\rho}\,\gamma^{ik}\,\epsilon , \tag{21}$$

where we have used (18a).

iii) Finally, the variation of ψ_μ in the Yukawa term gives

$$-\frac{i}{8} g\,T^{ij}\,\mathcal{F}_{\mu\nu j}{}^{k}\,\bar{\epsilon}\,\Gamma^{\mu\nu}\,(\gamma_i\chi_k + \gamma_k\chi_i) , \tag{22}$$

where we have used (18b).

It is not difficult to arrange the cancellation of these new terms by adding to the Lagrangian terms which are quadratic in spinor fields and linear in the T-tensor. However the supersymmetric variations of such terms as a by-product will produce further unwanted terms.

The procedure to handle this problem is by now well known: In addition to all possible quadratic fermion terms, one adds a potential which is proportional to g^2T^2, as well as terms to the fermionic supersymmetry transformation laws which are of the form gT. The coefficients of these modifications must then be fixed by the requirement that all the g-dependent terms that arise from the new and old variations of the total modified action vanish. In this way, we have found that the SO(4) gauged action of $N = 2$ theory is given by

$$\mathcal{L} = \mathcal{L}_0\,(F \to \mathcal{F}\,;\; G \to \mathcal{G}\,;\; P \to \widetilde{P}\,;\; Q \to \widetilde{Q}) + \mathcal{L}_g \quad , \tag{23}$$

where $\mathcal{L}_0$ is given by (11) and $\mathcal{F}$, $\mathcal{G}$, $\widetilde{P}$, $\widetilde{Q}$ are defined by Eqs.(13), (14), (16). $\mathcal{L}_g$ is

$$\begin{aligned} V^{-1}\mathcal{L}_g = {} & \frac{1}{16\sqrt{2}}\, g\, T\, \bar{\psi}_\mu \Gamma^{\mu\nu} \psi_\nu - \frac{1}{4\sqrt{2}}\, g\, T^{ij}\, \bar{\psi}_\mu \Gamma^\mu \gamma_i \chi_j \\ & + \frac{1}{2\sqrt{2}}\, g\, T^{ij}\, \bar{\chi}_i \chi_j - \frac{1}{16\sqrt{2}}\, g\, T\, \bar{\chi}_i\, (\gamma^i\gamma^j + \eta^{ij})\, \chi_j \\ & - \frac{1}{32}\, g^2\, (T_{ij}T^{ij} - \frac{1}{2}T^2) \quad , \end{aligned} \tag{24}$$

where $T = T_{ij}\,\eta^{ij}$. The supersymmetry transformation laws read:

$$\delta = \delta_0\,(F \to \mathcal{F}\,;\; G \to \mathcal{G}\,;\; P \to \widetilde{P}\,;\; Q \to \widetilde{Q}) + \delta_g \;, \tag{25}$$

where δ_0 is given by Eq.(12) and δ_g is given by

$$\delta_g \psi_\mu = \frac{i}{40\sqrt{2}}\, gT\, \Gamma_\mu \epsilon \;, \qquad \delta_g \chi_i = \frac{-i}{4\sqrt{2}}\, g\, (T_{ij} - \frac{1}{5}\eta_{ij}T)\, \gamma^j \epsilon \,. \tag{26}$$

7. In this section, we shall briefly discuss the issue of spontaneous compactification in our theory. Assuming that the background spinors, vector fields, as well as $\mathcal{G}_{\mu\nu m}$ and $\mathcal{G}_{\mu mn}$ vanish while $\mathcal{V}_\alpha{}^i$ are constants and $\mathcal{G}_{mnp} = e\,\mathcal{V}^{-2}\epsilon_{mnp}$ where $m = 5,6,7$, c is a constant and $e = (-\det g^{(3)}_{mn})^{1/2}$, we have found that the Einstein equation, together with the other equations of motion, implies

$$R_{\hat{\mu}\hat{\nu}} = \mathcal{V}^2\, \mathcal{G}_{\hat{\mu}\hat{\lambda}\hat{\tau}}\, \mathcal{G}_{\hat{\nu}}{}^{\hat{\lambda}\hat{\tau}} \quad , \tag{27}$$

where $\hat{\mu} = 1,\ldots,7$. This is a remarkable equation because it automatically yields

$$R^{(4)}_{\mu\nu} = 0 \quad ; \quad R^{(3)}_{mn} = -\,2c^2\,\mathcal{V}^{-2}\, g_{mn} \quad , \tag{28}$$

where, we recall that, $\mathcal{V}$ is assumed a constant. Thus, (28) means that, $d = 4$ space-time is Ricci flat (e.g. Minkowski) and the internal space is a three-dimensional compact constant curvature space (e.g. 3-sphere). Unfortunately however, although the $B_{\mu\nu}$ and $A_\mu{}^{\alpha\beta}$ field equations are solved as well by the above ansatz, the scalar field equation is not satisfied, because it gives

$$-\,8c^2\,\mathcal{V}^{-2}\,\eta^{ij} + (T^{ik}T^{kj} - \frac{1}{2}TT^{ij}) = 0 \quad . \tag{29}$$

It is not difficult to show that Eq.(29) cannot be fulfilled. Note that Eq.(27) implies that compactification on $S^1 \times S^2$ or $S^1 \times S^1 \times S^1$ are not possible either. More elaborate compactification schemes will be discussed elsewhere.

After this paper was written, we received a CERN preprint by P.K. Townsend and P. can Nieuwenhuizen who have considered $d = 7$, pure $N = 2$ supergravity multiplet (without matter) and gauged the associated SO(3) symmetry group. These authors have worked with the third rank antisymmetric tensor field as opposed to the treatment in this note which uses a (dual) second-rank antisymmetric tensor field $B_{\mu\nu}$.

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