INTRODUCTION

Double stripping reactions are very useful in the study of collective levels in nuclei and in extracting such information as spin and parities of the nuclei involved in the reactions. In particular, two neutron transfer reactions involving tritons have been used extensively in obtaining information on pairing correlations in nuclei.

The theoretical formalism for explaining two-nucleon transfer reaction (TNTR) cross-sections depends on whether the reaction is a direct one or not, and on whether the two nucleons are transferred between heavy ions, e.g. \( ^{16}_0 \text{O} \rightarrow ^{16}_0 \text{O} \) or between light projectiles as in \( \text{t} \rightarrow \text{p} \) or \( ^{2}_\text{He} \rightarrow \text{p} \) reactions. In the case of direct two-nucleon transfer involving light projectiles, the distorted wave Born approximation (DWBA) theory is fairly adequate in extracting information on the nucleus involved in the TNTR process \(^1,2\).

One of the several difficulties that could be encountered in the use of DWBA codes is the matching of the theoretical curve with the experimental curve, especially when all the microscopic features of the experimental curve have been adequately reproduced by the codes. Some phenomenological scaling factors have been employed to bring about agreement in some cases for both direct TNTR and direct single nucleon transfer reactions (SNTR). \(^3\) However, barring the use of an inappropriate optical potential or some other faults in the DWBA code, this matching difficulty could also be due to the use of highly simplified and inappropriate interactions or wave functions for the projectiles \((a,b)\), or the use of drastic approximations in these interactions and wave functions as employed in the code. Assuming no drastic approximations are made, the problem could thus be due to the use of inappropriate descriptions for the light particles.

For a direct, core excitation free TNTR \( B(b,a)A \), the nuclear matrix element which occurs in the DWBA transition amplitude can be evaluated approximately without using any optical potential \(^4\). In such a scheme, the transition amplitude can be factorized into (i) a term which is essentially a fractional parentage coefficient, (ii) a form factor which depends on the interactions and wave functions employed in the projectiles, and (iii) a term which is made up of various coupling coefficients that depend on the nuclear model used in describing the wave functions \( \psi_A \) and \( \psi_B \), and the angular momentum coupling scheme used in expanding \( \Phi_A \) in terms of \( \Phi_B \) and the wave function \( \Phi(\Omega_1,\Omega_2) \) of the two transferred nucleons. Thus in order to see the effects of various interactions and wave functions on the exchange-free TNTR cross-section one can compute the form factor term for each case and then compare the results.

---

* To be submitted for publication.

** On leave of absence from Department of Physics, University of Ife, Ile-Ife, Nigeria.

Present address: Physics Division, Argonne National Laboratory, Argonne.
In general, there are several descriptions of the tri-nucleon system in the literature, and one can therefore expect different results in the DWBA code calculations that employ these descriptions, especially since some of these descriptions give different results for the expectation value of the same operator for the same tri-nucleon. It is with this in view that the calculations in this work were embarked upon. These calculations for various descriptions of the light particles could give one a fore-hand knowledge of the relative magnitudes to be expected from a full DWBA code calculation of the TNTR cross-section, in which particular descriptions of the tri-nucleon are utilized.

However, for meaningful results one needs to use interactions and wave functions that represent the tri-nucleon very well, besides avoiding drastic approximations in these potentials and wave functions. For core-independent TNTR we have employed the "point-triton" approximation scheme of DWBA theory to obtain a spectrum of relative magnitudes of cross-sections to be expected from full code calculations.

II. "POINT-TRITON" FORMALISM OF DIRECT TNTR DWBA THEORY

In the absence of the spin-orbit contribution in the two-body interaction, the DWBA cross-section for double-stripping reaction \( 2b(\pi,\mu)A \) is

\[
\frac{d\sigma}{d\Omega} = \frac{A_h^e}{(2\pi)^2} \sum_{J_A, J_B, N, S} \sum_{J, L, S, I} \left| \langle J, L, S, I | P_{\alpha, \lambda}^{(\cos \theta)} | l_{\alpha, \lambda} \rangle \right|^2 \frac{1}{\lambda^2} \Omega_{AB}(\cos \theta)
\]

where \( P_{\alpha, \lambda} \) and \( \Omega_{AB} \) are the distorted waves in the exit and entrance channels, \( \Omega_{AB}^{(\ell)}(l_{\alpha, \lambda}) (\ell = 1,2) \) are shell model states of the two captured nucleons when they are bound in the residual nucleus. \( \pi \) is the position vector of particle \( J \) with respect to an arbitrary origin. \( F_0 \) is an operator which makes \( x \) interchanges between transferred particles 1 and 2; \( V_{\alpha, \lambda} \) is the interaction between a transferred nucleon and the outgoing projectile, while \( \phi_a \) and \( \phi_b \) are internal wave functions of \( a \) and \( b \).

The evaluation of \( \Omega_{AB}^{(\ell)}(l_{\alpha, \lambda}) \) is usually very difficult on account of the nine-fold, non-separable integral involved. Consequently, various approximations such as the local energy approximation or the Chant-Mangelson approximation with a finite range or zero range prescription are made in the evaluation.

Assuming low projectile incident energies, high momentum transfers and high \( Q \) values one may use a zero-range prescription to evaluate \( \Omega_{AB}^{(\ell)}(l_{\alpha, \lambda}) \) thus:

\[
\Omega_{AB}^{(\ell)}(l_{\alpha, \lambda}) = \sum_{\ell_1, \ell_2} \frac{\langle \ell_1, \ell_2 | F_0 | l_{\alpha, \lambda} \rangle}{\sqrt{2\pi}}
\]

where \( \ell_1, \ell_2 \) are the angular momenta of the light particle spectroscopic factor, while \( \eta_0 \) is a fractional parentage coefficient which is a measure of the overlap of the wave functions of \( A \) and \( B \) with the wave function of the transferred nucleons. \( C_{\lambda}(J, M, l, s, \Gamma) \) is a product of various coupling coefficients and transformation brackets.
I'll only attend depending on the quantum numbers \( n_1, l_1 \), \( l' \), \( C \) is a normalization constant to the square of which the TRIH cross-section is proportional. In arriving at Eq.(4), we have used \( \Psi_{l_1}(r) \rightarrow \Psi_{l_1}(r) \cdot \chi_{l_1}^{(c)}(r) \). A close look at Eqs.(3) and (4) shows that \( C_0 \) depends closely on the two-nucleon interaction strength as well as the structure of the tri-nucleon. Consequently, some of the effects of using various prescriptions for the tri-nucleon system on the DWBA code analysis of TRIH data can be studied by computing \( C_0 \) for each case.

III. REPRESENTATIONS OF THE TRI-NUCLEON SYSTEM AND THE POINT-TRITON NORMALIZATION CONSTANT

Various wave functions and two-body interactions have been applied with varying degrees of success to reproduce experimental data for the tri-nucleon systems \(^3\)H and \(^3\)He. In general, for a given nucleus, any two prescriptions that have reproduced one each of two distinct measured data very well may not necessarily give the same results for a third operator. The object of this work is therefore to apply some of the tri-nucleon descriptions found in the literature to determine \( C_0 \) and then compare the results with those from applying Gaussian interactions and wave functions. Thus, for situations where the Gaussian prescriptions do not reproduce the correct normalization or scaling factor, one may find some other suitable combination of interactions and wave functions in the spectrum of values obtained in this or similar analysis.

The Gaussian interaction (GI) employed in this work is

\[
V(r) = \frac{V_0 \rho^b}{\left(1 + \rho^b \right)^6},
\]

where \( V_0 = 22.98 \text{ MeV} \) and \( b = 0.16 \text{ fm}^{-2} \). We applied Gaussian wave functions (GW) of forms

\[
\Psi_{l_1}(r) = C \exp \left( -\frac{r^2}{2a^2} \right) \chi_{l_1}^{(c)}(r).
\]

The evaluation of Eq.(3) is still very formidable and in order to evaluate it one may assume that either the two-body interaction is of zero range [i.e. \( V_{ab} = \delta(P_{ab}) = \nabla(P_{ab}) \)] or that the wave function of the projectile \( b \) is of zero range in which case it can be represented by a delta function. The latter approximation is called "point-triton" approximation and is the approach employed in this work. Under this scheme the kinematical factor \( I_{l_1 l_1 l_1} \) reduces to a single integral, thus

\[
\int I_{l_1 l_1 l_1} = C_3 \int \chi_{l_1}^{(c)}(r) \cdot \chi_{l_1}^{(c)}(r) \cdot \chi_{l_1}^{(c)}(r) \cdot \frac{d^3r}{2^l_1}.
\]

where

\[
C_3 = \frac{2^l_1}{12} \frac{\rho^b}{A B} \left[ \left( \frac{a l_1}{\rho^b} \chi_{l_1}^{(c)}(\rho^{b_1} \chi_{l_1}^{(c)}(\rho^{b_2} \chi_{l_1}^{(c)}(\rho^{b_3} \chi_{l_1}^{(c)}(r)) \right) \left( \frac{2^l_1}{2^l_1} \right)^{l_1} \chi_{l_1}^{(c)}(l) \chi_{l_1}^{(c)}(l) \chi_{l_1}^{(c)}(l) \chi_{l_1}^{(c)}(l).
\]

(5)
The values of the length parameter employed are those that fit the experimental rms radii of $^3$H and $^3$He.

We also employed hard-core interactions (HCI) given by

$$V(r_{ij}) = \frac{1}{4}(2 + \sigma_i^2 \sigma_j^2) V_s(r_{ij}) + \frac{1}{4}(1 - \sigma_i^2 \sigma_j^2) V_s(r_{ij}),$$

(8)

where for even states

$$V_s(r_{ij}) = -A_k e^{-k(r_{ij} - r_c)} \quad \text{for} \quad r_{ij} > r_c = 0 \quad \text{for} \quad r_{ij} < r_c$$

with $k = t, s$ respectively for the triplet and singlet states. $A_k = 4.75, 0.44$ MeV, $\lambda = 235.41$ MeV, $\lambda_s = 2.52$ fm$^{-1}$, $\lambda_s = 2.03$ fm$^{-1}$ and $r_c = 0.4$ fm are the corresponding parameters that fit low-energy scattering data. The hard-core wave function (HWC) used is

$$\psi \propto \chi_{a} \chi_{s},$$

(9)

where

$$\chi_a = \frac{1}{\sqrt{2}} \left[ \alpha(z) \beta(z) - \alpha(z) \beta(z) \right] \alpha(z)$$

$$\psi_s = \begin{cases} N_s \{ e^{i k(r_{ij} - r_0)} - e^{-i k(r_{ij} - r_0)} \} \text{ for } r_{ij} > r_c \\ 0 \text{ for } r_{ij} < r_c \end{cases}$$

$N_s$ is as given in Ref. 6 and $\alpha = 0.4$ fm$^{-1}$, $\alpha = 0.5$ fm$^{-1}$ for $r_c = 0.4$ fm.

The Yukawa interaction (YI) applied is due to Rustgi and is of the form

$$V(r_{ij}) = -\frac{V_0 e^{-K r_{ij}}}{K r_{ij}} \left[ 1 - (y + z) + (y + z) e^{-K r_{ij}} \right],$$

(10)

where $K$ is the Bartlett operator and $(y + z) = 0.155$, $V_0/K = 78.74$, $K = 0.85$ fm$^{-1}$ for fitting low-energy scattering data.

The Irving wave function (IW) employed here is

$$\psi = N_I e^{-\frac{1}{2} (r_{ij}^2 + r_{13}^2 + r_{12}^2 + r_{23}^2)}$$

(11)

with $N_I = 0.0374 a^3$, $a = 1.84$ for $^3$H and $a = 2.27$ for $^3$He. These values reproduce the electromagnetic form factors very well and also give the best results for the Coulomb energies of these nuclei. The value $a = 1.301$ that gave a good result for the binding energy of the triton was also used.

The other wave function employed in this work for the triton is an exponential form due to Srivastava. It is of the form

$$\psi = N_s \left( e^{-\frac{1}{2} \sum r_{ij}^2} + A e^{-\frac{1}{2} \sum r_{ij}^2} \right)$$

(12)

with

$$N_s = \left( \frac{4}{7} \right)^{\frac{1}{2}} \left[ \frac{1}{a^2} + \frac{2/3}{a^2} \right]$$

(13)

The parameters used are $A = 0.1$, $a = 0.88$ fm$^{-1}$; $A = 1$, $a = 0.771$ fm$^{-1}$, $\delta = 1.105$ fm$^{-1}$; $A = 1.305$, $a = 0.732$, $\delta = 1.45$ fm$^{-1}$. They were obtained from variational calculations of the binding energy of the triton.

For most of the cases considered, transformations from the $(x_i, x_{12}, x_{23})$ co-ordinates to $(r_{13}, r_{12}, r_{23})$ co-ordinates are first affected in order to calculate $C_0$. Then another set of transformations is carried out followed by further transformations

$$s = r_{12} + r_{13}, \quad t = r_{12} - r_{13} \quad \text{and} \quad p = r_{23}$$

where

$$s = m, \quad p = \psi s \quad \text{and} \quad t = \psi s.$$

-7-
In the Gaussian interaction and wave functions analysis, the exact calculation (GOR) was carried out as well as another type in which the $F_{ij}$ term in the interactions was neglected as in the Chant-Mangelson approximation. The result in the latter case is referred to as GOCM in what follows.

\begin{align}
C_c(GGE) &= 8 \pi \eta (12 \pi^2)^3 \eta V_c \left( 2 \eta^2 + \beta \right) \left( \eta^2 + \beta^2 \right)^{-3/2}, \quad (14a) \\
C_c(GOCEM) &= 16 \pi \eta (12 \pi^2)^3 \eta V_c \left( 2 \eta^2 + \beta \right) \left( \eta^2 + \beta^2 \right)^{-3/2}. \quad (14b)
\end{align}

For the hard core interaction (HCI) and exponential type wave function we obtained

\begin{equation}
C_{\alpha k} = -32 \chi_k \left\{ \frac{1}{(\lambda \alpha)^9} \left[ \frac{2}{(\lambda \alpha)^2} + \frac{15}{\lambda \alpha} - \frac{7}{2} \right] - 1 \right\}
- \frac{1}{\lambda \alpha^3} \left[ \frac{1}{(\lambda \alpha)^2} + \frac{1}{\lambda \alpha} - \frac{2}{\lambda \alpha^3} \right] \right\}
\end{equation}

with $k = t$ or $s$, $\lambda = \lambda_t$ or $\lambda_s$ respectively for the triplet or singlet states. For the simple hard core wave function case i.e. $\nu = 0$, we have

\begin{equation}
C_{\alpha k} = C_{\alpha k}(\mu, \lambda) \text{ with } \chi_k = \frac{2 \mu^2}{3} N V_c e^{\lambda \mu^2} \chi_c. \quad (16)
\end{equation}

while for Srivastava's exponential wave function case

\begin{equation}
C_{\alpha k} = C_{\alpha k}(\mu) + A C_{\alpha k}(\nu) \text{ with } \chi_k = \frac{2 \mu^2}{3} N V_c e^{\lambda \mu^2} \chi_c. \quad (17)
\end{equation}

The combinations (1) of HCI and IX, IY and IW, GI and IW, GI and EW gave similar intermediate results for $C_{01}'$ as follows:

\begin{equation}
C_{01}' = \chi_1 \left[ \int_0^1 u^2 d \nu d \omega \left[ x_1(\nu, \omega) + x_2(\nu, \omega) \right] \right]. \quad (18)
\end{equation}

where with $q_+ = q_{\nu}(\nu, \omega) = 1 + \nu \omega$, $q_- = 1 - \nu \omega$, $q_0 = q_{\nu} q_-$ and $f = (1 + 2 \nu \omega + \nu^2) - 1/2$.

\begin{align}
x_{\pm} &= \frac{q_1}{q_{\nu} q_0} \left( \frac{\lambda \kappa q_{\nu} + \alpha f}{f + \frac{1}{2} \alpha^2 \kappa q_{\nu}} \right)^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c e^{\lambda \mu^2} \chi_c \text{ for HCI - IW} \\
x_{\pm} &= \frac{q_1}{q_{\nu} q_0} \left( \lambda \kappa q_{\nu} + \alpha f \right)^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c \text{ for IX - IW} \\
x_{\pm} &= q_{\nu} \left( \frac{\lambda q_{\nu} + \alpha f}{f + \frac{1}{2} \alpha^2 \kappa q_{\nu}} \right)^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c \text{ for YI - IW} \\
x_{\pm} &= q_{\nu} \exp(y_{\nu}) D_{\nu}(\pm \sqrt{y_{\nu}}) f^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c \text{ for YI - GW} \\
x_{\pm} &= Z_{\nu} \exp(y_{\nu}) D_{\nu}(\pm \sqrt{y_{\nu}}) f^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c \text{ for GI - IW} \\
x_{\pm} &= Z_{\nu} \exp(y_{\nu}) D_{\nu}(\pm \sqrt{y_{\nu}}) f^{2}, \quad \chi_k = \frac{1}{2} \pi \eta N V_c \text{ for GI - EW}
\end{align}

and $Z_{\nu} = q_{\nu} \gamma^6 \chi_{\nu}$ for both GI-IW and GI-EW. $D_{\nu}(u)$ and $D_{\nu}(u)$ are parabolic cylinder functions with

\begin{align}
D_{\nu}(u) &= \sqrt{\frac{\pi}{4}} \left[ e^{\frac{u^2}{4}} \left( 3 + 6 u^2 + u^4 \right) - 4 \frac{u^4}{\pi} \sum_{n=1}^{\infty} \frac{n(n-1)}{(2n-3)!!} \right] \\
D_{\nu}(u) &= -\sqrt{\frac{\pi}{5}} \left[ e^{\frac{u^2}{4}} \left( 15 + 10 u^2 + u^4 \right) - 4 \frac{u^4}{\pi} \sum_{n=1}^{\infty} \frac{n(n-1)}{(2n-3)!!} \right].
\end{align}
For the Yukawa potential and Srivastava's wave function

\[ C_0 = C_0(\alpha) = A C_0(\beta) \]

with

\[ C_0(x) = \mathcal{C} \left\{ \frac{\alpha}{g} + \frac{\beta}{K g^3} \right\} + \frac{1}{x^3} + \frac{3}{x(3x + K)} + \frac{1}{x^3} \left( \frac{2}{3} - \frac{1}{x} \right) \]

\[ + \frac{2}{h^2} \left( \frac{1}{xK} - \frac{2}{gK} + \frac{1}{g^3} \right) - \frac{1}{x^2} \left( \frac{2}{x} - \frac{1}{2} - \frac{1}{xK} \right) \}

(19)

with \( g = x + K \), \( h = x - K \), \( \mathcal{C} \approx \frac{8\pi^2 N}{3K^2} \) and \( x = a \) or \( b \).

For the hard core interaction and hard core wave function (HCW) with \( V \neq 0 \) and \( k = t \) or \( s \)

\[ C_{0k}(\lambda, \mu, \nu) = -2 \sum_{i=1}^{2} \sum_{j=1}^{2} \chi_{ik}(T_{ij} + P_{ij} + Q_{ij}) \]

(20)

with

\[ T_{ij} = \frac{1}{a_{ij} a_{j}} \left( \frac{b}{a_{ij}} + \frac{1}{a_{j} x_{j}} + \frac{1}{a_{ij} x_{i}} + \frac{1}{a_{ij} a_{j}} \right) \]

\[ P_{ij} = \frac{1}{a_{ij} a_{i}} \left( \frac{1}{a_{ij} b_{i}} - \frac{1}{a_{ij} a_{j}} - \frac{1}{a_{ij} a_{i}} \right) \]

\[ Q_{ij} = -\frac{1}{a_{ij} b_{ij}} \left( \frac{1}{b_{ij} b_{ij}} + \frac{2}{a_{ij} b_{ij}} + \frac{1}{a_{ij} b_{ij}}(\frac{1}{a_{ij} a_{j}} + \frac{1}{a_{ij} b_{ij}}) + \frac{2}{a_{ij} b_{ij}}(\frac{1}{a_{ij} b_{ij}}) \right) \]

with \( x_i = a_{ih} + a_{i} \), \( b_{ij} = a_{ij} + a_{j} \), \( \frac{1}{a_{ij} a_{j}} \), \( \frac{1}{a_{ij} b_{ij}} \).

The constants as well as the typical integral involved in evaluating

\[ C_{0k}(\lambda, \mu, \nu) \]

are as shown in the Appendix.

IV. RESULTS AND DISCUSSIONS

The point triton approximation constants obtained in this work are shown in Table I for \((t,p)\) reaction and in Table II for \((^3He,p)\) reaction.

The Gaussian interactions and wave functions (GG) results are for \( \eta = 0.24 \) for \((t,p)\) in Table I and for \( \eta = 0.26 \) for \((^3He,p)\) in Table II. For each reaction, the exact (GGE) and the Chant-Mangelson approximation (GGCM) results are fairly comparable for a given interaction strength and tri-nucleon size. This situation is in contrast to the zero-range interaction (ZRI) results in which a constant \( D^0 \), to which the TNTE cross-section is proportional, was found to be a factor 3 higher in the exact (GGE-ZRI) case than in the corresponding GG-ZRI-Chant-Mangelson approximation results. In other words, the point-triton (PT) approximation is not sensitive to the Chant-Mangelson approximation, especially for \((t,p)\) reaction. Even though it might seem that one would get drastically different cross-sections from using both PT and ZRI DWBA formalisms for the same reaction, the results in the two cases are equivalent as shown by Towner and Hardy.

The Yukawa interactions (YI) and Gaussian wave function results for both \((t,p)\) and \((^3He,p)\) are a factor 2 to 3 higher than the GG results in each case. The same potential coupled with Srivastava's exponential wave function have comparable results to the GGE results, especially for the case \( \alpha = 0.73, \beta = 1.3 \) and \( A = 0.305 \). It should be noted that these values of \( \alpha, \beta \) and \( A \) give the best result for the variational calculation of the binding energy of the triton in Srivastava's work. However, when YI was coupled with Gaussian wave functions, the results were a factor 6 to 8 less than the GGE results for each reaction.

\[ C_0 \] values for cases in which Gaussian interactions were used with the exponential and exponential wave functions are much too great to be meaningful. Consequently, it would be most inappropriate to analyse direct TNTE cross-sections with a DWBA code employing these combinations for describing the tri-nucleon system.

The Kikuta hard-core interaction and wave function (HCW) results in Table I are for \( \nu = 0 \) as well as for \( \nu \neq 0 \). The \( \nu = 0 \) result is a factor 5 greater than the exact Gaussian results, while the \( \nu = 4.5 \) \( fm^{-3} \) result is a factor 80 higher than the same results for \((t,p)\) reaction. The \( \alpha = 1.84 \) and \( \beta = 1.30 \) results from a combination of the same potential and Irving wave function are comparable to the GGE results for \((t,p)\), while the \( \alpha = 1.27 \) \( fm^{-3} \) result for \((^3He,p)\) is also comparable to the GGE result in Table II. On the other hand, the least result for this interaction and exponential wave functions
for (t,p) is for \( a = 0.71 \text{ fm}^{-1} \) and is a factor 2 greater than the Gaussian results.

VI. CONCLUSIONS

If the exact Gaussian results (GGE) are considered as standard for the point triton approximation constant \( C_0 \) for direct (t,p) and \((^3\text{He},p)\) reactions, then some combinations of interactions and wave functions reproduce this standard, while others do not. A consequence of the spectra of values of \( C_0 \) obtained for the two reactions considered is that one would require additional scaling or absolute normalization constants in order to fit exactly the direct TNTR cross-sections when one uses a DWBA code that reproduces all the microscopic details of the cross-sections. The spectrum of additional scaling factors required for, say, (t,p) reactions would be inversely proportional to the square of the above results, provided no approximations are made in triton structure and two-body interactions, and provided the same nuclear model is used in describing the parent and daughter nuclei for each combination indicated above.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where part of this work was done. The interest and support of Professor L. Fonda for this work is also sincerely appreciated.

APPENDIX

The typical integral involved in evaluating \( C_{0k}(\lambda,\mu,\nu) \) is of the form given below. The exact analytical expression for the integral is as given in the text.

\[
C_{0k} = -2 \sum_{i} \int_{-\infty}^{\infty} e^{-\alpha_{i}t} dt \int_{-\infty}^{\infty} e^{-\alpha_{i}t} \rho dt \sum_{i} (\delta_{i} e^{i \lambda t} + e^{-i \lambda t}) dt
\]

with

- \( a_{11} = a_{12} = a_{22} = a_{23} = a_{31} = a_{32} = a_{33} = a_{34} = 1 \),
- \( a_{13} = a_{24} = a_{35} = a_{4} = a_{53} = a_{63} = a_{73} = \lambda \),
- \( a_{44} = a_{54} = a_{64} = a_{74} = a_{84} = a_{94} = a_{104} = \frac{1}{2} (\lambda + \mu + \nu) \),
- \( a_{55} = a_{66} = \frac{1}{2} (\lambda - \mu - \nu) \),
- \( a_{66} = a_{77} = \frac{1}{2} (\lambda + \mu - \nu) \),
- \( a_{77} = a_{88} = \frac{1}{2} (\lambda + \mu + \nu) \),
- \( a_{88} = \frac{1}{2} (\lambda + 2\mu) ; a_{99} = \frac{1}{2} (\lambda + 2\nu) ; a_{1010} = \lambda / 4 \).

Using \( \delta \) and \( \epsilon \) as defined in the text.

\[
\gamma_{1k} = \delta \epsilon_{1k} ; \gamma_{2k} = \delta \epsilon_{2k} ; \gamma_{3k} = \delta \epsilon_{3k} ; \gamma_{4k} = \delta \epsilon_{4k} ; \gamma_{5k} = \delta \epsilon_{5k}
\]

\[
\gamma_{6k} = \gamma_{0k} = \gamma_{3k} ; \gamma_{7k} = \gamma_{4k} = \gamma_{5k} \]
Table I

<table>
<thead>
<tr>
<th>Interaction (I) and wave function (W)</th>
<th>$V_0$</th>
<th>Other parameters</th>
<th>$C_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) GI + CW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exact</td>
<td></td>
<td>$n = 0.242$, $p^2 = 0.379$</td>
<td>4034.44</td>
</tr>
<tr>
<td>Chant-Mangels approximation</td>
<td></td>
<td>$n = 0.242$, $p^2 = 0.379$</td>
<td>3723.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n = 0.242$, $p^2 = 0.46$</td>
<td>3783.88</td>
</tr>
<tr>
<td>(b) HCI + EW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{ot} = 475.04 , \text{MeV}$</td>
<td></td>
<td>$\alpha = 0.58$, $\delta = 0$, $A = 0$</td>
<td>22,992.66</td>
</tr>
<tr>
<td>$V_{ot} = 235.4$</td>
<td></td>
<td>$\alpha = 0.771$, $\delta = 1.105$, $A = 0$</td>
<td>11,533.34</td>
</tr>
<tr>
<td>$\lambda_c = 2.52$, $\lambda_y = 2.034$</td>
<td></td>
<td>$\alpha = 0.732$, $\delta = 1.415$, $A = 1.305$</td>
<td>21,461.94</td>
</tr>
<tr>
<td>(c) HCI + IW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As in (b)</td>
<td></td>
<td>$\alpha = 1.84$, $r_c = 0.4 , \text{fm}$</td>
<td>3152.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha = 1.301$, $r_c = 0.4 , \text{fm}$</td>
<td>3917.33</td>
</tr>
<tr>
<td>(d) GI + IW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As in (a)</td>
<td></td>
<td>$\beta^2 = 0.379$, $\alpha = 1.301$</td>
<td>4034.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^2 = 0.46$, $\alpha = 1.301$</td>
<td>&gt; 10$^{20}$</td>
</tr>
<tr>
<td>(e) TI + EW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K = 0.85$</td>
<td></td>
<td>$A = 0$, $\alpha = 0.58$</td>
<td>5701.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A = -1$, $\alpha = 0.77$, $\delta = 1.105$</td>
<td>2674.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A = -1.305$, $\alpha = 0.732$, $\delta = 1.415$</td>
<td>4942.78</td>
</tr>
<tr>
<td>(f) TI + CW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As in (e)</td>
<td></td>
<td>$\eta = 0.242$</td>
<td>15,605.35</td>
</tr>
<tr>
<td>(g) TI + IW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As in (e)</td>
<td></td>
<td>$\alpha = 1.84$</td>
<td>474.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha = 1.301$</td>
<td>610.68</td>
</tr>
<tr>
<td>(h) HCI + NOW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As in (b)</td>
<td></td>
<td>$\alpha = 0.4$, $\beta = 4.5$, $r_c = 0.4$</td>
<td>3.4517 \times 10^9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha = 0.4$, $\beta = 0$, $r_c = 0.4$</td>
<td>2.9999 \times 10^8</td>
</tr>
</tbody>
</table>
### Table I

<table>
<thead>
<tr>
<th>Interaction (I) and wave function (W)</th>
<th>( V_0 )</th>
<th>Other parameters</th>
<th>( C_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>-62.2</td>
<td>( n = 0.206, \beta^2 = 0.379 )</td>
<td>370.46</td>
</tr>
<tr>
<td></td>
<td>-72.98</td>
<td>( \beta^2 = 0.46 )</td>
<td>2984.65</td>
</tr>
<tr>
<td>Chant-Mangelson approximation</td>
<td>-62.2</td>
<td>( n = 0.206, \beta^2 = 0.379 )</td>
<td>3193.33</td>
</tr>
<tr>
<td></td>
<td>-72.98</td>
<td>( \beta^2 = 0.46 )</td>
<td>2458.52</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Interaction (I) and wave function (W)</th>
<th>( V_0 )</th>
<th>Other parameters</th>
<th>( C_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl + IW</td>
<td>As in (b) Table I</td>
<td>( n = 1.27, r_0 = 0.4 ) fm</td>
<td>3533.67</td>
</tr>
<tr>
<td>Y1 + GW</td>
<td>66.93</td>
<td>( K = 0.85, n = 0.271^1 )</td>
<td>8592.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n = 0.232^1 )</td>
<td>12,539.16</td>
</tr>
<tr>
<td>Y1 + IW</td>
<td>As in (c)</td>
<td>( K = 0.85, a = 1.27 )</td>
<td>623.29</td>
</tr>
</tbody>
</table>

---

**Table Captions**

Point triton normalization constants \( C_n \) for direct exchange-free, direct \((3p,3p)\) reaction as obtained with various interactions \( I \) and wave functions \( W \). The prefixes \( G, HC, I, Y \) and \( G, HC, I, Y \) stand respectively for Gaussian, hard core, Irving, Yukawa and exponential. The units of the various interaction and wave function parameters are as indicated in the text.

Point triton approximation normalization constants \( C_0 \) for direct exchange-free, direct \((3p,3p)\) reaction as obtained with various interactions \( I \) and wave functions \( W \). The prefixes \( G, HC, I, Y \) and \( G, HC, I, Y \) stand respectively for Gaussian, hard core, Irving, Yukawa and exponential. The units of the various interaction and wave function parameters are as indicated in the text.
CURRENT ICTP PUBLICATIONS AND INTERNAL REPORTS

V.G. GODWIN and E. TOGATI: Local field corrections to the binding energies of core excitons and shallow impurities in semiconductors.

M.P. KOGATAP, M.A. SENARY and M.A. AHMED: Magnetic susceptibility investigation of some antiferromagnetic F3" complexes.

I.A. AMIN: The exchange property of modulus.

K. MANDAVI-KHAYAT: Remarks concerning the running coupling constants and the unifying mass scale of grand unified gauge theories.

ISMAIL A. AMIN: On a conjecture of Erdos.

H. YUSOFF and A. MOOKERJEE: Phonon frequency spectrum in random binary alloys.

S. GOETTI: Anisotropic plasmon-phonon modes in degenerate semiconductors.

T.N. SNYDER: Supersymmetric extension of the SU(5) model.

M.S. BAKLINI: An SU(3) theory of electroweak interactions.

M.Y.A. HASSAN and M.M.M. MANSOUR: Relativistic calculation of polarized nuclear matter.


A. OSWAL: Nucleon-nucleon interaction in the three-nucleon system.

S. PERRAH: Superspace aspects of supersymmetry and supergravity.

M. STEILICKA and K. KEMP: Variation calculation of surface states for a three-dimensional array of δ-function potentials.

K.S. SINH and M.P. TOH: Relation between bulk compressibility and surface energy of electron-hole liquids.

A.R. HASSAN: Two-photon transitions to exciton polaritons.

G. AKHENTZ and A.O. BARUH: Gauge-invariant formulation of dyon Hamiltonian on the sphere S^3.

J.N. KOKA: Linear photon and two photon absorption by surface polaritons.

A. VITEIPOU and A. CORIOVETI: Dechanneling in the WKB approximation.

H. APSTOL: Finite size effects on the plasma frequency in layered electron gas.

F. DESTEPAPO and K. TAHIR SHAH: Quasi-catastrophes as a non-standard model and changes of topology.

A. OSWAL: Effect of Coulomb forces in the three-body problem with application to direct nuclear reactions.


A.A. EL SHA3LY, P.A. GANI and M.K. EL MOUSLY: Determination of optical absorption edge in amorphous thin films of selenium and selenium doped with sulphur.

M.M. MANSOUR: Variationally optimized muffin-tin potentials for band calculations.

W. WADIA and L. BALLOONAI: On the transformation of positive definite hermitian form to unit form.

G.S. DUBEY and D.K. CHOPRAVIRAI: Dynamical study of liquid aluminium.


M.M. BAKRI and H.M.M. MANSOUR: The relativistic two-fermion equations (I).

N.G. TOCHIY and J.C. BONKOV: On the s-d model for coexistence of ferromagnetism and superconductivity.

A.D. KANDIYAN, M.T. PRIMALTRAHA and V. ITCHUVA: Interface states in a class of heterojunctions between diatomic semiconductors.

W. CABKHVEN: Geometrical unification of gauge and Higgs fields.

M.Y.A. HASSAN and M.M.M. MANSOUR: The relativistic two-fermion equations (II).

S. MONTASSER and S. RAMADAN: On the thermal properties of polarized nuclear matter.

A. OSWAL: Two-, three-, and four-body correlations in nuclear matter.

J. BARNES and G.I. GHANDOUR: On quantizing gauge theories without constraints.

T.N. SNYDER: Higgs potential in the SU(5) model.

W.M. BAKRI and H.M.M. MANSOUR: The relativistic two-fermion equations (II).

J. NIEDERER: Supercavity.

T.N. SNYDER: Higgs potential in the SU(5) model.

W.M. BAKRI and H.M.M. MANSOUR: The relativistic two-fermion equations (II).

R.R. BASILY: The effect of the gate electrode on the C-V characteristics of the structure M-TeF\textsubscript{3}-SiO\textsubscript{2}-Si.

J. NIEDERER: Quantization as mapping and as deformation.


R. BUNI: Ion-acoustic holes in a two-electron temperature plasma.

* Internal Reports: Limited distribution

These preprints are available from the Publications Office, ICTP, P.O. Box 586, 34100 TRIESTE, ITALY.
IC/79/144: W. KRÓLIKOWSKI: Recurrence formulae for lepton and quark generations.


IC/79/146: C. TONÉ: Change of elastic constants induced by point defects in hcp crystals.


IC/79/154: RIAZUDDIN and PATTAZUDDIN: A model for electroweak interactions based on the left-right symmetric gauge group $U_L(2) \otimes U_R(2)$.

IC/79/155: B. JULIA and J.P. LUCIANI: Non-linear realizations of compact and non-compact gauge groups.


IC/79/162: P. BUDINI: Reflections and internal symmetry.


IC/80/5: RIAZUDDIN: Two-body D-meson decays in non-relativistic quark model.


IC/80/7: A.M. Kurbatov and D.P. Sankovich: On the one variational principle in quantum statistical mechanics.

IC/80/8: G. Stratan: On the alpha decay branching ratios of nuclei around $A = 110$.

IC/80/12: M.V. MinaIović and M.A. NAGARAJAN: A proposal for calculating the importance of exchange effects in rearrangement collisions.

IC/80/14: W. KRÓLIKOWSKI: Lepton and quark families as quantum-dynamical systems.
