GAUGE INTERACTIONS, ELEMENTARITY AND SUPERUNIFICATION

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1981 MIRAMARE-TRIESTE
ABSTRACT

Present thinking on the concept of elementarity of particles and forces is reviewed as are the ideas behind superunification of gravity and the electro-nuclear force.
1. **INTRODUCTION**

Particle Physics, as we know it today, began some ninety years ago with J.J. Thomson's discovery of the electron and Lorentz's bold extrapolation of Maxwell's electrodynamics down to the distances of the electron's "classical" radius. Assuming that the "family" concept currently employed to classify particles is correct, the companions of the electron, essentially constituting the First Family took around forty years of experimentation to identify, as did the strong and the weak nuclear forces, governing their mutual interactions. The second family began with the cosmic-ray discovery of the muon and required yet another forty years for its completion.

Contrast this relatively slow development, ranging over more than eighty years, with the revolutionary changes registered by the subject during the last decade. Not only was the second family completed and a third nearly so, but more important: the experimental work during the decade, made possible by availability of detection devices and higher accelerator energies, gave us confidence in the essential correctness of gauge ideas - the subject of this Conference - for describing elementary forces. The first result of this has been the pushing up of the energy frontier, over which it now appears possible to ask meaningful questions, from a few GeV to Planck energies of the order of $10^{19}$ GeV - with a corresponding pushing back of the time frontier from $10^{-7}$ seconds to $10^{-44}$ seconds, within the context of a big bang model of the early Universe. A second result has been the possible obliteration of the traditional distinction between electromagnetic, nuclear and gravitational forces.

The greatness of gauge ideas for phenomenological physics lies in the circumstance that through their use, two of the basic questions (1) of what are the elementary constituents of matter and (2) what are the elementary forces among them - get inter-related with each other through the concept of elementary charges. Describing elementary particles as the basic carriers of certain elementary charges - gravitational, electrical and nuclear - one finds that the gauge forces turn out (at the first approximation) to be proportional to these charges. A postulated symmetry among the charges, then, leads directly to a unification hypothesis among the elementary forces.*

This is important. But the real import of gauge theories goes deeper. The elementary charges mentioned above - and the field-theoretic currents associated with them - are rooted, according to our present ideas, within the symmetries of space and time and the symmetries of mysterious manifolds describing the internal structure of elementary particles. By focussing on these symmetries, gauge theories provide us with windows on topological (and other) structure of space and time as well as of the internal manifolds and appear to motivate an intimate synthesis between them.

A part of the package of these symmetry ideas is the study of the observed patterns of symmetry-breaking and in particular the breaking of symmetries spontaneously. Spontaneous symmetry breaking has the character of a transition phenomenon, with the possibility of symmetry restoration, revealed in suitable environments of temperature, space-time curvature, topology, or external electric and magnetic fields. An important part of our study relates to the energies - the mass scales - where such transitions occur.**

* Gauge theories, besides their role in describing and motivating a unification of elementary forces, have also revealed the possible existence of rich topological structures - like instantons and monopoles.

** The transition phenomena associated with the onset of spontaneous symmetry breaking and its restoration, at higher mass scales - as revealed by cosmological remnants of epochs gone by - have knit particle physics and cosmology more intimately together.
Our subject has thus been transformed during the last decade through the twin studies of gauge symmetries and their spontaneous breaking. But, these advances notwithstanding, we are still very far from the elucidation of what the nature of the elementary charges is or of the problems posed by the mass scales. During this talk, my first task is to consider in the light of the gauge ideas the question: Is the very concept of elementarity, of charges, forces and particles tied to the mass scale? My second task is more specific: to speak on a possible unification of the gravitational with the electro-nuclear force near Planck energies ($\sim 10^{19}$ GeV), through a gauging of a newly discovered - and before 1971 wholly unsuspected - symmetry between bosons and fermions, called supersymmetry. This is the Superunification in the title of this talk, which promises to achieve, not only a unified theory of matter and its interactions, but attempts to find a geometrical meaning for the elementary charges it employs within extended space-time.

2. TWO PERSPECTIVES ON ELEMENTARITY AND ELECTRONUCLEAR GAUGE THEORIES

2.1. The Concept of Elementarity

Consider first the concept of elementarity for particles and gauges. At least as far as the electronuclear phenomena are concerned, there are expressed at present two points of view. These are:

(a) We have discovered the ultimate elementary particles; they are the quarks and the leptons, represented by a renormalizable gauge theory effective over all energies, with no length parameter in the interaction - i.e. the field theoretic "radius" of the particles is zero. Intermediate mass scales, whose origin is obscure, are introduced as (Higgs) parameters in the Lagrangian. As energy increases, beyond successive intermediate mass scales, the symmetry of the theory also progressively increases.

(b) A contrasting point of view states that gauge symmetries are not golden calves to be worshipped; that there are stages of elementarity dependent on the energy; that quarks and leptons are composed of pre-quarks (preons), preons are possibly composites of pre-preons, pre-preons of pre-pre-preons ... At each energy stage effective Lagrangians exist; the symmetries relevant to effective Lagrangians for the light composites may differ in different energy regimes - in fact symmetries may even decrease as energy increases. The intermediate mass scales may correspond to the different levels of elementarity.

2.2. The First View, GUTS

The first point of view is exemplified by the Grand-Unifying Theories (GUTS) and I shall briefly review these, emphasizing in particular the intermediate mass scales, and the possibility that
symmetry-breaking phenomena are spontaneously realised.

(a) We have heard evidence that low-energy phenomena exhibit exact
\[ SU_3 \times U(1) \] symmetries of chromodynamics and quantum electrodynamics. EM
QCD is a most remarkable theory. Besides asymptotic freedom i.e. decreasing coupling parameter \( a \) for increasing energy,
\[ \left( g_s(q^2) = \frac{12\pi}{3q^2 - 1} \right) \] the theory is believed to
confine exactly, i.e. colour symmetry is an invisible symmetry in the physical spectrum.

(b) We have heard of good evidence, as energy increases beyond 100 GeV,
for the \( U(1) \) symmetry to increase to \( U(1) \times SU(2) \) of the electroweak
force. At this stage the expected symmetry group \( SU_3(3) \times U(1) \times SU(2) \)
is characterized by three independent parameters \( a, \alpha \) and \( \sin^2 \theta \)
(or alternatively \( a, \alpha \), \( \sin^2 \theta \) if \( a \) is dimensionally transmuted in
favour of the mass-parameter \( A_c \)). The standard model of three families
of quarks and leptons, with fifteen 2-component particles in each family,
employ a Higgs doublet to generate spontaneous symmetry-breaking of
\( SU(2) \times U(1) \) for energies below \( \approx 100 \) GeV. The model needs
at least 26 empirically determined parameters for its specification - a
daunting task for the eventual theory.

(c) For energies above \( \approx 250 \) GeV the symmetry represented by \( U(1) \)
may expand into \( U(1) \times SU(2) \), connoting a left-right symmetry. There is expected
an expansion in each family from fifteen two-component quarks and leptons to
sixteen (i.e. each \( v_L \) is accompanied by \( v_R \)) and new gauge bosons
\( W^\pm \) coupling with \( (Y + A) \) currents.

(d) Increasing of energies\(^\ast\) beyond \( \approx 10^5 - 10^9 \) GeV, may increase the
symmetry \([SU_3(3) \times U(1)] \times [SU_R(2) \times SU_L(2)]\) to \( SU_3(4) \times SU_R(2) \times SU_L(2)\).

\( ^\ast \) The Four Colour Symmetry \( SU_3(4) \) (Pati & Salam 1974) - which may supersede
\([SU_3(3) \times U(1)]\) beyond \( 10^7 - 10^9 \) GeV - would be the first/symmetry exhibiting
a fundamental quark-lepton unification, in the sense that quarks as well
as leptons would be described as members of one irreducible multiplet of a
single symmetry group \( SU_3(4) \). The left-right symmetric, \( SU_3(4) \times SU_R(2) \)
\( \times SU_L(2) \) would depend on two coupling parameters \( a_s \) and \( \sin^2 \theta \);
this is also the last unification stage where (on account of the non-Abelian
character of the groups concerned) all charges must appear quantised.
The spontaneously broken \( SU_3(4) \) permits proton decays (Pati & Salam 1973)
into three leptons (e.g. \( P + 3\nu + \pi^0 \), \( N + e^- + 2\nu + \pi^0 \)) as well as neutron-
anti-neutron oscillations at the level of \( \tau_{\nu,R} < 10^{-8} \) seconds - i.e.
at the level to which experiments are presently directed.

(e) The next step of grand unification - the increasing of symmetry
such that the theory registers just one gauge constant may come about in
two ways:

1) The "flavour" symmetries \([SU_L(2) \times SU_R(2)]\) for any one family may
become part of a flavour \( SU_5(4) \), with multiplets containing also mirror-
quarks and mirror-leptons: these need be no heavier than \( \approx 300 \) GeV. A
discrete flavour-colour symmetry between \( SU_3(4) \) and \( SU_5(4) \) would then
ensure one coupling parameter for a grand unifying symmetry \([SU_3(4) \times SU_5(4)]\)
emerging beyond \( 10^14 \) GeV. In this model, appropriate Higgs could bring
about proton decay in the mode \( P \rightarrow \bar{Z} \) (e.g. \( P \rightarrow e^+ + \nu^0, N \rightarrow e^+ + \nu^0 \)).

2) Or the symmetry \( SU_3(4) \times SU_R(2) \times SU_L(2) \) may be part of an \( SO(10) \)

\( ^\ast \) Note the vast separation between expected succession of mass scales \( \approx 250 \) GeV,
\( 10^7 - 10^9 \) GeV, ... . Even with the promise of techni-colour with its characteristic
mass scale \( \approx 1000 \) GeV, we are entering the age, either of a true pasimony
of nature for new phenomena, or of our theoretical bankruptcy in recognising
empirical important clues. Clearly we desperately need new experimental inputs.
which manifests itself for energies in excess of $10^{11}$ GeV. The 16-fold
spinor multiplet of SO(10) would contain left-handed quarks and leptons
as well as left-handed antiquarks and anti-leptons. (Alternatively there
may be no intermediate $SU(4) \times SU_8(2) \times SU_2(2)$ stage; the $SU(3) \times
SU(2) \times U(1)$ may expand directly into $SU(5)$ for energies exceeding
$10^{14}$ GeV.) The minimal $SU(5)$ and SO(10) models may find it difficult
to accommodate - without introducing extra intermediate mass scales through
extra Higgs - $N\bar{N}$ oscillations at the present level of
experimentation (assuming these are discovered), or $\nu_\mu$-mass of around
10 eV (assuming this is confirmed) or proton decay into a lepton plus
pions ($P + e^- + \pi^+ + \pi^0$).

ii) Finally there is the possibility of the maximal gauge symmetry being
realised: this is SU(16), the maximal symmetry that could hold for sixteen
two-component quarks and leptons and their anti-particles, belonging to
one family (Pati, Salam, Strathdee (1975,1980)). (Here again anomaly
cancellation makes mirror-particles mandatory). This symmetry could permit
the co-existence of four types of decay modes for the proton; $F \rightarrow e^+$,
$P \rightarrow e^-$, $P \rightarrow 3\pi$, $P \rightarrow 3\bar{H}$ alongside of $N\bar{N}$ oscillations, at the presently
planned level of experimentation. There will be three intermediate mass scales
($10^{14}$ GeV, $10^9-10^{10}$ GeV, and $10^4-10^{5}$ GeV):

<table>
<thead>
<tr>
<th>Allowed Processes</th>
<th>Appropriate Mass Scales</th>
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<tbody>
<tr>
<td>$P + e^+ + \pi^+ + \pi^0$</td>
<td>$10^{14}$ GeV</td>
</tr>
<tr>
<td>$e^- + e^+$</td>
<td>$10^9-10^{10}$ GeV</td>
</tr>
<tr>
<td>$e^- + \pi^+$</td>
<td>$10^4-10^5$ GeV</td>
</tr>
</tbody>
</table>

| $N + \bar{N}$ | Not allowed |

- This is the case for which the three couplings (for $SU(3)_C \times SU(2)_L \times U(1)_Y$)
  converge to a common value, using renormalisation group techniques. The assumptions
  which go in the computation of $\Lambda_0$ are: there are no new forces up to $\Lambda_0$
  (including forces which might differentiate between the three families), nor
  new particles (which might upset $\sin^2 \theta (\Lambda_0)$ from its value of 3/8, derived on
  the basis of known particles). Thus if we assume that there exists a desert of new
  phenomena up to $\Lambda_0$, renormalisation group then tells us that $\Lambda_0$ is high at $10^{14}$ GeV. Note
  that so long as a grand-unifying group $G$ descends with one mass scale $\Lambda_0$, down to
  $SU(3) \times SU(2) \times U(1)$, and so long as $\sin^2 \theta (\Lambda_0) = 3/8$, every $G$ (e.g. $SU(5)$ or SO(10) or SU(16)
  etc.) will give identical predictions for proton decay.
The important differentiation between SU(16) and SO(10), so far as proton decays of \( P \rightarrow e^+ + \pi^0 \) variety are concerned, lies in the circumstance that the decay is intrinsic in the SO(10) (or SU(5)) model; even beyond \( 10^{16} \text{ GeV} \) when the gauge particles concerned may have transitioned to masslessness, \( P \rightarrow uu + e^+ \) continues unabated. For the SU(16) on the other hand, where \( P \rightarrow e^+ \) it is a consequence of spontaneous symmetry breaking, the transition \( P \rightarrow uu + e^+ \) will cease, when symmetry is restored beyond \( 10^{16} \text{ GeV} \). Of course SU(16) permits in addition, decays of the type \( P \rightarrow e^+ + g^0, P \rightarrow 3v + n, P \rightleftarrows n \) forbidden for example in minimal SU(5).

2.3. The Second View on Elementarity and Gauges

A contrasting view to GUTS posits that there is no linear progression of increasing symmetry as energy increases; that intermediate mass scales do exist but that they represent new levels of elementarity. Quarks and leptons are composites, made of pre-quarks (preons); preons may be composites of pre-preons; pre-preons of pre-pre-preons and so on. (Pati, Salem & Strathdee 1975, 1980; Pati, Rajpoot & Salam, 1980 and references therein.) This view has surfaced because of discontent with:

(a) Far too many Higgs needed in Grand Unified Theories like SO(10) or SU(16), necessary if a number of intermediate mass scales exist.

(b) Far too many quarks and leptons (39 two-component ones already discovered at the last count; six more awaited), to qualify as an elementary set.

(c) Too many gauge bosons; too large symmetry groups.

(d) And finally, too widely spaced (technically "unnatural") intermediate mass scales - for example 100 GeV and \( 10^{16} \text{ GeV} \) in minimal SU(5).

* Presumably, after this stage (of SU(16) or its siblings) would come the uniting of families into "tribes". I shall not discuss this.

The simplest preonic model (Freund & Curtright 1979) with quarks and leptons as composites of preons) assumes eight preons: \( f_u^+, f_d^+, C_R^+, C_Y^+, C_B^+, F_1^+, F_2^+, F_3^+ \) - two flavons; \( f_u, f_d \) carrying flavour, three chromons \( C_R, C_Y, C_B \) carrying colour, and three fermions \( F_1^0, F_2^0, F_3^0 \). The light preons would correspond to an SU(8) symmetry, containing SU(5) X SU\(^2\) family (3).

An alternative which illustrates the notion of differing symmetries at the composite (quark-lepton) level compared to the symmetries of preonic theory is Harari's model (Harari 1979). There are 18 preons - Tohu's (T) and Vohu's (V) - with intrinsic symmetry \( SU(3) \times SU(3) \times [U(1)]^2 \), H.C.

\[ T_{L,R} = (3, 3), V_{L,R} = (3, 3), L_{L,R} \] Here SU(3) is a hyper-colour group with an appropriate \( A_{H.C.} \) in the TeV range. (Though Harari does not take this point of view the 18 preons may themselves be composites of six pre-preons \( (C_R, C_Y, C_B, H_L, H_R, H_D) \) if one is economy-conscious.)

Quarks and leptons - singlets of hypercolour - are \( T_T, T_V, TV, \text{ and } VV \) composites. As energy decreases below \( A_{H.C.} \) - i.e. at the composite (\( q, l \) level, the symmetry group is not \( SU(3) \times SU(3) \times [U(1)]^2 \), H.C. but

\[ SU(3) \times SU(2) \times SU(2) \times U(1) \], with the implication

that \( W_L^\pm, W_R^\pm \) are composite gauges; the corresponding charges are non-elementary. The \( W^\pm \) forces are Van-der-Waals forces between hyper-colour neutral composite objects.

The question now arises: why does the weak Van-der-Waals force (mediated by the \( W^\pm \)'s) exhibit such an elegant Yang-Mills character? A second and related question: why are quarks and leptons (composites of preons) so light compared with \( A_{H.C.} \)? What symmetry protects them against acquiring (heavy) masses?

A lore has developed in answer to both these questions tied to chirality as the protector of fermions against mass-acquisition and

*As we have heard at this Conference, in the context of super-techni-colour, supersymmetry for fermions and bosons can protect scalar companions of chiral fermions from mass acquisitions.
renormalisability as protector of spin one particles against non-Yang-Mills behaviour. A chiral spinor is massless; a vector meson theory with no associated mass scale is renormalisable if it is Yang-Mills and vice versa.

Let us envisage the following scenario. Let there be a succession of colour-like theories: colour, hypercolour, hyper-hypercolour ... with associated mass scales $\Lambda_c$, $\Lambda_{h.c.}$, $\Lambda_{h.h.c.}$ ... Why these mass scales, and how to determine one in terms of the next remains a mystery. Assume the theory derives all other masses dynamically from these. Let quarks and leptons be "elementary" below $\Lambda_{h.c.}$; quantitatively within the energy range $\Lambda_c - \Lambda_{h.c.}$, let these describe all physical phenomena through an "elementary" Lagrangian; preons play this role within the range $\Lambda_{h.c.} - \Lambda_{h.h.c.}$. The lengths $\Lambda_c^{-1}$, $\Lambda_{h.c.}^{-1}$, $\Lambda_{h.h.c.}^{-1}$ are the confinement (bag) radii of the singlet light composites of colour (hadrons), hypercolour (quarks and leptons), hyper-hypercolour (preons) ....

Thus we have the following picture:

Table 2

<table>
<thead>
<tr>
<th>Energy Regime</th>
<th>&quot;Elementary Entities&quot;</th>
<th>Light Composites</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_c \leftrightarrow \Lambda_{h.c.}$</td>
<td>(q, l)</td>
<td>hadrons; singlets of colour</td>
</tr>
<tr>
<td>$\Lambda_{h.c.} \leftrightarrow \Lambda_{h.h.c.}$</td>
<td>Preons</td>
<td>(q, l); singlets of hypercolour</td>
</tr>
<tr>
<td>$\Lambda_{h.h.c.} \leftrightarrow \Lambda_{h.h.h.c.}$</td>
<td>Prepreons</td>
<td>Preons; singlets of hyper-hypercolour</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

If in any energy regime, Physics can be described through a renormalisable field theory of "Elementary Entities" or equally through an effective Lagrangian of fields corresponding to composites (both light and heavy)

* It could be that leptons (or at least $\nu$ and $\bar{\nu}$) are "elementary entities" of a level different from that of quarks, and their role is only that of spectators (for anomaly cancellations) for the regime indicated in Table 2.

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made out of the "elementary entities" of the energy regime before (decoupling theorem), then the following ansatz should hold: (Veltman as quoted in Ellis, Gaillard & Zumino 1980) up to energies $\Lambda_n$:

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tation by Banks, Schwimmer, Yankelovicz (1980) and Bars and Yankelovicz (1981) except to remark that (surprisingly) the implementation involves representations of graded algebras.

Before concluding let me remark that Ellis, Gaillard, Maiani and Zumino (1980) have considered a preon supergravity model which I shall be discussing later. Anticipating however the elementarity and mass scales they might propose:

<table>
<thead>
<tr>
<th>Mass Scale</th>
<th>Elementary Entities</th>
<th>Symmetry Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>A⊙(10^14 GeV)</td>
<td>(4, 1) Preons</td>
<td>SU(5) X SU(3) family</td>
</tr>
<tr>
<td>Planck (10^{33} GeV)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Beyond 10^{39} GeV, Pati, Salam and Strathdee (1980) suggest that there may be a pre-preonic regime, of (analogous) electric and magnetic dynams.

The symmetry group is the humble U(1) family X U(1) with gravity itself an induced phenomenon. And these remarks finally bring us to gravity and its unification with the electromagnetic force.

3. **Unification of Electromagnetic Forces with Gravity, Eleven Space-Time Dimensions**

It is a vast extrapolation (some sixty orders of magnitude) to believe that Einstein's gravity theory, with its dimensional constant \( \kappa = \sqrt{\frac{G}{c^4}} \approx (10^{39} \text{ GeV})^{-1} \) - devised originally to describe long-range phenomena - will continue to hold down to distances \( \approx \lambda = 10^{-33} \text{ cm} \).

Assuming that such an extrapolation makes sense, the presence of the dimensional constant \( \kappa \) in Einstein's theory clearly sets it apart from the electro-nuclear where the gauge coupling is dimensionless. A unification of the electro-nuclear with gravitational theory must be construed in the sense that Einstein envisaged; the electro-nuclear charges must find a niche in the geometry of space-time, like the gravitational charge which in Einstein's theory found an association with the geometrical notion of space-time curvature.

Now just this type of unification was accomplished, in a remarkable theory, between Maxwell's electromagnetism and gravity by Th. Kaluza in 1921, followed by Klein in 1926. The suggestion was that the electric charge may be identified with the fifth component of momentum in a space-time extending to five dimensions. Formally Kaluza showed that the scalar curvature in a five dimensional space-time equals Einstein Lagrangian (which is scalar curvature in four dimensions) plus Maxwell's Lagrangian, in standard interaction with gravity, provided the electromagnetic potential is identified with the \( E_{\gamma\mu} \) component of the metric. More specifically, writing,

\[
E_{\gamma\mu} = \kappa^2 A_{\gamma\mu} A^\gamma + \kappa A_{\gamma} A^\gamma \equiv \kappa \, A_{\gamma} A^\gamma - \lambda A_{\gamma} \quad \text{if } \gamma, \mu = 0, 1, 2, 3
\]
the action
\[ S = -\frac{1}{16\pi^2} \int d^5 x \sqrt{g} \int d^4 x \sqrt{g_{\text{eff}}} R \]
equals the sum of the standard Einstein and Maxwell actions if 
\[ g_{\mu\nu} \text{ and } A_\mu \text{ are independent of the 5th coordinate } x_5. \]

Two types of objections were raised against this unification. Einstein objected; he could not see how other matter - and particularly now, spinor matter - could be geometrical. We shall see later that this objection is met today through supersymmetry which unites fermions with bosons. The second objection came from Pauli (1933); electricity and gravity had really separated like oil and water in this theory. Surely somewhere there ought to be new testable consequences of the unification suggested. One might indeed discern new consequences for charged spin \( \frac{1}{2} \) particles, but these appeared physically disastrous, at least so far as 1933 was concerned.

To see this, following Thirring (1972), one can write the Lagrangian for a spin-\( \frac{1}{2} \) fermion in five dimensions in the form:* 
\[ \mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \psi^\dagger \psi + g \epsilon_{\mu} \gamma_5 \psi^\dagger \gamma^5 \psi + \frac{1}{16 \pi^2} (\frac{\mathcal{M}^2}{\mathcal{N}} - 1) \mathcal{M} \right] \]  
where for the dependence on the fifth coordinate \( x_5 \), we have assumed that \( \psi(x, x_5) = \exp(i\mathcal{M}x_5) \psi(x) \). Here \( \mathcal{M} = \mathcal{M}_0 + \frac{\mathcal{M}}{\mathcal{N}} \), while \( \mathcal{M}_0 \) is the electric charge e. Note that:

(a) We have made the assumption that the five-dimensional manifold is a product \( \mathbb{R}^4 \times S^1 \), of the four dimensional Minkowski manifold \( \mathbb{R}^4 \) with the circle \( S^1 \) of size \( 2\pi/\mathcal{N} \). The fifth dimension has thus curled up to a size of the order of \( \sim 10^{-37} \text{ cm} \).

(b) The charged fermion mass \( \mathcal{M} \) is \( \sim 10^{19} \text{ GeV} \).

(c) The charged particle carries a non-zero electric dipole moment, which violates both P and T.

Since 1933 we have become used, not only to T-violation*, but also to particles of Planck mass. Pauli's objections do not have the same force today as then.

I shall not pursue fermions any further in connection with Kaluza-Klein theory, since later we shall be formulating supergravity theories which contain fermions in extended space-times. I shall here merely cite a recent remark of Witten (1971) which considers the problem of finding the manifold of minimum dimensionality which could support unification of \( SU(3) \times SU(2) \times U(1) \) with Einstein's gravity - in the Kaluza-Klein sense.

* Thirring (1972) suggested that to push up the magnitude of T-violation to the level observed in K-phenomena, one needs to consider seriously a spin two Kaluza-Klein theory of strong f-gravity (c.f. Isham, Salam and Strathdee (1971)). I would like to take this opportunity to emphasise the virtues of strongly interacting composites of spin 2 made up for example of "elementary" gluons or "hyper-gluons" or "hyper-hyper-gluons"... Since the only ghost-free spin-2 equation known is the Einstein equation, the fields representing such composites must satisfy this equation. Further these composites could acquire induced mass terms which are dynamically generated through a mechanism analogous to gluon-condensation and described by Salam and Strathdee (1976). The chief virtue of these spin 2 composites is that they would confine quarks or gluons within bags of the size of their inverse masses. (Salam and Strathdee 1973, Baaklini and Salam 1979). 

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Let $B$ be the internal space parameterized by $\phi^i$, $i = 1, \ldots, n$ for an internal symmetry $G$ (symmetry generators $T^a$, $a = 1, \ldots, N$), write the generalised Kaluza-Klein metric in the form

$$g_{AB}(x^\alpha, \phi^k) = g_{\mu\nu}(x^\alpha) \sum_a A^a_\mu(x^\alpha) K^a_\nu(\phi^k) + \gamma_{ij}^{\mu}(\phi^k).$$

Here $\gamma_{ij}^{\mu}$ is the metric of the internal space $B$ and $A^a_\mu(x^\alpha)$ are massless gauge fields of $G$, and $K^a_\mu$ are the appropriate Killing vectors.

What is the manifold $B$ of minimal dimensionality which can support $SU(3) \times SU(2) \times U(1)$?

Now $U(1)$ is the symmetry group of the circle $S^1$, with dimension one. The lowest dimension space with symmetry $SU(2)$ is the sphere $S^2$ with dimension two, while the lowest dimension space with symmetry $SU(3)$ is the complex projective space $\mathbb{CP}^2$ with dimension four. Thus the space $\mathbb{CP}^2 \times S^2 \times U(1)$ can support $SU(3) \times SU(2) \times U(1)$ and has dimension $4 + 2 + 1 = 7$. With four non-compact "space-time" dimensions, the total dimensionality of our world must then be $4 + 7 = 11$, if gravity as well as $SU(3) \times SU(2) \times U(1)$ are to be supported in a gauge fashion according to the ideas of Kaluza and Klein.

This is a remarkable result. We shall see later, eleven dimensions is probably the maximum for supergravity. This is because supergravities in higher dimensions most likely contain massless particles of spins greater than two. And the existence of such particles would contradict many of the fundamental assumptions of quantum theory of fields.

We may indeed have been living in eleven-dimensional space-time all the time, but no one knew till 1979 when $SU(3) \times SU(2) \times U(1)$ symmetry was first clearly established! Eleven, as a number, has the merit, that to my knowledge, nothing mystical has ever been associated with it.

We shall now go on to discuss supergravity theories with their twin unification of gravity with the electro-nuclear force as well as the unification of fermions with bosons.
Supersymmetry, Simple and Extended

Supersymmetry is the symmetry between fermions and bosons. That for a simple Bose-Fermi free theory:

\[ \mathcal{L} = \frac{1}{2} (\partial_{\mu} A)^2 - \frac{1}{2} (\partial_{\mu} B)^2 - \frac{i}{2} X \lambda \]

one can "rotate" bosons into fermions came as a profound surprise when Gelfand and Lichtman in 1971 (and Wess and Zumino independently in 1973) discovered this remarkable symmetry.*

The rotation is in a spinor-space extension of space-time:

\[ 4A = \frac{1}{2} \epsilon A, \quad 4B = \frac{1}{2} \epsilon Y A, \quad 4\lambda = \frac{1}{2} (A - \gamma_5 B) \epsilon \]

Here \( A \) and \( B \) are massless spin-zero, and \( \lambda \) is a Majorana field; with \( \epsilon \), the constant infinitesimal spinor parameter of rotation.

The Lagrangian \( \mathcal{L} \) is invariant up to a divergence. The symmetry-algebra closes on-shell i.e. one must use equations of motion to demonstrate closure, though by adjoining two auxiliary non-propagating fields to the physical set \((A, B, \lambda)\), one can secure closure even off-shell for this particular theory. This however is not always possible and poses one of the unsolved problems in the subject i.e. on-shell supersymmetric Lagrangians are often available, but not their off-shell counterparts.

Turning to interactions, one may deduce for example the Lagrangian for a Higgs (spin-zero) field from a Lagrangian for spin \( \frac{1}{2} \) quarks. Likewise gauge bosons (spin 1) would be accompanied by and interact with gauge-fermions (gauginos), and spin 2 gravitons by spin 3/2 gravitinos in a supersymmetric theory*.

Since supersymmetry transformations convert bosons into fermions, the supersymmetry generator - the supersymmetry charge \( Q_\alpha \) - must be a (Majorana) spinor. One can demonstrate the anti-commutator relation:

\[ [Q_\alpha, Q_\beta] = 2 (\gamma_\mu)_{\alpha \beta} P^\mu \]

Here \( P_\mu \) is the energy-momentum vector. Clearly supersymmetry represents an extension of Poincare's space-time symmetry.

Exceedingly important for physical applications are the introduced by generators

Extended Supersymmetries /Salam & Strathdee (1974) where the supersymmetric \( \mathcal{Q}_i \) (i = 1, 2, ..., \( N \)) correspond to the fundamental representation of an internal symmetry \( SO(N) \). The \( \mathcal{Q}_i \)'s satisfy**:

* A supersymmetric theory of gravity would thus realise Einstein's dream of elevating the "base wood" of (fermion) matter on the right-hand of his equation

\[ R_{\mu \nu} = \frac{1}{2} K_{\mu \nu} R = -\nabla_{\mu} \nabla_{\nu} \]

to the status of (spin 2 bosonic) "marble" of gravity on the left-hand side. As we shall see, this dream has come to be realised in a manner Einstein would have approved: not only can supersymmetric gravity be formulated; it happens to be the gauge theory of supersymmetry.

** More generally \( (\mathcal{Q}_i, \mathcal{Q}_j) = 2 (\gamma_\mu)_{ij} P^\mu + 3 \bar{\mathcal{Q}}_i \mathcal{Q}_j + \mathcal{Q}_i \mathcal{Q}_j \). Here the \( (n-1) \mathcal{Q}_i \)'s and \( (n-1) \bar{\mathcal{Q}}_i \)'s are the so-called central charges which, on-shell and in flat space-time commute with each other and with \( P_\mu \).
For the simple \( N = 1 \) supersymmetry, the massless super-multiplets consist of helicity states \((\pm \frac{1}{2}, 0^+, 0^-), (\pm 1, \pm \frac{1}{2}), (\pm \frac{1}{2}, \pm 1), \) or \((\pm 1, \pm i), (\pm 1, \pm \frac{1}{2}), or (\pm 2j, \pm |)\); for \( N = 2 \), the helicity content of the fundamental supermultiplet is \((\pm 1, 2 \times (\pm \frac{1}{2}), 2 \times 0)\); for \( N = 4 \) the helicity content is \((\pm 1, 8 \times (\pm \frac{1}{2}), 8 \times 0)\). Here \( \frac{1}{2} \times (\pm \frac{1}{2}) \) (for example) stands for four massless Majorana fields, representing \( \frac{1}{2} \times 2 \) physical degrees of freedom, making up a 4-fold of \( SO(4) \); likewise the six spin-zero objects represent a 6-fold of \( SO(4) \).

Extended supersymmetries corresponding to \( N = 2 \) and \( N = 4 \) are particularly interesting; the content of the super-multiplets is the same as of (compactified) \( N = 1 \) simple-supersymmetry super-multiplets in six and 10-dimensional space-time respectively.*

* To see this consider for example \( N = 1 \) simple-supersymmetry in 10 dimensions, represented by a 10-component field \( A_{\mu} \) and a sixteen-component spinor \( \psi_{\alpha} \). After compactification down to 4-dimensions, the 10-vector \( A_{\mu} \) appears as a 4-vector \( A_{\mu} \) plus six scalars \( A_{5}, A_{6}, \ldots A_{10} \). Likewise the 16-component spinor \( \psi_{\alpha} \) \( \equiv \psi_{\alpha}(1 = 1,2,3,4) \) has the content of four Majorana spinors in 4-dimensional space-time. This parentage of the \( N = 4 \) extended supersymmetry from ten dimensions (\( SO(9,1) \supset SO(4,1) \times SO(6) \)) anticipates that a hidden "internal" symmetry exists and that it is likely to be as large as \( SO(6) \approx SU(4) \) rather than just \( SO(4) \).

4.2 \( N = 4 \), Yang-Mills Theory and its Finiteness

A renormalisable on-shell supersymmetric Lagrangian for the \( N = 4 \) multiplet can be written down. If we introduce external non-Abelian local symmetry \( G = SU(k) \) such that the total symmetry is \( SU(k) \times (N = 4 \) supersymmetry)– we would be dealing with a Yang-Mills theory with \((k^2 - 1)\) helicity \( f_k, k(k^2 - 1) \) helicity \( \frac{k^2}{2} \) and \( 6(k^2 - 1) \) helicity zero fields, with a unique coupling parameter \( g \). Now a remarkable thing happens. It has been verified by direct calculations that up to three loops the renormalisation group \( \beta \)-function \( \beta(g) = N \frac{d}{dg} \) vanishes identically. If this result can be proved generally, for all loops, this would be the first finite infinity-free quantum field theory in Physics.*

Is this theory really finite to all orders?

A general proof for this miracle has been given by Schmutz and West (1981) and by Ferrara and Zumino (unpublished). The proof relies on a certain number of assumptions, plausible but not all quite demonstrated in a supersymmetric context. The proof relies on: (1) the identity \( g_{\mu}^{\nu} = \frac{\delta(g)}{2g^{3}} f_{\mu}^{|} f_{\nu}^{|} \), (2) the absence of all anomalies, (3) the structure of the supersymmetric conformal anomaly-multiplet and (4) the hidden \( SU(k) \) "internal" symmetry this particular theory exhibits and mentioned already.

* The Green's functions of this theory may exhibit gauge-dependent infinities; to see that such infinities are inconsequential, remark that in the supersymmetric analogue of the axial gauge, even these would be absent, if \( \delta(g) \) vanishes.
For the one-loop case an alternative proof has been given by Curtwright (1981). The Yang-Mills current can be split into convective and magnetic parts; their contributions to the $S$-function are:

$$S = M \frac{d}{d\theta} \theta^2 \frac{1}{\theta} (1 - \theta^2)^{-2S}$$

where $C$ is the appropriate quadratic invariant for the gauge group representation carried by the particle. From the helicity content of the $N = \frac{1}{2}$ theory discussed above, it is easy to verify that:

$$\sum_{\text{helicities}} (-1)^{2S} = 0 = \sum_{\text{helicities}} (-1)^{2S}$$

Thus the convective and the magnetic infinities vanish individually.

Each time the "magnetic" Yang-Mills coupling acts, one may perhaps expect an additional factor of $S$ in the expression for $S$ in the Curtwright formula. If this is the case, its one-loop generalisation may read:

$$g = a + a \frac{S}{2} + \ldots + 1$$

and the individual vanishing of convective and magnetic infinities may not carry over to more than one loop. Such a cancellation, we recall, was a consequence of the following identities for extended supermultiplets:

$$\sum_{\text{helicities}} (-1)^{2S} [S(j)]^n_D(j) = 0 \quad k = 0, 1, \ldots, N-1$$

$$\sum_{\text{helicities}} (-1)^{2S} [S(j)]^n_C(j) = 0 \quad k = 0, 1, \ldots, N-3$$

$$\sum_{\text{helicities}} (-1)^{2S} [S(j)]^n_F(j) = 0 \quad k = 0, 1, \ldots, N-5$$

Clearly for $N = 1$, these formulae cannot take us beyond one loop. For two and three loops, either there are other sources of infinity cancellation or, as has sometimes happened with infinities, we are following a red-herring. This is bound to be unpopular but there appear to be some conjectural reasons why the 4th loop may be infinite.

4.3. Extended Supergravities

$N = 1$ supergravity is the gauge-theory of simple supersymmetry in the same sense that Einstein's gravity is the gauge theory of the Lorentz-Poincare-symmetry. To motivate this, note that for $N = 1$ supersymmetry, the anti-commutator for charges

$$(\theta_{\alpha}, \theta_{\beta}) = 2(\gamma_{\mu})_{\alpha\beta} \delta^N(x)$$

may be expected to generalise to

$$(\theta_{\alpha}, J_{\beta\alpha}(x)) = 2(\gamma_{\mu})_{\alpha\beta} \delta^N(x)$$

where $J_{\beta\alpha}(x)$ is the current corresponding to the charge $Q_{\alpha}$ and $\delta^N(x)$, the energy-momentum tensor is the current of $P^\mu$. Clearly a supersymmetric generalisation of the gauge theory which associates a spin 2 graviton with $\delta^N(x)$ must be a theory which also associates a gravitoon of spin 3/2 with the super-symmetry current $J_{\beta\alpha}(x)$.

Such a theory was written down by Freedman, Van Nieuwenhuisen and Ferrara (1976) and independently by Deser and Zumino in (1976). The Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{Einstein}}(e, \omega) + \mathcal{L}_{\text{Harita-Schwinger}}(\phi, \psi, \omega)$$

where $e, \omega, \phi$ are the vierbein, spin-connection and spin -3/2 Harita-Schwinger fields, with the covariant derivative $D_p$ and $\omega$ defined as

$$D_p = \partial_p + \frac{1}{4} \epsilon_{mn} Q_m^p$$

with

$$\epsilon_{mn} = \epsilon_{mp}(e) + \frac{1}{4} (\epsilon_{\mu
u} - \omega_{\mu
u}) - m + \frac{1}{2} (\epsilon_{\mu
\nu} - \omega_{\mu\nu}).$$
Of fundamental importance for a programme of superunification of gravity with the electro-nuclear force are the extended supergravity theories for \( N = 2, 3, \ldots, 8 \). (Theories with \( N > 8 \) would contain spins \( \geq 5/2 \) for which consistent gravitationally coupled Lagrangians do not exist. Thus \( N = 8 \) represents the maximal supergravity theory there can be on present ideas).

A supersymmetric supermultiplet in an extended theory consists of a set of \( SO(N) \) multiplets of different spins. Thus the content of a massless supermultiplet for the \( N = 8 \) extended theory, with maximum helicity \( \pm 2 \) is as follows:

<table>
<thead>
<tr>
<th>Helicity</th>
<th>( \pm 2 )</th>
<th>( \pm 3/2 )</th>
<th>( \pm 1 )</th>
<th>( \pm \frac{1}{2} )</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SO(8) ) Content</td>
<td>( 1 )</td>
<td>( 8 )</td>
<td>( 28 )</td>
<td>( 56 )</td>
<td>( 70 )</td>
</tr>
</tbody>
</table>

The total number of physical states of integer (half-integer) helicity is 128. Lagrangians for massless supermultiplets (with maximum helicity \( \pm 2 \)) can be written down up to and including \( N = 8 \). Naturally these Lagrangians would sport just one coupling parameter (the gravitational), and are expected to be globally \( SO(N) \) invariant. Noting that the spin-one fields in the \( N = 8 \) supermultiplet (28 fields) correspond to the adjoint representation of the "internal" global \( SO(8) \), the question may be raised: can one add supersymmetry preserving terms to the Lagrangian which might convert \( SO(8) \)-global to \( SO(N) \)-local, with adjoint \( SO(N) \) spin-one fields (already contained in the supermultiplet) as Yang-Mills gauges. This would then permit us to include a second coupling parameter \( g \), of the type (and magnitude) familiar in electro-nuclear theory. The final unified theory would not be a uni-constant theory, but it would be a uni-supermultiplet theory, certainly for the case of \( N = 8 \).

The answer to this question appears to be in the affirmative, though the actual construction has so far been carried out up to and including \( N = 5 \) only. A characteristic of all such Lagrangians is the appearance of a cosmological term of the form \( \lambda h(\phi) \) with the parameter \( |\lambda| \sim g^2 \kappa^{-2} \) and a spin 3/2 "mass" term of the general form \( g^2 \kappa_{\mu\nu} \phi_{\mu} \phi_{\nu} \). Here \( \phi \)'s are the scalar fields in the supermultiplet. Thus \( N \)-extended supergravity theories which are also locally \( SO(N) \) Yang-Mills, contain two parameters the gravitational \( \kappa \) and \( g \) or equivalently \( \kappa \) and \( \lambda \) (the cosmological constant). As remarked earlier the cosmological constant \( \lambda \sim g^2 \kappa^{-2} \) is some 66 orders of magnitude larger than the empirical cosmological upper limit would permit. Notwithstanding the attractiveness of local \( SO(N) \), I shall in the sequel set \( g = 0 \), and speak only of pure gravitational super-Lagrangians.

In this context the most important result is the construction of the \( N = 8 \) super-Lagrangian by Cremmer and Julia in 1979 from which Lagrangians for \( N < 8 \) can be derived by suitable contractions. Cremmer and Julia started with the remark that the \( N = 8 \) supergravity supermultiplet in 4-dimensions has the same physical content as the \( N = 1 \) simple supergravity multiplet in 11-dimensions, provided that in 11-dimensions the fields introduced correspond to the elf-bein \( e_{\mu} \), the spinor field \( \psi_{\alpha} \) and a three-index anti-symmetric tensor \( A_{\alpha\beta\gamma} \). The independent physical degrees of freedom on reduction to 4-dimensions can be checked to be 128 for bosons as well as for fermions. We are back, once again to 11 dimensional space-time like Kaluza and Klein.

Now the exciting part of the Cremmer-Julia construction was the discovery of hidden (on-shell) symmetries for the equations of motion as well as (off-shell) symmetries for the Lagrangian. The on-shell symmetries were
found to constitute a non-compact $E_7$ with 133 generators; the off-shell symmetries are $SU(8)$, rather than the humble "internal" $SO(8)$ we started from. The construction uses a scalar $56 \times 56$ matrix field $V$ of the $E_7$ algebra. Writing $V^\dagger V^{-1}$ in the form \[
abla \nu \nu \nu^{-1}, \]
the $Q_\nu$ piece can be considered as 63 auxiliary spin-one objects, which occur in characteristic combinations like $(D - Q)$; (e.g. the spin $3/2$ terms read:
\[\epsilon_{\lambda\mu} \tau_\nu \nabla_\nu (D_{\lambda\mu} - Q_{\lambda\mu}) \]
where $\tau_\nu$ is the Rarita-Schwinger $SU(8)$ octet). Thus the 63-fields comprised in $Q_\nu$ might act as Yang-Mills gauge fields for an internal $SU(8)$, if these fields possessed a propagation character.

It is important to realise that the $Q_\nu$ are not endowed with a basic kinetic energy term in the Cremmer-Julia Lagrangian*. They made the conjecture that $Q_\nu$'s may be quantum-completed, acquiring a propagation character through quantum loops. Here Cremmer and Julia draw an analogy with the well-known $O(9)$ model in 2-dimensions. This model starts with a Lagrangian for scalar fields $\phi_i$ containing a non-propagating auxiliary field $V_\nu$, \[
\mathcal{L} = -\frac{1}{4} (\partial_{\nu}^2 - 1) (\partial_{\nu} V_\nu)^2. \]
The Lagrangian exhibits a $U(1)$ symmetry:
\[
\partial_{\nu} \phi_i = \epsilon_{a(x)}^{a(x)} \partial_{\nu} \phi_i \quad \partial_{\nu} V_\nu = 1/2 \epsilon_{a(x)}^{a(x)} \phi_i^a + V_\nu - \partial_{\nu} \phi_i. \]

These fields represent an $SU(8)$ "gauge" in the same sense as the antisymmetric part of the vierbein field $\epsilon_\mu^a$ does – the analogy of $E_7$ being with $GL(4,\mathbb{R})$, and of $SU(8)$ being with $O(3,1)$ in the 4-dimensional gravity theory of Weyl-Sciama-Kibble. (Note that $GL(4,\mathbb{R})$ has dimension 16-6 = 10; this gives the count of the number of components of the physical graviton field in four dimensions. Likewise the co-set $E_7/SU(8)$ with dimension 133-63 = 70 represents the 70 spin-zero physical fields in $N = 8$ extended supergravity).

There is however no kinetic energy term for $V_\nu$ in the Lagrangian itself. It can be shown that this field $V_\nu$ does propagate but as a consequence of radiative effects; that it then acts both as a confining and a binding field among basic scalars of the theory, and the spectrum of the composite states exhibits an $SU(n)$ symmetry. For the $N = 8$ supergravity theory of Cremmer and Julia, the corresponding conjecture would be that the sixty-three $Q_\nu$ fields do acquire $SU(8)$ Yang-Mills propagation in a similar manner; that they provide electro-nuclear type of binding and confining forces, and that the composites which arise in this theory, make up an infinite dimensional unitary representation of the non-compact $E_7$, whose maximal compact subgroup is $SU(8)$.

4.4. $N = 8$ Supergravity as a Superunified Theory of Preons

The conjecture that $N = 8$ supergravity theory of Cremmer and Julia represents a preonic Lagrangian has been made by Ellis, Gaillard, Maiani and Zumino (EGMZ). They started with the remark that the attempt to use the $N = 8$ supermultiplet, with 28 spin one and 56 spin $3/2$ objects had come to rapid grief when one realised that $SO(8) = SU(3) \times SU(2) \times U(1)$. An aspect of this failure is that when we decompose $SO(8)$ relative to $SU(3)$ and electric charge, we obtain:

\[
28 = 8(0) + 3(1) + 3(-1) + 3(2) + 3(-2) + 3(3) + 3(-3) + 3(4) + 3(-4) + 56. \]

Thus, the $N = 8$ supermultiplet, if identified with physical particles, might, at best, accommodate $u,d,s,c,(colour-triplets of quarks),a colour-\text{octet of quarks}\ b, a neutral spin $\frac{1}{2}$ octet, the electron and two neutrinos, in its spin-$\frac{3}{2}$ sector, plus coloured gluons, the photon, the $Z^0$ and
fractionally charged superheavy gauge bosons among the spin-one particles. There, however, are no $\nu^i$, no $\nu$, $\nu^i$, nor $t$: these would have to emerge as composites.

Now instead, assume that the entire $N = 6$ supermultiplet consists of preons with the exception of the SU(8) singlet - the graviton; assume that preons bind into heavy composites through the operation of forces represented by the $N = 6$ super-Lagrangian and into "light" composites, through the effective electronuclear type of force propagated by the composite gauges $Q_{\mu}$ of SU(8). Assume that this SU(8) will contain (and also spontaneously break into) the physical SU(5) x SU(3) family. The question now is: what are the "light" preonic composites? Since the composite $63 Q_{\mu}^B$ are expected to be massless gauge particles, clearly the other light composites should belong to the supermultiplet to which these 63 particles can be assigned. One could then examine what else would be contained in the supermultiplet of which $Q_{\mu}^B$ are members; does it contain, in particular, light spin $\frac{3}{2}$ composites, identifiable with three fermion families of $5 + \overline{10}$ of SU(5). Is this supermultiplet unique?

Now EGMZ have conjectured that the following may be the supermultiplet to which $Q_{\mu}^B$ belong:

$$\left[ \frac{1}{2} \right]_{A}^{A}, \left[ 1 \right]_{B}^{A}, \left[ \frac{3}{2} \right]_{[BC]}^{A}, \left[ 0 \right]_{[BC]}, \ldots \left[ -\frac{3}{2} \right]_{A}^{A}$$

+ TCP conjugate $$\left[ \frac{1}{2} \right]_{A}^{1}, \left[ 2 \right]_{A}, \left[ \frac{3}{2} \right]_{[BC]}^{A}, \ldots \left[ -\frac{3}{2} \right]_{A}^{1}$$

This multiplet contains a whole variety of objects of spins greater than one. Using the preonic ansatz stated earlier, we shall right-away assume that all composites of spins greater than one are superheavy (Planck mass). To select out the light spin $\frac{3}{2}$ composites from among the irreducible SU(8) representations $5 \oplus 56 \oplus 216 \oplus 216$ which are contained in this supermultiplet EGMZ start with assuming that the "trace parts" of the multiplet ($5 \oplus 56$) are also superheavy. Out of the remaining $216 \oplus 216$, they then select the maximal SU(5) anomaly free set, such that colour and electric charge are vector-like. Using these and certain other criteria, they claim that finally within $216 \oplus 216$ there are left just three SU(5) multiplets ($10 + \overline{5}$), which may qualify as light spin $\frac{3}{2}$ composites - and which just correspond to the three known families of quarks and leptons.

EGMZ have been criticised by Derendinger, Ferrara and Savoy (1981), who find no convincing reason why for example the "trace" multiplets were left out of consideration, nor why SU(8) should break into SU(5) x SU(3). They themselves, adopting somewhat different criteria, motivate a two-family set of light-composites of spin-$\frac{3}{2}$ emerging from a rather peculiar set of SU(8) multiplets $\frac{5}{2} + (\frac{1}{2} + \frac{3}{2}) + (\frac{3}{2} + \frac{1}{2})$ with five $\overline{5}$'s. Ellis, Gaillard and Zumino (unpublished) have attempted to show that those spin-$\frac{3}{2}$ objects which are contained in the $216 \oplus 501$, and which they had earlier discarded are in fact swallowed up by higher spin representations, to give to the latter, their (large) masses. And there the matter rests at present, with surely more to come in this exciting $N = 8$ supergravity preonic story.

1.5 Infinites in Extended Supergravity Theories

We saw that one of the attractive features of supersymmetric theories is the mildness of their infinities, as exemplified by the vanishing of the three loop $\gamma s$ for $N = 4$ extended Yang-Mills. What is the situation for extended supergravities?

* An alternative descent of SU(8) into a singlet-family, grand-unifying SU(4)$_{ flavour } \times$ SU(4)$_{ colour }$ mentioned in 2.2(e) may also be envisaged.

** Is the photon a composite field? Is charge conservation spontaneously violated? Does the photon have a mass and if so, is the mass related to $R^{-1} \approx 10^{-14}$ GeV or to the energy scale where the eleven dimensions compactify to four? Why does this compactification take place?
There are three types of infinities which have been investigated.

(a) On-shell S-matrix elements. These are one-loop finite for all
N ≤ 8, and most likely also two-loop finite. (This is assuming that
duality transformations of the theory continue to hold, notwithstanding
quantum corrections and there are no unexpected anomalies). For eight loops or
higher, there do exist counter terms which may signal the possible
existence of infinities for N = 8. The issue of whether such infinities
are absent or not can unfortunately be decided only by an actual calculation.*

(b) Assume that for all N, a Yang-Mills supersymmetric coupling of
the spin-one fields in the theory can be carried through. (As stated
before, such theories have explicitly been constructed up to and including
N = 5; the new couplings (parameter g) include a cosmological term with
λ = g^2/α^2. In β(g) = 0 for such theories; equivalently is there no
infinite renormalisation of the cosmological constant? If there is not, the
empirically desirable value λ = 0 is stable against renormalisation.

This problem was first addressed by Christensen, Duff, Gibbons and
Rocek (1980) and then by Curtright (1981). Their one-loop result is that
β = 0 for N = 5, 6, 7, 8. Curtright's proof has already been given when
we were discussing N = 8 Yang-Mills extended supersymmetry. To apply
his formulae, say for N = 8 note that:

\[ B = \sum_{S} \frac{2}{96n^2} \frac{1}{2} \text{ helicity } C \text{ C}(S) (1 - 12S^2) (-1)^{2S} \]

the summation being over the quadratic Casimirs C(S) of the
appropriate SU(8) multiplets as well as over the helicities, comprised
in the N = 8 supermultiplet. The appropriate C(S)'s are given in the

* As Kallosh has shown, counter terms at three loop level exist for the linear
N = 8 theory; they may however disappear when the full non-linear theory is
considered.

following table:

<table>
<thead>
<tr>
<th>N = 8</th>
<th>helicity (S)</th>
<th>D(S)</th>
<th>C(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>±2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>±3/2</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>±1</td>
<td>28</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>±3/2</td>
<td>56</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>70</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>±8</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Curtright finds in fact that for N > 4 any supermultiplet gives vanishing
convective and magnetic contributions individually to β for all internal
SO(N) (and also for any "hidden" internal SU(N), like SU(8) of Cremer and Julia).
This means that one-loop β = 0, also for the Ellis, Gaillard, Malan, Fan,
Sumino composite super-multiplet.

(c) A third type of one-loop infinity investigated by Duff and Van
Nieuwenhuizen (1980) is the Euler infinity which may arise as a renormal-
isation of the Euler number

\[ x = \frac{1}{32\pi^2} \int d^8 x \sqrt{g} \left( R_{\mu\nu\rho} R^{\mu\nu\rho} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \]

This infinity is connected with the trace anomaly in supergravity theories.
The result of the calculations shows that one loop infinity is absent for
all N > 3. The important remark (for example for N = 8 extended super-
gravity) is that a naive calculation would not have given a zero result.
One must take proper account of the Lorentz-character of the scalar fields
in the theory. To explain, when a descent is made from 11 dimensions to
two, the 70 spin-zero fields really appear as 63 + 7\delta_{uv} + 14_{uv} where
$\delta_{\mu}^\nu$ is two-index antisymmetric and $\delta_{[\mu}^\nu$ three-index anti-symmetric. The two-index anti-symmetric $\delta_{\mu}^\nu$ can be shown to be equivalent to a scalar field for all purposes except for the computation of its trace-anomaly; likewise $\delta_{[\mu}^{\nu]}$ is trivial except for its anomaly contribution. Once this is properly taken into account, the overall $N=8$ trace-anomaly vanishes and with it the possible infinity associated with the renormalisation of the Euler Number. One can but marvel how supergravities manage to defeat infinities, in the examples considered. This must be connected with the essential geometry behind the supergravity theories - a subject which we are painfully and slowly beginning to understand and one which I could not emphasise in this brief report.*

Clearly one's first reaction at the absence of infinities in supergravity is one of rejoicing. One must remember however that in a conventional renormalisable theory, the structure of the infinities and the high-energy behaviour of a renormalisable theory are intimately related. Now even if the S-matrix in the $N=8$ extended supergravity theory is loop by loop finite, it is unlikely that its high-energy behaviour for 1-loops would have been drastically improved from what one expects for normal gravity theory (i.e. for large $\mathcal{N}$ S-matrix elements $\propto e^{-1}\mathcal{O}^{1/3}$, $\mathcal{N}$ = number of loops, $\mathcal{O}$ = number of external lines for the graph.)

If one believes that all theories, including supergravity, should exhibit Froissart boundedness for cross-sections - and this may be questioned - either a loop-summation should now be carried out, or one must hope that the "running" gravitational constant $\kappa(E)$ - if this can be defined in a renormalisation group sense - runs like $\frac{1}{E}$ for large $E$. In this case, S-matrix elements would indeed behave in the Froissart manner we have come to expect for normal theories (i.e. $\kappa(E) \propto E^{\mathcal{O}^{1/3}} = E^{1-\mathcal{O}}$).

How can one use renormalisation group technology for estimating the running constant $\kappa(E)$? Is the use of such a technology even necessary? Could one devise other methods for summing successive loop contributions? The renormalisation group approach to gravity theories was motivated some while back by Juive and Tonin (1978) and by Salam and Strathdee (1978). In the language of supergravity, one may write an (extended) conformal supergravity Lagrangian which contains $g^2 R^2$ like terms plus a Poincare supergravity term $R/K^2$.

On a power counting basis, as is well known, such a theory is conventionally renormalisable; its failing is the presence of ghosts. These can be made arbitrarily massive by letting the coupling constant $g$ (in the $g^2 R^2$ like term) tend to zero after one has solved the renormalisation group equations. One may ask under what conditions is this limit $g \rightarrow 0$ permissible? Interestingly, one of these conditions would be $\delta(g) = 0$.

What I am saying is that we would welcome extended conformal supergravity theories to ensure that this particular S-function vanishes. The $g/e^2$ term which acts like a mass-term when added to such a theory may still need a renormalisation of $e^2$. We conjecture that the renormalisation group machinery may then show, that $\kappa(E) \propto \frac{1}{E}$. I realise that there is much tortuousness and wishful thinking in this conjecture but it may be interesting all the same to compute one-loop corrections for an extended conformal supergravity theory to see if there is any basis for entertaining the hope that the relevant

* See Salam 1978 Tokyo Conference, where a review of the fermionic extensions of space-time (superspace) in relation to supergravity is presented.)

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$\mathcal{E}(g)$ vanishes. At the very least the limit $g \to 0$, which can then be taken, will act as a regulariser for the physical theory.*

To conclude, supergravity theories are attractive as field theories, and

on account of their superior finiteness. The $N = 8$ supergravity is attractive in
combining in one gauged multiplet, the elementary particles and the elementary
forces. Most important of all, it is attractive because it seeks the meaning of

elementary charges it employs, within the still more elementary construction of
an extended space-time structure with eleven bosonic dimensions. Among these
charges are included the "fermionic charges" for which the appropriate space time
extension may be the fermionic dimensions of a superspace. There is, however just
dynamical

one mass scale in the theory ($M_{\text{Planck}}$); the severe/problem of deducing all the
other masses in terms of it, is left to the future.

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* What, if any, are the direct experimental tests of supergravities? One such
test was suggested by J. Scherk; antigravitational force of repulsion between
all matter, caused by spin-one partners (gravity,photons) of gravitons. Such a
force would be short-range if gravitophotons are massive. If however this mass
is tiny, anti-gravity might manifest itself over laboratory distances. After
examining records of all experiments performed to verify Newtonian law of
gravitation, and also examining the limits that could come from the known accuracy
of the equivalence principle experiments, Scherk concluded that antigravitational
effects may indeed exist with a range of $10^2$-10$^3$ m. For details, see the
tragically posthumous record of Scherk's talk given at the Europhysics Study
Conference held in Erice, Sicily, March 1980.

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5. EXPERIMENTAL OUTLOOK FOR PARTICLE PHYSICS

I wish to end with a remark on the experimental outlook for testing
the ideas we have been expressing. And one must confess that it is bleak.

There are four types of experiments which are presently yielding data
on particle physics:

(a) Accelerator Experiments; (b) Cosmic Ray Experiments; (c) Non-Accelerator
Experiments and finally (d) Cosmological Data. Consider the prospects for
each in turn.

(a) Accelerators: Let us assume the FF-collider, the Tevatron,
Isabelle and Lep are available for experimentation during at least part
of the decade. We shall then be well off in the TeV range of energies.
In the decade after, between 1990-2005, one may envisage the possible
installation of a FF collider in the Lep tunnel and the construction of
the supertevatron. With superconducting technology these might optimist-
ically reach 10 TeV, centre of mass. What happens to the subject twenty-
five years from now, around 2005, when most of you in the audience would
still be in your prime?

For definiteness, let us consider reaching 100 TeV - the presently
accepted inverse radius of the muon, as revealed by limits on $\nu \to e + \gamma$.
With present accelerator technology we shall have reached a saturation
(1) in the CERN and Fermi-laboratory sites (2) in available funds and
(3) most crucially in ideas for further machine design. Which, let us
gratefully recall were created for our generation by far-seeing men
twentyfive years ago.

We desperately need, on a 25-year perspective, new ideas on
accelerator design. To emphasise this point, let us remember that
present designs are limited by the gradients of accelerating fields,
These presently attain values around ~ 1.2 MV/metre and will improve to ~ 5 MV/metre with superconducting magnets. If a credible design using lasers, for example, could be made available, \( E_{\text{acc}} \) could register values of the order of 6V/metre. (Willis at CERN has considered collective ion effects, which promise field gradients of the order 30V/metre; Palmer estimates 20V/metre using surface effects of a grating; this figure rising to 200V/metre if gratings were permitted to be destroyed at each pulse.)

If such designs could become reality - and one must not underestimate their difficulties - (laser wave-lengths are in the micron region) - a 100 TeV accelerator need be no longer than ~ 30 KM; perhaps even as compact as 5 KM.

What I am trying to emphasize is that accelerators may become extinct as dinosaurs in twenty-five years, unless our Community takes heed now and invests effort on new design.

(b) Cosmic Ray Experiments: The highest possible cosmic-ray energies on earth unfortunately do not exceed 100 TeV (centre of mass). The global cosmic-ray detection effort produces no more than 300 events/year at this energy and no more than 2000 events/year at 10 TeV (centre of mass). These numbers would increase by a factor of 10 if there was a 100 KM coverage with detection devices - certainly worthwhile until a 100 TeV accelerator becomes available, but no substitute for investment in new accelerators and their design.

(c) Non-Accelerator Experiments which include (i) search for proton-decays (ii) search for M-H oscillations (iii) neutrino mass and oscillation experiments, involving reactors and (iv) search (also geo-chemical) for neutrino-less double \( \beta \) decay are likely to provide some of the most eagerly awaited information on the distribution of intermediate mass scales. For example, each of the proton-decay modes

\[ P \rightarrow e^+ + \nu^0, P \rightarrow e^- + \nu^+ + \nu^-, P \rightarrow 3\nu + \nu^0 \text{ and } P \rightarrow 3\nu + \nu^+ \]
REFERENCES


Klein, O. 1926, Z. Phys. 37, 895.


Pauli, W., 1933 Rev. dcer Phys. 18, 337.


F.A. KUMAJA and A.T. KATCHELSK - The experimental study of establishing local order in binary metallic solid solutions.

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A. QAUD - A criticism of Tivari's paper on coupled zero mass and electromagnetic fields.

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G. PASCOE and G. SENATORE and M.P. TOSI - Electric resistivity and structure of liquid alkali metals and alloys as electron-ion plasmas.