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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

DYNAMICAL MASS GENERATION IN $(U_{\gamma}(1) \times U_{p}(1))^{2}$ *

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ABSTRACT

The mass matrix for a pair of fermions interacting through chiral gauge fields is computed by solving approximate Dyson-Schwinger equations.

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We attempt to compute the mass matrix including mixing angles, for a pair of fermions interacting with gauge fields. What we have in mind is a system of two families say the e and the p families of fermions. Each is gauged separately e.g. $(SU(3) \times SU(2) \times U(1))_{\mathbb{S}^{\times}} (SU(3) \times SU(2) \times U(1))_{\mathbb{S}^{\times}}$ At the present low energies only the diagonal sum of the two types of gauges manifests itself. We wish to compute the general fermion mass matrix dynamically, the only input being chiral symmetry for the sauge mesons; e.g. our colour group is not just $SU(3) \ge SU_{1}(3)$ but $[SU_{1}(3) \times SU_{2}(3)] \ge xe + u$. To simplify the discussion we consider just two fermions, and the colour group as U(1). No Higgs field is coupled to the fermion fields and the assumed gauge symmetry precludes an elementary mass term for the fermions. In such a system one can hope to find (dynamical) masses only by going outside the framework of perturbation theory. Now it is well known that perturbative approximations to the effective action preserve the symmetry of the classical action (apart from the superficial complications associated with gauge fixing) so that if fermionic masses are excluded by symmetry considerations at the classical level then the same will be true at every finite order of a perturbative calculation. One must go to infinite order and, in practice. this means attempting to solve a Dyson equation. Apart from the technical difficulty there is also a problem of ultraviolet divergences: parameters like the fermion masses tend to be divergent and therefore uncomputable. However, as was shown in a recent note there do occur examples of gauge theories where the divergences cancel and the fermion masses become in principle computable. The technical difficulties remain severe, of course, in particular the matter of gauge dependence. Since the Dyson equations operate with offshell and therefore gauge dependent quantities, propagators and verter functions, it is a highly non-trivial problem to extract any gauge-independent information from them. We ignore this problem here.

The two Dirac spinors ψ_1 and ψ_2 interact with gauge fields W_{1L} , W_{1R} , W_{2L} , W_{2R} . In terms of two-component spinors

$$\Psi_{L} = \begin{pmatrix} \Psi_{1} \\ \Psi_{2} \end{pmatrix}_{L} , \qquad \Psi_{R} = \begin{pmatrix} \Psi_{1} \\ \Psi_{2} \end{pmatrix}_{R}$$

and gauge fields

$$W_{\underline{L}} = \begin{pmatrix} w_{\underline{L}} & o \\ o & W_{\underline{L}} \end{pmatrix} , \qquad W_{\underline{R}} = \begin{pmatrix} w_{\underline{L}R} & o \\ o & W_{\underline{L}R} \end{pmatrix}$$

the Lagrangian takes the form

$$d = \Psi_{L} \left(* \vartheta - g W_{L} \right) \psi_{L} + \Psi_{R} \left(* \vartheta - g W_{R} \right) \psi_{R}$$

$$-\frac{1}{4} \operatorname{Tr} \left(W_{L\mu\nu}^{2} + W_{R\mu\nu}^{1} \right) + \operatorname{gauge} \operatorname{hxing}$$

$$+ \frac{M_{1}^{2}}{4} \left(W_{L} - W_{LR} \right)^{2} + \frac{M_{1}^{2}}{4} \left(W_{2L} - W_{2R} \right)^{2} + \frac{M_{2}^{2}}{4} \left(W_{LL} + W_{1R} - W_{2L} - W_{2R} \right)^{2} .$$

$$(1)$$

Here the gauge fixing terms have not been specified. It is assumed that the gauge symmetry breaks spontaneously to U(1) so that three of the four gauge fields acquire mass by the Higgs mechanism. We are not interested in the details of the Higgs system and have suppressed all scalar field-containing terms. (In general there can be scalar vector mixing terms which affect the structure of the vector propagators and should therefore be taken into account. Here, for simplicity, we shall adopt the Landau gauge where such couplings do not occur.)

The gauge propagators are easily deduced from those of the orthogonal combinations

$$A_{1} = \frac{1}{\sqrt{2}} (W_{1R} - W_{1R})$$

$$A_{2} = \frac{1}{\sqrt{2}} (W_{2L} - W_{2R})$$

$$V_{3} = \frac{1}{2} (W_{1L} + W_{1R} - W_{2L} - W_{2R})$$

$$V_{0} = \frac{1}{2} (W_{1L} + W_{1R} + W_{2L} + W_{2R})$$
(2)

which invert to give

$$W_{1L} = \frac{1}{\sqrt{2}} A_1 + \frac{1}{2} V_3 + \frac{1}{2} V_0$$

$$W_{2L} = \frac{1}{\sqrt{2}} A_2 - \frac{1}{2} V_3 + \frac{1}{2} V_0$$

$$W_{1R} = -\frac{1}{\sqrt{2}} A_1 + \frac{1}{2} V_3 + \frac{1}{2} V_0$$

$$W_{2R} = -\frac{1}{\sqrt{2}} A_2 - \frac{1}{2} V_3 + \frac{1}{2} V_0 \qquad (3)$$

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In the Landau gauge, then

$$\langle T W_{1L\mu} W_{1R\nu} \rangle = -\frac{i}{2} \langle T A_{1\mu} A_{c\nu} \rangle + \frac{i}{4} \langle T V_{3\mu} V_{3\nu} \rangle + \frac{i}{4} \langle T V_{c\mu} V_{a\nu} \rangle$$

$$= \frac{\hbar}{4} \left(\eta_{\mu\nu} - \frac{k_{\mu} k_{\nu}}{k^{2}} \right) \left(-\frac{i}{2} \frac{i}{k^{2} - M_{1}^{2}} + \frac{i}{4} \frac{1}{k^{2} - M_{3}^{2}} + \frac{i}{4} \frac{1}{k^{2}} \right)$$

$$= \frac{\hbar}{4} \left(\eta_{\mu\nu} - \frac{k_{\mu} k_{\nu}}{k^{2}} \right) \frac{1}{k^{2}} \left(\frac{-M_{1}^{2}/2}{k^{2} - M_{1}^{2}} + \frac{M_{3}^{2}/4}{k^{2} - M_{3}^{2}} \right)$$

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$$= -\frac{1}{4} \langle T V_{3\mu} V_{3\nu} \rangle + \frac{1}{4} \langle T V_{0\mu} V_{0\nu} \rangle$$
$$= \frac{1}{4} \left(\eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^{2}} \right) \left(-\frac{-M_{3}^{2}/4}{k^{2} - M_{3}^{2}} \right) \frac{1}{k^{2}}$$

$$\langle TW_{2L\mu} W_{2L\nu} \rangle = -\frac{1}{2} \langle TA_{*\mu} A_{2\nu} \rangle + \frac{1}{4} \langle TV_{3\mu} V_{3\nu} \rangle + \frac{1}{4} \langle TV_{0\mu} V_{0\nu} \rangle$$

$$= \frac{t}{i} \left(\eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^{i}} \right) \left(-\frac{1}{2} \frac{1}{k^{2} - M_{s}^{i}} + \frac{1}{4} \frac{1}{k^{2} - M_{s}^{i}} + \frac{1}{4} \frac{1}{k^{2}} \right)$$

$$= \frac{t}{i} \left(\eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^{i}} \right) \frac{1}{k^{2}} \left(-\frac{M_{s}^{2}/2}{k^{2} - M_{s}^{i}} + \frac{M_{s}^{2}/4}{k^{2} - M_{s}^{i}} \right) .$$

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Since the object of the calculation is to compute the fermion mass matrix, μ , we must use this matrix in the parametrization of the fermion propagator. To obtain the appropriate expressions for the propagator components consider the effective Lagrangian

 $\mathcal{I}_{\text{Fermi}} = \overline{\Psi}_{\text{L}} \not = \Psi_{\text{L}} + \overline{\Psi}_{\text{R}} \not = \Psi_{\text{R}} - \overline{\Psi}_{\text{L}} + \Psi_{\text{R}} - \overline{\Psi}_{\text{R}} + \Psi_{\text{L}} - .$

The mass matrix can be represented in the canonical form

$$\nu = UmV^{-1}, \qquad (5)$$

(6)

where U and V are unitary and m is real and diagonal. It is straight-foward to derive the expressions

$$\left\langle T \ \psi_{L} \ \overline{\psi}_{L} \right\rangle = -\frac{\hbar}{i} \frac{(1+iY_{3})}{2} \ \mathcal{P} \quad \mathcal{U} \left(p^{2} - m^{2} \right)^{-1} \mathcal{U}^{-1}$$

$$= -\frac{\hbar}{i} \frac{(1+iY_{3})}{2} \ \mathcal{P} \quad \left(p^{2} - \mu \mu^{4} \right)^{-1}$$

$$\left\langle T \ \psi_{L} \ \overline{\psi}_{R} \right\rangle = -\frac{\hbar}{i} \frac{(1+iY_{3})}{2} \ \mathcal{U} \quad m \left(p^{2} - m^{2} \right)^{-1} \mathcal{V}^{-1}$$

$$= -\frac{\hbar}{i} \frac{(1+iY_{3})}{2} \ \mu \left(p^{2} - \mu^{4} \mu \right)^{-1}$$

$$= -\frac{\hbar}{i} \frac{(1+iY_{3})}{2} \ \left(p^{2} - \mu^{4} \mu \right)^{-1} \mu ,$$

etc. In the approximation where corrections to the vector propagators and the vertices are discarded, the Dyson equation for the fermion propagator takes the form

$$\Sigma(\mathbf{p}) = -\frac{y^{2}\mathbf{h}}{\epsilon} \int \frac{d^{\mathbf{a}}\mathbf{k}}{(2\pi)^{\mathbf{a}}} \gamma_{\mathbf{p}} \mathbf{t}^{\mathbf{a}} \left(\mathbf{p} - \mathbf{k} - \Sigma \right)^{\mathbf{a}} \gamma_{\mathbf{p}} \mathbf{t}^{\mathbf{b}} D_{\mathbf{p}\mathbf{p}}^{\mathbf{a}\mathbf{b}} \left(\mathbf{k} \right).$$
(7)

In this equation the vertices derive from the interaction Lagrangian

in which the matrices t^a generally include the chiral projector $(1 \pm i\gamma_5)/2$. The free vector propagators (4) are included in the general expressions

$$\langle T W_{\mu}^{a} W_{\nu}^{b} \rangle = -\frac{\hbar}{i} D_{\mu\nu}^{ab}(k)$$
 (8)

The complete fermion propagator is written

$$\langle \mathfrak{T} \psi \overline{\psi} \rangle = -\frac{\hbar}{1} (\not p - \mathfrak{I}(p))^{-1}$$
 (9)

The diagonal terms (a = b) in the vector propagator $D_{\mu\nu}^{ab}(k)$ must be $O(k^{-2})$ for $k \neq \infty$ and they will be associated with ultraviolet divergent terms in the Dyson equation. Off-diagonal terms D^{ab} will be $O(k^{-4})$ if the corresponding element of the mass matrix $(M^2)^{ab}$ is non-vanishing, otherwise they will decrease even faster. All such terms contribute finite pieces to the right-hand side of (7). If the equation is decomposed into chiral pieces then the mass term Σ_{LR} will be given by a convergent integral if there is no component among the vectors which couples to both left and right spinors, i.e. if the gauging is fully chiral. (The kinetic terms Σ_{LL} and Σ_{RR} must inevitably diverge and be subtracted by wave function renormalizations.)

If the mass matrix is determined by a convergent integral equation then, at least in principle, one can compute it. That is one can express the fermion masses in terms of a set of given vector boson masses. This is a more modest programme than the fully self-consistent generation of dynamical symmetry breakdown in which the vector masses are also to be computed by exploiting the appropriate Dyson equations.

To obtain an estimate for the mass matrix μ we set p = 0 in (7) and in the integrand let Σ be represented just by the mass matrix itself,

$$\Sigma(k) \simeq \frac{1-i\gamma_5}{2} \mu + \frac{1+i\gamma_5}{2} \mu^{\dagger} \ . \label{eq:sigma}$$

It is a simple matter to eliminate the Dirac matrices and extract the equation

$$\mu = -\frac{q^{2}\hbar}{i} \int \frac{d^{2}k}{(2\pi)^{4}} t^{a} \mu (k^{2} - \mu^{\dagger}\mu)^{-1} t^{b} D^{ab}_{\mu \mu}(k)$$
(10)

$$\mu = U m V^{-1}$$

$$= \begin{pmatrix} \cos\theta_{L} & -\sin\theta_{L} \\ \sin\theta_{L} & \cos\theta_{L} \end{pmatrix} \begin{pmatrix} m_{L} & 0 \\ 0 & m_{2} \end{pmatrix} \begin{pmatrix} \cos\theta_{R} & \sin\theta_{R} \\ -\sin\theta_{R} & \cos\theta_{R} \end{pmatrix}$$
(13)

(any complex phases in μ can be absorbed by appropriately re-defining ψ_L and ψ_R). It is convenient to introduce the rotated coupling matrices

$$t^{\frac{1}{2}}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} t^{\frac{1}{2}} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
$$\begin{pmatrix} \cos^{2}\theta & -\sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin^{2}\theta \end{pmatrix}$$

and

$$t^{2}(\theta) = \begin{pmatrix} \sin^{2}\theta & \sin\theta\cos\theta \\ \\ \sin\theta\cos\theta & \cos^{2}\theta \end{pmatrix}.$$
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Eq.(12) now takes the form

$$m = \frac{3}{9} \frac{\alpha}{\pi} \left[t^{i}(\theta_{L}) \frac{2 M_{i}^{2} m}{M_{i}^{2} - m^{2}} l_{n} \frac{M_{i}^{2}}{m^{2}} t^{i}(\theta_{R}) + t^{2}(\theta_{L}) \frac{2 M_{i}^{2} m}{M_{i}^{3} - m^{2}} l_{n} \frac{M_{i}^{2}}{m^{1}} t^{2}(\theta_{R}) - (t^{i}(\theta_{L}) - t^{2}(\theta_{L})) \frac{M_{i}^{2} m}{M_{j}^{2} - m^{2}} l_{n} \frac{M_{i}^{2}}{m^{3}} (t^{i}(\theta_{R}) - t^{2}(\theta_{R})) \right],$$
(15)

where m is a diagonal matrix. There are four distinct equations here for the four unknowns, m_1 , m_2 , θ_L and θ_R . To solve them we shall make some approximations. Thus, we shall assume that the fermion masses are small relative to the vector masses

$$m/M \ll 1 \qquad (16)$$

Then (15) reduces to

where it is understood that
$$t^a$$
 (a = 1,2) refers to W_L and t^b (b = 1,2) to W_R . The propagator components D^{ab} needed for (10) are just those listed in (4). On making the substitutions,

$$\begin{split} \mu &= 3 \frac{q^{2}k}{i} \int \frac{d^{4}k}{(2\pi)^{4}} \left[t^{1} \mu \left(k^{2} - \mu^{\dagger} \mu \right)^{-1} t^{1} - \frac{1}{k^{2}} \left(-\frac{-M_{1}^{2}/2}{k^{2} - M_{1}^{2}} + \frac{M_{3}^{2}/4}{k^{2} - M_{3}^{4}} \right) + \right. \\ &+ \left(t^{1} \mu \left(k^{2} - \mu^{\dagger} \mu \right)^{-1} t^{2} + t^{2} \mu \left(k^{2} - \mu^{\dagger} \mu \right)^{-1} t^{1} \right) \frac{1}{k^{2}} \left(-\frac{-M_{3}^{2}/4}{k^{2} - M_{3}^{3}} \right) + \\ &+ t^{2} \mu \left(k^{2} - \mu^{\dagger} \mu \right)^{-1} t^{2} - \frac{1}{k^{2}} \left(-\frac{-M_{2}^{2}/2}{k^{2} - M_{2}^{3}} + \frac{M_{3}^{2}/4}{k^{2} - M_{3}^{3}} \right) \right] . \end{split}$$

$$(11)$$

Make a Wick rotation $k_0\to ik_{l_1}$ and integrate out the angles, i.e. write $d^{l_1}k=2\pi^2k^2dk^2$ to obtain

$$\mu = \frac{3 q^{2} k}{9 \pi^{2}} \int_{0}^{\infty} dk^{2} \left[t^{1} \mu \left(k^{2} + \mu^{t} \mu \right)^{-1} t^{1} \left(\frac{M_{1}^{2}/2}{k^{2} + M_{1}^{2}} - \frac{M_{2}^{2}/4}{k^{2} + M_{3}^{2}} \right) \right. \\ \left. + \left(t^{1} \mu \left(k^{1} + \mu^{t} \mu \right)^{-1} t^{2} + t^{2} \mu \left(k^{2} + \mu^{t} \mu \right) t^{1} \right) \frac{M_{2}^{2}/4}{k^{2} + M_{3}^{2}} \right. \\ \left. + t^{2} \mu \left(k^{2} + \mu^{t} \mu \right) t^{2} \left(\frac{M_{1}^{2}/2}{k^{2} + M_{3}^{2}} - \frac{M_{2}^{2}/4}{k^{2} + M_{3}^{2}} \right) \right] \right] \\ = \frac{3}{9} \frac{\alpha'}{\pi} \left[t^{1} \mu \frac{2M_{1}^{2}}{M_{1}^{2} - \mu^{t} \mu} \ln \frac{M_{1}^{2}}{\mu^{t}} t^{1} + t^{2} \mu \frac{2M_{2}^{2}}{M_{2}^{2} - \mu^{t} \mu} \ln \frac{M_{2}^{2}}{\mu^{t} \mu} t^{2} - \left. - (t^{1} - t^{2}) \mu \frac{M_{3}^{2}}{\mu^{2}_{3} - \mu^{t} \mu} \ln \frac{M_{3}^{2}}{\mu^{t}_{3} - \mu^{t} \mu} \right] ,$$

$$(12)$$

where $\alpha = g^2 \hbar/4$. Alternatively, this equation can be given in terms of eigenvalues and mixing angles. Quite generally, the mass matrix can be expressed in the form

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$$s_{\rm L}^2 = s_{\rm R}^2$$
 (19)

For convenience, in the remaining equations we shall adopt a particular solution

$$\theta_{\rm L} = -\theta_{\rm R} = \theta$$
, (20)

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where 0 is given by

$$0 = -s_{c} (m_{1} + m_{1}) ln \frac{M_{1}M_{2}}{M_{2}^{2}} - s(m_{1} - m_{2}) ln \frac{M_{1}}{M_{2}},$$

i.e.

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$$\cos 2\theta \cdot \frac{m_1 + m_2}{m_1 - m_2} = - \frac{\ln \frac{m_1}{M_2}}{\ln \frac{M_1 M_2}{m_3^*}}$$
(21)

The remaining two equations are

$$m_{1} = \frac{3}{9} \frac{\alpha'}{\pi} \left[m_{1} \ln \frac{M_{1}M_{2}}{m_{1}^{*}} + 2 c m_{1} \ln \frac{M_{1}}{M_{2}} + (c^{2}m_{1} - s^{2}M_{2}) \ln \frac{M_{1}M_{2}}{M_{2}^{*}} \right]$$

$$m_{2} = \frac{3}{9} \frac{\alpha}{\pi} \left[m_{2} \ln \frac{M_{1}M_{2}}{m_{2}^{*}} - 2 c m_{2} \ln \frac{M_{1}}{M_{2}} + (-s^{2}m_{1} + c^{2}m_{2}) \ln \frac{M_{1}M_{2}}{M_{2}^{*}} \right]$$
(22)

To simplify these we make an assumption about the relative magnitudes of the vector masses,

$$\left| \ln \frac{M_1 M_2}{M_3^2} \right| \ll \left| \ln \frac{M_1}{M_2} \right|$$
(23)

since this serves to decouple the equations (22). They give

$$m_1^2 = M_1^{1+2C} M_2^{1-2C} e^{-\frac{9\pi}{3\alpha}}$$

 $m_1^2 = M_1^{1-2C} M_2^{1+2C} e^{-\frac{9\pi}{3\alpha}}$

$$m = \frac{3}{3} \frac{\alpha'}{\pi} \left[t_{L}^{1} 2m \ln \frac{M_{1}^{2}}{m^{2}} t_{R}^{1} + t_{L}^{2} 2m \ln \frac{M_{2}^{2}}{m^{2}} t_{R}^{2} - (t^{2}-t^{2})_{L} m \ln \frac{M_{3}^{2}}{m^{2}} (t^{1}-t^{2})_{R} \right]$$

or, in detail,

$$\begin{pmatrix} m_{1} & o \\ 0 & m_{2} \end{pmatrix} = \frac{3}{9} \frac{\alpha}{\pi} \begin{pmatrix} m_{1} \ln \frac{M_{1}M_{2}}{m_{1}^{2}} + (C_{L}+C_{R})m_{1}\ln \frac{M_{1}}{M_{2}} + (C_{L}C_{R}m_{1}+S_{L}S_{R}m_{2})\ln \frac{M_{1}M_{2}}{M_{3}^{2}} \\ (-S_{L}C_{R}M_{1}+C_{L}S_{R}m_{2})\ln \frac{M_{1}M_{2}}{M_{3}^{2}} - (S_{L}m_{1}+S_{R}m_{2})\ln \frac{M_{1}}{M_{2}} \\ (-C_{L}S_{R}m_{1}+S_{L}C_{R}M_{3})\ln \frac{M_{1}M_{2}}{M_{3}^{2}} - (S_{R}m_{1}+S_{L}m_{2})\ln \frac{M_{1}}{M_{2}} \\ m_{2}(m\frac{M_{1}M_{2}}{m_{2}^{2}} - (C_{L}+C_{R})m_{2}\ln \frac{M_{1}}{M_{3}} + (S_{L}S_{R}m_{3}+C_{L}C_{R}M_{3})\ln \frac{M_{1}}{M_{2}} \end{pmatrix}$$

$$(17)$$

where $c_L = \cos 2\theta_L$, $c_R \neq \cos 2\theta_R$,...,etc. Consider, firstly,the two off-diagonal components

$$0 = \left(-S_{L}C_{R}\ln\frac{M_{1}M_{1}}{M_{3}^{*}} - S_{L}\ln\frac{M_{1}}{M_{2}}\right)^{M_{1}} + \left(C_{L}S_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{R}\ln\frac{M_{1}}{M_{2}}\right)^{M_{2}}$$
$$0 = \left(-C_{L}S_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{R}\ln\frac{M_{1}}{M_{2}}\right)^{M_{1}} + \left(S_{L}C_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{L}\ln\frac{M_{1}}{M_{2}}\right)^{M_{2}}$$

If m_ and/or m_ is non-vanishing then compatibility requires

$$0 = \left(-S_{L}C_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{L}\ln\frac{M_{1}}{M_{2}}\right) \left(S_{L}C_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{L}\ln\frac{M_{1}}{M_{2}}\right) - \left(-C_{L}S_{R}\ln\frac{M_{1}M_{2}}{M_{3}^{*}} - S_{R}\ln\frac{M_{1}}{M_{2}}\right) \left(C_{L}S_{R}\ln\frac{M_{1}M_{1}}{M_{3}^{*}} - S_{R}\ln\frac{M_{1}}{M_{2}}\right) - \left(c_{L}^{1}S_{R}^{*} - S_{L}^{*}C_{R}^{*}\right)\left(n^{2}\frac{M_{1}M_{2}}{M_{3}^{*}} + \left(S_{L}^{1} - S_{R}^{1}\right)\ln^{2}\frac{M_{1}}{M_{2}}\right) - \left(c_{L}^{1}S_{R}^{*} - S_{L}^{*}C_{R}^{*}\right)\left(n^{2}\frac{M_{1}M_{2}}{M_{3}^{*}} + \left(S_{L}^{1} - S_{R}^{1}\right)\ln^{2}\frac{M_{1}}{M_{2}}\right) - \left(s_{L}^{1} - S_{R}^{*}\right)\left(-Ln^{2}\frac{M_{1}M_{2}}{M_{3}^{*}} + Ln^{2}\frac{M_{1}}{M_{2}}\right).$$
(18)

Since the vector masses are presumably independent we must take

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(24)

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where c is to be obtained by solving the transcendental equation which results on substituting (24) into (21)

$$C = \frac{(M_1/M_2)^{c} + (M_2/M_1)^{c}}{(M_1/M_2)^{c} - (M_2/M_1)^{c}} = -\frac{l_m \frac{M_1}{M_2}}{l_m \frac{M_1M_2}{M_1}}$$
(25)

An amusing feature of this calculation is the emergence of the relation (19) $\sin^2 2\theta_L = \sin^2 2\theta_R$

For the two family system treated here, we started with three spin-one masses and recovered three parameters for the fermion mass matrix. For three families, the number of spin-one masses will be five, while the Fermi mass matrix which we recover contains seven parameters. Thus the results of the calculation will give testable constraints, if the individual fermions are replaced by the known e, μ , τ families.

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- IC/80/64 W. KLONOWSKI: Living protein macromolecule as a non-equilibrium INT.REP. metastable system.

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IC/80/66 A.N. ERMILOV, A.N. KIREEV and A.M. KURBATOV: Spin glasses with non-INT.REP. Gaussian distributions. Frustration model.

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