IC/77/80
INTERNAL REPORT

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

"NATURALNESS" OF ATOMIC PARITY CONSERVATION WITHIN LEFT-RIGHT SYMMETRIC UNIFIED THEORIES *

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MIRAMARE - TRIESTE
August 1977

* To be submitted for publication.

** Supported in part by National Science Foundation Grant No.GP43662X.
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ABSTRACT

The question of "naturalness" of atomic parity conservation for left-right symmetric unified theories is examined. It is shown that the previously proposed patterns of spontaneous symmetry breaking do not offer a "natural" solution for such parity conservation. It may, however, be possible to secure this is left-right symmetry breaking in the neutral sector has a dynamically radiative origin.

1. Results of recent atomic parity experiments [1], when confronted with the present theoretical calculations [2], appear to show that the strength of atomic parity violation in neutral-current interactions may perhaps be one to two orders of magnitude smaller than G_F (if not smaller still), in contrast to charged current interactions where the magnitude is known to be of order G_F. Such a dichotomy between charged and neutral current interactions is not permissible within the simple "left-handed" $SU(2)_L \times U(1)$ theory. [3] However it can find a simple explanation (as can the entire body of presently known neutral current data) within the left-right symmetric theory [4] $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$, proposed some time ago with the primary motivation that Nature must be <u>intrinsically</u> symmetric between left versus right.

The left-right symmetric theory $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ (as well as <u>all</u> its quark-lepton unifying extension, e.g. the one based on [4] $SU(2)_L \times SU(2)_R \times SU(4)'_{L+R}$ or [5] $[SU(4)]^4$) have the distinguishing feature that for every left-handed (V-A) current coupled to the gauge particles $(W_L^{\pm}, W_L^3)$, there must exist a <u>parallel</u> (V+A) current coupled to a <u>distinct</u> set of gauge particles $(W_R^{\pm}, W_R^3)$ with equal strength $(g_L^{(0)} = g_R^{(0)})$. Parity violation at low energies arises in this class of theories due to spontaneously induced mass splittings between W_L's and W_R's.

The dichotomy between the degree of parity violation in the charged versus neutral-current sectors can arise within this theory, if the spontaneously induced mass asymmetry between the charged gauge particles $(W_L^{\pm}, W_R^{\pm})$ is large, while at the same time the mass asymmetry between the neutral members $(W_{L,R}^3)$ is small or "zero". To see how this may come about [6], consider Higgs fields $E_R = (1, \underset{\sim}{3}, Y=0)$ and $E_L = (\underset{\sim}{3}, 1, Y=0)$ transforming as <u>vectors</u> under $SU(2)_{L,R}$. The appropriate vacuum expectation values contribute only to charged W' masses, but <u>not</u> to the masses of the neutral ones. Introduce also the scalar fields $B = (1, \underset{\sim}{2}, Y = +1)$ and $C = (\underset{\sim}{2}, 1, Y = +1)$ transforming as spinors under $SU(2)_{L,R}$. These contribute (through their VEV) to the neutral as well as the charged W masses. Thus with

$$\langle E_R \rangle \gg \langle E_L \rangle \qquad (1)$$

but

$$\langle B \rangle \sim \langle C \rangle \quad ,$$

one would obtain a large mass asymmetry between the charged W's, even though that between the neutral ones ($W^3_{L,R}$) may be small or "zero".[7] Correspondingly parity violation in the charged sector would be large ($\mathcal{O}(g_L^2/8m^2_{W^+_L}) \equiv \mathcal{O}(G_F/\sqrt{2})$), while that in the neutral sector would be vanishingly small. In the limit $\langle B\rangle = \langle C\rangle$ and with $g_L = g_R$, neutral-current interactions would acquire the effective parity-conserving form (VV + AA). Allowing for <u>finite</u> $\mathcal{O}(\alpha)$ radiative corrections[4),8)] to $(g_L-g_R)/g_L$ parity violation in neutral-current processes would arise (in this case) in order $G_F^{(N)}\alpha$ (where $G_F^{(N)}/\sqrt{2} \equiv g_L^2/8m^2_{N_1}$ and m_{N_1} is the mass of the lightest neutral weak gauge particles). (Note, unlike standard SU(2) × U(1) vector-like theories[9)], an interaction possessing VV as well as AA pieces would distinguish between neutrinos (ν_L) and antineutrinos ($\bar{\nu}_R$), even though it conserves parity[10)] simply because the available neutrinos are left-handed, while the antineutrinos are right-handed - produced as they are by dominantly (V-A) charged current interactions. Thus the theory would still predict ($\sigma^{NC}_{\nu_L P} \neq \sigma^{NC}_{\bar{\nu}_R P}$), as observed experimentally[11)].

Given the left-right symmetric theory, and the picture of spontaneous symmetry breaking as outlined above, it is natural to ask: Is the solution of <u>vanishing</u> left-right mass asymmetry in the neutral sector ($\langle B\rangle - \langle C\rangle = 0$) "natural"? <u>In other words, is this solution radiatively stable despite the mass asymmetry in the charged sector</u> ($m_{W^+_R} \gg m_{W^+_L}$), <u>in the sense that loop corrections induce at most finite and therefore calculable order α corrections to the relevant asymmetry parameter ($\langle B\rangle^2 - \langle C\rangle^2)/\langle C\rangle^2$</u> ? The question is at the same level as the one which arises when we try to achieve a "natural" understanding of isospin conservation[12)] ($m_n - m_p = \mathcal{O}(\alpha)\, m_n$) within unified theories. The purpose of this note is to examine the zeroth-order condition $\langle B\rangle = \langle C\rangle \neq 0$ and to remark that it is not natural in the above sense.

Now it appears that if one wishes to obtain $\langle B\rangle = \langle C\rangle \neq 0$ with $\langle E_R\rangle \neq \langle E_L\rangle$ in the zeroth order of spontaneous symmetry breaking, one has to impose the following restrictions on the relevant Higgs potential: (a) the mass parameters of B and C be equal in the bare Lagrangian ($\mu_b^{(0)2} = \mu_c^{(0)2}$) ; (b) their quartic couplings also be equal (this is required by natural $L \leftrightarrow R$ symmetry[8)]); (c) the invariant quartic coupling $(BB^+ - CC^+)(E_RE_R^+ - E_LE_L^+)$, <u>though allowed by the gauge as well as $L \leftrightarrow R$ symmetry</u>, be absent in the bare Lagrangian, and (d) the invariant term $(E_L^+E_L)(E_R^+E_R)$ be present. (This last term is essential to generate

$\langle E_L\rangle \neq \langle E_R\rangle$ with $\mu^{(0)2}_{E_R} = \mu^{(0)2}_{E_L}$.) The point we wish to make is that, at the least, the condition (c) cannot be maintained when we consider the perturbative radiative corrections involving W_L^+ and W_R^+ loops. These, re-introduce, with <u>infinite</u> strength the omitted quartic coupling $(BB^+ - CC^+)(E_RE_R^+ - E_LE_L^+)$. The infinities may, of course, be absorbed, at the expense however of introducing corresponding counter-terms into the bare Lagrangian. This makes the <u>renormalized</u> value of the parameter $(\langle B\rangle^2 - \langle C\rangle^2)/\langle C\rangle^2$ in general non-vanishing and incalculable within the theoretical framework as presently available. The implications of this observation and a possible resolution are noted at the end of this note.

2. To see the result stated above, we first write down the general Higgs potential involving B, C, E_R, E_L fields consistent with renormalizability and "natural" $L \leftrightarrow R$ symmetry. (Note that "natural" $L \leftrightarrow R$ symmetry as defined in Ref.8, requires that $L \leftrightarrow R$ discrete symmetry must be preserved everywhere, except possibly for scalar mass terms, so that radiative corrections to $(g_L-g_R)/g_L$ are finite and of order α.) The general potential subject to the discrete symmetry $E_{L,R} \to E_{L,R}$ is given by

$$
\begin{aligned}
V(E_R,E_L;B,C) = & -\mu_B^{(0)2}\,(B^+B) - \mu_C^{(0)2}\,(C^+C) \\
& + \lambda_{B1}^{(0)}((B^+B)^2 + (C^+C)^2) + \lambda_{B2}^{(0)}\,(B^+B)\,(C^+C) \\
& - \mu^{(0)2}_{E_R}\,E_R^+E_R - \mu^{(0)2}_{E_L}\,E_L^+E_L \\
& + \lambda_{E1}^{(0)}\,((E_R^+E_R)^2 + E_L^+E_L)^2) + \lambda_{E2}^{(0)}\,(E_R^+E_R)\,(E_L^+E_L) \\
& + \kappa_s^{(0)}\,(E_R^+E_R + E_L^+E_L)\,(B^+B + C^+C) + \kappa_a^{(0)}\,(E_R^+E_R - E_L^+E_L)\,(B^+B - C^+C) \quad . \qquad (2)
\end{aligned}
$$

We do not exhibit the presence of other fields such as A = (2,2, Y = 0) which must be present to give masses to fermions. The presence of such fields does not influence the issue of naturalness. (The terms $\lambda_E^{(0)}\{(E_R^+\,t_iE_R)(E_R^+t_iE_R) + R \to L\}$ and $\lambda_{EB}^{(0)}\{(E_R^+t_iE_R)(B^+\tau_iB) + (E_L^+t_iE_L)(C^+\tau_iC)\}$ have been dropped for ease of writing. These would not contribute to the extremum conditions $\partial V/\partial B^+ = 0$, $\partial V/\partial C^+ = 0$ upon substitutions for the vacuum expectation values for $E_{L,R}$.)

Insisting on complete $L \leftrightarrow R$ symmetry in the basic Lagrangian, one must set the scalar mass terms to be $L \leftrightarrow R$ symmetric ($\mu_B^{(0)} = \mu_C^{(0)}$ and $\mu_{E_R}^{(0)} = \mu_{E_L}^{(0)}$). It can be shown, following Ref.13, that even with a completely $L \leftrightarrow R$ symmetric potential involving all four fields (B, C, E_L, E_R), it is possible to obtain a solution $\langle E_R \rangle \neq \langle E_L \rangle$ and thereby $m_{W_R^+} \neq m_{W_L^+}$ for a range of values of the parameters subject to $\mu_i^2 > 0$ $(i = B, C, E_R, E_L)$ and $\lambda_{E2}^{(0)} > 2\lambda_{E1}^{(0)}$. Thus to proceed with let us set $\mu_B^{(0)} = \mu_C^{(0)}$; $\mu_{E_R}^{(0)} = \mu_{E_L}^{(0)}$. (As it will be clear later, our conclusions will not depend upon this restriction.)

We are asking the question: Is the following pattern of zeroth order vacuum expectation values:

$$\langle B \rangle = \langle C \rangle = \begin{pmatrix} 0 \\ b \end{pmatrix} \neq 0 \quad ,$$

$$\langle E_R \rangle = \begin{pmatrix} 0 \\ \epsilon_R \\ 0 \end{pmatrix} \text{ and } \langle E_L \rangle = \begin{pmatrix} 0 \\ \epsilon_L \\ 0 \end{pmatrix} \neq \langle E_R \rangle \quad , \tag{3}$$

radiatively stable and therefore a "natural" solution for the minimum of the potential for a range of values of the parameters defining the zeroth-order potential?

To answer this question, first write the extremum conditions for the zeroth-order potential ($\partial V/\partial B^+ = 0$; $\partial V/\partial C^+ = 0$):

$$\Big[\mu_b^{(0)2} + 2\lambda_{B1}^{(0)} B^+B - \lambda_{B2}^{(0)+} (C^+C) + \kappa_s^{(0)} (E_R^+E_R + E_L^+E_L) + \kappa_a^{(0)} (E_R^+E_R - E_L^+E_L)\Big] B = 0 \quad , \tag{4}$$

$$\Big[\mu_c^{(0)2} + 2\lambda_{B1}^{(0)} C^+C - \lambda_{B2}^{(0)} (B^+B) + \kappa_s^{(0)} (E_R^+E_R + E_L^+E_L) - \kappa_a^{(0)} (E_R^+E_R - E_L^+E_L)\Big] C = 0 \quad . \tag{5}$$

Substituting the pattern of vacuum expectation values (3) into (4) and (5) and taking the difference between the two equations, we obtain (with $\mu_B^{(0)2} = \mu_C^{(0)2}$)

$$\kappa_a^{(0)} (\epsilon_R^2 - \epsilon_L^2) = 0 \quad . \tag{6}$$

Since $\epsilon_R \neq \epsilon_L$, we see that a necessary condition for the pattern $\langle B \rangle = \langle C \rangle \neq 0$ with $\langle E_R \rangle \neq \langle E_L \rangle$ is that

$$\kappa_a^{(0)} = 0 \quad , \tag{7}$$

i.e. the term $(E_R^+E_R - E_L^+E_L)(B^+B - C^+C)$ must be absent in the bare Lagrangian. This term, though odd under the interchange $B \leftrightarrow C$, is even under the simultaneous interchange $(B \leftrightarrow C,\ E \leftrightarrow F)$, and thus allowed by discrete $L \leftrightarrow R$ symmetry. It is of course also allowed by the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$. Thus, as might be expected, even if one did not introduce such a term into the bare Lagrangian, it is induced by loop diagrams calculated perturbatively with respect to the symmetric vacuum (see Figs.1 and 2)

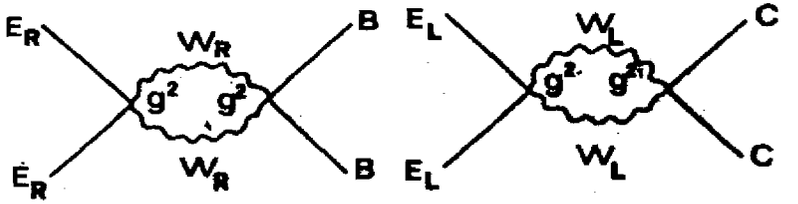

Fig.1

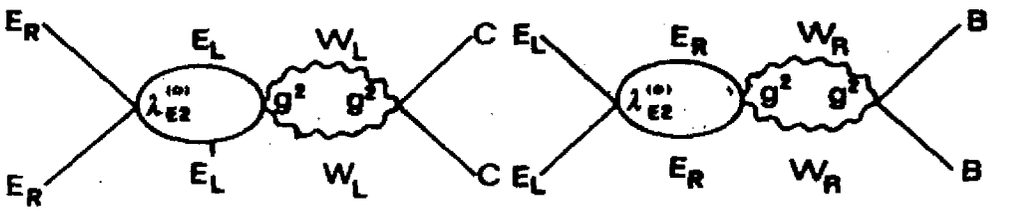

Fig.2

Note that both Fig.1 and Fig.2 are logarithmically divergent. Thus they generate (since their strengths are unequal) both the $B \leftrightarrow C$ symmetric $(B^+B + C^+C)(E_R^+E_R + E_L^+E_L)$ and the $B \leftrightarrow C$ antisymmetric term $(B^+B - C^+C)(E_R^+E_R - E_L^+E_L)$ with infinite strengths. The infinities can be absorbed only if we allow the presence of corresponding counter-terms in the bare Lagrangian. Hence, insisting on renormalizability, we must choose $\kappa_s^{(0)} \neq 0$, $\kappa_a^{(0)} \neq 0$ in the bare Lagrangian. The renormalized value of κ_a is thus a free parameter in the theory which cannot be computed. To this

extent the renormalized value of $(\langle B\rangle^2 - \langle C\rangle^2)/\langle C\rangle^2$ as well is not calculable. It thus follows that the zeroth-order solution $\langle B\rangle = \langle C\rangle \neq 0$ together with $\langle E_R\rangle \neq \langle E_L\rangle$ is not a "natural" solution of the theory (in the technical sense).

Note that the same conclusion is reproduced if we examine the minimum of the effective potential calculated with respect to the symmetric vacuum by including the effect of all one-loop corrections to order g^4, which inevitably reproduces the κ_a term through Fig.1.

Note, for the sake of generality, that if we had chosen $\mu_B^{(0)} \neq \mu_C^{(0)}$ (and even if this had permitted $\langle B\rangle = \langle C\rangle \neq 0$) we would obtain [14] from the difference between (4) and (5) the equation $(\mu_b^{(0)2} - \mu_c^{(0)2}) + 2\kappa_a^{(0)}(\epsilon_R^2 - \epsilon_L^2) = 0$ instead of (6). This can only be satisfied for a specific value of the parameter $\kappa_a^{(0)} = -(\mu_b^{(0)2} - (\mu_c^{(0)})^2)/(\epsilon_R^2 - \epsilon_L^2)$. Thus one more parameter is needed for the calculation of $(\langle B\rangle^2 - \langle C\rangle^2)/\langle C\rangle^2$. This is contrary to the conventional concept of naturalness.

Now suppose it happens that with continuing improvements in experimental measurements and theoretical calculations, it is established that the effective strength of parity violation in atoms is not just one but two orders of magnitude smaller than G_F. This observation can, of course, be accommodated within the left-right symmetric theory [4] by assuming that the renormalized values of the parameters $\langle B\rangle$ and $\langle C\rangle$ are nearly equal. Correspondingly, there would be several testable predictions (in particular those involving e^-e^+ forward-backward asymmetry measurements [15] and likewise measurements involving dilepton production by hadrons [16]). However, one could face a dilemma calling for a natural understanding of this dramatic situation. Below we present briefly a possible resolution of such a possible dilemma.

We have so far followed the pattern of spontaneous symmetry breaking proposed in earlier works [4),7),8),13] and have posed the question whether within such a pattern the zeroth-order parity-conserving solution $\langle B\rangle = \langle C\rangle \neq 0$ is radiatively stable with $\langle E_R\rangle \neq \langle E_L\rangle$. Note the distinctive feature of this pattern that all gauge particles (charged as well as neutral) acquire mass in the zeroth order.

Now consider an alternative solution. Allowing for all possible invariant terms in the potential (Eq.(2)) consistent with renormalizability and discrete $L \leftrightarrow R$ symmetry [4], choose the signs of B and C-(mass)^2 terms, so that in the zeroth order, minimization of the potential yields:

$$\langle B\rangle = \langle C\rangle = 0 \quad ; \quad \langle E_L\rangle = 0 \quad .$$

But

$$\langle E_R\rangle \neq 0 \quad . \tag{8}$$

Note that the vanishing of the $\kappa_a^{(0)}$ term is no longer necessary (vide Eqs. (4) and (5)) once $\langle B\rangle = \langle C\rangle = 0$ (rather than $\langle B\rangle = \langle C\rangle \neq 0$). The solution (8) implies that in the zeroth order of spontaneous symmetry breaking (i.e. barring loop corrections) only the charged $W_R^\pm$ acquire a mass, all other gauge particles $(W_L^\pm, W_L^3)$, W_R^3 as well as the U(1) field remain massless. The symmetry $\mathcal{G} = SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times (P)$ thereby descends to $SU(2)_L \times U(1)_R \times U(1)_{L+R}$ (where P denotes discrete $L \leftrightarrow R$ symmetry).

But now allowing for radiative corrections [17], both $\langle B\rangle$ and $\langle C\rangle$ can develop at the one-loop level non-zero vacuum expectation values. However, this time there is the important bonus that both $\langle B\rangle^2$ and $\langle C\rangle^2$ are $O(\alpha)$ compared with $\langle E_R\rangle^2$. In turn the (mass)^2 of the left-handed gauge particles $(W_L^\pm)$ mediating (V-A)-interactions and the (mass)^2 of the two neutral gauge particles (N_1 and N_2) are of order $\alpha m_{W_{R^+}}^2$. The difference $(\langle B\rangle^2 - \langle C\rangle^2)$, however, is

$$\{[O(\alpha^2) + O(\kappa_a^{ren}, \lambda_1^{ren})]/(2\lambda_{B1}^{ren} - \lambda_{B2}^{ren})\}\langle E_R\rangle^2 \quad .$$

The $O(\alpha)$-contribution to $(\langle B\rangle^2 - \langle C\rangle^2)$ vanishes in this case due to left-right symmetry of the basic Lagrangian. The parity violating parameter [15] $x \equiv (b^2 - c^2)/c^2$ from this mechanism is expected to be naturally small [18], implying a small atomic parity violation compared with the $SU(2)_L \times U(1)$ value and a light neutral gauge particle [15] N_1 (with mass $\approx m_{W_L^+}$). Such a picture may provide an attractive possibility of a natural hierarchy for the gauge masses [19] and deserves study in its own right. This would be pursued in a subsequent note.

Two of us (H.S.M. and J.C.P.) would like to acknowledge the hospitality of the International Centre for Theoretical Physics, Trieste.

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4) The first suggestion of the left $\leftrightarrow$ right symmetric theory $SU(2)_L \times SU(2)_R \times U(L)_{L+R}$ comprising all matter (quarks as well as leptons) was made by J.C. Pati and Abdus Salam, Phys. Rev. Letters 31, 661 (1973); ibid. Phys. Rev. D10, 275 (1974). In this work two alternative patterns of spontaneous symmetry breaking were proposed, one of them permitting the possibility of two relatively light neutral gauge bosons. It is this alternative, which is of relevance to present atomic parity experiments (Ref.1), and has been pursued recently by several authors (Ref.7).

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6) See Sec.IV and footnote 21 of Ref.4.

7) The consequences of the zeroth-order solution $\langle B \rangle = \langle C \rangle \neq 0$ arising within the model proposed in Ref.4 has been examined by H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976) and more recently by R.N. Mohapatra and D.P. Sidhu, Phys. Rev. Letters 38, 667 (1977) and B.N.L. Preprint (22561). The more general case comprising $\langle B \rangle \sim \langle C \rangle$ and $\langle B \rangle = \langle C \rangle$ has been examined by J.C. Pati, S. Rajpoot and Abdus Salam, Imperial College, London, preprint ICTP/76/11 (to be published in Phys. Rev. D), and Imperial College, London, preprint ICTP/76/15. For a somewhat different treatment of spontaneous symmetry breaking of the group structure proposed in Ref.4 see, A. De Rujula, H. Georgi and S.L. Glashow, Harvard preprint, 1977.

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10) It needs to be stressed that contrary to common impression $(\sigma_{\nu_L p} \neq \sigma_{\bar{\nu}_R p})_{NC}$ does not imply parity non-conservation since $\nu_L \not\leftrightarrow \bar{\nu}_R$ under parity. Such a distinction between ν_L and $\bar{\nu}_R$ cross-sections eliminates only the class of parity-conserving theories, which are vector-like with no AA piece.

11) A. Benvenuti et al., Phys. Rev. Letters 37, 1039 (1976); J. Blietschau et al., Preprint CERN/EP/Phys. 76-55; B.C. Barish, Cal. Tech. preprint CALT-68-544. The clearest distinction is shown by measurements of $\sigma(\bar{\nu}_R p \to \bar{\nu}_R p)$ versus $\sigma(\nu_L p \to \nu_L p)$. See D. Clive et al., Phys. Rev. Letters 37, 252, 648 (1976), and W. Lee et al., Phys. Rev. Letters 37, 186 (1976).

12) That the left-right symmetric gauge structure $SU(2)_L \times SU(2)_R \times U(1)$ provides a solution for natural conservation of isospin was first observed by S. Weinberg, Phys. Rev. Letters 29, 1698 (1972), who introduced as a semirealistic model the symmetric gauge structure for quarks, but not leptons.

13) G. Senjanovic and R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).

14) Whether $\langle B \rangle = \langle C \rangle \neq 0$ is an allowed zeroth-order solution for the minimum of the potential with $\mu_b^{(0)^2} \neq \mu_c^{(0)^2}$, needs to be examined. But this is not relevant to our conclusion.

15) J.C. Pati, S. Rajpoot and Abdus Salam (Ref.7).

16) H.S. Mani, J C Pati, S. Rajpoot and Abdus Salam, ICTP, Trieste, preprint IC/77/88.

17) S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

18) Such a calculation is in progress. The quantities $\langle B\rangle^2$ and $\langle C\rangle^2$ would be of order $(\alpha m^2_{W^+_R})$ multiplied by appropriate log factors, if $m_{W^+_R}$ is the controlling heaviest mass in the theory. Given that the $(\text{mass})^2$-terms for B and C fields acquire the "desirable" sign through $O(\alpha)$-radiative corrections (for non-zero vacuum expectation values), it may be argued that $(\langle B\rangle^2 - \langle C\rangle^2)$ should vanish for some critical value of the coupling constant $g^2 = g_c^2 = O(\kappa_a, \lambda_i)$, i.e. $(\langle B\rangle^2 - \langle C\rangle^2) \propto (g^4 - g_c^4)$ with $0 \le g_c^4 \le g^4$. This implies that $\{(\langle B\rangle^2 - \langle C\rangle^2)/\langle C\rangle^2\}$ would in this case be naturally $O(\alpha)$.

19) Also this question may acquire a new complexion if we imbed $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ or $SU(2)_L \times SU(2)_R \times SU(4)'_{L+R}$ into a bigger group like $[SU(4)]^4$.