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SUPERSYMMETRIC FIELD THEORY OF MONOPOLES *

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ABSTRACT

We supersymmetrize Zwanziger's local field theory of monopoles and charges.

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Zwanziger's [1] renormalizable [2] Lagrangian field theory of charges and monopoles (even if its utility for practical calculation is limited) represents perhaps the most accessible field-theoretic construct for a general discussion of the properties of Dirac magnetic poles of spin zero or of spin 1/2.

The theory starts with two gauge fields A_μ and A'_μ, with kinetic terms made up from field tensors $F_{\mu\nu}$ and $F'_{\mu\nu}$. A gauge-invariant term containing a string parameter n_μ is then introduced, which mixes A_μ with A'_μ. This has the remarkable effect of reducing the four degrees of freedom corresponding to the two gauge fields to just two degrees of freedom corresponding to one photon. The Lagrangian reads:

$$\mathcal{L}_Z = \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_1 = -\frac{1}{8} F_{\mu\nu} F_{\mu\nu} - \frac{1}{8} F'_{\mu\nu} F'_{\mu\nu}$$

$$\mathcal{L}_2 = \frac{1}{4} n_\mu (F_{\mu\nu} + \tilde{F}'_{\mu\nu}) n_\rho (F_{\rho\nu} + \tilde{F}'_{\rho\nu}) + \frac{1}{4} n_\mu (\tilde{F}_{\mu\nu} - F'_{\mu\nu}) n_\rho (\tilde{F}_{\rho\nu} - F'_{\rho\nu}) .$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$ and n_μ is a fixed four-vector, remnant of the Dirac string.

This is at the level of free fields. The interaction with matter is introduced in the conventional manner, the field A_μ interacting with electric charge (g) and A'_μ with magnetic charge (h). Brandt, Neri and Zwanziger [3] then show that the full Green functions of the gauge invariant operators (specifically the electric and magnetic currents) are n_μ-independent, provided Dirac's quantization condition is satisfied, i.e.

$$\frac{gh}{4\pi} = \frac{j}{2} ; \qquad j \text{ integer } (0, \pm 1, \pm 2, \ldots) .$$

In this note we wish to supersymmetrize this Lagrangian. Supersymmetrization will introduce two gauge Majorana fermions, i.e. ("photinos") λ, λ' which, on account of their being supersymmetric partners of the Zwanziger photon pair A_μ, A'_μ, should describe just two physical states.

Our immediate motivation for supersymmetrization stems from a recently proposed theory of preons [4]. In its simpler version, this theory postulates that there are three types of preons; f = (g,0), C = (0,h) and S = (-g,-h) where g and h signify (analogue) "electric" and "magnetic" types of charges. Quarks and leptons are considered as (fCS) <u>neutral</u> composites of these preons. The C-preons correspond to a <u>4</u> representation of the colour group $SU_C(4)$, the f's are (2,1) + (1,2) of $SU_L(2) \times SU_R(2)$ and S is a singlet of these internal symmetry groups. The full symmetry of the theory is $[SU_C(4) \times SU_L(2) \times SU_R(2)] \times U(1) \times U(1)$. Here $U(1) \times U(1)$ give the (analogue) "electric" and "magnetic" Zwanziger-type forces which bind the preons together into quarks, leptons, Higgs and gauge particles of $SU(4) \times SU(2) \times SU(2)$.

Without supersymmetry, either all preons are spin 1/2 or just one of them, the other being spinless. Noticing that chiral supersymmetric fields possess the remarkable group property that a product of any number of (left/right chiral) superfields is again a (left/right chiral) superfield, it was suggested [5] that preons should be represented by supersymmetric scalar chiral fields. Thus the quark and lepton composites are also scalar chiral superfields, the supersymmetry guaranteeing that a simple product of such fields does not generate higher spin composites. It was further assumed that supersymmetry is broken at energies ≈ 100 TeV. This is intermediate to the characteristic electroweak energy scale of around 1 TeV, and the dissociation energy of the quark and leptonic composites.

Assuming thus that there do exist chiral superfields representing preons we wish to construct a supersymmetric theory which can describe their $U(1) \times U(1)$ analogue "electric" and "magnetic" type of binding.

Consider free fields first. The construction of a supersymmetric extension of Zwanziger's Lagrangian proceeds by introducing the following chiral spinor superfield [6]:

$$\Psi_a = - \frac{i}{2\sqrt{2}} \bar{D}_+ D_- \left(e^{-2V} D_{a+} e^{2V}\right) ,$$

where V is a general real superfield containing the gauge field A_μ, while $D = \left(\frac{\partial}{\partial\bar\theta} - \frac{i}{2} \not\partial\theta \right)$. In the Wess-Zumino gauge in which V takes the form

$$V = \frac{1}{4} \bar\theta i \gamma_\nu \gamma_5 \theta A_\nu + \frac{1}{2\sqrt{2}} \bar\theta\theta\bar\theta\gamma_5\lambda + \frac{1}{16} (\bar\theta\theta)^2 \partial^2 \phi ,$$

Ψ becomes

$$\Psi = \exp\left(- \frac{1}{4} \bar\theta \not\partial \gamma_5 \theta\right) \left[\lambda_+ + \frac{i}{\sqrt{2}} (\partial^2\phi + \frac{1}{2} \sigma_{\mu\nu} F_{\mu\nu}) \theta_+ + \frac{1}{2} \bar\theta_- \theta_+ (-i\not\partial\lambda_-)\right]$$

(the conventional notation for $\partial^2\phi$ is D). We also introduce a primed chiral spinor superfield Ψ' with corresponding components.

The supersymmetric extension of Zwanziger's Lagrangian is given by

$$- \frac{1}{8} \left[\bar\Psi \not\partial \not n \not\partial \left(i \frac{n\cdot\partial}{\partial^4}\right) \Psi + \bar\Psi' \not\partial \not n \not\partial \left(i \frac{n\cdot\partial}{\partial^4}\right) \Psi' + \bar\Psi \not\partial \not n \not\partial \left(\frac{n\cdot\partial}{\partial^4}\right) \Psi' - \bar\Psi' \not\partial \not n \not\partial \left(\frac{n\cdot\partial}{\partial^4}\right) \Psi\right]_D ,$$

where D signifies the coefficient of $[\frac{1}{32} (\bar\theta\theta)^2]$.

First, examine its bosonic sector. Given that the gauge fields enter the Lagrangian through gauge invariant combinations, the resulting theory is gauge invariant. In order to fix the gauge we choose the gauge fixing condition $\partial\cdot A = \partial\cdot A' = 0$ or its supersymmetric version $\bar D_- D_+ \Psi = \bar D_- D_+ \Psi' = 0$. Using the transversality of A_μ and A'_μ the bosonic sector reduces to the local form:

$$\frac{1}{2} \Big[(A_\mu (n\cdot\partial)^2 A_\mu + A'_\mu (n\cdot\partial)^2 A'_\mu - \epsilon_{\sigma\rho\mu\tau} A_\rho n_\mu (n\cdot\partial) \partial_\sigma A'_\tau +$$

$$+ \epsilon_{\sigma\rho\mu\tau} A'_\rho n_\mu (n\cdot\partial) \partial_\sigma A_\tau +$$

$$+ \phi (n\cdot\partial)^2 \partial^2 \phi + \phi' (n\cdot\partial)^2 \partial^2 \phi' - 2\phi (n\cdot\partial) \partial^2 (n\cdot A') + 2\phi' (n\cdot\partial) \partial^2 (n\cdot A) \Big] .$$

For the free field case, the ϕ and ϕ' are easily eliminated using their equations of motion. We obtain the Zwanziger Lagrangian (up to surface terms) in the equivalent form:

$$\mathcal{L}_Z = - \frac{1}{2} n_\mu F_{\mu\rho} n_\nu F_{\nu\rho} - \frac{1}{2} n_\mu F'_{\mu\rho} n_\nu F'_{\nu\rho} - \frac{1}{2} n_\mu F_{\mu\rho} n_\nu \tilde{F}'_{\nu\rho} + \frac{1}{2} n_\mu F'_{\mu\rho} n_\nu \tilde{F}_{\nu\rho} .$$

In momentum space the various propagators are:

$$(AA)_{\mu\nu} = (A'A')_{\mu\nu} = - \frac{g_{\mu\nu}}{K^2} + \frac{K_\mu K_\nu}{K^4}$$

$$(AA')_{\mu\nu} = -(A'A)_{\mu\nu} = - \frac{\epsilon_{\mu\nu\rho\sigma} n_\rho K_\sigma}{K^2 (n\cdot K)}$$

$$(\phi\phi) = (\phi'\phi') = \frac{1}{K^4}$$

$$(\phi A')_\nu = - (\phi' A)_\nu = -i \frac{n\cdot K}{K^4} \left[n_\nu - \frac{n\cdot K}{K^2} K_\nu\right]$$

The $(\phi\phi)$, $(\phi'\phi')$, $(\phi A')$ and $(\phi' A)$ propagators are a new feature, arising on account of supersymmetry. (The $(AA)_{\mu\nu}$ and $(A'A')_{\mu\nu}$ propagators differ from Zwanziger's in the gauge dependent terms on account of the differing gauge choice.)

The Lagrangian for the "photinos" reads:

$$\bar\lambda_- \frac{(n\cdot\partial)^2}{\partial^2} i\not\partial\lambda_- + \bar\lambda'_- \frac{(n\cdot\partial)^2}{\partial^2} i\not\partial\lambda'_- + \bar\lambda'_- \frac{n\cdot\partial}{\partial^2}[(n\cdot\partial)\not\partial-\partial^2\not n]\lambda_- - \bar\lambda_- \frac{n\cdot\partial}{\partial^2}[(n\cdot\partial)\not\partial-\partial^2\not n]\lambda'_- \cdot$$

The propagator matrix it leads to is given by:

$$\begin{bmatrix} \lambda_-\bar\lambda_- & \lambda_-\bar\lambda'_- \\ \lambda'_-\bar\lambda_- & \lambda'_-\bar\lambda'_- \end{bmatrix} = \begin{bmatrix} 1 & i - \frac{i\not n\not P}{n\cdot P} \\ -i + \frac{i\not n\not P}{n\cdot P} & 1 \end{bmatrix} \frac{1}{\not P} \ .$$

There is just one pole at $P^2 = 0$, which has as residue an 8×8 matrix in Dirac and λ,λ' space. This has rank two, which guarantees that the (λ,λ') pair of fermions describes only two states, as it should from supersymmetry.

We now turn to the interacting fields and couple matter to the "electromagnetic" field. This coupling is conventional through the superfields V and V', and presents no new problems. Thus if ϕ_+, ϕ_- are "electrically" charged chiral superfields and ϕ'_+, ϕ'_- magnetically charged chiral superfields, the matter field coupling takes the form

$$\frac{1}{4}\left[\phi_+^* e^{2V}\phi_+ + \phi_-^* e^{-2V}\phi_- + \phi_+'^* e^{2V'}\phi'_+ + \phi_-'^* e^{-2V'}\phi'_-\right]_D \ .$$

(For dyons, $\phi_+ = \phi'_+$, $\phi_- = \phi'_-$ and the coupling is

$$\frac{1}{4}\left[|\phi_+|^2 e^{2V+2V'} + |\phi_-|^2 e^{-2V-2V'}\right]_D \ . \Big)$$

We expect the final theory to be Lorentz invariant when the Dirac condition is satisfied in the same sense as Zwanziger's, since it represents a mere (?) supersymmetrization of Zwanziger's theory.

It is possible to break the U(1) gauge invariance of the theory, with the "photon" field acquiring a mass, by giving a vacuum expectation value to the scalar monopole. Supersymmetry is not broken but in this manner one may simulate a Meissner-type of Nielsen-Olesen screening of magnetic and electric preons.

It should be emphasized that supersymmetrization has dynamical consequences. The ultraviolet divergences become less severe and in particular mass counterterms are finite and calculable (presumably to each supersymmetric order). From this point of view our theory resembles the solitonic non-Abelian monopole and may be considered as its field-theoretic representation, in the energy region below its (dynamically generated) mass, where the non-Abelian monopole can be considered as an elementary entity.

Since freedom from anomalies as well as electric and magnetic charge conservation require that left as well as right chiral fields $\phi_+(\phi'_+)$ and $\phi_-(\phi'_-)$ occur in the theory, one is tempted to consider extension of the theory to $N = 2$ supersymmetry of which the pair ϕ_+, ϕ_- (as well as ϕ'_+,ϕ'_-) constitutes a multiplet [7].

For a conventional U(1) theory, such an extension is carried through in the following manner. In terms of $N = 1$ supermultiplets, the $N = 2$ "gauge" multiplet is made up of the real non-chiral superfield V together with a chiral field S_+ which couples with the matter pair (ϕ_+,ϕ_-) through an F-type coupling $g(\phi_+ S_+ \phi_-^*)_F$ of magnitude *) g. The fermion ζ_+ contained inside S_+ together with the gauge fermion λ_- then form a Dirac 4-component fermion, characteristic of $N = 2$ supersymmetry. According to Ref.9 the resulting off-shell theory exhibits a global SU(2) invariance.

We have succeeded in writing a supersymmetric Lagrangian for S_+ and S'_+ fields which guarantees that ζ_+, ζ'_+ (contained inside S_+ and S'_+) do indeed join λ_- and λ'_- to form appropriate Dirac fermions. **) However the free Lagrangian does not appear to exhibit the off shell SU(2) of Ref.9. We therefore do not know if this is the appropriate $N = 2$ extension.

Our interest in extended supersymmetries stems from the dilemma posed by the Dirac relation: is it $\frac{gh}{4\pi} = \frac{1}{2}$ or is $\frac{g_R h_R}{4\pi} = \frac{1}{2}$ (where g_R and h_R are the renormalized charges)? Certainly in Zwanziger's theory g and h renormalize in the same manner [2], i.e. $g_R = Z^{1/2}g$; $h_R = Z^{1/2}h$. We hope that extended supersymmetrization of Zwanziger's theory may lead [8] to $Z = 1$.

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*) $N = 2$ supersymmetric theories have the merit of making the wave function renormalization for matter fields finite [8]. The argument for this is simple. The non-renormalization of the F coupling of S_+, i.e. $(\phi_+ S_+ \phi_-^*)_F$ implies $g_R = Z_S^{1/2} Z_\phi g_0$. Also from Ward identities $g_R = Z_V^{1/2} g_0$ implying $Z_S^{1/2} Z_V^{-1/2} Z_\phi = 1$. However V and S_+ belong to the same $N = 2$ multiplet, i.e. $Z_V = Z_S$. Thus $Z_\phi = 1$.

**) A necessary condition for the existence of such a theory appears to be that charged matter must be dyonic in character with equal electric and magnetic type charges.