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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ON THE MONTONEN-OLIVE CONJECTURE

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It has been conjectured by Montonen and Olive[1] that gauge theories might possess dual forms analogous to the Kramers-Wannier dual lattice theories[2] or the Sine-Gordon-Thirring duality. According to this conjecture a spontaneously broken gauge theory gives rise to magnetic monopole states which fill the adjoint representation of a dual group. In the dual theory these monopoles are associated with elementary vector fields; there is a spontaneous breakdown of the dual symmetry and solitonic states appear which correspond to the elementary gauge particles of the original theory. A U(1) gauge particle, the photon, appears as an elementary particle on both sides.

A partial realization of this conjecture has been constructed by Freund[3]. From an $O(3)_{el.} \times O(3)_{mag.}$ gauge theory with Higgs fields in the adjoint representation Freund projected two invariant field strength tensors which could then be treated like the $F_{\mu\nu}$ and $F'_{\mu\nu}$ of Zwanziger[4]. That is, the electric $F_{\mu\nu}$ and magnetic $F'_{\mu\nu}$ were combined in such a way as to share a single photon. Unlike the U(1) x U(1) model of Zwanziger, however, the $F_{\mu\nu}$ and $F'_{\mu\nu}$ of Freund are non-linear in the elementary fields of the system. This means not only that the model is un-renormalizable (as Freund points out), but also that the covariance proof of Brandt, Neri and Zwanziger[5] is not applicable. The non-covariant n_μ-dependence of Freund's Lagrangian appears in the vertices as well as the propagator whereas B.N.Z. had to contend only with non-covariant photon propagators.

Apart from the covariance problem, the Freund model does not exhibit the duality envisaged by Montonen and Olive. Thus, apart from the common photon, the electric- and magnetic-type gauge fields of O(3) x O(3) both appear as elementary fields in the same Lagrangian, i.e. on the same footing. The M-O conjecture was that two distinct Lagrangians, one electric and one magnetic, should describe the same physical system.

It might be possible to generalize the Freund type of model in such a way that all the gauge particles are shared, like the photon. The gauge fields of the electric-magnetic group G x G are each paired in the Zwanziger fashion so that each pair accommodates only one quantum. This might be brought about by a condensation of the Higgs fields. For example, in the effective term,

$$F^{\alpha}_{\mu\nu} F^{\beta'}_{\mu\rho} \nabla_\nu \phi^\alpha \nabla_\rho \phi^{\beta'} \ ,$$

we might suppose that

$$\nabla_\nu \phi^\alpha \nabla_\rho \phi^{\beta'} \simeq \langle \nabla_\nu \phi^\alpha \nabla_\rho \phi^{\beta'} \rangle \simeq n_\nu n_\rho \delta^{\alpha\beta'} \ .$$

However, quite apart from the difficulty of justifying such a particular condensation, this kind of model suffers the same renormalizability and covariance problems as does Freund's.

More generally, one might conjecture that the dual gauge multiplet of Montonen and Olive is to be identified with the original multiplet. The model would be "self-dual". If this could be realized then the gauge particles would carry both electric and magnetic charges. They would be dyons - or else neutral. Presumably all the charged matter in such a model would have to be dyonic.

A conceivable motivation for the self-dual type of model is the charge distortion caused by CP-violation. In CP-violating theories the electric charge of a particle receives a contribution proportional to its magnetic charge*. All monopoles are then necessarily dyons. The

* Charge quantization in units of $e\hbar$ as given by gauge theories refers to the charge as eigenvalues of a number operator. A second definition of charge, using Gauss' theorem relates it to the long range behaviour of the electric field. It was observed by Witten[6] that the two definitions can disagree in CP-violating theories.

formula of Witten[6] gives

$$q_{elec} = n \frac{e\hbar}{4\pi} + \beta\, q_{mag}$$

where e is the coupling constant and $\beta \sim \theta\, e^2\hbar/16\pi^2$ measures the CP violation. The magnetic charge is topological in origin,

$$q_{mag} = \frac{4\pi}{e}$$

The Dirac condition as generalized by Schwinger[7] and Zwanziger[8]

$$q_e\, q_m' - q_m\, q_e' = n\hbar ,$$

is not violated provided the electric charges of all dyons in the theory <u>differ</u> only by integer multiples of $e\hbar/4\pi$.

The argument that all monopoles are necessarily dyons in a CP violating theory can presumably be reversed if the theory is also self-dual: all <u>charges</u> are necessarily dyons.

The Witten formula also has some relevance to the interpretation of the Julia-Zee dyon[9]. This state appears in the SO(3) model of Georgi and Glashow. At least it is supposed to be indicated by the existence of a finite energy solution to the classical equations of motion. In the B.P.S. limit the solution is given explicitly[10],

$$< D \mid W_o^a \mid D > = \frac{1}{e} \hat{r}_a \,\mathrm{sh}\gamma \left(-\frac{1}{r} + \frac{\beta}{\mathrm{th}\beta r}\right)$$

$$< D \mid W_i^a \mid D > = \frac{1}{e} \varepsilon_{ain} \hat{r}_n \left(\frac{1}{r} - \frac{\beta}{\mathrm{sh}\beta r}\right)$$

$$< D \mid \phi^a \mid D > = \frac{1}{e} \hat{r}_a \,\mathrm{ch}\gamma \left(-\frac{1}{r} + \frac{\beta}{\mathrm{th}\beta r}\right)$$

The long range parts of the radial components of the field strength tensor $F_{\mu\nu}^{a}$, viz. $E_r^a \sim \hat{r}_a/r^2$ and $B_r^a \sim \hat{r}_a/r^2$ define the electric and magnetic charges of the dyon state, $| D >$. According to Gauss' theorem

$$q_{elec} = \frac{1}{e} \frac{\mathrm{sh}\gamma}{4\pi} ,$$

$$q_{mag} = \frac{4\pi}{e} .$$

(If the theory includes ordinary charged particles with $q_e = ne\hbar/4\pi$ and $q_m = 0$, then the Dirac condition is not violated). The parameter $\mathrm{sh}\gamma$ is just an integration constant at the classical level but it can be related to CP-violation if the Witten formula is assumed to apply. Thus,

$$q_{elec} = \frac{1}{e} \frac{\mathrm{sh}\gamma}{4\pi}$$

$$= \frac{ne\hbar}{4\pi} + \beta \frac{4\pi}{e} ,$$

$$\text{i.e.} \qquad -\mathrm{sh}\gamma = ne^2\hbar + 16\pi^2\beta$$

$$= (n + \theta)\, e^2\hbar .$$

where $e^2\hbar$ is the fine structure constant. (The presence of $\hbar$ in the formula for $\mathrm{sh}\gamma$ indicates its quantum mechanical origin and, hence, the incompleteness of the purely classical description of the state, $| D >$).

Finally, a remark on the Osborn formula for the mass spectrum in $N = 4$ super Yang Mills theory[11]. By a canonical quantization Osborn is able to obtain the anti-commutator

$$\{Q , \bar{Q}\} = P_\mu \gamma_\mu + \text{central charges}$$

where the central charges are given by surface integrals in 3-space. There are six of magnetic type

$$T_n^{\;A} = \oint dS_i \gamma_5 A_n^\alpha H_i^\alpha \quad , \quad T_n^{\;B} = \oint dS_i B_n^\alpha H_i^\alpha \quad ,$$

and six of electric type

$$Q_n^{\;A} = \oint dS_i A_n^\alpha E_i^\alpha \quad , \quad Q^B = \oint dS_i \gamma_5 B_n^\alpha E_i^\alpha \quad .$$

In these formulae A_n^α, B_n^α, denote scalar fields; $n = 1,2,3$ and α spans the adjoint representation of the gauge group. Osborn embeds a B.P.S. solution ($V(A,B) = 0$) with $E_i^\alpha = 0$ and shows - for this finite energy solution - that

$$P_\mu^2 = \Sigma \text{ (central charges)}^2$$

This makes it plausible that the B.P.S. monopole belongs to a supermultiplet of the same dimension as the elementary particles of the $N = 4$ theory, and hence lends some support to the Montonen-Olive conjecture.

It should be noted that the central charges which appear in this mass formula are determined by the long range parts of the fields (Gauss theorem). If it is possible to include CP-violation then these central charges would have the Witten form. Thus, the electric-type central charges, instead of vanishing as Osborn assumes would receive CP-violating contributions proportional to the magnetic-type central charges. Since supersymmetry leaves the Lagrangian invariant only up to surface terms one would expect the addition of a CP-violating surface term $\sim \beta F_{\mu\nu}\tilde{F}_{\mu\nu}$ to do no damage to the argument. However, since the commutation rules are sensitive to surface terms, it would be necessary to check that the Osborn formula with Witten's expression for the charges for $\{Q, \bar{Q}\}$ persists.

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