IC/83/7

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS * LIBRARY
1312/83
22 MAR 1983

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

COMPACTIFICATION OF SUPERGRAVITY PLUS YANG-MILLS IN TEN DIMENSIONS

S. Randjbar-Daemi

Abdus Salam

and

J. Strathdee

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1983 MIRAMARE-TRIESTE

IC/83/7

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

COMPACTIFICATION OF SUPERGRAVITY PLUS YANG-MILLS IN TEN DIMENSIONS *

S. Randjbar-Daemi
Institut für Theoretische Physik, Boltzmanngasse 5, A-1090 Vienna, Austria,

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy, and Imperial College, London, England,

and

J. Strathdee
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We investigate the criteria which determine supergravity-induced compactification of a supersymmetric Yang-Mills theory in ten dimensions down to spaces of the type (Minkowski) x $(\frac{G}{H})$.

MIRAMARE - TRIESTE
January 1983

* To be submitted for publication.

In the search for a realistic model of spontaneous compactification it is important to distinguish the cases where chiral flavour dynamics can emerge in the 4-dimensional (low-energy) sector. Thus, for example, in the 11-dimensional supergravity [1], where compactification must produce a 7-dimensional internal space, it is inevitable that the fermions will belong to real representations of the symmetry group of this space [2] - whatever it turns out to be. This means that the flavour symmetries must be looked for elsewhere in this particular model. (For instance, one may hope that the Cremmer-Julia "local" chiral SU(8) can be elevated by dynamical effects into a true gauge symmetry.) However, there do exist models where chirality emerges directly in the 4-dimensional theory in the sense that the zero-mode fermions belong to a complex representation of the vacuum symmetry. In particular, a vector-like gauge theory in ten dimensions can compactify to a chiral theory in four dimensions [3]. (This is due to the existence of Majorana-Weyl spinors in ten dimensions.) Recently, following Bergshoeff et al. [4] Chapline and Manton [5] have constructed the Lagrangian for the coupling of 10-dimensional supergravity and Yang-Mills systems. In this note we investigate some of the possibilities for spontaneous compactification in this theory.

The bosonic part of the Chapline-Manton Lagrangian [5] is

$$\mathcal{L} = e\left[-\frac{1}{2\kappa^2}R(e,\omega(e)) - \frac{3}{4}\phi^{-3/2}F_{ABC}{}^2 - \frac{9}{16\kappa^2}\left(\frac{\nabla_A\phi}{\phi}\right)^2 - \frac{1}{4}\phi^{-3/4}\vec{F}_{AB}{}^2\right], \quad (1)$$

where

$$\vec{F}_{AB} = \nabla_A\vec{A}_B - \nabla_B\vec{A}_A + g\,\vec{A}_A\times\vec{A}_B , \quad (2a)$$

$$F_{ABC} = \nabla_{[A}A_{BC]} - \frac{\kappa}{\sqrt{2}}\left[\vec{A}_{[A}\cdot\vec{F}_{BC]} - \frac{2}{3}g\,\vec{A}_{[A}\cdot\vec{A}_B\times\vec{A}_{C]}\right] . \quad (2b)$$

The bosonic fields include: gravity $e_M{}^A$, antisymmetric tensor A_{AB}, scalar ϕ and gauge vector $\vec{A}_A$. Since we are concerned only with the classical background here, we shall not be considering the various fermionic fields. We should like to find solutions of the classical field equations wherein the 10-dimensional geometry factorizes into $M^4\times B^6$ where M^4 is 4-dimensional Minkowski spacetime and B^6 is a compact 6-dimensional Riemannian space. Chapline and Slansky [3] have argued - in a somewhat different context - that the spaces S^6, $G_2/SU(3)$, $\mathbb{C}P^2\times S^2$ and $\mathbb{C}P^3$ could be physically relevant. We shall examine these candidates in the light of the equations of motion generated by (1).

We require that the vacuum shall be invariant under Poincaré × G, where G represents the symmetry group of the internal space, B^6. (For simplicity we shall assume that B^6 is a quotient space, G/H.) The scalar field, ϕ, must therefore be a constant. Unfortunately, this is already too much to ask for. Indeed, by varying ϕ in (1), one obtains the equation

$$0 = \frac{9}{8}\phi^{-5/2} F_{ABC}{}^2 + \frac{3}{16}\phi^{-7/4} \vec{F}_{AB}{}^2 + \text{terms in } \partial\phi \quad . \tag{3}$$

Poincaré invariance requires that any non-vanishing components of F_{ABC} and $\vec{F}_{AB}$ must be in the tangent space of B^6. Hence the first two terms in (3) are non-negative. If $\partial\phi = 0$ then we have only the trivial case, $F_{ABC} = \vec{F}_{AB} = 0$, and no compactification. At the strictly classical level this rules out any G-invariant solution on G/H. In order to proceed one must either look for solutions with less symmetry or else modify the Lagrangian.

An interesting example of the first alternative is the solution with $\vec{F}_{AB} = 0$ but F_{ABC} non-vanishing on a 3-sphere, O(4)/O(3), [6]. Instead of the maximal O(4) it has only O(3) symmetry since ϕ is not constant. Of course the resulting vacuum geometry, $AdS^7 \times S^3$, is not very desirable.

Here we shall adopt the second alternative. If the theory is to have any relevance to low energy physics, then the geometry $M^4 \times B^6$ must emerge. In order to get such solutions we add to the Lagrangian a supersymmetry breaking term

$$-e\,V(\phi) \quad . \tag{4}$$

Perhaps this modification, or something equally procrustean, will eventually be justified by a dynamical calculation but, for the present, we take it as something given. Its contribution to Eq.(3) gives

$$\frac{\partial V}{\partial\phi} = \frac{9}{8}\phi^{-5/2} F_{ABC}{}^2 + \frac{3}{16}\phi^{-7/4} \vec{F}_{AB}{}^2 \tag{5a}$$

for constant ϕ. This could be solved for ϕ once the form of $V(\phi)$ has been chosen.

The other equations of motion are

$$R_{AB} = -\kappa^2 \Big[\frac{9}{2}\phi^{-3/2}(F_{ACD}F_{BCD} - \frac{1}{12}g_{AB}F^2) + \phi^{-3/4}(\vec{F}_{AC}\cdot\vec{F}_{BC} - \frac{1}{16}g_{AB}\vec{F}^2) + \frac{1}{4}g_{AB}V(\phi)\Big] \quad , \tag{5b}$$

$$\nabla_A F_{ABC} = 0 \quad , \tag{5c}$$

$$D_A \vec{F}_{AB} + \frac{\kappa}{\sqrt{2}} F_{BCD}\vec{F}_{CD} = 0 \quad . \tag{5d}$$

In addition to the familiar Bianchi identity satisfied by the Yang-Mills 2-form, $\vec{F}_2 = d\vec{A} + g\vec{A}\wedge\vec{A}$, there is a new one, peculiar to this model,

$$dF_3 = \frac{\kappa}{\sqrt{2}}\,\mathrm{Tr}(F_2\wedge F_2) \quad , \tag{6}$$

where F_3 denotes the 3-form corresponding to the field strength (2b). We now show that, of the four candidate spaces listed above, only two, S^6 and $G_2/SU(3)$ are capable of satisfying (6). Considering them in turn:

1) $S^6 = SO(7)/SO(6)$

On this space we must have $F_3 = 0$ since no SO(6) invariant exists among the components of the antisymmetric tensor of third rank. On the other hand, there does exist an SO(7)-invariant solution of (5d) given by the ansatz [7]

$$F_2 = \frac{1}{g} e^\alpha \wedge e^\beta Q_{\alpha\beta} \qquad (\text{or } F_{\alpha\beta} = \frac{1}{2}\frac{1}{ga^2}Q_{\alpha\beta}) \quad , \tag{7}$$

where the 1-forms e^α define an orthonormal basis on S^6, of radius a, and $Q_{\alpha\beta}$ denote the generators of SO(6) contained in the gauge algebra. Since $\mathrm{Tr}(F_2\wedge F_2) \sim e^\alpha\wedge e^\beta\wedge e^\gamma\wedge e^\delta\,\mathrm{Tr}(Q_{\alpha\beta}Q_{\gamma\delta}) = e^\alpha\wedge e^\beta\wedge e^\gamma\wedge e^\delta(\delta_{\beta\gamma}\delta_{\alpha\delta} - \delta_{\alpha\gamma}\delta_{\beta\delta}) = 0$, it follows that the identity (6) is satisfied trivially.

Poincaré invariance implies $R_{aB} = 0$, $\vec{F}_{aB} = 0$ and $F_{aBC} = 0$ for $a = 0,\ldots,3$. Hence Eq.(5b) reduces to

$$0 = \frac{3}{8}\phi^{-3/2} F_{\alpha\beta\gamma}{}^2 + \frac{1}{16}\phi^{-3/4}\vec{F}_{\alpha\beta}{}^2 - \frac{1}{4}V(\phi) \quad , \tag{8a}$$

$$-\frac{1}{\kappa^2} R_{\alpha\beta} = \frac{9}{2}\phi^{-3/2} F_{\alpha\gamma\delta} F_{\beta\gamma\delta} + \phi^{-3/4}\vec{F}_{\alpha\gamma}\cdot\vec{F}_{\beta\gamma} \quad . \tag{8b}$$

In the case of S^6 we have

$$F_{\alpha\beta\gamma} = 0, \quad \vec{F}_{\alpha\gamma}\cdot\vec{F}_{\beta\gamma} = \frac{5}{2}\frac{1}{g^2 a^4}\delta_{\alpha\beta} \quad \text{and} \quad R_{\alpha\beta} = -\frac{5}{a^2}\delta_{\alpha\beta}$$

which, together with (5a), (8a) and (8b), imply the relations,

$$a^2 = \frac{\kappa^2}{2g^2}\phi^{-3/4} \quad , \tag{9a}$$

$$V = \frac{15 g^2}{\kappa^4}\phi^{3/4} \quad , \tag{9b}$$

$$\frac{\partial \ln V}{\partial \ln \phi} = \frac{3}{4} \quad . \tag{9c}$$

Of these equations, (9a) and (9b) determine the values of ϕ and a, while (9c) gives a constraint on the parameters in V. (For example, with $V = \lambda + (m^2/2\kappa^2)(\ln\phi)^2$ one obtains a constraint on the 10-dimensional cosmological constant λ. This constraint arises from our insistence on vanishing 4-dimensional curvature.)

2) $G_2/SU(3)$

With the isotropy group SU(3) embedded in the tangent space group SO(6) such that the 6-vector branches into $3 + \bar{3}$, it is useful to group the six basis 1-forms accordingly. Thus we have e^α, $\alpha = 1,2,3$, transforming as an SU(3) triplet, and the complex conjugates are denoted e_α. Since $e^\alpha \wedge e^\beta \wedge e^\gamma$ is an SU(3) singlet, we automatically obtain a G_2-invariant solution of (5c),

$$F_3 = C\,\varepsilon_{\alpha\beta\gamma}\, e^\alpha \wedge e^\beta \wedge e^\gamma + \text{c.c.} \quad , \tag{10}$$

where C is an arbitrary constant. It is possible to solve (5d) by means of the ansatz of Ref.7,

$$F_2 = \frac{1}{g} e^\alpha \wedge e_\beta\, Q_\alpha{}^\beta \quad , \tag{11}$$

where $Q_\alpha{}^\beta$ denotes the SU(3) generators in the gauge algebra. (The non-vanishing components of F_2 and F_3 are such that the second term in (5d) makes no contribution.)

The Bianchi identity (6) is non-trivial in this case. Both dF_3 and $\mathrm{Tr}(F_2 \wedge F_2)$ are non-vanishing. The former can be evaluated with the help of Maurer-Cartan equations, $de^\alpha \sim \varepsilon^{\alpha\beta\gamma} e_\beta \wedge e_\gamma$. One obtains from (6) the value of the constant in (10), $C \sim \kappa/g^2$.

Finally, the scale of the space would be determined by inserting the components of F_2 and F_3 into Eqs.(5a) and (5b).

3) $\mathbb{C}P^2 \times S^2 = SU(3) \times SU(2)/SU(2) \times U(1) \times U(1)$

Here the tangent space 6-vector decomposes under the isotropy group $SU(2) \times U(1) \times U(1)$ according to $6 = 2_{1,0} + 2_{-1,0} + 1_{0,1} + 1_{0,-1}$, where the subscripts denote the $U(1) \times U(1)$ quantum numbers. Let us label the basis 1-forms accordingly; $e^{a+} \sim 2_{1,0}$, $e^{a-} \sim 2_{-1,0}$, $e^{0+} \sim 1_{0,1}$, $e^{0-} \sim 1_{0,-1}$.

It is not possible to make an $SU(2) \times U(1) \times U(1)$-invariant 3-form out of the e^α. Hence $F_3 = 0$. On the other hand, F_2 satisfying (5d) is given by

$$F_2 = e^{a+} \wedge e^{b-}\left((\tau_2)_{ab}\, Q_8 + (\tau_k \tau_2)_{ab}\, Q_k \right) + e^{0+} \wedge e^{0-}\, T_3 \quad , \tag{12}$$

where $Q_{1,\ldots,8}$ and $T_{1,2,3}$ are SU(3) and SU(2) generators, respectively, contained in the gauge algebra. One can easily verify that $\mathrm{Tr}(F_2 \wedge F_2)$ does not vanish and hence that this space cannot be a solution*).

4) $\mathbb{C}P^3 = SU(4)/SU(3) \times U(1)$

In this case the 6-vector decomposes relative to $SU(3) \times U(1)$ into $6 = 3_1 + \bar{3}_{-1}$. Again it is not possible to make an $SU(3) \times U(1)$ invariant 3-form so there can be no SU(4) invariant solution to (5c) except $F_3 = 0$.

*) It should also be noted that the gravitino, a spinor which is neutral with respect to the Yang-Mills gauge group could not be defined on the $\mathbb{C}P^2$ part of the manifold.

However, there is a non-vanishing SU(4) invariant solution to (5d),

$$F_2 = \frac{1}{g} (e^\alpha \wedge e_\alpha B + e^\alpha \wedge e_\beta Q_\alpha{}^\beta) \quad , \tag{13}$$

where $Q_\alpha{}^\beta$ and B denote the generators of SU(3) × U(1). Since $\mathrm{Tr}(F_2 \wedge F_2)$ does not vanish this solution fails to satisfy the Bianchi identity (6).

To conclude, compactification of the 10-dimensional supergravity theory – with a symmetry-breaking term adjoined – could yield the spaces $M^4 \times S^6$ or $M^4 \times G_2/SU(3)$. The zero-mode content has yet to be analyzed but, at the least there would result Yang-Mills fields associated with the groups O(7) or G_2 with coupling strengths $\sim \kappa/a$. In addition there could remain some unbroken part of the original Yang-Mills group, viz. all those gauge transformations which leave invariant the vacuum solution, $\langle F_2 \rangle$. Their coupling strength, g, would also be of order κ/a. However, the existence of such residual gauge symmetries may be indicative of instability. In this context we have recently examined [8] the compactification of $M^4 \times S^2$. We find that stable O(3) invariant solutions are obtainable only in U(1) gauge theory. (It so happens that in this theory the residual gauge symmetry was just U(1).) Any larger Yang-Mills gauge group leads to tachyons if O(3) invariance is assumed [9]. Extrapolating to the case of ten dimensions it would seem to us that gauge groups larger than the minimal O(6) (or SU(3)) might fail to yield O(7) (or G_2) invariant vacua.

The super-Yang-Mills matter multiplet contains Majorana-Weyl fermions in the adjoint representation. To get realistic fermions in four dimensions, it is necessary that the fermionic zero-modes belong to a complex representation of the residual symmetry, and this symmetry should include at least SU(3) × SU(2) × U(1). For these reasons Chapline and Slansky argued for E_8 as the Yang-Mills group. They showed that the adjoint representation of this group could contain the usual quark-lepton families. However, from the remarks above ([7], [8]) there is the danger that such a large structure might be unstable.

S.R.D. acknowledges partial support from the Einstein Memorial Foundation.

REFERENCES

[1] E. Cremmer and B. Julia, Nucl. Phys. B159, 141 (1979).

[2] E. Witten, Nucl. Phys. B186, 412 (1981).

[3] G.F. Chapline and R.S. Slansky, Los Alamos preprint, LA-UR-82-1076 (1982) (to appear in Nucl. Phys. B.).

[4] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nulc. Phys. B195, 97 (1982).

[5] G.F. Chapline and N.S. Manton, Phys. Lett. 120B, 105 (1983).

[6] M. Duff, P.K. Townsend and P. van Nieuwenhuizen, CERN preprint TH.3465 (1982).

[7] S. Randjbar-Daemi and R. Percacci, Phys. Lett. 117B, 41 (1982).

[8] S. Randjbar-Daemi, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/82/208 (to appear in Nucl. Phys. B.).

[9] S. Randjbar-Daemi, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/83/6.