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MAXIMAL EXTENDED SUPERGRAVITY THEORY IN SEVEN DIMENSIONS *

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ABSTRACT

A maximal extended supergravity Lagrangian is constructed in seven dimensions, which exhibits USp(4) $\approx$ SO(5) local and SL(5,R) global invariances. We find that the antisymmetric second rank tensor fields must possess a generalized gauge invariance in order that the theory is consistent and supersymmetric. We conjecture that the theory might admit SO(5) × SO(5) Yang-Mills symmetries in d = 7 and SO(4) × SO(5) × SO(5) in d = 4 (with the last SO(5), gauging composite fields) after suitable compactification.

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I. Supergravity theories in higher dimensions are of interest because they can lead to the construction of more complicated four-dimensional theories, the discovery of their hidden symmetries [1] hints for the auxiliary fields [2] and for the breaking of the supersymmetries [3]. Examples studied so far are theories in eleven [4], ten [5),6] and five [7] dimensions. Viewed purely as Kaluza-Klein theories, supergravity theories in higher dimensions have the merit of possessing within them a unique embedding of fermions and scalars [8]. They also offer the possibility of studying varieties of spontaneous compactifications [9),10] (possibly hierarchical), thereby accommodating progressively larger and physically acceptable unifying groups.

In this note we present construction of the maximal extended supergravity Lagrangian in seven dimensions [*]. The content of the theory we work with is derived from the pioneering work of Cremmer and Julia [1),11] who on the basis of their construction of N = 1 supergravity in d = 11, have given the possible content of such maximally extended supergravity theory in d = 7 and conjectured that the theory would possess USp(4) local and SL(5,R) global symmetries [**]. We construct a Lagrangian which actually exhibits these (off-shell) symmetries. We have discovered that the anti-symmetric tensor fields in the theory possess a generalized gauge invariance, consistent with supersymmetry. Of all lower than 11-dimensional theories, the one in d = 7 is unique because only in this theory the count (10) of vector fields matches the count needed to gauge the maximal compact subgroup, SO(5), of the global SL(5,R) symmetry of the Lagrangian. Such gauging might be carried out in 7-dimensions (analogous to the Nicolai-de Wit construction [13] which however exists in 4-dimensions only). In this case the theory may exhibit an overall Yang-Mills SO(5) × SO(5) symmetry in d = 7, with the first SO(5) gauged by the ten vector fields in the theory and the second SO(5) gauged by composites [1] of scalar fields.

The theory contains five second-rank antisymmetric tensor fields. We conjecture also that after a suitable compactification from d = 7 to d = 4 these fields may provide the 15-fold of gauge fields needed for a

*) "Maximal extended", in the sense that, the particle content is read off from the maximal N=1, d=11 theory when reduced to d=7.

**) In this context, one may recall that non-trivial solutions of 11-dimensional field equations are known, which lead to a spontaneous compactification of d = 11, to anti-de Sitter [10)-12] $d = 4 \times S^7$ or anti-de Sitter $d = 7 \times S^4$. Which one of these solutions is energetically the more favourable is not known at present. Even without this motivation, we believe that the richness of the 7-dimensional supergravity structure warrants an independent study of this theory.

further Yang-Mills gauging of a hidden SO(6). Thus in 4-dimensions, the theory may altogether exhibit a Yang-Mills gauging of an SO(5) x SO(6) symmetry plus the gauging of three U(1)'s, corresponding to a total of 28 vector fields in the theory when reduced to $d = 4$. Such gaugings may be important for phenomenological prospects of the theory. The variety of these gaugings and the compactification patterns (including those relevant to Kaluza-Klein) enhance the value of this theory as a theoretical laboratory.

II. One way to construct the $d = 7$ theory is to perform ordinary dimensional reduction of the $d = 11$ theory. This would lead to a fairly complicated theory in 7-dimensions with global SL(4,R) (as a result of $d = 11$ general co-ordinate) and a local SO(4) (as a result of local $d = 11$ Lorentz) invariances. According to Cremmer and Julia's conjecture, these symmetries can be enlarged to global SL(5,R) and local SO(5) by field re-definitions and duality transformations. Guided by this and the structure of the known $d = 4$ and 5, $N = 8$ theories, we start the construction <u>directly</u> in 7-dimensions, thereby avoiding the highly elaborate field re-definitions. Due to the presence of the antisymmetric tensor fields, some terms that do not arise in 4- or 5-dimensions will be present. To deal with them we shall be guided by the results of the ordinary dimensional reduction from 11-dimensions. With these considerations in mind, we assign the fields of $d = 7$ theory to the irreducible representations of global SL(5,R) and local SO(5) (which is isomorphic to USp(4)). The field content and representation assignments are given below.

	V_μ^r	ψ_μ^a	χ^{abc}	$A_\mu^{\alpha\beta}$	$B_{\mu\nu\,\alpha\beta}$	$\mathcal{V}_{\alpha\beta}^{ab}(\phi)$
Global SL(5,R)	1	1	1	10	$\bar{5}$	$\bar{5}$
Local USp(4)	1	4	16	1	1	5

The SL(5,R) indices $\alpha,\beta,\ldots$, and the USp(4) indices $a,b,\ldots$ run from 1 to 4. The 16-spinor fields χ^{abc} have mixed symmetry and they are symplectic traceless

$$\chi^{abc} = \chi^{[ab]c}, \quad \chi^{[abc]} = 0, \quad \chi^{abc}\,\Omega_{ab} = 0 \,. \qquad (1)$$

Here $\Omega_{ab} = \text{antidiag}(\mathbb{1}, -\mathbb{1})$. ψ_μ^a and χ^{abc} are 8-dimensional symplectic

Majorana spinors which obey *)

$$\psi_\mu^a = C\,\bar{\chi}^{Ta}, \quad \chi^{abc} = C\,\bar{\chi}^{Tabc} \quad , \qquad (2)$$

where $\bar{\psi}_\mu^a = (\psi_{\mu a})^\dagger\,\Gamma_0$, and similarly for $\bar{\chi}^{abc}$. The ten vectors and five antisymmetric tensor fields have the following symmetries and reality properties:

$$A_\mu^{\alpha\beta} = A_\mu^{(\alpha\beta)} \qquad (A_\mu^{\alpha\beta})^* = A_\mu^{\alpha'\beta'}\,\Omega_{\alpha'\alpha}\,\Omega_{\beta'\beta} \qquad (3a)$$

$$B_{\mu\nu\,\alpha\beta} = B_{\mu\nu[\alpha\beta]|} \qquad (B_{\mu\nu\,\alpha\beta})^* = -\Omega^{\alpha\alpha'}\Omega^{\beta\beta'}\,B_{\mu\nu\,\alpha'\beta'} \qquad (3b)$$

where ( ) denotes (unit strength) symmetrization and [ ]| denotes (unit strength) sympletic traceless antisymmetrization. The reason for choosing $A_\mu^{\alpha\beta}$ and $B_{\mu\nu\,\alpha\beta}$ to transform as cogradient $\underset{\sim}{10}$ and contragradient $\bar{\underset{\sim}{5}}$, respectively, as well as their opposite pseudo-reality properties is a consequence of the symmetries of the Lagrangian (see discussion after Eq.(8)). $\mathcal{V}_{\alpha\beta}^{ab}$ is SL(5,R) valued 5 x 5 matrix. Its symmetries and reality properties are

$$\mathcal{V}_{\alpha\beta}^{ab} = \mathcal{V}_{[\alpha\beta]|}^{[ab]|} \; ; \; \left(\mathcal{V}_{\alpha\beta}^{ab}\right)^* = \Omega^{\alpha\alpha'}\,\Omega^{\beta\beta'}\mathcal{V}_{\alpha'\beta'\,ab} \qquad (3c)$$

$\mathcal{V}$ is parametrized by 24 scalar fields. Only 14 of them are physical due to the local USp(4) invariance. It is an essential feature (apparently common to all extended supergravity theories) that scalar fields are described by a non-linear σ-model-type Lagrangian. Accordingly, $\mathcal{V}_{\alpha\beta}^{ab}$ transforms under global SL(5,R) from the left and local USp(4) from the right. The infinitesimal action of SL(5,R) can be described as follows:

$$\delta\mathcal{V}_{\alpha\beta}^{\;\;ab} = \left(\Sigma_{\alpha\beta}^{\;\;\gamma\delta} + 2\,\Lambda_{[\alpha}^{\;\;[\gamma}\,\delta_{\beta]}^{\;\;\delta]|}\right)\mathcal{V}_{\gamma\delta}^{\;\;ab} \;, \qquad (4)$$

where $\Sigma_{\alpha\beta\,\gamma\delta} = \Sigma_{\alpha\beta}^{\;\;\gamma'\delta'}\Omega_{\gamma'\gamma}\,\Omega_{\delta'\delta} = \Sigma_{[\alpha\beta]|\,[\gamma\delta]|} = \Sigma_{\gamma\delta\,\alpha\beta}$ are the 14 generators of SL(5,R)/USp(4) coset, and $\Lambda_{\alpha\beta} = \Lambda_\alpha^{\;\;\delta}\,\Omega_{\delta\beta} = \Lambda_{(\alpha\beta)}$ are the ten generators of the USp(4) subgroup of SL(5,R). To verify this, we write (4) as

*) In our convention $\eta_{rs} = (+ - - - - - -)$. We can choose all Γ-matrices to be real except Γ_0 which is imaginary. In this case $C = \mathbb{1}$ and $\Gamma_\mu^T = -\Gamma_\mu$ 14) $(\{\Gamma_\mu, \Gamma_\nu\} = 2\,\eta_{\mu\nu})$.

$$\delta\mathcal{V} = \left(P_\mu^{(ij)}\,\Sigma^{ij} + Q_\mu^{[ij]}\,\Lambda^{ij}\right)\mathcal{V}, \qquad (5)$$

where

$$\Sigma^{ij} = \Gamma^{(i} \otimes \Gamma^{j)} - \frac{1}{5}\delta^{ij}\,\Gamma^k \otimes \Gamma^k \quad , \qquad (6a)$$

$$\Lambda^{ij} = \Gamma^{ij} \otimes \mathbb{1} + \mathbb{1} \otimes \Gamma^{ij} \quad . \qquad (6b)$$

The index i runs from 1 to 5. The Γ matrices are the usual 4×4 Dirac matrices including γ_5: $\Gamma^a = \gamma^a\gamma_5$ $(a = 1,\ldots,4)$, $\Gamma^5 = \gamma_5$ and $\Gamma^{ij} = \Gamma^{[i}\,\Gamma^{j]}$. It is clear from (6) that Σ^{ij} and Λ^{ij} obey the algebra of SL(5,R), Cartan decomposed with respect to its maximal subalgebra USp(4): $[\Sigma,\Sigma] \supset \Lambda$, $[\Sigma,\Lambda] \supset \Sigma$ and $[\Lambda,\Lambda] \supset \Lambda$.

In order to describe the kinetic term for the scalars and furthermore to construct a connection for local USp(4), we consider the derivative of the scalar matrix $\mathcal{V}^{ab}_{\alpha\beta}$. Since $\mathcal{V}^{-1}\,\partial_\mu\mathcal{V}$ lies in the algebra of SL(5,R) it has the following decomposition:

$$\mathcal{V}^{\alpha\beta}{}_{ab}\,\partial_\mu\,\mathcal{V}_{\alpha\beta}{}^{cd} = P_{\mu\,ab}{}^{cd} + 2Q_{\mu[a}{}^{[c}\,\delta_{b]}{}^{d]} \quad , \qquad (7)$$

P_μ lies in the SL(5,R)/USp(4) coset, and transforms homogeneously under local USp(4); it can therefore be used in the construction of the scalar kinetic term. $Q_{\mu\,ab}$ lies in USp(4), and it transforms inhomogeneously under local USp(4); it can be used as a USp(4) connection. P_μ and Q_μ satisfy identities which have been used in the construction of the Lagrangian. All these aspects are similar to those encountered already in the construction of the E(6) global $\otimes$ USp(8) local theory in five dimensions and we shall skip details [15]. Before we write the Lagrangian we note the following points:

1) Besides supersymmetry, general co-ordinate, local Lorentz and SL(5,R) global $\otimes$ USp(4) local invariances, the theory is expected to have local U(1) invariances associated with the vector fields and the anti-symmetric tensor fields. These latter deserve special attention due to the fact that the coupling of the antisymmetric tensor fields to other fields may present consistency problems [16]. To solve these problems, we generalize the work of Scherk and Schwarz [3], who have shown that in a dimensional reduction from d = 11 to d = 7, there would arise the following field strengths:

$$G_{\mu\nu\rho\alpha} = \partial_\mu B_{\nu\rho\alpha} + 2\,F_{\mu\nu}{}^{\beta}\,B_{\rho\alpha\beta} + 2 \text{ perms} \quad , \qquad (8a)$$

$$G_{\mu\nu\rho\sigma} = 4\,\partial_{[\mu}\,B_{\nu\rho\sigma]} + 12\,F_{[\mu\nu}{}^{\alpha}\,B_{\rho\sigma]\alpha} \quad . \qquad (8b)$$

Here $B_{\mu\nu\alpha}$ and $B_{\mu\alpha\beta} = B_{\mu[\alpha\beta]}$ are related to the well-known third rank anti-symmetric tensor field of the d = 11 theory while $F_{\mu\nu}{}^{\alpha} = (\partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha)$, where A_μ^α is essentially the off-diagonal component of the d = 11 metric. These field strengths are invariant under a set of generalized antisymmetric gauge transformations. These transformations result from the reduction (to seven dimensions) of the simple d = 11 law: $\delta\hat{B}_{\hat\mu\hat\nu\hat\rho} = \partial_{\hat\mu}\,\hat\Lambda_{\hat\nu\hat\rho} + 2$ perms ($\hat\mu = 1,\ldots,11$) and are given by

$$\delta B_{\mu\alpha\beta} = \partial_\mu\Lambda_{\alpha\beta} \qquad (9a)$$

$$\delta B_{\mu\nu\alpha} = \partial_\mu\Lambda_{\nu\alpha} - \partial_\nu\Lambda_{\mu\alpha} + 2\,F_{\mu\nu}{}^{\beta}\,\Lambda_{\beta\alpha} \quad , \qquad (9b)$$

$$\delta B_{\mu\nu\rho} = \partial_\mu\Lambda_{\nu\rho} + 2F_{\mu\nu}{}^{\alpha}\Lambda_{\alpha\rho} + 2 \text{ perms} \quad . \qquad (9c)$$

To supplement this, we note that under the general co-ordinate transformations in the internal four directions, A_μ^α transform as:

$$\delta A_\mu^\alpha = \frac{1}{2}\,\partial_\mu\,\xi^\alpha \quad . \qquad (9d)$$

Eqs.(8a) and (8b) suggest the following ansatz:

$$G_{\mu\nu\rho[\alpha\beta]|} = \partial_\mu B_{\nu\rho[\alpha\beta]|} - i\,F_{\mu\nu[\alpha}{}^{\delta}\,A_{\rho\;\beta]|\,\delta} + 2 \text{ perms} \quad , \qquad (10)$$

with the transformation laws (9a) to (9d) combining as:

$$\delta A_\mu^{(\alpha\beta)} = \partial_\mu\,\Lambda^{(\alpha\beta)} \quad , \qquad (11a)$$

$$\delta B_{\mu\nu\,[\alpha\beta]|} = \partial_\mu\,\Lambda_{\nu[\alpha\beta]|} - \partial_\nu\Lambda_{\mu[\alpha\beta]|} + i\,F_{\mu\nu[\alpha}{}^{\delta}\,\Lambda_{\beta]|\delta} \quad . \qquad (11b)$$

The conjectured form of the FA term in (10) will be justified by the requirement of supersymmetry of the Lagrangian *) (see later), which also

*) A similar FA term was found in the construction of Maxwell-Einstein supergravity theory in d = 10. [6]

specifies the coefficient in front of the FA term to be "-i". (This "-i" in turn dictates the pseudo-reality (3b) of $B_{\mu\nu}$, from the hermicity requirements on the Lagrangian.)

2) The contraction, as well as the raising and lowering of indices in the FA and FA terms of (10) and (11) are through $\Omega_{\alpha\beta}$. No scalar matrix $\mathcal{V}^{ab}_{\alpha\beta}$ is needed in the FA and FA terms because $\underset{\sim}{10} \otimes \underset{\sim}{10}$ of SL(5,R) already contains a $\underset{\sim}{\bar{5}}$ (i.e. $G_{[\alpha\beta]|}$ on the left-hand side). Actually, it is essential that the scalar matrix $\mathcal{V}$ does not appear in (10) and (11), otherwise $G_{\mu\nu\rho[\alpha\beta]|}$ would not be invariant under (11). This also explains our choice of assigning $B_{\mu\nu}$ and A_μ to $\underset{\sim}{\bar{5}}$ and $\underset{\sim}{10}$ of SL(5,R).

3) We can now write all the kinetic as well as Pauli and Yukawa terms. In addition to these terms there is need from supersymmetry, of bosonic cubic terms which may be traced to the well-known $\epsilon^{\hat\mu_1\ldots\hat\mu_{11}} \hat F_{\hat\mu_1\ldots\hat\mu_4} \hat F_{\hat\mu_5\ldots\hat\mu_8} \hat A_{\hat\mu_9\ldots\hat\mu_{11}}$ coupling of the $d = 11$ theory. From dimensional reduction it is seen that a term of the form: $\epsilon^{\mu_1\ldots\mu_7} G_{\mu_1\ldots\mu_3} G_{\mu_4\ldots\mu_6} A_{\mu_7}$ could arise. However, this alone will not be invariant under (11) and we find that one must add a term like $\epsilon^{\cdots}$ GFB with an appropriate coefficient, for invariance.

III. The final expressions for the $d = 7$ Lagrangian (up to the quartic fermion terms) and the supersymmetry transformations (up to quadratic fermion terms) are as follows ($\kappa = 1$):

$$
\begin{aligned}
V^{-1}\mathcal{L} = &-\tfrac{1}{4} R - \tfrac{1}{8}\,\mathcal{U}^{ab}_{\alpha\beta}\,\mathcal{U}_{\gamma\delta,ab}\,F^{\alpha\beta}_{\mu\nu} F^{\mu\nu,\gamma\delta} - \tfrac{1}{12}\,\mathcal{V}^{\alpha\beta\ ab}\,\mathcal{V}^{\gamma\delta}_{ab}\,G_{\mu\nu\rho\alpha\beta}\,G^{\mu\nu\rho}_{\gamma\delta} \\
&- \tfrac{1}{4}(D_\mu \mathcal{V}^{ab}_{\alpha\beta})(D^\mu \mathcal{V}^{\alpha\beta}{}_{ab}) - \tfrac{i}{2}\bar\psi^a_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_{\rho a} + \tfrac{i}{8}\bar\chi^{abc}\Gamma^\mu D_\mu \chi_{abc} \\
&- \frac{iV^{-1}}{90\sqrt2}\,\epsilon^{\mu\nu\rho\sigma\lambda\tau\kappa}\,\Omega^{\gamma\delta}\left[G_{\mu\nu\rho\alpha\gamma}\,G_{\lambda\tau\sigma\ \delta\beta}\,A^{\alpha\beta}_\kappa - 6\,G_{\mu\nu\rho\alpha\gamma}\,B_{\lambda\tau\delta\beta}\,F^{\alpha\beta}_{\sigma\kappa}\right] \\
&- \tfrac{1}{2} P^{abcd}_\mu\,\bar\psi_{\nu d}\,\Gamma^\mu\Gamma^\nu\chi_{abc} - \frac{1}{4\sqrt2}\,\mathcal{U}^{ab}_{\alpha\beta} F^{\alpha\beta}_{\mu\nu}\Big[\bar\psi^\lambda_a\,\Gamma_{[\lambda}\Gamma^{\mu\nu}\Gamma_{\tau]}\,\psi^\tau_b \\
&- i\,\bar\psi^c_\lambda\,\Gamma^{\mu\nu}\Gamma^\lambda\chi_{bca} + \tfrac{1}{2}\bar\chi_a{}^{dc}\,\Gamma^{\mu\nu}\chi_{bcd}\Big] \\
&+ \tfrac{1}{12}\,G_{\mu\nu\rho\alpha\beta}\,\mathcal{V}^{\alpha\beta\ ab}\Big[\bar\psi^\lambda_a\,\Gamma_{[\lambda}\Gamma^{\mu\nu\rho}\Gamma_{\tau]}\,\psi^\tau_b + \tfrac{i}{2}\,\bar\psi^c_\lambda\,\Gamma^{\mu\nu\rho}\Gamma^\lambda\chi_{abc} \\
&+ \tfrac{1}{4}\,\bar\chi^{cd}{}_a\,\Gamma^{\mu\nu\rho}\chi_{cdb}\Big] \quad . \qquad (12)
\end{aligned}
$$

The action is invariant under the following supersymmetry transformations laws:

$$
\begin{aligned}
\delta V^r_\mu &= -i\,\bar\epsilon^a\gamma^r\psi_{\mu a} \\
\mathcal{V}^{\alpha\beta}{}_{cd}\,\delta\mathcal{V}_{\alpha\beta\ ab} &= -\tfrac{1}{2}\left[\bar\chi_{abc}\,\epsilon_d - \tfrac14\,\Omega_{cd}\,\chi_{abe}\,\epsilon^e + ab \leftrightarrow cd\right] \\
\mathcal{U}^{ab}_{\alpha\beta}\,\delta A^{\alpha\beta}_\mu &= -\sqrt2\left[\bar\epsilon^{(a}\psi_\mu{}^{b)} - \tfrac{i}{2}\,\bar\epsilon^c\,\Gamma_\mu\,\chi_c{}^{(ab)}\right] \\
\mathcal{V}^{\alpha\beta\ ab}\,\delta B_{\mu\nu\ \alpha\beta} &= -2i\,\mathcal{U}^{c[a}_{\alpha\beta}\,A^{b]|}_{[\mu c}\ \delta A^{\alpha\beta}_{\nu]} \\
&\quad + 2\left[\bar\epsilon^{[a}\Gamma_{[\mu}\psi^{b]|}_{\nu]} + \tfrac{i}{8}\,\bar\epsilon^c\,\Gamma_{\mu\nu}\,\chi^{ab}{}_c\right] \\
\delta\psi_{\mu a} &= D_\mu\epsilon_a - \frac{i}{10\sqrt2}\,F_{\rho\sigma\ ab}\left(\Gamma^{\rho\sigma}{}_\mu + 8\,\Gamma^\rho\,\delta^\sigma_\mu\right)\epsilon^b \\
&\quad + \tfrac{i}{15}\,G_{\rho\sigma\tau\ ab}\left(\Gamma^{\rho\sigma\tau}{}_\mu + \tfrac{9}{2}\,\Gamma^{\rho\sigma}\,\delta^\tau_\mu\right)\epsilon^b \\
\delta\chi_{abc} &= 2i\,P_{\mu abcd}\,\gamma^\mu\epsilon^d + \frac{1}{2\sqrt2}\left(F_{bc}\,\epsilon_a - F_{ac}\,\epsilon_b\right) \\
&\quad + \frac{1}{5\sqrt2}\left[\Omega_{ab}\,F_{cd} - \tfrac12\,\Omega_{bc}\,F_{ad} - \tfrac12\,\Omega_{ca}\,F_{bd}\right]\epsilon^d \\
&\quad + \tfrac{1}{9}\left[G_{ab}\,\epsilon_c - \tfrac12\,G_{bc}\,\epsilon_a - \tfrac12\,G_{ca}\,\epsilon_b\right] \\
&\quad + \tfrac{1}{45}\left[\Omega_{ab}\,G_{cd} - \tfrac12\,\Omega_{bc}\,G_{ad} - \tfrac12\,\Omega_{ca}\,G_{bd}\right]\epsilon^d \quad . \qquad (13)
\end{aligned}
$$

In (13), $F_{ab} \equiv \Gamma^{\mu\nu} F_{\mu\nu}{}^{\alpha\beta}\,\mathcal{U}_{\alpha\beta\ ab}$, $G_{ab} \equiv G_{\mu\nu\rho\ \alpha\beta}\,\mathcal{V}^{\alpha\beta}_{ab}$ and $\Gamma^{\mu_1\ldots\mu_n} = \Gamma^{[\mu_1}\Gamma^{\mu_2}\ldots\Gamma^{\mu_n]}$ with unit normalization. D_μ is Lorentz and $\mathcal{U}$Sp(4) covariant derivative. For instance, $D_\mu\epsilon_a = \left[\left(\partial_\mu + \tfrac14\,\omega_{\mu rs}\,\Gamma^{rs}\right)\delta^b_a + Q_{\mu a}{}^b\right]\epsilon_b$. $\mathcal{U}^{ab}_{\alpha\beta} = \mathcal{U}^{(ab)}_{(\alpha\beta)}$ is a 10 x 10 matrix which is made of a product of two $\mathcal{V}$'s

$$\mathcal{U}^{ab}_{\alpha\beta} = \mathcal{V}^{ac}_{\alpha\gamma}\,\mathcal{V}^{db}_{\delta\beta}\,\Omega_{cd}\,\Omega^{\gamma\delta} \quad . \qquad (14)$$

Clearly $\mathcal{U}$ transforms as $\overline{\underset{\sim}{10}}$ under SL(5,R) since $\mathcal{V}$ transforms as $\underset{\sim}{5}$. The covariant derivative of $\mathcal{U}$ satisfies the following relation *):

$$D_\mu \mathcal{U}^{ab}_{\alpha\beta} = 2\, P^{bcda}_\mu \,\mathcal{U}_{\alpha\beta\, cd} \qquad (15)$$

This relation was crucial in proving supersymmetry. We now wish to make the following comments on (12) and (13):

1) $\mathcal{U}$ matrices in the F^2 term, and $\mathcal{V}$ matrices in the G^2 term are needed for SL(5,R) invariance **) ($\underset{\sim}{10} \otimes \underset{\sim}{10} \not\supset \underset{\sim}{1}$ and $\underset{\sim}{5} \otimes \underset{\sim}{5} \not\supset \underset{\sim}{1}$). In the G^2A and GBF terms there are no scalars and the contractions are with $\Omega_{\alpha\beta}$, because $\underset{\sim}{5} \otimes \underset{\sim}{\bar{5}} \otimes \underset{\sim}{10} \supset \underset{\sim}{1}$.

2) We emphasize that our Lagrangian is invariant under the generalized antisymmetric gauge transformations given in (11) (actually, G and F are separately invariant under (11)) which also determines the relative coefficient between the GGA and GBF terms. It is due to this gauge invariance that the supersymmetric variation of the cubic bosonic terms is greatly simplified.

3) The first term in $\mathcal{V}\delta B$ in (13) is noteworthy. It is determined by requiring that the variation of $G_{\mu\nu\rho\;\alpha\beta}$ does not contain $\partial_\mu [(\delta A_\nu) A_\rho]$. Such terms could not have been cancelled without losing supersymmetry. It is amusing that in Maxwell-Einstein supergravity theory in d = 10 similar terms have been discovered [6].

4) $\mathcal{V}^{-1}\delta\,\mathcal{V}$ lies in the SL(5,R)/USp(4) coset. This is similar to the situation encountered in d = 4 and 5 theories. It implies that Q_μ is supercovariant (i.e. its supersymmetric variation does not contain $\partial_\mu \epsilon$).

5) Comparing our Lagrangian with the 4- and 5-dimensional ones, we see that there are other similarities. For instance in the Pauli and Yukawa terms (and also in the transformation laws) similar combinations of Γ matrices occur. ***)

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*) In proving this, we have used (7) (which can be read as $D_\mu \mathcal{V}^{ab}_{\alpha\beta} = P^{abcd}_\mu \mathcal{V}_{\alpha\beta cd}$) and the fact that a third rank totally anti-symmetric, symplectic traceless USp(4) tensor identically vanishes (Schouten identity).

**) The action of SL(5,R) on $F^{\alpha\beta}_{\mu\nu}$ can be described by defining $F^{\alpha\beta}_{\mu\nu} = F^{[\alpha\gamma]|\;[\delta\beta]|}_{\mu\nu}\,\Omega_{\gamma\delta} = -F^{[\delta\beta]|\;[\alpha\gamma]|}_{\mu\nu}\,\Omega_{\gamma\delta}$ and using the transformation law which is contradgradient to that of Eq.(4). Note also that $\mathcal{U}^{ab}_{\alpha\beta} F^{\alpha\beta}_{\mu\nu} = \frac{1}{2} \mathcal{V}^{c(a}_{\alpha\gamma} \mathcal{V}^{\;b)}_{\delta\beta c} F^{\alpha\gamma\;\delta\beta}_{\mu\nu}$ due to the identity $F^{\alpha\beta\;\gamma\delta}_{\mu\nu} = \frac{1}{2}\Omega^{\beta\gamma} F^{\alpha\delta}_{\mu\nu} + \frac{1}{2}\Omega^{\alpha\delta} F^{\beta\gamma}_{\mu\nu} - (\alpha \longleftrightarrow \beta)$.

***) Some of these features can presumably be explained by the group manifold approach [17]. Note also that $\bar{\chi}_a{}^{cd} G^{ab} \chi_{bcd} = 0$ (this can be proved by using the Schouten identity), while $\chi_a{}^{dc} F^{ab} \chi_{bcd} = \chi_a{}^{cd} F^{ab} \chi_{bcd} - \frac{1}{2} \chi^{cd}{}_a F^{ab} \chi_{cdb}$.

6) The quartic terms in the Lagrangian and quadratic fermion terms in the transformation laws can, as usual, be constructed by requiring the closure of the supersymmetry transformation laws and supercovariance of the fermion field equations. One standard step [6] in this direction is to supercovariantize the spin connection ω_μ^{rs}, the field strengths $F^{\alpha\beta}_{\mu\nu}$, $G_{\mu\nu\rho\;\alpha\beta}$, and $P_{\mu abcd}$. We leave this problem for the future.

To conclude, in this note we have completed the construction of the ungauged maximal extended supergravity theory in d = 7 (up to quartic fermion terms). The problem of Yang-Mills gauging of this theory (again in d = 7) requires further study.

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