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Subsequent to the printing of this note it has been pointed out to us by Professor P. Furlan that F. Englert has essentially set forth the same ideas in his lectures at the Cargèse Summer School, 1975, published by Plenum Press, New York and London, edited by M. Levy, J-L. Basdevant, D. Speiser and R. Gastmans. Englert was reporting on the work done by F. Englert, J-M. Frère and P. Nicoletopoulos, Phys. Letters 52B, 433 (1974).

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ABSTRACT

Fermion mass generation is demonstrated through a dynamical mechanism which exhibits a mass hierarchy.

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Fermionic mass is commonly represented in Lagrangian field theories either by direct mass terms or by Yukawa couplings to scalar fields which have the potential to develop non-zero expectation values. Both types involve a priori undetermined parameters. In gauge theories there arises the further problem of explaining why some of the fermions are relatively light (the gauge hierarchy problem [1]). The purpose of this note is to suggest a mechanism, appropriate to a class of gauge theories, which might go some way to solving these problems. Our considerations though couched for a particular group, are more generally applicable.

Consider a gauge theory with a chiral structure such as $SU(3)_L \times SU(3)_R$ (colour) or $SU_L(2) \times SU_R(2)$ (flavour). It is possible to envision a spontaneous breakdown, for example, of the local colour symmetry to $SU(3)_{L+R}$ such that the axial vector gauge fields acquire mass. In bringing this about a number of Higgs fields are of course involved. But suppose that there is no mass term for the fermions. This could be for various reasons: no suitable Higgs field, a discrete symmetry, or simply the absence unrelated to symmetry of a bare mass or Yukawa coupling in the Lagrangian. In all such cases the vanishing fermionic mass will persist in finite orders of perturbation theory. However, it would be unrealistic to trust this perturbative behaviour. Our suggestion is that one must sum an infinite selection of graphs to get a more realistic view. The simplest way to do this is by approximating to the Dyson-Schwinger equations.

Now the crucial point of this note is that the graphs which contribute to fermion mass must contain a line representing the mixed propagator $\langle T\, W_{L\mu}\, W_{R\nu} \rangle$. The components of this propagator decrease like $O(k^{-4})$ at large momentum and, in consequence, the integrals are ultraviolet convergent. No subtraction is needed and the fermion mass is computable in principle *). Of course one meets the usual difficulties of approximation schemes which are not strictly perturbative. In particular, gauge invariance tends to be lost. **)

*) The convergence of the self-mass calculation can also be seen as the result of a partial cancellation between vector and axial vector contributions to the fermion's self energy. This fact was noted by Budini [2] who brought Ref.2 to our notice after the completion of this work.

**) Lack of gauge invariance is a serious weakness of the approximation scheme and must of course be overcome before applying it to realistic models. The gauge problem presents itself for situations where Green's functions are involved; e.g. for many of the renormalization group calculations. A "gauge technique" to overcome this problem was elaborated in Ref.3. However in this note, this technique has not been applied.

To illustrate the main point we use the example of a pair of chiral triplets ψ_L and ψ_R interacting with the colour $SU_L(3) \times SU_R(3)$ gauge fields W_L and W_R. A matrix of Higgs fields, ϕ, in the representation $(3,\bar{3})$ serves to break the symmetry to $SU(3)_{L+R}$. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} = {} & \bar\psi_L \gamma_\mu \left(i\partial_\mu - gW_{L\mu}\right)\psi_L + \bar\psi_R \gamma_\mu\left(i\partial_\mu - gW_{R\mu}\right)\psi_R \\ & + 2\,\mathrm{Tr}\left[\partial_\mu\phi + ig\, W_{L\mu}\phi - ig\phi\, W_{R\mu}\right]^2 - V(\phi,\phi^\dagger) \\ & + 2\,\mathrm{Tr}\left[-\frac{1}{4} W^2_{L\mu\nu} - \frac{1}{2\beta_L}\left(\partial_\mu W_{\mu L}\right)^2 - \frac{1}{4} W^2_{R\mu\nu} - \frac{1}{2\beta_R}\left(\partial_\mu W_{\mu R}\right)^2\right] ,\end{aligned}$$

where $W_L = W_L^\alpha \lambda^\alpha/2$ with the standard SU(3) matrices. The parameters β_L and β_R define the gauge. We assume that the potential V develops a stable minimum at

$$\phi = \frac{M}{g\sqrt{2}} \times \text{unit matrix}$$

so that the axial vector combinations $\frac{1}{\sqrt{2}}(W_L - W_R)$ acquire the mass M while the vectors $\frac{1}{\sqrt{2}}(W_L + W_R)$ remain massless. Note that the Yukawa coupling, $\bar\psi_L\,\phi\,\psi_R$ + h.c., has been omitted. In every finite order of perturbation theory the fermions will remain without mass.

It is straightforward to obtain the gauge field propagators from the above Lagrangian. They are

$$-\frac{i}{\hbar}\left\langle T\, W_{L\mu} W_{L\nu}\right\rangle = \frac{k^2 - \frac{M^2}{2}}{k^2(k^2 - M^2)}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) - \beta_L \frac{k_\mu k_\nu}{k^4}$$

$$-\frac{i}{\hbar}\left\langle T\, W_{L\mu} W_{R\nu}\right\rangle = \frac{-\frac{M^2}{2}}{k^2(k^2 - M^2)}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)$$

$$-\frac{i}{\hbar}\left\langle T\, W_{R\mu} W_{R\nu}\right\rangle = \frac{k^2 - \frac{M^2}{2}}{k^2(k^2 - M^2)}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) - \beta_R \frac{k_\mu k_\nu}{k^4} .$$

It is amusing that in this family of (Feynman-like) gauges the mixed components $\langle T\, W_L W_R\rangle$ do not depend on the parameters β_L and β_R and to the order we are working (see below) the gauge problem is circumvented. However this circumstance is not repeated in other types of gauge.

The fermion self-energy $\Sigma(p) = A(p^2) + \not p\, B(p^2)$ appears in the propagator,

$$S = (\not p - \Sigma(p))^{-1} .$$

It is to be obtained by solving an integral equation. The approximate Dyson-Schwinger equation is presented graphically in Fig.1. Write, for the mixed components,

$$D_{\mu\nu}(k) = \frac{-\frac{M^2}{2}}{k^2(k^2 - M^2)}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) .$$

Then the amplitude $A(p^2)$ is given by the convergent projection of the full equation:

$$-\frac{1+i\gamma_5}{2}\Sigma(p)\frac{1-i\gamma_5}{2} = \frac{\hbar}{i}\int \frac{d^4k}{(2\pi)^4}\, g\gamma_\mu \frac{1-i\gamma_5}{2}\frac{\lambda^\alpha}{2}(\not p - \not k - \Sigma)^{-1} g\gamma_\nu \frac{1+i\gamma_5}{2}\frac{\lambda^\alpha}{2} D_{\mu\nu}(k)$$

in the form: *)

$$-A(p^2) = \frac{g^2\hbar}{i}\int \frac{d^4k}{(2\pi)^4}\frac{A((p-k)^2)}{(p-k)^2(1-B)^2 - A^2}\frac{4}{3}D_{\mu\mu}(k) .$$

To obtain an estimate of the fermion mass, m, we reduce this to an algebraic problem by making the simplifying assumptions

$$A(p^2) = m \quad , \quad B(p^2) = 0$$

and taking $p = 0$.

$$-m = \frac{g^2\hbar}{i}\int \frac{d^4k}{(2\pi)^4}\frac{m}{k^2 - m^2}\frac{\frac{3}{2}M^2}{k^2(k^2 - M^2)}\frac{4}{3} .$$

The result is

$$1 = \frac{\alpha_s}{\pi}\frac{M^2}{M^2 - m^2}\ln\frac{M^2}{m^2} ,$$

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*) With the standard normalizations of the SU(3) matrices λ^α there occurs the group-theoretical factor $\Sigma\frac{\lambda^\alpha}{2}\frac{\lambda^\alpha}{2} = \frac{4}{3}$.

where $\alpha_s = g^2\hbar/4\pi$. With the approximations of this note the fermion mass is therefore given by

$$m \approx M \exp(-\pi/2\alpha_s) \quad .$$

Thus the axial gluon mass M is of the order of 10^7 GeV if $\alpha_s \approx \frac{1}{10}$, and the fermionic mass m is ≈ 1 GeV.

In view of the approximations which went into the derivation of this result, and in view of the fact that the starting model itself is not fully realistic *) the numbers above are only a rough guide to the orders of magnitude involved. Also, as noted earlier, the ratio m/M is sensitive to gauge through the factor $D_{\mu\mu}(k)$ in the Dyson-Schwinger equation. (Presumably one is not allowed to replace, as we have done, a full vertex Γ_ν by the point approximation $g\,\gamma_\nu(1 + i\gamma_5)/2$ while at the same time keeping the full propagator $(\not p - \Sigma)^{-1}$ for the fermion.) However, in spite of this serious technical difficulty we believe that the basic computability of fermion mass in this type of gauge theory is a fact of some importance. Further, the particular form of our result lends credence to the idea that light fermions can coexist with heavy gauge mesons in gauge theories. Similar ideas can be used to compute, in principle, Cabibbo and other mixing angles finitely and self-consistently. This will be shown in another note. We wish to thank Professor Jogesh C. Pati for stimulating discussions.

*) For example, we have left out of consideration extra fermions needed to cancel anomalies.

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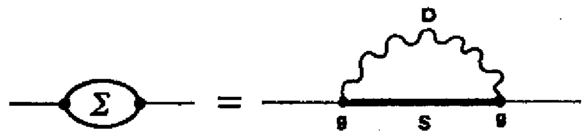

Fig.1 Approximate Dyson-Schwinger equation for the fermion propagator. The fermion self-energy Σ is here expressed in terms of the unmodified vertices $g\,\gamma_\mu(1 \pm i\gamma_5)/2$ and vector propagator $D_{\mu\nu}$, but with the full fermion propagator $S = (\not p - \Sigma)^{-1}$.