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S. Randjbar-Daemi

Abdus Salam

and

J. Strathdee

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SPONTANEOUS COMPACTIFICATION IN SIX-DIMENSIONAL EINSTEIN-MAXWELL THEORY *

S. Randjbar-Daemi **
International Centre for Theoretical Physics, Trieste, Italy,

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy,
and
Imperial College, London, England,

and

J. Strathdee
International Centre for Theoretical Physics, Trieste, Italy.

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** Address after 1st December 1982: Institute for Theoretical Physics, University of Vienna, Boltzmanngasse 5, Vienna, Austria.

ABSTRACT

A discrete set of solutions to the classical Einstein-Maxwell equations in six-dimensional spacetime is considered. These solutions have the form of a product of four-dimensional constant curvature spacetime with a 2-sphere. The Maxwell field has support on the 2-sphere where it represents a monopole of magnetic charge, $n = \pm 1, \pm 2, \ldots$ The spectrum of massless and massive states is obtained for the special case of the flat 4-space, and the solution is shown to be classically stable. The limiting case where the radius of the 2-sphere becomes small is considered and a dimensionally reduced effective Lagrangian for the long range modes is derived. This turns out to be an SU(2) x U(1) gauge theory with chiral couplings.

I. INTRODUCTION

To illustrate the phenomenon of spontaneous compactification in theories of the Kaluza-Klein type, a six-dimensional model was constructed by Horvath. Palla, Cremmer and Scherk [1),2)]. In this model the gravitational field is coupled to an Abelian gauge field. [*)] Three independent parameters are involved, the gravitational and gauge coupling constants and a cosmological constant. Among the possible solutions it was shown that there occurs a candidate ground state in which the geometry factorizes into the product of a four-dimensional spacetime of constant curvature with a two-dimensional sphere. In this solution the Maxwell field is non-vanishing only on the 2-sphere where it assumes a magnetic monopole configuration. This non-vanishing ground state value of the Maxwell field contributes to the stress-energy tensor on the right-hand side of the six-dimensional Einstein equations and, in effect, generates the curvature of the 2-sphere. In view of the non-zero "magnetic" charge of the proposed vacuum configuration, it was conjectured that the solution would be a stable one. One of our aims in this article is to test the solution for stability against small perturbations.

Our main purpose, however, is to examine in some detail the Kaluza-Klein mechanism which operates in this model. In particular, we shall compute the excitation spectrum for the special case where the four-dimensional spacetime is flat. It will be shown that the spectrum comprises several series of massive states with spins 0, 1 and 2, and ten massless states: the graviton (helicities ± 2) and gauge vectors (helicities ± 1) corresponding to the local symmetry $SU(2) \times U(1)$. Perhaps surprisingly, there are no massless scalars.

A secondary aim is to illustrate the use of harmonic expansions [4)] on the 2-sphere in analyzing the spectrum. This is not entirely trivial and, indeed, we shall find, contrary to the usual assumption, that the SU(2) Yang-Mills 4-vector does not lie entirely in the six-dimensional metric, but is partly in the Maxwell 6-vector as well. In addition, we shall find that its coupling to massless fermions is chiral [**)].

An important feature of this model is the presence of two independent scales. In the vacuum geometry the curvature of the 2-sphere can be varied independently of the four-dimensional spacetime curvature. This enables us to consider the special case where spacetime is flat [***)] and it further enables

*) A class of non-Abelian generalizations has recently been obtained [3)].

**) A general discussion of the chirality question can be found in G. Chapline and R. Slansky, Los Alamos preprint LA 82-1076.

***) This is to be contrasted with the interesting solution of Freund and Rubin [5)] to the 11-dimensional supergravity system where the four-dimensional anti-de Sitter space has curvature of comparable magnitude to that of the "internal" 7-space.

us to extract in a meaningful way the effective four-dimensional theory of the zero modes by allowing the radius of the 2-sphere to shrink to zero. *)

If the four-dimensional curvature is not adjusted to zero, the candidate ground state would have either de Sitter ($\Lambda > 0$) or anti-de Sitter ($\Lambda < 0$) geometry. It has been shown by Sezgin [6], using the approach of Abbot and Deser [7] that the anti-de Sitter solution is stable in that a conserved, non-negative, energy functional can be defined. Here we consider only the flat case because the interpretation of particle spectra is more straightforward.

This paper is arranged as follows. Sec.II sets out the model and the possible ground state solutions are reviewed. Sec.III is devoted to the fluctuation spectrum. Fields are expanded around their background values and the linearized equations of motion are obtained. These are solved in the presence of external sources in a suitable gauge. By this means the fluctuation Green's functions are in effect obtained. The singularities of these Green's functions define the spectrum. It is verified that no tachyons or ghosts are contained. For this investigation it is necessary to distinguish physical states from gauge modes and we do this by using appropriately conserved sources. Dimensional reduction is considered in Sec.IV where an effective Lagrangian for the zero modes is obtained by shrinking the 2-sphere. Fermions are introduced in Sec.V where it is shown that zero modes occur whose SU(2) quantum numbers are governed by the Chern class of the monopole background.

II. GROUND STATE BACKGROUND

The six-dimensional Einstein-Maxwell theory with cosmological constant is characterized by the action

$$S = - \int d^6z \sqrt{-g} \left[\frac{1}{\kappa^2} R + \frac{1}{4} F_{MN} F^{MN} + \lambda \right] \quad , \tag{2.1}$$

where R denotes the curvature scalar **), and

*) It should be observed, however, that the shrinking of the 2-sphere leads to a regime of strong couplings for the gauge fields. The classical approximation which we employ cannot be reliable in this limit.

**) Upper case indices take the values 0,1,2,3,5,6. In the following, lower case latin indices will take the values 0,1,2,3, while lower case greek indices are restricted to 5,6. That is, we write $z^M = (x^m, y^\mu)$. Early alphabet letters will be used as frame labels, while the mid-alphabet is used for world indices. We follow the conventions of Deser and van Nieuwenhuizen [8]: The signature is $-+\ldots+$. The Riemann tensor is defined such that $R^K{}_{LMN} = \partial_M \Gamma^K_{LN} - \partial_N \Gamma^K_{LM} - \ldots$, and the Ricci tensor is $R_{LM} = R^K{}_{LMK}$.

$$F_{MN} = \partial_M A_N - \partial_N A_M \quad . \tag{2.2}$$

The field equations are

$$R_{MN} - \frac{1}{2} g_{MN} R = - \frac{\kappa^2}{2} (T_{MN} - \lambda g_{MN}) \quad , \tag{2.3}$$

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} F^{MN}) = 0 \quad , \tag{2.4}$$

where the stress energy tensor takes the usual Maxwell form,

$$T_{MN} = F_{ML} F_N{}^L - \frac{1}{4} g_{MN} F^2 \quad . \tag{2.5}$$

The most symmetric solution to the equations (2.3) and (2.4) would of course be six-dimensional Minkowski space with vanishing Maxwell field. However, such a solution could hardly be relevant to the description of a four-dimensional world. We shall therefore restrict our attention to the most symmetric solutions with the structure $M^4 \times M^2$, viz. the product of four- and two-dimensional spaces of constant curvature. Thus, we take

$$g_{MN} dz^M dz^N = g_{mn}(x) dx^m dx^n + g_{\mu\nu}(y) dy^\mu dy^\nu \quad , \tag{2.6}$$

$$A_M dz^M = A_\mu(y) dy^\mu \quad , \tag{2.7}$$

where $g_{mn}(x)$ corresponds to de Sitter or anti-de Sitter space and $g_{\mu\nu}(y)$ to a 2-sphere. The 1-form, $A_\mu dy^\mu$ is required to be invariant (up to a gauge transformation) under rotations of the 2-sphere. We believe that physical interpretations are less vexed in the anti-de Sitter case (having a global time-like Killing vector and positive energy unitary representations) than they are for the de Sitter world and we shall see that it is possible to exclude the de Sitter solution by appropriate restrictions on the parameters κ, λ and e. Moreover, it can be shown that the anti-de Sitter solution is classically stable [6].

It is convenient to use spherical polar co-ordinates on the 2-sphere,

$$g_{\mu\nu} dy^\mu dy^\nu = a^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \quad , \tag{2.8}$$

where a, the radius is to be determined. It is not an integration constant.

The rotation invariant Maxwell field is given by

$$A_\mu(y)\, dy^\mu = \frac{n}{2e} (\cos\theta \mp 1)\, d\varphi \quad , \qquad (2.9)$$

where n is a positive or negative integer and e is the U(1) gauge coupling constant. This coupling constant does not appear in the Lagrangian (2.1), where all fields are neutral, but one should keep in mind the possibility of charged matter fields to which A would couple. An example of this will be dealt with in Sec.V. In effect we are asking that the theory should be invariant under the gauge transformations

$$A_M \to A_M + \frac{1}{e} \partial_M \Lambda \quad ,$$

where $\exp(i\Lambda)$ should be single-valued. In (2.9) the monopole configuration is necessarily specified on two patches ($0 \leq \theta < \pi$ and $0 < \theta \leq \pi$, $0 \leq \varphi < 2\pi$, respectively). Where these neighbourhoods overlap, say at $\theta = \pi/2$, $0 \leq \varphi \leq 2\pi$, the two representations of A must be related by a single-valued gauge transformation, $\exp(in\varphi)$. Hence the requirement that n be a positive or negative integer. The field strength corresponding to (2.9) is given by the 2-form,

$$F = dA = -\frac{n}{2ea^2}\, a\, d\theta \wedge a \sin\theta\, d\varphi \quad , \qquad (2.10)$$

where a is the radius of the 2-sphere. It is clearly rotation invariant.

Corresponding to the field strength (2.10) the components of the stress energy tensor (2.5), in an orthonormal basis, are given by

$$T_{ab} = -\frac{n^2}{8e^2a^4}\, \eta_{ab} \quad , \qquad a,b = 0,1,2,3$$

$$T_{\alpha\beta} = 0$$

$$T_{\alpha\beta} = \frac{n^2}{8e^2a^4}\, \delta_{\alpha\beta} \quad , \qquad \alpha,\beta = 5,6 \qquad (2.11)$$

The non-vanishing parts of the Riemann tensor, in the same basis are,

$$R_{abcd} = \frac{\kappa^2\Lambda}{6} (\eta_{ac}\, \eta_{bd} - \eta_{ad}\, \eta_{bc})$$

$$R_{\alpha\beta\gamma\delta} = \frac{1}{a^2} (\delta_{\alpha\gamma}\, \delta_{\beta\delta} - \delta_{\alpha\delta}\, \delta_{\beta\gamma}) \quad , \qquad (2.12)$$

where the four-dimensional cosmological constant, Λ, is to be determined along with the radius a, by substituting (2.12) into the Einstein equations (2.3), or the equivalent form

$$R_{AB} = -\frac{\kappa^2}{2} \left\{ T_{AB} - \frac{1}{4}\eta_{AB} T + \frac{1}{2}\eta_{AB}\lambda \right\} \quad .$$

One finds two algebraic equations for Λ and a^2,

$$\Lambda = -\frac{n^2}{16e^2a^4} + \frac{\lambda}{2}$$

$$\frac{2}{\kappa^2a^2} = \frac{3n^2}{16e^2a^4} + \frac{\lambda}{2} \quad . \qquad (2.13)$$

They are solved by

$$\frac{1}{a^2} = \frac{16e^2}{3n^2\kappa^2} \left[1 \pm \sqrt{1 - \frac{3n^2}{32}\frac{\kappa^4\lambda}{e^2}} \right] \quad , \qquad (2.14)$$

$$\Lambda = \frac{2}{3}\lambda - \frac{32}{9n^2}\frac{e^2}{\kappa^4} \left[1 \pm \sqrt{1 - \frac{3n^2}{32}\frac{\kappa^4\lambda}{e^2}} \right] \quad . \qquad (2.15)$$

If $\lambda < 0$ then, for each value of the monopole charge, n, there is one positive solution for a^2. This solution corresponds to $\Lambda < 0$ (anti-de Sitter world). On the other hand, if $\lambda > 0$, then there are two positive solutions for a^2, provided n^2 is not too large,

$$n^2 \leqslant \frac{32}{3}\frac{e^2}{\kappa^2\lambda} \quad .$$

At the upper end of this range, however, one finds $\Lambda > 0$ (de Sitter world) which should probably be excluded. To have $\Lambda < 0$ we require

$$n^2 \leqslant 8\, \frac{e^2}{\kappa^4\lambda} \quad . \qquad (2.16)$$

Of particular interest for our purposes is the case of flat 4-space, $\Lambda = 0$. This occurs when the parameters of the six-dimensional theory are adjusted such that

$$e^2 = \frac{n^2}{8}\kappa^4\lambda \qquad (2.17)$$

and the radius then takes the value

$$a^2 = \frac{n^2}{8}\frac{\kappa^2}{e^2} \quad . \tag{2.18}$$

We shall examine the fluctuations on this background in Sec.III.

We conclude by remarking that the four-dimensional theory based on this vacuum will be invariant under the transformations of Poincaré x $(SU(2) \times U(1))_{local}$. The local SU(2) corresponds to x-dependent rotations of the 2-sphere with appropriate U(1) transformations in the tangent space as explained in Ref.4. The local U(1) corresponds to the four-dimensional part of the Maxwell gauge symmetry.

III. FLUCTUATION SPECTRUM ($\Lambda = 0$)

The classical test for stability would be to compute the total energy carried by fluctuations on the background solution and show that it is positive. Such an analysis has been carried out for $\Lambda \leqslant 0$ and these solutions are indeed stable, at least for weak fluctuations [6]. Our purpose here is somewhat different in that we would like to see what kinds of states are produced in this theory. In particular, we shall derive the spectrum of masses for the scalar, vector and tensor excitations on the $\Lambda = 0$ background. In the course of this we shall verify that there are no tachyons or negative metric ghosts: the background is therefore at least perturbatively stable.

In order to obtain the spectrum one must substitute into the action functional (2.1) the expressions

$$\begin{aligned} g_{MN} &= \bar{g}_{MN} + \kappa\, h_{MN} \\ A_M &= \bar{A}_M + V_M \quad , \end{aligned} \tag{3.1}$$

where $\bar{g}_{MN}$, $\bar{A}_M$ denote one of the ground state solutions obtained above, and h_{MN}, V_M represent the fluctuations. Terms linear in h, V will vanish by virtue of the field equations satisfied by $\bar{g}$, $\bar{A}$. Terms bilinear in h, V will govern the propagation of the weak disturbances. These we retain. Higher order terms, representing interactions will be discarded. In an orthonormal basis the bilinear terms reduce to

$$\begin{aligned} S_2 = \int d^6z \sqrt{-g}\, \Big[& -\frac{1}{4} h_{AB;C}\, h_{AB;C} + \frac{1}{8} h_{AA;C}\, h_{BB;C} \\ & + \frac{1}{4}\left(\bar{R} + \frac{\kappa^2}{4}\bar{F}^2\right)\left(h_{AB}h_{AB} - \frac{1}{2} h_{AA}h_{BB}\right) \\ & - \frac{1}{2}\bar{R}_{BC}\left(h_{AB}h_{AC} - h_{AA}h_{BC}\right) \\ & - \frac{\kappa^2}{2}\bar{F}_{BD}\,\bar{F}_{CD}\left(h_{AB}h_{AC} - \frac{1}{2} h_{AA}h_{BC}\right) \\ & + \frac{1}{2}\left(R_{ACBD} - \frac{\kappa^2}{2}\bar{F}_{AC}\bar{F}_{BD}\right) h_{AB}h_{CD} \\ & - \frac{1}{2} V_{A;B}\, V_{A;B} + \frac{1}{2}\bar{R}_{AB}\, V_A V_B \\ & + \kappa\left(h_{AB}\bar{F}_{BC} - \frac{1}{4} h_{BB}\bar{F}_{AC}\right)\left(V_{C;A} - V_{A;C}\right) \\ & - \frac{1}{2} T_{AB}\, h_{AB} + J_A V_A \Big] \quad , \end{aligned} \tag{3.2}$$

where external sources T_{AB} and J_A have been included. The semicolon notation indicates covariant derivatives using the background connection. In general, the bilinear part of the action would reflect the general co-ordinate invariance and U(1) gauge invariance of the original action as invariance under the transformations

$$\begin{aligned} \delta h_{AB} &= \frac{1}{\kappa}\left(\xi_{A;B} + \xi_{B;A}\right) \\ \delta V_A &= \omega_{;A} - \bar{F}_{AB}\,\xi_B \quad . \end{aligned} \tag{3.3}$$

We have taken account of this by imposing the gauge conditions

$$\begin{aligned} h_{AB;B} &= \frac{1}{2} h_{BB;A} \\ V_{A;A} &= 0 \quad . \end{aligned} \tag{3.4}$$

It should now be observed that the linear equations of motion for h, V obtained from (3.2) are compatible only if the sources are suitably constrained, viz.

$$\begin{aligned} T_{AB;B} &= \kappa\,\bar{F}_{AB}\,J_B \\ J_{A;A} &= 0 \quad . \end{aligned} \tag{3.5}$$

In a complete theory these conservation laws would follow from the equations of motion of those fields which contribute to the sources. Here we must view them as a necessary characterization of "physical" sources (i.e. on-shell matrix elements of current operators).

The next step is to solve the linear equations for h and V in terms of T and J. The solutions are then substituted into the functional (3.2). The result reduces to

$$I(T,J) = \frac{1}{2}\int d^6z \sqrt{-g}\left[\frac{1}{2} T_{AB}h_{AB} + J_A V_A\right] , \qquad (3.6)$$

where h and V are functional of J and T. This functional defines the components of the propagator,

$$S_2 = \frac{1}{2}\int d^6z \sqrt{-g}\, d^6z' \sqrt{-g'}$$

$$(T_{AB}(z)\; J_A(z)) \begin{pmatrix} \frac{1}{4}\langle h_{AB}(z)\, h_{CD}(z')\rangle & \frac{1}{2}\langle h_{AB}(z)\, V_C(z')\rangle \\ \frac{1}{2}\langle V_A(z)\, h_{CD}(z')\rangle & \langle V_A(z)\, V_C(z')\rangle \end{pmatrix} \begin{pmatrix} T_{CD}(z') \\ J_C(z') \end{pmatrix} . \qquad (3.7)$$

In practice, the computation would be rather awkward because of the curved background associated with the 2-sphere. To cope with this we have found it useful to expand all fields and sources in spherical harmonics on the 2-sphere. The invariance of the sphere under SO(3) implies that the propagators will be diagonal in an isospin-like quantum number $\ell = 0,1,2,\dots$ In particular, poles of the four-dimensional Fourier transform will be labelled by ℓ. We shall obtain in this way several sequences of masses, the excitation spectrum.

The expansion method is discussed in detail in Ref.4. Here, in order to fix the notation, we summarize the main features. To begin, viewing the 2-sphere as a quotient space, SU(2)/U(1) introduce the SU(2) boosts, $L_{\theta\varphi}$, to parametrize it.

$$L_{\theta\varphi} = e^{-\varphi Q_3} e^{-\theta Q_2} e^{\varphi Q_3} \qquad (3.8)$$

(excluding *) $\theta = \pi$) where $[Q_1,Q_2] = Q_3$, etc. The action of an SU(2) element, g, on this space is defined by

$$g\, L_{\theta\varphi} = L_{\theta'\varphi'}h , \qquad (3.9)$$

where $h = \exp\zeta Q_3$ depends on θ,φ and g. The 1-form, $L^{-1}dL$, belongs to the algebra of SU(2) and its components comprise a covariant basis, $E^{\pm}$, and the U(1) connection 1-form, e^3,

$$L^{-1}dL = \frac{1}{a}E^{+}\frac{Q_1 - iQ_2}{\sqrt{2}} + \frac{1}{a}E^{-}\frac{Q_1 + iQ_2}{\sqrt{2}} + e^3 Q_3 , \qquad (3.10)$$

where

$$E^{\pm} = \pm\frac{a}{i\sqrt{2}} e^{\mp i\varphi}(d\theta \mp i\sin\theta\, d\varphi)$$

$$e^3 = -d\varphi(\cos\theta - 1) . \qquad (3.11)$$

The frames $E^{\pm}$ are related to those of Sec.II by

$$E^{\pm} = \frac{1}{\sqrt{2}}(E^5 \pm iE^6) . \qquad (3.12)$$

From (3.9) and (3.10) one finds that, under the action of SU(2), the frames and connection transform according to

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*) Strictly, one should employ two patches. The parametrization (3.8) is suitable for $0 \le \theta < \pi$, $0 \le \varphi < 2\pi$. For the other patch, $0 < \theta \le \pi$, $0 \le \varphi < 2\pi$ one should write $L_{\theta\varphi} = e^{-\varphi Q_3} e^{-\theta Q_2} e^{-\varphi Q_3}$. The action of $L_{\theta\varphi}$ can be seen by embedding S^2 in R^3 where spherical polar co-ordinates give the parametrization

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix} = D(L_{\theta\varphi})\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} ,$$

where

$$D(L_{\theta\varphi}) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

$$E^{\pm}(y) \to E^{\pm}(y') = E^{\pm}(y)\, e^{\pm i\zeta}$$

$$e^3(y) \to e^3(y') = e^3(y) - d\zeta \quad . \tag{3.13}$$

Defining the covariant differential of $L^{-1}_{\theta\varphi}$,

$$\begin{aligned} \nabla L^{-1} &= dL^{-1} + e^3 Q_3 L^{-1} \\ &= E^+ \nabla_+ L^{-1} + E^- \nabla_- L^{-1} \end{aligned} \tag{3.14}$$

one finds, by simply rearranging (3.10), the useful formula

$$\nabla_{\pm} L^{-1} = -\frac{1}{a} \frac{Q_1 \mp iQ_2}{\sqrt{2}} L^{-1} \quad . \tag{3.15}$$

The behaviour (3.13), in conjunction with the identification (3.12) means that rotations of the 2-sphere must be associated with tangent space rotations in the 5–6 plane in order to preserve the form of the background (2.9). Observe that the background Maxwell field is proportional to $e^3(y)$. Therefore, fields to be expanded must first be decomposed into irreducible representations of these SO(2) rotations, labelled by the "iso-helicity", λ. Let $\phi_\lambda(x,\theta,\varphi)$ be a typical one. It can then be shown that the harmonic expansion of ϕ_λ takes the form

$$\phi_\lambda(x,\theta,\varphi) = \sum_{\ell \geq |\lambda|} \sqrt{2\ell+1} \sum_m D^{\ell}_{\lambda m} (L^{-1}_{\theta\varphi})\, \phi^{\ell}_{\lambda m}(x) \quad , \tag{3.16}$$

where $D^{\ell}_{\lambda m}$ belongs to the $2\ell+1$-dimensional unitary irreducible representation of SU(2).

By virtue of the formula (3.15) the computation of covariant derivatives reduces to algebraic manipulation, *)

$$\nabla_{\pm} D^{\ell}_{\lambda m}(L^{-1}) = -\frac{1}{a\sqrt{2}} D^{\ell}_{\lambda m}(Q_{1\mp i2} L^{-1})$$

$$= + \frac{i}{a} \sqrt{\frac{(\ell \mp \lambda)(\ell \pm \lambda + 1)}{2}}\, D^{\ell}_{\lambda \pm 1, m}(L^{-1}) \quad . \tag{3.17}$$

This feature leads to considerable simplications in the equations of motion.

Among the fields h_{AB} and V_A we have the following SO(2) irreducible pieces:

$$\begin{aligned} h_{++} &= \frac{1}{2}(h_{55} - h_{66} - 2ih_{56}) \ , & \lambda &= 2 \\ h_{a+} &= \frac{1}{\sqrt{2}}(h_{a5} - ih_{a6}) \ , & \lambda &= 1 \\ h_{+-} &= \frac{1}{2}(h_{55} + h_{66}) \ , & \lambda &= 0 \\ V_+ &= \frac{1}{\sqrt{2}}(V_5 - iV_6) \ , & \lambda &= 1 \end{aligned} \tag{3.18}$$

plus h_{ab}, V_a with $\lambda = 0$ and the complex conjugates $h_{--} = h^*_{++}$, etc. Each of these has an expansion of the form (3.16). The sources, T_{AB} and J_A are likewise expanded.

The equations of motion for h and V derived from (3.2) are

$$\nabla^2(h_{AB} - \frac{1}{2}\eta_{AB} h_{CC}) + (\bar{R}_{AC} h_{CB} + \bar{R}_{BC} h_{CA})$$

$$+ \kappa \bar{F}_{BC}(\nabla_A V_C - \nabla_C V_A) + \kappa \bar{F}_{AC}(\nabla_B V_C - \nabla_C V_B) - \kappa \eta_{AB} \bar{F}_{CD} \nabla_C V_D = -T_{AB}$$

$$\nabla^2 V_A + R_{AB} V_B + \kappa \nabla_C(h_{AB}\bar{F}_{BC} + \bar{F}_{AB} h_{BC} + \frac{1}{2} h_{BB} \bar{F}_{CA}) = -J_A \quad . \tag{3.19}$$

Considerable simplifications have been brought about in these equations by use of the background solution. The only non-vanishing components of $\bar{R}$ and $\bar{F}$ are, in the basis (3.11),

$$\begin{aligned} \bar{R}_{+-} &= \bar{R}_{-+} = -\frac{1}{a^2} \\ \bar{F}_{+-} &= -\bar{F}_{-+} = \frac{i}{a}\frac{\sqrt{2}}{\kappa} \quad . \end{aligned} \tag{3.20}$$

Introducing harmonic expansions for the various components into (3.19) and using (3.17) to evaluate the covariant derivatives $\nabla_{\pm}$, one can immediately extract the equations for the harmonic components. These equations are, suppressing the iso-spin labels ℓ,m,

*) We are using antihermitian generators, Q, with phase conventions [9] such that

$$\langle\lambda| Q_{1-i2} = \frac{1}{i}\sqrt{(\ell-\lambda)(\ell+\lambda+1)}\ \langle\lambda+1|$$

$$\langle\lambda| e^{\zeta Q_3} = e^{-i\lambda\zeta} \langle\lambda| \quad .$$

Under the tangent space U(1) associated with the global action of an element, g, of SU(2) on S^2,

$$\phi'_\lambda(x,\theta',\varphi') = e^{-i\lambda\zeta}\,\phi_\lambda(x,\theta,\varphi)$$

This implies invariance of $E^+\phi_+$, etc., as it should.

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)(h_{ab} + \eta_{ab}h_{+-}) + \eta_{ab}\frac{\sqrt{\ell(\ell+1)}}{a^2}(V_+-V_-) = -T_{ab} + \frac{1}{2}\eta_{ab}T_{cc}$$

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)h_{aa} + \frac{4}{a^2}h_{+-} + 2\frac{\sqrt{\ell(\ell+1)}}{a^2}(V_+-V_-) = 2T_{+-}$$

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)V_a + \frac{\sqrt{\ell(\ell+1)}}{a^2}(h_{a+}-h_{a-}) = -J_a$$

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)h_{a\pm} \mp i\frac{\sqrt{2}}{a}\partial_a V_+ \mp \frac{\sqrt{\ell(\ell+1)}}{a^2}V_a = -T_{a\pm}$$

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)V_\pm \pm i\frac{\sqrt{2}}{a}\partial_b h_{b\pm} \mp \frac{\sqrt{\ell(\ell+1)}}{a^2}(h_{+-} - \frac{1}{2}h_{bb}) = -J_\pm$$

$$\left(\partial^2 - \frac{\ell(\ell+1)}{a^2}\right)h_{\pm\pm} = -T_{\pm\pm} \quad . \qquad (3.21)$$

The solutions of these equations are to be substituted into the expression

$$I(T,J) = \frac{1}{2}\int d^6z\,\sqrt{-g}\,\left(\frac{1}{2}T_{AB}h_{AB} + J_A V_A\right)$$

$$= 2\pi a^2\int d^4x \sum_{\ell m}\Big[\frac{1}{2}\bar{T}_{ab}h_{ab} + \frac{1}{2}\bar{T}_{++}h_{++} + \frac{1}{2}\bar{T}_{--}h_{--}$$

$$+ \bar{T}_{a+}h_{a+} + \bar{T}_{a-}h_{a-} + \frac{1}{4}\bar{T}h + \bar{J}_a V_a + \bar{J}_+V_+ + \bar{J}_-V_-\Big] \quad . \qquad (3.22)$$

Because of the reality of h_{AB} and V_A and the properties of $D^\ell_{\lambda m}$ [9] it follows that the harmonic components are subject to the reality condition

$$\overline{\phi^\ell_{\lambda m}(x)} = (-)^{\lambda-m}\,\phi^\ell_{-\lambda,-m}(x) \quad . \qquad (3.23)$$

This ensures the reality of $I(T,J)$.

To solve the set (3.21) and substitute the result into (3.22) is a straightforward though lengthy computation. We illustrate the approach by finding part of the dependence *) of I on T_{++}. The contribution of h_{++} can be found quite easily since h_{++} decouples from the rest. Thus, from (3.21)

$$h_{++} = -\left(\partial^2 - \frac{\ell(\ell+1)-2}{a^2}\right)^{-1}T_{++} \quad .$$

The contribution to I is then

*) The full dependence of I on T_{++} can only be obtained by solving the complete set of equations (3.21) The result is contained I_0, given below, cf. (3.26).

$$2\pi a^2\int d^4x \sum_{\ell m}\left(\frac{1}{2}\bar{T}_{++}h_{++} + \frac{1}{2}\bar{T}_{--}h_{--}\right)$$

$$= 2\pi a^2\int d^4x \sum_{\ell m}\left[-\frac{1}{2}\bar{T}_{++}\left(\partial^2 - \frac{\ell(\ell+1)-2}{a^2}\right)^{-1}T_{++} + \text{h.c.}\right]$$

$$= 2\pi a^2\int d^4x \sum_{\ell\geq 2}\sum_m \bar{T}_{++}\left(-\partial^2 + \frac{\ell(\ell+1)-2}{a^2}\right)^{-1}T_{++} \quad .$$

To interpret this result one can Fourier analyze T_{++} obtaining

$$2\pi a^2\int\frac{d^4p}{(2\pi)^4}\sum_{\ell\geq 2}\sum_m \frac{|\tilde{T}_{++}(p)|^2}{p^2 + \frac{\ell(\ell+1)-2}{a^2}} \quad .$$

One sees that it describes the exchange of particles of mass $(\ell(\ell+1)-2)/a^2$ carrying quantum numbers appropriate to the vertex factor $\tilde{T}^\ell_{++m}(p)$, i.e. $2\ell+1$-dimensional multiplets of scalar particles. The overall factor $2\pi a^2$ is not significant, it can be scaled away.

It is important to make use of the conservation laws (3.5) since only the physical part of the spectrum is of interest. With the help of (3.17) one can extract from (3.5) the conservation laws for the harmonic components,

$$\partial_a T_{ab} + \frac{i}{a}\sqrt{\frac{\ell(\ell+1)}{2}}\,(T_{b+} + T_{b-}) = 0$$

$$\partial_a T_{a\pm} + \frac{i}{2}\sqrt{\frac{\ell(\ell+1)}{2}}\,T_{-+} + \frac{i}{a}\sqrt{\frac{(\ell+2)(\ell-1)}{2}}\,T_{\pm\pm} = \pm i\frac{\sqrt{2}}{a}J_\pm$$

$$\partial_a J_a + \frac{i}{a}\sqrt{\frac{\ell(\ell+1)}{2}}\,(J_+ + J_-) = 0 \quad . \qquad (3.24)$$

The solution of (3.21) is given in the appendix. Here we exhibit only the pole terms which result when the substitution into (3.22) is made. The expressions are simplified by referring them to an appropriate Lorentz frame. To see the massive states it is best to use the rest frame, $p_a = (p_0,0,0,0)$. For massless states we use instead the frame $p_a = (p_0,0,0,p_3)$. The spin of the exchanged particle is signalled by the source currents appearing in the residue.

Taking first the massive excitations one can show that the pole terms in $I(T,J)$ arrange themselves into six towers,

$$I = \sum_{\ell \geq 0} \left(I_0^{(+)} + I_0^{(-)} + I_2\right) + \sum_{\ell \geq 1} \left(I_1^{(+)} + I_1^{(-)}\right) + \sum_{\ell \geq 2} I_0 \quad , \tag{3.25}$$

where the subscripts 0,1,2 indicate the spin of the exchanged particles which contribute. The explicit forms are

$$I_0 = \frac{1}{p^2 + M_0^2} \frac{1}{2\ell(\ell+1)} |T_{++} - T_{--}|^2$$

$$I_0^{(\pm)} = \frac{1}{p^2 + M_{0\pm}^2} \frac{1}{4\sqrt{1+12\ell(\ell+1)}} \Bigg| \sqrt{\frac{4}{8}} \left[5 \pm \sqrt{1+12\ell(\ell+1)}\right]^{1/2} \left(\frac{3}{4} T_{00} + \frac{1}{4} T_{kk}\right) +$$

$$+ \left[\frac{\pm (3\ell(\ell+1)-2) \sqrt{1+12\ell(\ell+1)} - 9\ell(\ell+1)-2}{\ell(\ell+1) - 2}\right]^{1/2} T_{+-}$$

$$+ i \left[\frac{\ell(\ell+1)}{\ell(\ell+1)-2} \left(5 \mp \sqrt{1 + 12\ell(\ell+1)}\right)\right]^{1/2} (J_+ + J_-)\Bigg|^2$$

$$I_1^{(\pm)} = \frac{1}{p^2 + M_{1\pm}^2} \left|T_{k+} - T_{k-} \pm \sqrt{2}\, J_k\right|^2$$

$$I_2 = \frac{1}{p^2 + M_2^2} \left|T_{jk} - \frac{1}{3} \delta_{jk} T_{ii}\right|^2 \quad , \tag{3.26}$$

where the non-zero masses are given by

$$M_0^2 = (\ell-1)(\ell+2)/a^2 \quad , \qquad \ell \geq 2$$

$$M_{0\pm}^2 = \left(2\ell(\ell+1) + 1 \pm \sqrt{1 + 12\ell(\ell+1)}\right)/2a^2 \quad , \qquad \ell \geq 0$$

$$M_{1\pm}^2 = \left(\ell(\ell+1) \pm \sqrt{2\ell(\ell+1)}\right)/a^2 \quad , \qquad \ell \geq 1$$

$$M_2^2 = \ell(\ell+1)/a^2 \quad , \quad \ell \geq 0 \quad . \tag{3.27}$$

It must be emphasized that these mass formulae have been derived by using the rest frame so that only the non-zero values belong to the spectrum. It will be observed that $M_{0-} = 0$ for $\ell = 0,1$, $M_{1-} = 0$ for $\ell = 1$, and $M_2 = 0$ for $\ell = 0$. These zeroes are artefacts of the computational procedure. To find the true massless states it is necessary to repeat the computation in a more appropriate frame, $p_a = (p_0,0,0,p_3)$. When this is done, one finds the terms

$$I(M = 0) = \frac{1}{p^2} \Bigg\{ \left[\frac{1}{4} |T_{11}-T_{22}|^2 + |T_{12}|^2 + |J_1|^2 + |J_2|^2\right]_{\ell=0}$$

$$+ \left[|T_{1+}+T_{1-} - \sqrt{2}\, J_1|^2 + |T_{2+}+T_{2-} - \sqrt{2}\, J_2|^2\right]_{\ell=1} \Bigg\} \tag{3.28}$$

indicating a total of ten massless states: the graviton (helicities $\lambda = \pm 2$), a "photon" ($\lambda = \pm 1$), and a Yang-Mills triplet ($\lambda = \pm 1$, $\lambda = 1$). Note, in particular, the absence of zero-helicity massless states.

Finally, we observe that there are no tachyons and, since the residues all have the same sign, no negative metric states. This means that the ground state is at least perturbatively stable

IV. ZERO-MODE ANSATZ

The purpose of this section is to derive an effective four-dimensional Lagrangian for the long-range sector of the six-dimensional system. That is, we shall suppose that the radius a tends to zero while discarding from the theory all those harmonic components corresponding to masses $\sim 1/a$. This is not an entirely satisfactory kind of limit: it corresponds, in fact, to letting the Maxwell gauge coupling grow without limit,

$$e = \frac{n}{2\sqrt{2}} \frac{\kappa}{a} \to \infty \quad .$$

Of course, the model cannot pretend to be realistic, and with the limited set of parameters available we cannot avoid this difficulty. Our aim here is only to illustrate the procedure of "dimensional reduction" in the context of a particularly simple model.

One of the features to be brought out is the rather complicated placing of the SU(2) gauge field in the six-dimensional system. As mentioned in Sec.I, the Yang-Mills vector turns out to be a mixture of the six-dimensional metric and Maxwell fields. This is contrary to the common assumption that Yang-Mills fields are purely metrical in origin. We shall argue, however, that such mixing effects are to be expected whenever the "matter" fields which participate in the spontaneous compactification carry the appropriate quantum numbers.

Since we know from the analysis of Sec.III that no scalar zero-modes are present we do not need to introduce any scalar fields in the zero-mode ansatz. We need to account only for the graviton and the SU(2) × U(1) vectors.

We also know from Sec.III that the helicity -1 triplet is a mixture of metric and Maxwell fields. In order to take account of this mixing we shall start by introducing two independent, $\ell = 1$, 4-vectors in the ansatz. One combination will reveal itself as massive and we shall then discard it. Thus, our starting ansatz, in the notation of 1-forms *), reads

$$
\begin{aligned}
E^a(x,y) &= dx^m\, E_m{}^a(x) \\
E^a(x,y) &= a\, dy^\mu\, e_\mu{}^\alpha(y) - \kappa\, dx^m\, W_m{}^{\hat\alpha}(x)\, D_{\hat\alpha}{}^\alpha(L_y) \\
&= a\, e^\alpha(y) - \kappa\, W^{\hat\alpha}(x)\, D_{\hat\alpha}{}^\alpha(L_y) \\
A(x,y) &= dx^m\, V_m(x) + \frac{n}{2e}\left[dy^\mu\, e_\mu{}^3(y) - \frac{\kappa}{a}\, dx^m\, U_m{}^{\hat\alpha}(x)\, D_{\hat\alpha}{}^3(L_y)\right] \\
&= V(x) + \frac{n}{2e}\left[e^3(y) - \frac{\kappa}{a}\, U^{\hat\alpha}(x)\, D_{\hat\alpha}{}^3(L_y)\right] ,
\end{aligned}
\tag{4.1}
$$

where $a = 0,1,2,3$; $\alpha = +,-$; $\hat\alpha = 1,2,3$ and $y^\mu = (\theta,\varphi)$. At this stage the system includes the graviton field, $E_m{}^a(x)$, a U(1) vector, $V_m(x)$, and two SU(2) vectors, $W_m{}^{\hat\alpha}(x)$ and $U_m{}^{\hat\alpha}(x)$.

The symmetry group of the background includes rotations of the 2-sphere coupled to appropriate frame rotations and U(1) gauge transformations. The frame rotations are determined by the invariance of $e^\alpha(y)$ and the U(1) transformations by the invariance of $e^3(y)$. As discussed in Sec.III, a left translation, g, of the quotient space, SU(2)/U(1) (parametrized by L_y), induces a U(1) rotation, h, such that $g\, L_y = L_{y'} h$. On writing $h = \exp\zeta Q_3$ one finds

$$
\begin{aligned}
e^\pm(y') &= e^\pm(y)\, \exp(\pm i\zeta) \\
e^3(y') &= e^3(y) - d\zeta ,
\end{aligned}
\tag{4.2}
$$

where $\zeta = \zeta(y,g)$. Invariance is obtained if the frames $E^\pm$ are rotated through ζ and the Maxwell field A undergoes a gauge transformation with parameter $\Lambda = n\zeta/2$. This much is enough to give invariance under x-independent left translations. In order to get local invariance it is necessary to consider the behaviour of $W^{\hat\alpha}$ and $U^{\hat\alpha}$. Although these fields are associated with excitations on the background, their transformations include inhomogeneous

*) The 1-forms $e^\pm(y)$ which appear here are related to $E^\pm(y)$ defined in Eq.(3.11) by $a e^\pm(y) = E^\pm(y)$.

terms whose contributions are needed in order to make manifest the background invariance with respect to local left translations on S^2. The local version *) of (4.2) is

$$
\begin{aligned}
e^\alpha(y') &= e^\beta(y)\, D_\beta{}^\alpha(h^{-1}) + (g^{-1}\, dg)^{\hat\beta}\, D_{\hat\beta}{}^\alpha(L_y h^{-1}) \\
e^3(y') &= e^3(y) - d\zeta + (g^{-1}\, dg)^{\hat\beta}\, D_{\hat\beta}{}^3(L_y) ,
\end{aligned}
\tag{4.3}
$$

where $g^{-1}\, dg = dx^m\, g^{-1}\, \partial_m g$ belongs to the algebra of SU(2). Taken in conjunction with the ansatz (4.1) these identities allow one to deduce the transformation properties of V,W,U. The group acts on E^α and A such that

$$
\begin{aligned}
E^\alpha(x,y) &\to E'^\alpha(x,y') = E^\beta(x,y)\, D_\beta{}^\alpha(h^{-1}) \\
A(x,y) &\to A'(x,y') = A(x,y) - \frac{n}{2e}\, d\zeta .
\end{aligned}
\tag{4.4}
$$

Using (4.3) it is a simple matter to show that the form of the ansatz is maintained provided

$$
\begin{aligned}
U'^{\hat\alpha}(x) &= U^{\hat\beta}(x)\, D_{\hat\beta}{}^{\hat\alpha}(g^{-1}) - \frac{a}{\kappa}\, (g\, dg^{-1})^{\hat\alpha} \\
W'^{\hat\alpha}(x) &= W^{\hat\beta}(x)\, D_{\hat\beta}{}^{\hat\alpha}(g^{-1}) - \frac{a}{\kappa}\, (g\, dg^{-1})^{\hat\alpha} ,
\end{aligned}
\tag{4.5}
$$

That is, both U and W transform as SU(2) gauge fields. (The U(1) vector, V(x), is invariant.) It is now clear that the difference, U-W, transforms homogeneously and could therefore be expected to acquire a mass. This indeed happens. A straightforward computation gives the field strength 2-form, F = dA,

$$
\begin{aligned}
F = &-\frac{1}{2}\, dx^m \wedge dx^n \left\{V_{mn} - \sqrt{2}\left[W^{\hat\alpha}_{mn} + \nabla_m (U-W)^{\hat\alpha}_n - \nabla_n (U-W)^{\hat\alpha}_m\right] D_{\hat\alpha}{}^3(L)\right\} \\
&+ dx^m \wedge E^\beta\, \frac{\sqrt{2}}{a}\, (U-W)^{\hat\alpha}_m\, D_{\hat\alpha}{}^\gamma(L)\, \epsilon_{\beta\gamma 3} \\
&- \frac{1}{2}\, E^\alpha \wedge E^\beta\, \frac{n}{2ea^2}\, \epsilon_{\alpha\beta 3} ,
\end{aligned}
\tag{4.6}
$$

where ∇_m denotes the SU(2) covariant derivative

$$
\nabla_m (U-W)^{\hat\alpha}_n = \partial_m (U-W)^{\hat\alpha}_n - \frac{\kappa}{2}\, W^{\hat\beta}_m (U-W)^{\hat\gamma}_n\, \epsilon_{\hat\beta\hat\gamma\hat\alpha}
\tag{4.7}
$$

*) Ref.4, Appendix 3.

and W_{mn} the Yang-Mills field strength. When the components, F_{AB}, are substituted into the Lagrangian and the integration over y is performed, there results the expression,

$$-\frac{1}{4} V_{ab}^2 - \frac{1}{6}\left\{W_{ab} + \nabla_a(U-W)_b - \nabla_b(U-W)_a\right\}^2$$

$$-\frac{1}{6}\frac{4}{a^2}(U_a - W_a)^2 - \frac{n^2}{8e^2a^4} \quad . \qquad (4.8)$$

On the other hand, the vector W appears in E^α and so will contribute to the six-dimensional curvature scalar, R_6. The contribution of R_6 to the four-dimensional Lagrangian has been discussed in the literature [10]. It gives

$$-\frac{1}{\kappa^2} R_4 - \frac{1}{6} W_{ab}^2 + \frac{1}{\kappa^2 a^2} \qquad (4.9)$$

On combining (4.9) with (4.8) and introducing the new combinations,

$$A = \frac{1}{\sqrt{3}}(U + W), \qquad X = \frac{1}{\sqrt{3}}(U - W) \qquad (4.10)$$

one finds,

$$-\frac{1}{\kappa^2} R_4 - \frac{1}{4} V_{ab}^2 - \frac{1}{8}\left[A_{ab} - (\nabla_a X_b - \nabla_b X_a) - \frac{\kappa}{a}\frac{\sqrt{3}}{2} X_a \times X_b\right]^2$$

$$-\frac{1}{8}\left[A_{ab} + (\nabla_a X_b - \nabla_b X_a) + \frac{3\kappa}{a}\frac{\sqrt{3}}{2} X_a \times X_b\right]^2$$

$$-\frac{1}{2}\frac{4}{a^2} X_a^2$$

$$= -\frac{1}{\kappa^2} R_4 - \frac{1}{4} V_{ab}^2 - \frac{1}{4} A_{ab}^2 - \frac{1}{4}(\nabla_a X_b - \nabla_b X_a)^2 - \frac{1}{2}\frac{4}{a^2} X_a^2 + \dots \;, \qquad (4.11)$$

where A_{ab} denotes the Yang-Mills field strength made out of A and the covariant derivative ∇_a is now defined by

$$\nabla_a X_b = \partial_a X_b - \frac{\sqrt{3}}{2}\frac{\kappa}{a} A_a \times X_b \quad . \qquad (4.12)$$

It now appears that A_a is the true Yang-Mills vector, while X_a is massive. (It corresponds to the mass $M_{1+}(\ell=1)$ of Eq.(3.27).) In passing, we note that the SU(2) coupling constant is given by

$$f = \frac{\sqrt{3}}{2}\frac{\kappa}{a} = \frac{\sqrt{6}}{n} e \quad . \qquad (4.13)$$

where e is the U(1) coupling.

This concludes our discussion of the bosonic part of the long-range sector of this model. In the next section we consider the introduction of fermions and show how fermionic zero modes can arise.

V. FERMION SPECTRUM

At the classical level fermion fields do not affect the ground state solution and the question of its stability. However, after establishing the properties of the classical ground state one is free to consider how it affects the propagation of fermionic excitations. In particular, one can demonstrate the presence of zero-mode solutions of the Dirac equation.

Fermion fields should be treated as scalars under the group of general co-ordinate transformations and as spinors under the tangent space group SO(1,5). The simplest examples are 4-spinors and we shall treat only these. The relevant Clifford algebra is generated by eight-dimensional matrices, Γ_A, which satisfy

$$\frac{1}{2}\{\Gamma_A, \Gamma_B\} = \eta_{AB} = \mathrm{diag}(-1, 1, 1, 1, 1, 1) \quad . \qquad (5.1)$$

(On occasion we shall also use the combinations $\Gamma_5 \mp i\Gamma_6 = \sqrt{2}\,\Gamma_\pm$ appropriate to the spherical basis on the 2-sphere, i.e. $\eta_{+-} = \eta_{-+} = 1$, $\eta_{++} = \eta_{--} = 0$.) It is convenient to employ a particular realization of the algebra (5.1) which is adapted to the reduction, $SO(1,5) \to SO(1,3) \times SO(2)$. In this realization we have

$$\Gamma_a = \gamma_a \times \tau_1 \,, \qquad a = 0,1,2,3$$
$$\Gamma_5 = \gamma_5 \times \tau_1$$
$$\Gamma_6 = 1 \times \tau_2 \qquad , \qquad (5.2)$$

where γ_a and γ_5 denote the usual four-dimensional matrices,

$$\frac{1}{2}\{\gamma_a, \gamma_b\} = \eta_{ab}, \qquad \gamma_5^2 = 1 \quad . \qquad (5.3)$$

The generators of SO(1,5) are given by

$$-\frac{1}{2}\ \Gamma_{AB} = -\frac{1}{4}\{\Gamma_A, \Gamma_B\} \tag{5.4}$$

and they evidently commute with τ_3. The fundamental representations, ψ and $\bar{\psi}$, correspond to the eigenvalues +1 and -1 of τ_3, respectively.

Let $\psi(x,y)$ belong to the representation 4, i.e.

$$\tau_3\psi = \psi . \tag{5.5}$$

From (5.2), (5.4) we see that the SO(2) group of rotations in the 5-6 plane is generated by

$$-\frac{1}{2}\Gamma_{56} = -\frac{i}{2}\gamma_5\tau_3 . \tag{5.6}$$

Acting on the chiral components of ψ it gives

$$-\frac{1}{2}\Gamma_{56}\ \psi_L = \frac{i}{2}\psi_L$$
$$-\frac{1}{2}\Gamma_{56}\ \psi_R = -\frac{i}{2}\psi_R , \tag{5.7}$$

where ψ_L and ψ_R correspond to $\gamma_5 = -1$ and $\gamma_5 = +1$, respectively.

Now suppose that $\psi(x,y)$ carries the U(1) charge, e. Under local U(1) transformations defined by

$$A \rightarrow A - \frac{1}{e}\, d\Lambda \tag{5.8}$$

we have

$$\psi \rightarrow e^{i\Lambda}\,\psi , \tag{5.9}$$

where $\Lambda = \Lambda(x,y)$ is a real scalar. Derivatives of ψ which are covariant with respect to both the frame rotations, SO(1,5), and the U(1) gauge transformations are contained in the 1-form

$$\nabla\psi = d\psi + B\psi + ie\, A\psi , \tag{5.10}$$

where B denotes the spin connection 1-form

$$B = \frac{1}{2}\, dz^M\, B_{M[AB]}\left\{-\frac{1}{2}\,\Gamma_{AB}\right\} . \tag{5.11}$$

For the purposes of this section, where only fermion bilinears are to be retained, it will be sufficient to use background values for B and A, viz.

$$\bar{B} = -\frac{1}{2}\Gamma_{56}\, e^3$$
$$\bar{A} = \frac{n}{2e}\, e^3 , \tag{5.12}$$

where e^3 is the 1-form, $-d\varphi(\cos\theta-1)$, given in Sec.III. In effect, the background covariant derivative of ψ is given by

$$\bar{\nabla}\psi = d\psi + \frac{i}{2}(n + i\Gamma_{56})\, e^3\psi$$
$$= d\psi + i\left(\frac{1+\gamma_5}{2}\,\frac{n-1}{2} + \frac{1-\gamma_5}{2}\,\frac{n+1}{2}\right) e^3\psi . \tag{5.13}$$

The invariance group of the ground state involves the simultaneous action of left translations on the 2-sphere, rotations of the E^5 and E^6 frames and a U(1) gauge transformation. In the notation of (3.13), the tangent space rotation through the angle $\zeta = \zeta(\theta,\varphi,g)$ must be accompanied by a gauge transformation with $\Lambda = +\frac{n}{2}\zeta$ in order to preserve the form of $\bar{A}$, as discussed in Sec.IV. With respect to this background symmetry the effective iso-helicities of ψ_R and ψ_L become $\frac{1}{2} - \frac{n}{2}$ and $-\frac{1}{2} - \frac{n}{2}$, respectively, as indicated in (5.13).

Now consider the Dirac operator (using $\tau_3\psi = \psi$),

$$\Gamma^A\nabla_A\psi = (\gamma^a\tau_1\partial_a + \gamma^5\tau_1\nabla_5 + \tau_2\nabla_6)\psi$$
$$= \tau_1(\partial\!\!\!/ + \gamma_5\nabla_5 + i\nabla_6)\,\psi$$
$$= \tau_1\left(\partial\!\!\!/ + \frac{1+\gamma_5}{2}\,\sqrt{2}\,\nabla_- - \frac{1-\gamma_5}{2}\sqrt{2}\,\nabla_+\right)\psi .$$

It appears that the mass term for the 4-spinor takes the form

$$\sqrt{2}\,\left(\bar{\psi}_L\nabla_-\psi_R - \bar{\psi}_R\nabla_+\psi_L\right) . \tag{5.14}$$

To evaluate the covariant derivatives in this expression we must introduce the harmonic expansions,

$$\psi_{R,L} = \sum_{\ell \geq |\lambda_{R,L}|} \sqrt{2\ell+1}\, \sum_m D^{\ell}_{\lambda m}(L^{-1}_{\theta\varphi})\ \psi_{R,Lm}^{\ \ \ell}(x) , \tag{5.15}$$

where the relevant iso-helicities are

$$\lambda_R = \frac{1-n}{2} \qquad \text{and} \qquad \lambda_L = -\frac{1+n}{2} \quad . \tag{5.16}$$

In terms of the harmonic components the mass term (5.13) becomes

$$\frac{i}{a} \sum_{\ell m} \sqrt{\left(\ell + \frac{1}{2}\right)^2 - \frac{n^2}{4}} \; (\bar{\psi}_L \psi_R - \bar{\psi}_R \psi_L) \quad . \tag{5.17}$$

There is evidently one massless multiplet with

$$\ell = -\frac{1}{2} + \frac{|n|}{2} \quad .$$

This value of ℓ contributes to ψ_R but not to ψ_L if n is a positive integer. Conversely, if n is negative then ψ_L contains the massless mode*).

The coupling of the massless fermion field to the boson fields V, A and E can be derived by substituting the zero-mode ansatz into the six-dimensional Dirac Lagrangian

$$i\,\bar{\psi}\,\Gamma^A\,E_A{}^M(\partial_M + B_M + ieA_M)\,\psi \tag{5.18}$$

and integrating over y. Because of the local SU(2) × U(1) symmetry, the result must be

$$i\bar{\psi}_R \gamma^a E_a{}^m \left\{\partial_m + \frac{1}{4} B_{m[bc]}\,\gamma^{bc} - f\,A_m^{\hat{\alpha}}\,I_{\hat{\alpha}} + ieV_m\right\} \psi_R \tag{5.19}$$

if ψ_R is the massles field ($n > 0$). The fields ψ_R belong to the right-handed doublet of SO(1,3) and to the n-dimensional representation of SU(2) (with generators denoted $I_{\hat{\alpha}}$). The SO(1,3) spin connection is denoted $B_{m[bc]}$.

One may add more than one spinor field to the Lagrangian: for example, consider two fields ψ, ψ' with U(1) couplings Y and Y'. From the iso-helicity formulae for ψ:

$$\lambda_R = \frac{1-nY}{2} \quad , \quad \lambda_L = -\left(\frac{1+nY}{2}\right)$$

and with similar formulae for ψ' with Y replaced by Y', we would obtain for the $n = 1$ case, a massless iso-singlet ψ_R for $Y = 1$, and a massless iso-doublet ψ'_L for $Y' = -2$. For $n = 2$, with $Y = 1$, $Y' = -1$, we would obtain two "iso-doublets" ψ_R and ψ'_L, both massless. For $n = 2$, the formulae for the ratio of U(1) and SU(2) couplings, $\tan\theta_0 = \frac{n}{\sqrt{6}}$ (cf. Eq.(4.13) yields $\sin^2\theta = \frac{2}{5}$ - a value very close to the conventional unrenormalized value $\frac{3}{8}$. A determination of "renormalized" $\sin^2\theta$ with $\sin^2\theta_0 = \frac{2}{5}$ as input in a usual GUT context would give a 3% derivation from the value deduced with $\sin^2\theta_0 = \frac{3}{8}$ as input. This accidental agreement is perhaps devoid of physical significance, since we do not expect ψ_R and ψ'_L to correspond to the leptonic or quark doublets. Even so, the fact that $\sin^2\theta$ can in principle be determined by the n-quantum number (corresponding to the monopole-charge in the internal space) is an important consequence of the Kaluza-Klein picture of the influence of extra dimensions on physics.

*) The 2n zero-modes of $\Gamma_+V_- + \Gamma_-V_+$ associated with the magnetic charge, n, are of course predicted by the Atiyah-Singer index theorem.

VI. CONCLUSIONS.

The model discussed in this article, six-dimensional Einstein-Maxwell theory (with fermions), is not intended as a realistic physical theory. We have analyzed it in some detail because it seems to represent the simplest possible version of a Kaluza-Klein mechanism which yet contains some of the features to be hoped for in more elaborate systems: compactification arises as a consequence of dynamics, the internal space has a non-Abelian symmetry, massless chiral fermions are available, and the zero-mode sector includes no scalars. This model suffers the defects which are generally expected as well: it is not renormalizable, and the gauge couplings grow unacceptably large in the very limit which is needed to drive away the towers of massive modes. Another defect is the need for an "unnatural" adjustment of parameters in the parent Lagrangian in order to flatten the four-dimensional spacetime. The presence of such adjustable parameters is both a defect and a virtue. One would like to be able to start from a less arbitrary theory, a theory in which the "matter" content and its couplings are fixed. For this reason, the many-dimensional supergravities are attractive. Up to now, however, such theories have not yielded any physically acceptable ground state solution. Either the internal space is flat [10], with Abelian symmetry or, if it is curved, it generates four-dimensional curvature of the same magnitude [5]. There are too few adjustable parameters. One hopes that this is a temporary impasse.

Finally, we believe that our approach to the phenomenon of dimensional reduction, via the computation of spectra, illuminates an important matter. This is the question of what four-dimensional fields should be included in the zero-mode ansatz, and how they should be arranged. Without first going through the spectral analysis one cannot foresee the correct form of the ansatz. Although the gauge vectors are guaranteed by the background symmetry, their placement in the ansatz has to be found. The very presence or absence of massless scalars seems to be model dependent (except when the background is supersymmetric, in which case they are certain to be present). Quite generally, the underlying mechanism whereby "matter" fields generate curvature on the internal space would seem to imply a non-trivial mixing between matter and metrical field components. This mixing has to be correctly represented in the zero-mode ansatz.

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APPENDIX

The solution to the linear equations (3.19) which was used to obtain the pole structures (3.26)-(3.28) is listed here for reference

$$h_{ab} - \frac{1}{4}\eta_{ab}\, h_{cc} = \frac{1}{p^2 + M_2^2}\left(T_{ab} - \frac{1}{4}\eta_{ab}\, T_{cc}\right)$$

$$h_{aa} = \frac{1}{(p^2+M_2^2)(p^2+M_{0+}^2)(p^2+M_{0-}^2)}\left[-2(p^2+M_{1-}^{\;2})(p^2+M_{1+}^2)\,T_{+-} + \frac{p^2}{a^2}T_{aa} + \frac{2}{a}M_2(p^2+M_0^2)(J_+ - J_-) + \frac{2M_0M_2}{a^2}(T_{++} + T_{--})\right]$$

$$h_{+-} = \frac{1}{(p^2+M_{0+}^2)(p^2 + M_{0-}^2)}\left[\frac{1}{4}(p^2+M_2^2)(2T_{+-} - T_{aa}) - \frac{3}{2}\frac{M_2}{a}(J_+ - J_-) - \frac{3}{2}\frac{M_0M_2}{a^2(p^2+M_0^2)}(T_{++} + T_{--})\right]$$

$$h_{++} = \frac{1}{p^2 + M_0^2}\,T_{++}$$

$$V_a = \frac{\sqrt{2}\,M_2 p_a}{a^2(p^2+M_2^2)(p^2+M_{1+}^2)(p^2+M_{1-}^2)}\left[\frac{-M_0}{a(p^2 + M_0^2)}(T_{++} - T_{--}) - (J_+ + J_-)\right] + \frac{1}{(p^2+M_{1+}^2)(p^2+M_{1-}^{\;2})}\left[\frac{M_2}{a}(T_{a+} - T_{a-}) + (p^2 + M_2^2)\,J_a\right]$$

$$2V_- = \frac{1}{(p^2+M_{0+}^2)(p^2+M_{0-}^2)}\left[\frac{M_2}{a}\left(T_{+-} - \frac{1}{2}T_{aa}\right) - \left\{p^2+M_2^2 + \frac{1}{a^2}\right\}(J_+ - J_-) + \frac{(p^2+M_{0+}^2)(p^2+M_{0-}^2)}{p^2 + M_2^2}(J_+ + J_-) - \frac{M_0}{a}\,\frac{p^2+M_2^2 + 1/a^2}{p^2+M_0^2}(T_{++} + T_{--}) + \frac{M_0}{a}\,\frac{(p^2+M_{0+}^2)(p^2+M_{0-}^2)}{(p^2+M_0^2)(p^2+M_2^2)}(T_{++} - T_{--})\right]$$

$$2h_{a-} = \frac{-\sqrt{2}\,M_2 p_a}{(p^2+M_{0+}^2)(p^2+M_{0-}^2)}\left[T_{+-} + \frac{1}{a^2(p^2+M_2^2)}\,T_{bb}\right] + \frac{1}{a(p^2+M_{1+}^2)(p^2+M_{1-}^2)}\left[-2M_2\, J_a + \sqrt{2}\,p_a\,(J_+ + J_-)\right]$$

$$+ \frac{\sqrt{2}\, M_0 p_a}{a^2(p^2+M_{1+}^2)\,(p^2+M_{1-}^2)\,(p^2+M_0^2)} \,(T_{++} - T_{--})$$

$$- \frac{M_0}{\sqrt{2}} \; \frac{(p^2+M_2^2+1/a^2)p_a}{(p^2+M_{0+}^2)\,(p^2+M_{0-}^2)\,(p^2+M_0^2)} \,(T_{++} + T_{--})$$

$$+ \left[\left(\frac{-(p^2+M_2^2)}{(p^2+M_{1+}^2)\,(p^2+M_{1-}^2)} + \frac{1}{p^2+M_2^2} \right) \eta_{ab} - \frac{(p^2+M_2^2)+1/a^2)p_a p_b}{(p^2+M_{0+}^2)\,(p^2+M_{0-}^2)\,(p^2+M_2^2)} \right] T_{b+}$$

$$+ \left[\left(\frac{(p^2+M_2^2)}{(p^2+M_{1+}^2)\,(p^2+M_{1-}^2)} + \frac{1}{p^2+M_2^2} \right) \eta_{ab} - \frac{(p^2+M_2^2+1/a^2)p_a p_b}{(p^2+M_{0+}^2)\,(p^2+M_{0-}^2)\,(p^2+M_2^2)} \right] T_{b-} \, .$$

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