IC/79/43

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1979 MIRAMARE-TRIESTE

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MIRAMARE - TRIESTE

May 1979

* Work supported in part by the Istituto Nazionale di Fisica Nucleare, Sezione di Trieste.
Submitted for publication.

ABSTRACT

A detailed investigation of the small time deviations of the quantum non-decay probability from a pure exponential is made to study their physical consequences. Specific consideration is given to the problem of the dependence of the lifetime from the characteristics of the measuring apparata whose possible occurrence has recently been pointed out. In particular we investigate the problem of the indefinite increase of the lifetime when the frequency of the measurement processes tends to infinity, an effect referred to as Zeno's paradox in Quantum Mechanics.

It is shown that if the uncertainty relations are properly taken into account the arguments leading to the paradox are not valid. Moreover, by the same kind of arguments, it is shown that the dependence of the measured lifetime on the frequency of the measurement processes, even though present in principle, is practically not detectable. To verify experimentally that the reduction process is effective one must then resort to the comparison of an experiment with reductions with one in which no interactions with the environment take place.

1. INTRODUCTION

In the quantum description of an unstable system a crucial role is played by the non-decay probability P(t), which is given by

$$P(t) = |A(t)|^2, \tag{1.1}$$

where

$$A(t) = \langle u | e^{-iHt} | u \rangle, \tag{1.2}$$

$|u\rangle$ being the unstable state wave function and H the Hamiltonian of the system. As is well known, P(t), as defined by (1.1), cannot be a pure exponential for all times for any choice of $|u\rangle$ and H, provided some essential physical requirements are satisfied.[1] By expanding $|u\rangle$ on the improper eigenstates of H and calling $\omega(E)$ the energy form factor of $|u\rangle$, one has from (1.2)

$$A(t) = \int_0^\infty |\omega(E)|^2 e^{-iEt} \, dE. \tag{1.3}$$

In writing (1.3) we have assumed the threshold energy to be zero.

From general arguments it can be proved that P(t) exhibits deviations from the exponential both for small and large times[1]. The small time deviations from the exponential are particularly relevant from a physical point of view. In fact, it has recently been pointed out that in almost all practical cases, a decaying system cannot be considered as evolving undisturbed, but it is repeatedly subjected at random times to interactions with the environment, which can be accounted for as yes-no experiments which ascertain whether the system is decayed or not reducing its wave function to the pure decay products or to the pure unstable state, respectively.(*) The experimentally measured quantity then is not the non-decay probability P(t), but the probability F(t) of surviving to all measurements up to the time t. This survival probability F(t) turns out to be practically a pure exponential for all times and the experimentally measured lifetime turns out to depend from P(t) and from the mean frequency λ of the measurement processes [5]. Since the mean frequency is in fact large, the time interval during which the system evolves undisturbed from the unstable state is small, so that it is the small time behaviour of P(t) which is relevant in determining the experimentally measured lifetime.

In this paper we give first of all a quantitative estimate of the order of magnitude of the small time deviations of P(t) from an exponential. This will be done by using definite expressions for the energy form factor, which, however, can be considered appropriate quite in general, as discussed below. The obtained non-decay probability P(t) is then used to discuss a peculiar effect which has been known for a long time [6] and has recently been discussed and referred to as "Zeno's Paradox in Quantum Mechanics" [7]. It consists in the fact that the evolution equations for a quantum system subjected to repeated measurements ascertaining whether the system is decayed or not, imply formally that, when the frequency of the measurements tends to infinity, the unstable system will never be found decayed. Here we shall show that, when arguments based on the uncertainty relations are taken into account, the unlimited increase in the lifetime formally implied by the evolution equations cannot occur. Moreover, the arguments we shall use show that,

(*) The fact that the occurrence of interactions with the environment modify the evolution law of an unstable system had already been pointed out in Refs.2 and 3. A more accurate quantitative treatment of the effect has been given in Ref. 4, while the consequences of this type of description have been fully exploited in Ref.5.

in practice, it is very difficult to induce and therefore to detect experimentally the dependence of ν on λ. The general conclusion is then that, to verify experimentally that the reduction process is effective, one must resort to the comparison of an experiment with reductions with one in which no interactions with the environment take place.

2. SMALL TIME BEHAVIOUR OF THE NON-DECAY PROBABILITY

As stated in the introduction, we are interested in studying the small time behaviour of the quantum non-decay probability P(t). As is well known, the pure exponential $e^{-\gamma t}$ is obtained when the energy form factor has a pure Breit-Wigner shape and the integral (1.3) is extended from $-\infty$ to $+\infty$. The fact that the energy spectrum is actually bounded from below gives rise to deviations from the exponential law. Furthermore, the pure Breit-Wigner energy form factor has too slow a decrease for high energies, so that the corresponding mean value of the energy turns out to be infinite. It is then necessary, for physical reasons, to depress the form factor for large energy values by superimposing on the Breit-Wigner resonance a cut-off function. The presence of such a cut-off also gives rise to deviations from the exponential law. More precisely, general theorems on the Fourier transforms allows us to prove, under the sole assumption that the mean value of the energy be finite [1], that P(t) has a vanishing derivative at the origin,

$$\left.\frac{dP(t)}{dt}\right|_{t=0} = 0 \quad , \qquad (2.1)$$

implying a non-exponential behaviour for small times. Fleming [8] has obtained in a very elegant way a lower bound for P(t) at small times, assuming that the energy spread

$$\Delta E = \left[\langle u | H^2 | u \rangle - \langle u | H | u \rangle^2 \right]^{1/2} \tag{2.2}$$

be finite. More precisely, he has shown that for the non-decay probability amplitude A(t), the inequality

$$|A(t)| \geq \cos(\Delta E \cdot t) \tag{2.3}$$

holds for t such that

$$0 \leq t \leq \frac{\pi}{2 \cdot \Delta E}, \tag{2.4}$$

independent of the choice of the normalized state $|u\rangle$ used to evaluate A(t). We are interested in the study of A(t) for a quantum state $|u\rangle$ whose energy form factor exhibits a narrow Breit-Wigner resonance of width γ, for which, in a large time interval, $P(t) \simeq e^{-\gamma t}$. In order that the Breit-Wigner resonance is not essentially altered by the modulating cut-off one must assume $\Delta E \gg \gamma$. Expressing ΔE in units of γ, and t in units of $1/\gamma$,

$$\frac{\Delta E}{\gamma} = D, \quad \gamma t = \tau, \tag{2.5}$$

Eq.(2.3) becomes

$$\cos^2(D\tau) \leq P(\tau) \leq 1, \quad 0 \leq \tau \leq \frac{\pi}{2D}. \tag{2.6}$$

By considering the value $\bar{\tau}$ of τ in which $\cos^2(D\tau)$ intersects $e^{-\tau}$, we can say that P(t) exhibits relevant deviations from the pure exponential at least up to times of order $\bar{\tau}$, provided the obtained $\bar{\tau}$ is less than $\pi/2D$. The above formula (2.6) shows that the relevant parameter governing the small time deviations is the ratio of ΔE to the width γ. The choice of $e^{-\gamma t}$ as the comparison function is the proper one since, on one side it coincides for a large time interval with the quantum mechanical non-decay probability, and on the other, as we shall see, it expresses the experimentally observed survival probability F(t). It is easy to see that, for $D > 1$, the solution of the equation $\cos^2 D\bar{\tau} = e^{-\bar{\tau}}$ is given to a great accuracy by

$$\bar{\tau} = \frac{1}{D^2}, \quad \text{i.e.} \quad \bar{t} = \frac{\gamma}{(\Delta E)^2}. \tag{2.7}$$

According to (2.7), $\bar{\tau} \leq \frac{\pi}{2D}$ in the considered range of values of D (to be precise, for $D > \frac{2}{\pi}$), so that the deviations from the exponential extend at least up to times given by (2.7).

Another simple way of getting the result (2.7) is the following. Consider the power series expansion of A(t) around t = 0

$$A(t) = 1 - i\langle H \rangle t - \frac{1}{2}\langle H^2 \rangle t^2 + O(t^3), \tag{2.8}$$

from which one gets

$$P(t) = 1 - (\Delta E)^2 t^2 + O(t^4). \tag{2.9}$$

If we correspondingly write

$$e^{-\gamma t} = 1 - \gamma t + \frac{1}{2}\gamma^2 t^2 + O(t^3), \tag{2.10}$$

and we look for the time $\bar{t}$, for which $P(\bar{t}) = e^{-\gamma \bar{t}}$, we get

$$\bar{t} = \frac{2\gamma}{2(\Delta E)^2 + \gamma^2}, \tag{2.11}$$

which coincides with (2.7) for $\Delta E \gg \gamma$. The above estimate of the times up to which relevant deviations occur is actually confirmed by calculations on explicit models (see e.g. Refs.9, 10 and 13).

In the next section we shall study explicitly the function P(t) for various physically reasonable choices of the energy form factor of the state $|u\rangle$. It will turn out that the behaviour of P(t) is not relevantly influenced by the specific form factor. On the contrary, it is determined essentially by the parameter $\Delta E/\gamma$, at least as far as the time interval in which we are interested is concerned. The purpose of having explicit forms for P(t) is to use them to investigate physically relevant questions, such as the dependence of the experimentally observed lifetime from the characteristics of the measuring apparatus. The above remark, that the behaviour of P(t) in the relevant time region is rather insensitive to the specific cut-off function superimposed on the Breit-Wigner resonance, allows us to consider the obtained results as giving quite in general an appropriate estimate of the effects which are involved.

Another property of P(t) can be used to enforce the above conclusions. In fact, there is a "continuous" dependence of P(t) from the energy form factor of the state $|u\rangle$, in the following sense [1]. Let us consider two states $|u_1\rangle$ and $|u_2\rangle$, with form factors $\omega_1(E)$ and $\omega_2(E)$, respectively. Then the corresponding non-decay probability amplitudes $A_1(t)$ and $A_2(t)$ satisfy the relation

$$|A_1(t)-A_2(t)| = \left|\int_0^\infty \left[|\omega_1(E)|^2-|\omega_2(E)|^2\right] e^{-iEt}\, dE\right| \leq$$

$$\leq \int_0^\infty \left|\,|\omega_1(E)|^2-|\omega_2(E)|^2\right| dE \,, \qquad (2.12)$$

from which one gets

$$\begin{aligned}|P_1(t)-P_2(t)| &= \left|\,|A_1(t)|-|A_2(t)|\,\right|\cdot\left[|A_1(t)|+|A_2(t)|\right] \leq \\ &\leq 2\left|\,|A_1(t)|-|A_2(t)|\,\right| \leq \\ &\leq 2\,|A_1(t)-A_2(t)| \leq \\ &\leq 2\int_0^\infty \left|\,|\omega_1(E)|^2-|\omega_2(E)|^2\right| dE\,.\end{aligned} \qquad (2.13)$$

It follows that when $\omega_1(E)$ and $\omega_2(E)$ are sufficiently near, e.g. when $\int_0^\infty \left|\,|\omega_1(E)|^2-|\omega_2(E)|^2\right| dE < \eta$, then for all times $P_1(t)$ differs from $P_2(t)$ by less than 2η.

Actually it is this property that allows one to speak of an unstable state when its energy form factor is Breit-Wigner-like for an energy region which is large with respect to the width γ. In fact in such a case, extending the integral (1.3) from $-\infty$ to $+\infty$, we get the purely exponential decay law $e^{-\gamma t}$ with an accuracy which can be evaluated by using (2.13) to estimate the error introduced by the addition of the integral from $-\infty$ to 0.

In the explicit examples of the next section we shall consider various modulating cut-offs. The results obtained for P(t) will subsequently be used to investigate the problem of the dependence of the experimental lifetime on the characteristics of the measuring apparatus.

3. EVALUATION OF P(t) FOR SOME SPECIFIC CASES

As already stated, in this section we shall evaluate explicitly the non-decay probability for particular choices of the form factor of the unstable state $|u\rangle$. More precisely we consider the following three choices for the energy form factor $|\omega(E)|^2$:

A. $|\omega(E)|^2$ is given by a Breit-Wigner function of width γ modulated by another Breit-Wigner function of larger width μ:

$$|\omega(E)|^2 = \frac{\mu\gamma(\mu+\gamma)}{8\pi}\left[(E-E_r)^2+\frac{\gamma^2}{4}\right]^{-1}\cdot\left[(E-E_r)^2+\frac{\mu^2}{4}\right]^{-1}. \tag{3.1a}$$

B. $|\omega(E)|^2$ is given by a Breit-Wigner function of width γ modulated by a Gaussian factor of width Δ:

$$|\omega(E)|^2 = \frac{\gamma\, e^{-\gamma^2/4\Delta^2}}{2\pi\left[1-\mathrm{erf}\left(\frac{\gamma}{2\Delta}\right)\right]}\;\frac{e^{-(E-E_r)^2/\Delta^2}}{(E-E_r)^2+\frac{\gamma^2}{4}}, \tag{3.1b}$$

erf(x) denoting the error function.(*)

C. $|\omega(E)|^2$ is given by a Breit-Wigner function of width γ in the energy region $|E-E_r|<\delta$ and is zero outside this region:

$$|\omega(E)|^2 = \begin{cases} \dfrac{\gamma}{4\arctan\left(\frac{2\delta}{\gamma}\right)}\left[(E-E_r)^2+\dfrac{\gamma^2}{4}\right]^{-1} & \text{for } |E-E_r|\le\delta, \\ 0 & \text{otherwise}. \end{cases} \tag{3.1c}$$

We now give the values of ΔE and the expressions of P(t) for the three cases considered above.

Case A

If we put

$$\frac{\mu}{\gamma} = m, \quad \gamma t = \tau, \tag{3.2a}$$

we have

(*) For the special functions we follow the Handbook of Mathematical Functions, ed. by M. Abramowitz and I.A. Stegun (Dover Publ., New York 1968).

$$\frac{\Delta E}{\gamma} = \frac{\sqrt{m}}{2} \tag{3.3a}$$

and

$$P_A(\tau) = \frac{1}{(m-1)^2}\cdot\left[m^2 e^{-\tau} + e^{-m\tau} - 2m\, e^{-\frac{m+1}{2}\tau}\right]. \tag{3.4a}$$

Case B

Let us put

$$\frac{\Delta}{\gamma} = \mathfrak{M}, \quad \gamma t = \tau. \tag{3.2b}$$

We have

$$\frac{\Delta E}{\gamma} = \frac{1}{2}\left\{\frac{2\mathfrak{M}\, e^{-1/4\mathfrak{M}^2}}{\sqrt{\pi}\left[1-\mathrm{erf}(1/2\mathfrak{M})\right]} - 1\right\}^{1/2} \tag{3.3b}$$

and

$$P_B(\tau) = \frac{1}{4\left[1-\mathrm{erf}(1/2\mathfrak{M})\right]^2}\left\{e^{-\frac{\tau}{2}}\left[1-\mathrm{erf}\left(\frac{1}{2\mathfrak{M}}-\frac{\mathfrak{M}\tau}{2}\right)\right] + e^{\frac{\tau}{2}}\left[1-\mathrm{erf}\left(\frac{1}{2\mathfrak{M}}+\frac{\mathfrak{M}\tau}{2}\right)\right]\right\}^2 \tag{3.4b}$$

Case C

Putting

$$\frac{\delta}{\gamma} = M, \quad \gamma t = \tau, \tag{3.2c}$$

we have

$$\frac{\Delta E}{\gamma} = \frac{1}{2}\left[\frac{2M}{\arctan(2M)} - 1\right]^{1/2}, \tag{3.3c}$$

and

$$P_c(\tau) = \frac{1}{4\,[\arctan(2M)]^2} \Big\{ e^{\frac{\tau}{2}} \,\mathrm{Im}\, E_1\big(\tfrac{\tau}{2} + iM\tau\big) - e^{-\frac{\tau}{2}} \Big[\pi + \mathrm{Im}\, E_1\big(-\tfrac{\tau}{2} + iM\tau\big) \Big] \Big\}^2 , \qquad (3.4c)$$

where $E_1(z)$ is the exponential integral.

A comment about cases A and B is necessary. In normalizing the energy form factors (3.1) and in calculating the energy spreads (3.3) and the non-decay probabilities (3.4) we have extended the energy integrals from $-\infty$ to $+\infty$ for the sake of simplicity. This is the reason why the P(t)'s obtained decrease exponentially at infinity, contrary to the power-like behaviour implied by the boundedness from below of the energy spectrum . However, this approximation is irrelevant for the small time behaviour, provided the resonance is far from the threshold, as can be seen by using the argument of the previous section to evaluate the difference between the correct P(t) and the P(t) as calculated above. In fact, if we introduce the quantity

$$\eta = \int_{-\infty}^{0} |w(E)|^2\, dE \quad , \qquad (3.5)$$

the properly normalized energy form factor is $|w(E)|^2 / (1-\eta)$. According to the argument of the previous section, the difference between the correct and the calculated P(t)'s is less than twice the integral

$$\int_{-\infty}^{\infty} \Big| \vartheta(E) \big\{ |w(E)|^2/(1-\eta) \big\} - |w(E)|^2 \Big| \, dE = 2\eta \quad , \qquad (3.6)$$

where $\vartheta(E)$ is the step function. We have in case A

$$\eta < \frac{1}{2\pi}\, \frac{\gamma}{E_r}\, \frac{m+1}{m} \quad , \qquad (3.7)$$

and in case B

$$\eta < \frac{1}{2\pi}\, \frac{\gamma}{E_r}\, \frac{1}{1 - \mathrm{erf}(1/2\mathcal{M})} \quad . \qquad (3.8)$$

Since $\Delta E \gtrsim \gamma$ implies $m \gtrsim 4,\ \mathcal{M} \gtrsim 3.85$, the last factor in both (3.7) and (3.8) is bounded and of the order of unity. Therefore, when the considered resonance is narrow and far from the threshold, i.e. $\frac{\gamma}{E_r}$ is small, Eqs. (3.7) and (3.8) show that Eqs. (3.4a) and (3.4b) are very good approximations to the correctly calculated P(t)'s. Obviously, this result is significant only in the small time region, where P(t) and $e^{-\gamma t}$ are much larger than γ/E_r , while it does not give any information for large times where P(t) is of the same order or smaller than γ/E_r.

In Fig. 1, we have drawn the curves $P(\tau)$ (*) up to $\tau = 5$ for the cases A, B and C corresponding to $\frac{\Delta E}{\gamma} = \frac{1}{2}, 1, 2$ together with $e^{-\tau}$. Fig.2 is simply an amplification of Fig. 1, up to $\tau = 1$. For $\Delta E/\gamma = 1,2$ it can be seen that the three curves have similar trends, and for $\Delta E/\gamma = 4$ that the three curves are already almost undistinguishable from the exponential in the considered scale . Since we are interested in the small time behaviour or even in the limit for $\tau \to 0$ we have drawn the small time region more and more enlarged for various values of $\Delta E/\gamma$ in Figs.3 and 4. The grouping of the lines with the same $\Delta E/\gamma$ shows that the behaviour of $P(\tau)$ depends essentially on $\Delta E/\gamma$, while the particular choice of the energy form factor does not make significant differences.

In the next sections we shall investigate the physical consequences of the small time deviations from the exponential on the determination of the lifetime. Since we are only interested in estimating the orders of magnitude of the effects which are involved, we can choose any of the considered $P(\tau)$'s to

(*) We have considered also the case $\Delta E/\gamma = 1/2$ to show the large effect at small times when the width of the modulating factor becomes comparable to the width of the Breit-Wigner resonance. Note that also for $\Delta E/\gamma = 1/2$ the modulating factor has a width equal or larger than γ .

investigate them. Among the three forms of $P(\tau)$ derived above, the simplest to handle is $P_A(\tau)$, since it has a simpler mathematical structure, which greatly simplifies the calculations of Sec. 5.

4. DISCUSSION OF THE DEPENDENCE OF THE LIFETIME ON THE MEASURING APPARATUS

As we have already said in the introduction, it has been stressed recently [4,5] that, when the actual physical situation occurring in experiments devised to measure the non-decay probability is taken into account, what one is actually measuring is not the quantum mechanical function $P(t)$. This is due to the fact that a decaying system cannot be considered as evolving undisturbed, but is subjected to repeated interactions with the environment, occurring at random times, with a mean frequency λ. What one is actually measuring is then the probability $F(t)$ of survival of the unstable system to all measurements which it has suffered up to time t. It can be proved [5] that for λ appreciably larger than γ $F(t)$ turns out to be practically a pure exponential for all times:

$$F(t) = e^{-\nu t} \quad , \tag{4.1}$$

where ν is determined by the relation

$$\lambda \int_0^\infty e^{-\lambda t} \, e^{\nu t} \, P(t) \, dt = 1 \quad . \tag{4.2}$$

It turns out that this equation has a unique positive solution which is smaller than λ.

Equation (4.2) implies the occurrence of a dependence of the experimentally measured lifetime $1/\nu$ on the frequency λ of the reduction processes. Some physical consequences of Eq. (4.2) have already been discussed in Refs. 9 and 11. In what follows, besides discussing some conceptual implications of Eq. (4.2), we shall study explicitly, using the form (3.4a) for $P(t)$, this dependence of ν on the other parameters.

Before coming to the explicit study of ν, let us make some comments on Eq. (4.2). First of all, Eq. (4.2) shows that if $P(t)$ were a pure exponential for all times, $P(t) = e^{-\gamma t}$, then $\nu = \gamma$. Therefore it is the non-purely-exponential nature of $P(t)$ which is primarily responsible for the fact that $1/\nu$ is different from the quantum mechanical life-time $1/\gamma$ and that ν is a function of λ. Secondly, since a precise analysis of the actual physical situation shows [1] that $\lambda \gg \gamma$, Eq. (4.2) implies that in the determination of ν a crucial role is played by the small time behaviour of $P(t)$. Since for small times $P(t)$ is always larger than the exponential (see Sec. 2), the resulting experimental lifetime $1/\nu$ turns out to be larger than $1/\gamma$, so that

$$0 \leq \nu \leq \gamma < \lambda \quad . \tag{4.3}$$

Equation (4.2), for a given $P(t)$, implies that for $\lambda \to \infty$, $\nu \to 0$, so that the unstable system becomes stable. In fact, if

$$P(t) = 1 + \sigma(t) \quad , \tag{4.4}$$

a well known theorem on the asymptotic behaviour of the Laplace transform [12] implies, for large λ,

$$\nu = \lambda \, \sigma\left(\frac{1}{\lambda - \nu}\right) . \tag{4.5}$$

From Eq. (4.2) one sees that it is impossible that $\nu \to \lambda$ for $\lambda \to \infty$. Eqs. (4.3) and (4.5) therefore actually imply $\nu \to 0$ under the limit. The physical meaning of this feature should be obvious: if we measure whether the system is decayed or not with increasing frequency we are exploring the behaviour of P(t) for very small times, and since the derivative of P(t) vanishes at t = 0, in the limit of $\lambda \to \infty$ the system can never be found decayed. This effect (which seems to be an unavoidable consequence of any description of the decay processes taking into account the occurrence of repeated reduction processes) has been known for a long time[6] and has recently been discussed and referred to as "Zeno's Paradox in Quantum Mechanics"[7]. The occurrence of this indefinite increase of the lifetime is relevant both from a conceptual and a practical point of view. In particular, its real existence would imply (provided we are able to perform experiments in which the mean frequency of the measurements is sufficiently large) that the dependence of ν on λ could be shown experimentally. It is then quite appropriate to make a critical investigation of the arguments leading to Zeno's paradox.

A remark which has already been made[13] is that the limit $\lambda \to \infty$ is unphysical since the environment, which is responsible for the reduction processes, has an atomic structure, so that a really continuous measurement is impossible in any experiment one can devise in practice. But there is a deeper argument against the existence of the quantum Zeno paradox. We remark that the result that $\nu \to 0$ when $\lambda \to \infty$ has been derived using the assumption that the quantum non-decay probability P(t) depends only on the internal dynamics of the unstable system, so that it is independent of λ. This assumption cannot be correct when the limit $\lambda \to \infty$ is taken. In fact P(t) depends on the state $|u\rangle$ which is produced by the reduction process. The state $|u\rangle$ is essentially unique, in the sense that its energy form factor

has the Breit-Wigner shape over a large interval around the mean energy E_r of the unstable state; but its shape for energies far away from E_r is not determined by the internal dynamics of the unstable state and can very well depend on the reduction process. Such a process has not been analysed in a proper way, as far as its possible influence on the setting up of Zeno's paradoxical situation is concerned. In fact one must observe that no measurement can take place instantaneously in quantum mechanics. If we denote by T the actual time interval which is necessary for a measurement to be performed, we obviously have that T must be a lower bound for the mean time interval $1/\lambda$ between two consecutive measurements;

$$1/\lambda > T . \tag{4.6}$$

In other words, if we want to increase λ indefinitely, we must also affect the modalities of each individual measurement, making it last for a shorter and shorter time interval. A naive use of the time-energy uncertainty relations shows then that this in turn would affect the energy spread of the produced state, making it unavoidably larger and larger. On the other hand, we know that the small time behaviour of P(t) depends critically on the energy spread ΔE. Moreover, since the region of appreciable deviations from the exponential shrinks when ΔE is increased (see Sec. 2), we can suspect that Zeno's paradox and the related possibility of revealing the dependence of ν on λ by increasing λ could not exist.

The above argument, being based on an uncritical use of time-energy indeterminacy relations, needs to be discussed more critically (see, for instance, Refs. 14 and 15). We shall do this in the next section, showing that it is actually impossible to make T tend to zero without introducing a larger and larger energy spread. We can then conclude quite in general that the arguments leading to Zeno's paradox are not valid.

Let us come back to the study of Eq. (4.2). To get explicit quantitative estimates of the dependence of ν on λ, we shall use the form (3.4a) for P(t). It is useful to measure ν and λ in units of γ, so we put

$$\frac{\lambda}{\gamma} = \ell \quad , \quad \frac{\nu}{\gamma} = m \, . \tag{4.7}$$

Equation (4.2) can be written:

$$\ell \int_0^\infty e^{-\ell\tau} e^{m\tau} P(\tau) d\tau = 1 . \tag{4.8}$$

Using the form (3.4a) for P(t) and performing the integration we get

$$\frac{\ell}{(m-1)^2}\left[\frac{m^2}{\ell-m+1} + \frac{1}{\ell-m+m} - \frac{4m}{2\ell+m+1-2m}\right] = 1 . \tag{4.9}$$

Equation (4.9) when expressed in terms of the relative change (with respect to γ) of the frequency of the decays

$$x = \frac{\nu-\gamma}{\gamma} = m-1 , \tag{4.10}$$

becomes

$$f(x) = ax^3 + bx^2 + cx + d = 0 \quad , \tag{4.11}$$

where

$$\begin{aligned} a &= 2 \quad , \\ b &= -[4\ell + 3(m-1)] < 0 \quad , \\ c &= 2\ell(\ell-1) + (m-1)(3\ell+m-1) > 0 \, , \\ d &= \ell(2\ell+2m-1) > 0 \, , \end{aligned} \tag{4.12}$$

the inequalities always being satisfied since we are interested in the limit for large ℓ. One gets

$$\begin{aligned} f(0) &= d > 0 \, , \\ f'(0) &= c > 0 \, , \\ f''(x) &= 6ax + 2b < 0 \, , \quad \text{for } x \leq 0 . \end{aligned} \tag{4.13}$$

It follows that f(x) vanishes once and only once for negative x and that such a root is larger than - d/c, i.e.

$$0 < -x < \frac{\ell(2\ell+2m-1)}{2\ell(\ell-1)+(m-1)(3\ell+m-1)} \, . \tag{4.14}$$

Under the usual assumption that $\Delta E/\gamma > 1$, it is easily checked that the corresponding ν satisfies the condition (4.3), so that the considered solution of Eq. (4.11) is that corresponding to the solution of Eq. (4.2).

We note that the indefinite increase of the lifetime yielding Zeno's paradox corresponds to $x \to -1$ (see Eq. (4.10)). In accordance with the previous discussion about the possible dependence of ΔE on $1/\lambda$, we remark that in our model this dependence is expressed as m being a function of ℓ. We must then distinguish three cases:

a) $\quad m/\ell \xrightarrow[\ell\to\infty]{} 0 .$

In such a case from Eq. (4.12) one easily gets that the only acceptable solution of (4.11) is

$$x = -1 \quad , \tag{4.15}$$

which implies the occurrence of Zeno's paradox.

b) $\quad m/\ell \xrightarrow[\ell\to\infty]{} k > 0 .$

In such a case Eq. (4.12) implies, in the limit

$$x = -\frac{1}{1+k/2} \quad , \tag{4.16}$$

so that x remains strictly larger than - 1 and the infinite increase of the lifetime cannot occur.

c) $\ell/m \xrightarrow[\ell\to\infty]{} 0$.

In such a case Eq. (4.14) implies x = 0 and the observed lifetime $1/\nu$ coincides with the quantum mechanical one $1/\gamma$. As we shall see in the next section, one can prove that when $\ell\to\infty$, m goes to ∞ too, at least as fast as ℓ, so that we are in cases b) or c) above and Zeno's paradox is excluded.

5. DISCUSSION OF ZENO'S PARADOX

We want to show now that an unavoidable increase of the energy spread of the state produced by the measurement corresponds to the increase of the frequency of the reduction processes and we want to study explicitly this dependence of ΔE on λ. For this purpose, let us first of all use naively the time energy indeterminacy relation within our model. From

$$\Delta E \cdot \frac{1}{\lambda} > \Delta E \cdot T \gtrsim 1 \qquad (5.1)$$

we get, using Eq.(3.3a) and the definition (4.7),

$$\frac{\sqrt{m}}{2\ell} \gtrsim 1\,, \quad \text{i.e.} \quad m \underset{\ell\to\infty}{\simeq} \ell^2\,, \qquad (5.2)$$

which implies that we are in case c) of the previous section. Zeno's paradox is then excluded. As already stated, however, the use of the time energy indeterminacy relations can be criticized since, as shown by the deep analysis of Refs. 14 and 15, there is no fundamental principle in quantum mechanics which forbids us to make a measurement in an arbitrarily short time without inducing a large energy spread. However, it must be remarked that to violate $\Delta E \cdot T \gtrsim 1$, one must use (as discussed

in great detail in Ref.14) properly devised and operated measuring apparata. In decay experiments, the measuring apparatus is constituted essentially by the environment of the system and the experimenter cannot monitor the exact time in which the measurement takes place or the modalities of the energy exchanges between system and apparatus. Also in Ref.15, when criticizing the use of the $\Delta E \cdot T \gtrsim 1$ relation, the author states that this relation "...applies just to what we may call static experiments..." In these cases T is not a preassigned or externally imposed parameter of the apparatus but is on the contrary determined dynamically through the workings of the Schrödinger equation, which describes the passage of the wave packet through the analysing system. The wave may perhaps be said to interact with the apparatus during the time interval T and in this sense $\Delta E \cdot T \gtrsim 1$, is valid". We stress that the situation described above is exactly that occurring in decaying experiments where the interaction of the system with the environment is induced by Schrödinger evolution. The above conclusions about the non-occurrence of Zeno's paradox can then be considered correct. If one wants a more direct proof of the dependence of ΔE on λ, one must enter more specifically into the details of the reduction mechanism. To do this one must recall that, as exhaustively discussed in Ref.1, the yes-no experiments performed by the apparatus correspond, in practice, to a localization of the decay fragments within a relative distance R, characteristic of the experimental set up. The time T_R spent by the decay fragments to travel the distance R must be much smaller than the quantum lifetime $1/\gamma$, in order that the treatment leading to (4.2) be appropriate [5]. We can then make a very simple sketch of the actual physical situation. We have an unstable system very well localized around the origin of the reference frame which develops, by Hamiltonian evolution, components on decay states propagating away from the origin. Since we want to have the possibility of reducing the time T taken by the localization process at R, we must discuss in detail the operation of the detecting devices. For this purpose we consider a radial thickness ε, representing the distance over which the detector has efficiency one. To shorten the time T we must then make ε smaller and smaller. Moreover, since we want to avoid the occurrence of infinite mean energies associated with sharp localizations,(*) we shall assume that the localization consists in a smooth reduction process. Therefore, when the system is found undecayed, the reduction is accounted for by the application to the state ψ of the system before the reduction, of an operator L defined as follows:

$$[L\psi](\underline{r}) = L(r)\psi(\underline{r}) \quad , \tag{5.3}$$

where

$$L(r) = \begin{cases} 1 & \text{for } r \leq R \\ 0 & \text{for } r \geq R+\varepsilon \end{cases} \tag{5.4}$$

and L(r) varies continuously from 1 to 0 in the interval $R < r < R + \varepsilon$.

We want to relate the energy spread of the reduced state $L\psi$ to the time T. For this purpose let us consider the projection operator P_ε on the interval $(R, R+\varepsilon)$, i.e.

$$[P_\varepsilon\psi](\underline{r}) = \begin{cases} \psi(\underline{r}) & \text{for } R \leq r \leq R+\varepsilon \\ 0 & \text{everywhere else} \end{cases} \quad , \tag{5.5}$$

(*) Note that if $\langle H\rangle = \infty$ the argument leading to $\left.\frac{dP}{dt}\right|_{t=0} = 0$ is no longer valid.

and let us calculate the indeterminacy $(\Delta P_\varepsilon)^2$ in the reduced state $L\psi$. We have

$$(\Delta P_\varepsilon)^2 = \frac{1}{\|L\psi\|^2}\langle L\psi|P_\varepsilon^2|L\psi\rangle - \frac{1}{\|L\psi\|^4}\langle L\psi|P_\varepsilon|L\psi\rangle^2 \quad , \tag{5.6}$$

so that

$$\frac{1}{\|L\psi\|^2}\langle L\psi|P_\varepsilon^2|L\psi\rangle \geq (\Delta P_\varepsilon)^2 \; . \tag{5.7}$$

Taking into account that $P_\varepsilon^2 = P_\varepsilon$ we obtain

$$\langle L\psi|P_\varepsilon^2|L\psi\rangle = \int_{(R,R+\varepsilon)} d^3r\, L^2(r)|\psi(\underline{r})|^2 < \int_{(R,R+\varepsilon)} d^3r\, |\psi(\underline{r})|^2 \quad , \tag{5.8}$$

where we have used the shorthand notation $\int_{(R,R+\varepsilon)} d^3r = \int_R^{R+\varepsilon} r^2 dr \int d\Omega$.
From (5.7) and (5.8) we get

$$\frac{1}{\|L\psi\|^2}\int_{(R,R+\varepsilon)} d^3r\, |\psi(\underline{r})|^2 \geq (\Delta P_\varepsilon)^2 \; . \tag{5.9}$$

We can then write

$$\frac{1}{\|L\psi\|^2}(\Delta E)^2\int_{(R,R+\varepsilon)} d^3r\, |\psi(\underline{r})|^2 \geq (\Delta P_\varepsilon)^2(\Delta E)^2 \geq \frac{1}{4}\,\frac{|\langle L\psi|[H,P_\varepsilon]|L\psi\rangle|^2}{\|L\psi\|^4} \quad , \tag{5.10}$$

where we have denoted by ΔE the energy spread in the state $L\psi$ and we have used the general form of the uncertainty relation for the two observables H and P_ε.

Due to the fact that R is larger than the range of the forces acting between the fragments, we can identify H with the kinetic energy operator. Using the fact that $L(R)=1, L(R+\varepsilon)=0$, the matrix element of the commutator $[H, P_\varepsilon]$ appearing in (5.10) is easily calculated, giving

$$\langle L\psi|[H,P_R]|L\psi\rangle = -i\Phi_R \quad , \tag{5.11}$$

where Φ_R is the flux associated with ψ through the sphere of radius R, i.e.

$$\Phi_R = \int_{\Sigma_R} \frac{\underline{j}\cdot\underline{R}}{R}\, d\Sigma \quad , \tag{5.12}$$

with $\underline{j}$ the current density. Since $\|L\psi\| \leqslant 1$, from (5.10) and (5.11) we get

$$(\Delta E)^2 \int_{(R,R+\varepsilon)} d^3r\, |\psi(\underline{r})|^2 \geqslant \frac{1}{4}\Phi_R^2 \ . \tag{5.13}$$

In classical terms the time T, necessary to complete the measurement process, is given by

$$T = \frac{\varepsilon}{V_r} \quad , \tag{5.14}$$

V_r being the radial velocity of the fragments. Eq. (5.14) implies

$$\rho(R)\,\varepsilon = T\,\rho(R)\,V_r = T\,\frac{\underline{j}\cdot\underline{R}}{R} \ . \tag{5.15}$$

Integrating this expression over Σ_R we have

$$\int_{\Sigma_R} \rho(R)\,\varepsilon\, d\Sigma = T \int_{\Sigma_R} \frac{\underline{j}\cdot\underline{R}}{R}\, d\Sigma \ . \tag{5.16}$$

For small ε the quantum analogue of (5.16) is evidently

$$\int_{(R,R+\varepsilon)} |\psi(\underline{r})|^2 d^3r = T\,\Phi_R \quad , \tag{5.17}$$

which can be assumed as a definition in the quantum case of the

time T necessary to perform a measurement process. Using (5.17), Eq. (5.13) becomes

$$T\,(\Delta E)^2 \geqslant \frac{1}{4}\Phi_R \ . \tag{5.18}$$

Since, as already remarked, the mean frequency $1/\lambda$ between two reductions must be larger than T, one also has

$$\frac{1}{\lambda}(\Delta E)^2 \geqslant \frac{1}{4}\Phi_R \ . \tag{5.19}$$

Equation (5.19) shows that, for $\lambda \to \infty$, ΔE must diverge, because the r.h.s. of Eq. (5.17) remains different from zero in the limit. In fact the flux Φ_R through the sphere of radius R at time t can be written as:

$$\Phi_R(t) = -\frac{\partial}{\partial t}\langle\psi_0|\, e^{iHt} P_R\, e^{-iHt}|\psi_0\rangle \quad , \tag{5.20}$$

where $|\psi_0\rangle$ is the state present at time t = 0, and P_R is the projection operator on $r \leqslant R$. We note that the expectation value on the r.h.s. of (5.20) is not the non-decay probability P(t), since P_R is different from $|\psi_0\rangle\langle\psi_0|$. Contrary to what happens for P(t), one can then easily show that the quantity $\langle\psi_0|e^{iHt}P_R\,e^{-iHt}|\psi_0\rangle$ does not have a vanishing derivative at t=0, a fact that should also be obvious on physical grounds. We remark that this means that it would be inconsistent to identify $\Phi_R(t)$ with the derivative of P(t) and, in evaluating the limit of the r.h.s. of Eq.(5.19) for $\lambda \to \infty$, to use the argument that $\left.\frac{dP}{dt}\right|_{t=0}=0$, as one could naively think of doing.

From (5.19), using expression (3.3a) for ΔE in our model and the definition (4.7), we see that m diverges at least as fast as ℓ, so that we are in case b) or c) of the previous section and Zeno's paradox cannot occur. We remark that the result now obtained, without being in contradiction with Eq. (5.2) obtained by the straightforward use of time-energy indeterminacy relations, is weaker, since it guarantees only the increasing of m with ℓ instead of ℓ^2. However, the obtained result is sufficient to exclude the occurrence of Zeno's paradoxical situation.

To summarize the previous discussion, we remark that in any case Zeno's paradox cannot occur. Moreover, there are good reasons(*) which make it quite plausible that $\ell/m \to 0$ for $\ell \to \infty$. In such a case we get from (4.14), $\nu = \gamma$, which is exactly the opposite of Zeno's paradox: for a very high frequency of the reductions the observed lifetime $1/\nu$ tends to the quantum mechanical one $1/\gamma$. In any case we want to stress that in practical cases one works with an almost fixed value of λ/γ and one has, in the cases in which P(t) can be determined experimentally, a very small γ. Typical figures for λ and γ are, as discussed in Ref.11, $\lambda = 10^3\gamma$ and $\gamma \simeq 10^{-7}$eV for a lifetime of 10^{-8} sec. Due to the smallness of γ, and the practical impossibility of making wave packets too narrow in energy, one always has $m \gg \ell$, implying, through Eq. (4.14), $x \simeq 0$, i.e. the differences between ν and γ are undetectable in practice.

All the above results have been obtained under the assumption that all reductions lead to an energy form factor of type (3.1a). However, the general argument stressing that one cannot take the $\lambda \to \infty$ limit keeping P(t) unchanged is without doubt correct. Moreover, since the very small time behaviour of P(t) is essentially determined by the energy spread (as explicitly shown in Sec.3) and is largely independent of the particular form of the cut-off function, and since the use of the uncertainty relations is the crucial point in the previous discussion, our result can safely be considered to hold completely in general.

(*) Besides the general argument leading to $\Delta E \cdot T \gtrsim 1$, a very sketchy evaluation of $(\Delta E)^2$ for the reduced state $|L\psi\rangle$ indicates that actually $(\Delta E)^2 \propto \epsilon^{-3}$ for very small values of ϵ, so that $\ell/m \to 0$ for $\ell \to \infty$.

We can then conclude that the occurrence of Zeno's paradox is conceptually forbidden and that it is practically hopeless to try to detect any dependence of the lifetime on λ by making λ large. The mechanism of repeated reductions therefore has the only effect of yielding a purely exponential decay law instead of the quantum mechanical law P(t), while the observed lifetime turns out to be practically equal to $1/\gamma$ to a great accuracy. To ascertain that the reduction mechanism is effective, the only crucial experiment at small times would be to measure P(t) in vacuum, and to show the deviations from the exponential. Repeating the experiment with the reductions taking place, one should then get a purely exponential survival probability F(t).

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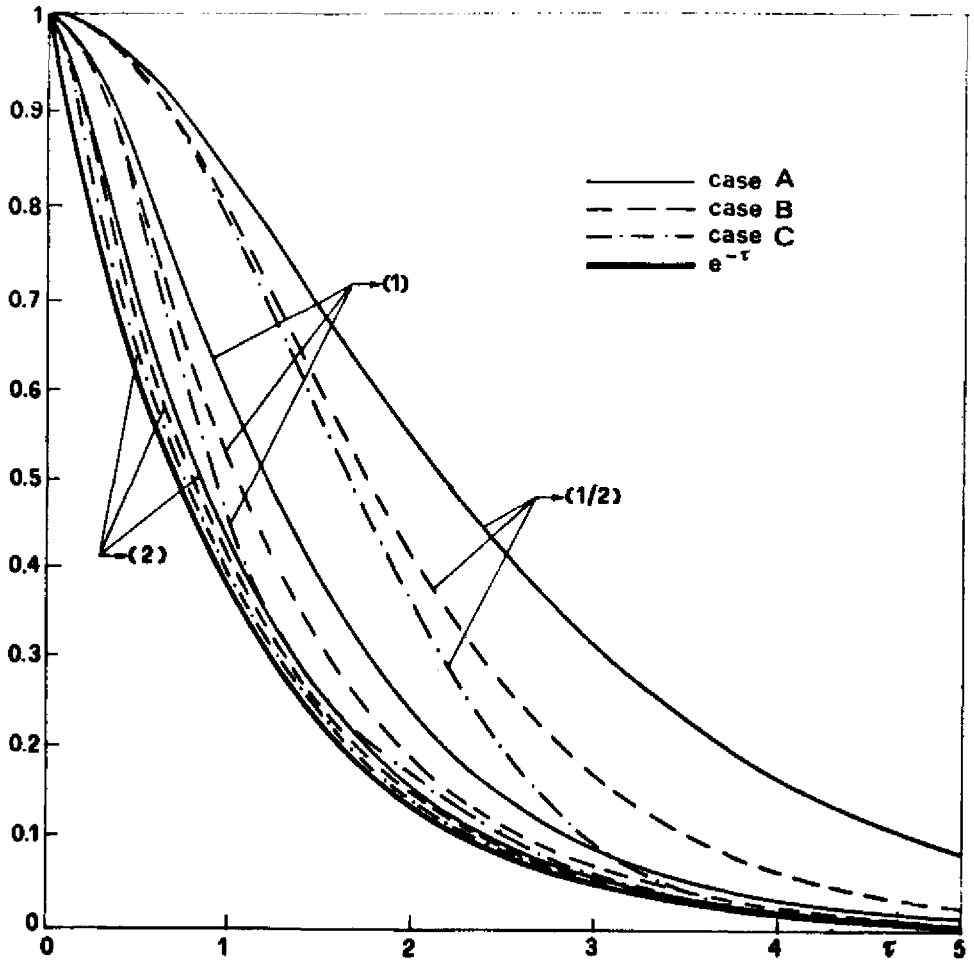

Fig. 1 The quantum non-decay probability $P(\tau)$'s for cases A, B and C of the text, up to $\gamma t = \tau = 5$. The numbers in parentheses are the values of $\Delta E/\gamma$. The corresponding values of the parameters $m, \mathfrak{M}$ and M (see text) are:

$m = 1, \mathfrak{M} = 1.155, M = 1.166$ for $\Delta E/\gamma = \frac{1}{2}$;

$m = 4, \mathfrak{M} = 3.850, M = 3.580$ for $\Delta E/\gamma = 1$;

$m = 16, \mathfrak{M} = 14.497, M = 13.026$ for $\Delta E/\gamma = 2$.

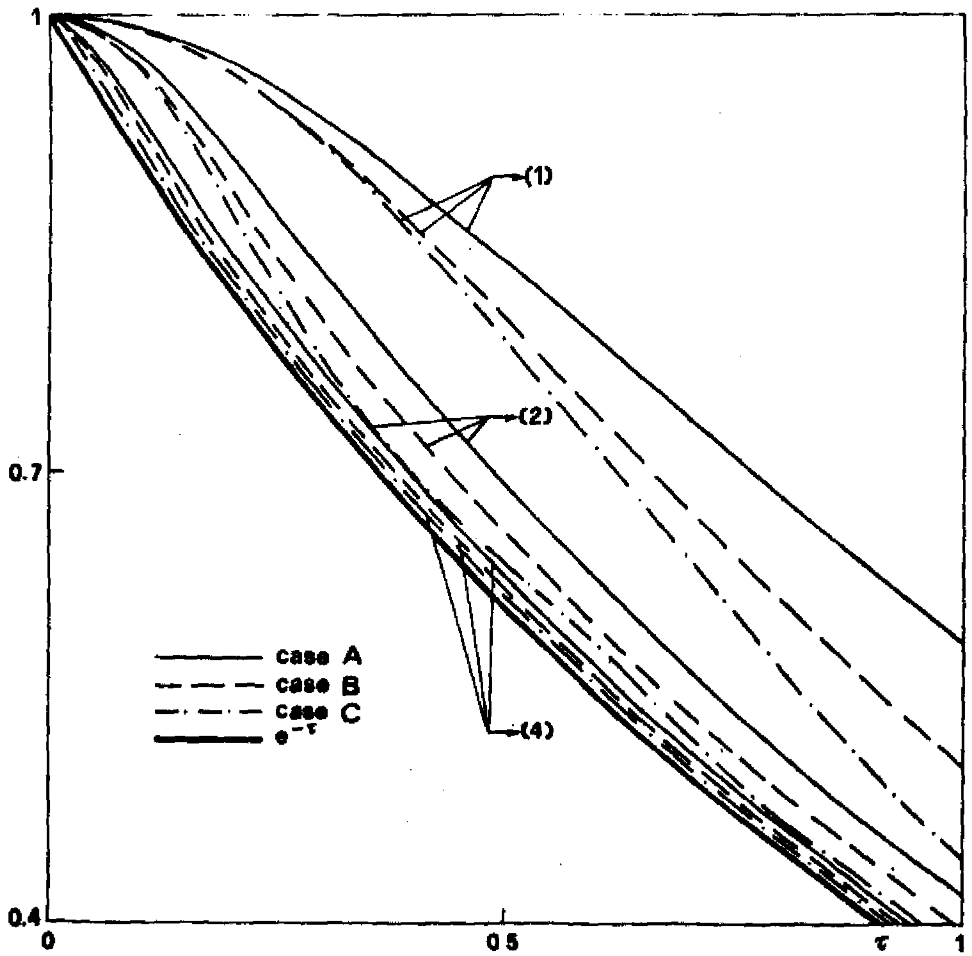

Fig. 2 The same as in Fig.1 1, up to $\tau = 1$.
For $\Delta E/\gamma = 4$ we have m = 64, $\mathcal{M}$ =
= 57.039, M = 50.731.

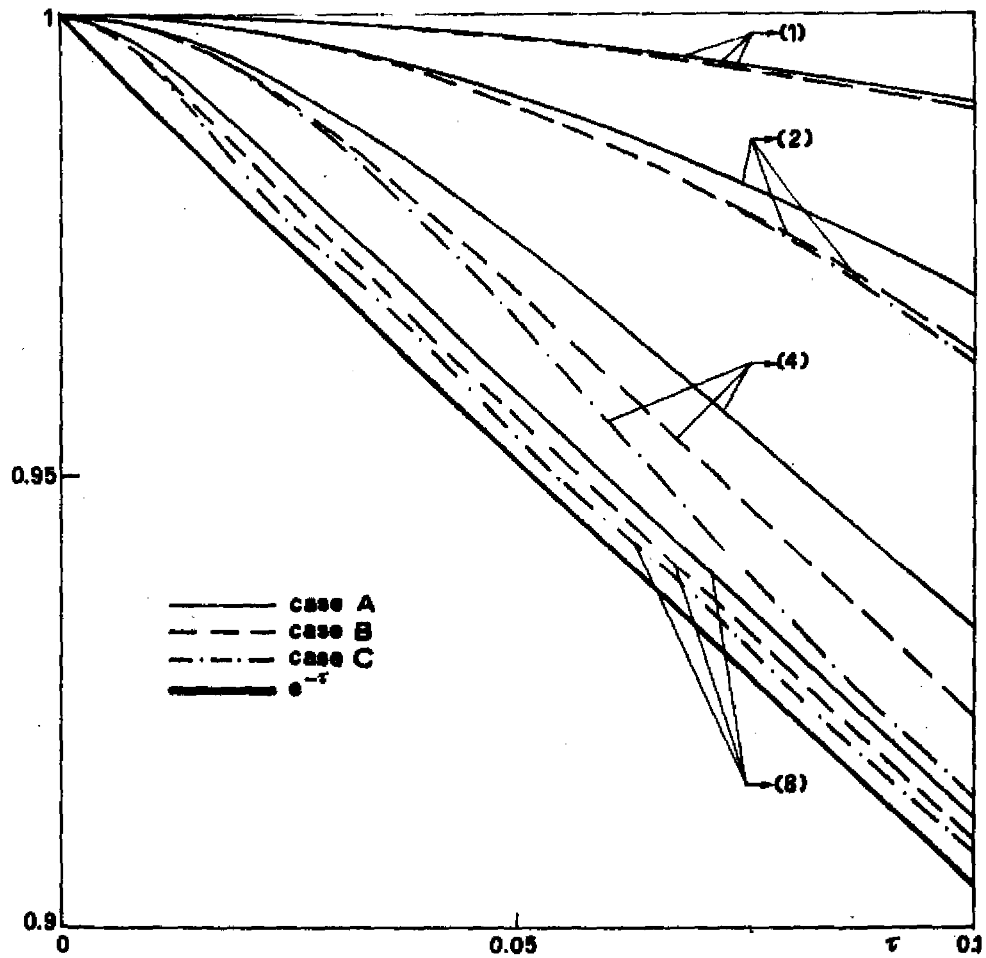

Fig. 3 The same as in Fig. 1, up to $\tau = 0.1$.
For $\Delta E/\gamma = 8$ we have m = 256, $\mathcal{M}$ =
= 227.196, M = 201.528.

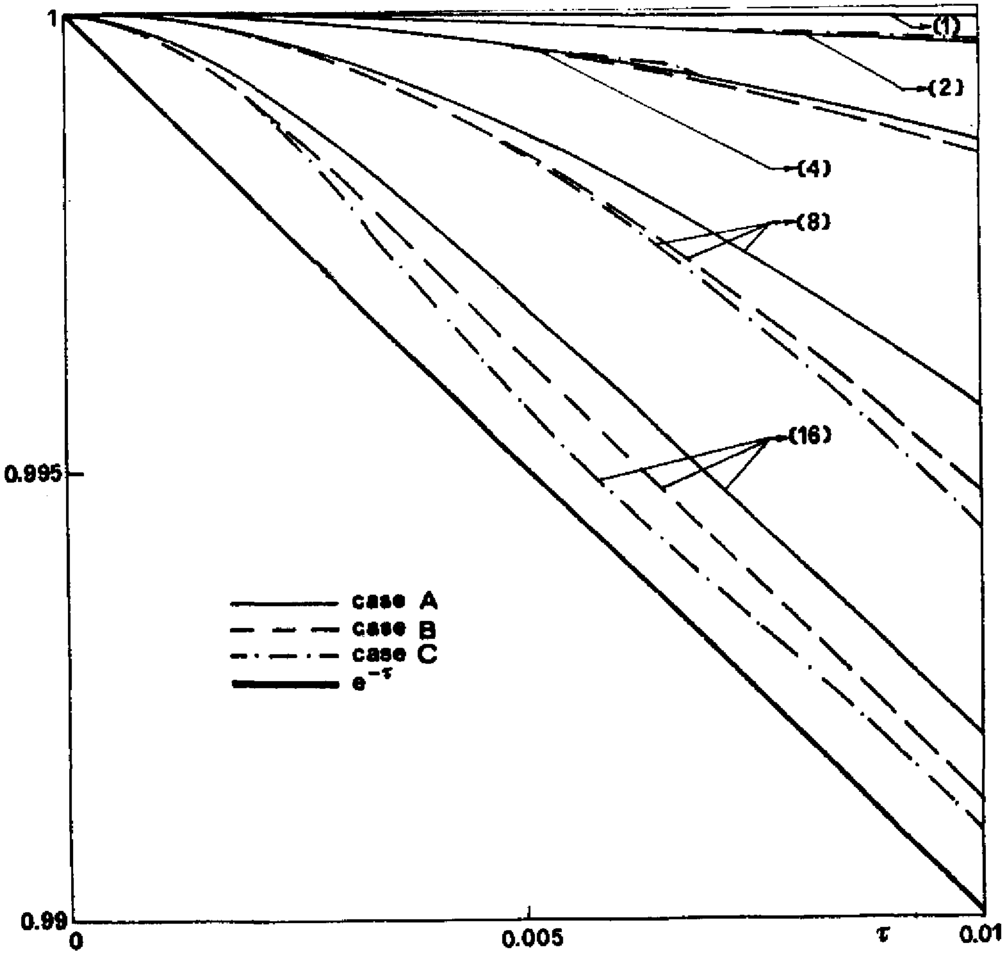

Fig. 4 The same as in Fig. 1, up to $\tau = 0.01$. For $\Delta E/\gamma = 16$ we have $m = 1024$, $\mathfrak{M} = 907.818$, $M = 804.715$.

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