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A STEEPLY-RISING POTENTIAL IN TENSOR GAUGE THEORY

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INTERNATIONAL ATOMIC ENERGY AGENCY

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ADDENDUM

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ADDENDUM

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p.5 : Add footnote ***) to the last line.

***) It has been shown that in addition to the classical solution of the mass-modified Einstein equation used in this note, there are possibly other classical Yukawa-behaved solutions, corresponding to massive f-quanta. [3] We have not so far been able to obtain analytical expressions for these, but if the associated potentials rise initially ($\approx r^2$) before the eventual Yukawa fall-off ($\approx e^{-Mr}/r$), such solutions may be the ones more relevant for describing physical hadrons. This is because they would give (1) partial - as opposed to exact - confinement; (2) Regge trajectories which rise at first but then bend over; and most important (3) an <u>inter-composite</u> effective f-potential vanishing for large r . This would obviate any need for a mechanism (for example, a quantum number, topological or otherwise) which is specifically designed to distinguish, in the context of f-potentials, quarks from their composites.

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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A STEEPLY-RISING POTENTIAL IN TENSOR GAUGE THEORY *

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ABSTRACT

Massive tensor gauge mesons produce a steeply rising "potential" of the type M^2r^2 in a non-perturbative classical solution of an Einstein-like equation. Such a potential is shown to give rising Regge trajectories.

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1. It has been suggested in Ref.1, that hadrons interact strongly through the exchange of spin-two particles which are governed by a gauge principle. The corresponding tensor field satisfies modified Einstein equations containing a strong coupling parameter as well as a mass term. The gauge group consists of the general co-ordinate transformations of Einstein and we shall assume the operation of a dynamical mechanism which causes their spontaneous breakdown [2] and at the same time generates a mass for the gauge tensor. (For the purpose of this note, however, a simple Pauli-Fierz-like mass term will be assumed.)

An exact classical solution of the mass-modified equations has recently been found [3]. This solution describes the tensor field produced by a spin-zero source (quark) at rest. (The source could carry an internal symmetry charge as well.) The solution can be interpreted as an effective "potential" which contains a term proportional to M^2r^2 for large r where M is the tensor mass. With this potential as input we solve a Klein-Gordon equation describing the motion of a (spin-zero) particle (another quark or anti-quark) in the field produced by the (spin-zero) source quark. The eigensolutions are discrete and fall on Regge trajectories rising proportionally to mass. We interpret these solutions as bound state wave functions of a composite system. Both the motion of the source and the reaction of the test particle are neglected. Subject to these reservations, however, we propose this picture as an idealized two quark, or quark-anti-quark system. (For simplicity we are ignoring the important vector gluon contributions which would distinguish qq and $\bar{q}q$.)

The main point we wish to make is that the trajectories rise indefinitely. There is no ionization limit: the source and test particles are confined in their mutual well. This confinement persists even when the test particle mass as well as the source "mass"-parameter are neglected. The crucial parameter in the model is the mass of the spin-two gauge particle which determines the form of the potential at large separations.

2. The modified Einstein system is characterized by the phenomenological Lagrangian

$$\mathcal{L} = \frac{1}{16\pi G_S}\sqrt{-g}\ R(g) + \frac{1}{16\pi G_N}\sqrt{-f}\ R(f) + \mathcal{L}_{fg} \quad ,$$

where the first term is the usual Einstein Lagrangian for the gravity field $g_{\mu\nu}$ and the second is its strong analogue for the tensor $f_{\mu\nu}$. In the second term the coupling parameter $G_S \sim 1\ \mathrm{GeV}^{-2}$ replaces the Newtonian constant G_N. The third term is needed to give the f particle a mass as well as a gravitational interaction. We choose the form

$$\mathcal{L}_{fg} = \frac{1}{32\pi G_S} \sqrt{-g} \left\{f^{\mu\nu} - g^{\mu\nu}\right\} \left\{f^{\kappa\lambda} - g^{\kappa\lambda}\right\} \left\{g_{\kappa\mu} g_{\lambda\nu} - g_{\kappa\lambda} g_{\mu\nu}\right\} ,$$

which is a scalar density with respect to general co-ordinate transformations acting simultaneously on $f_{\mu\nu}$ and $g_{\mu\nu}$. (As with the gravity field $g_{\mu\nu}$ we define $f = \det f_{\mu\nu}$ and $f^{\mu\nu}$ = matrix inverse of $f_{\mu\nu}$.) M is spin-two mass.

The spherically symmetric solution, recently found [3),*)], is given in spherical polar co-ordinates by the form

$$f_{\mu\nu}\, dx^\mu\, dx^\nu = \frac{2}{3} (1+\alpha)^{-1} \left[(1-p)\, dt^2 - 2\sqrt{p(p+\alpha)}\, dt\, dr - (1+\alpha+p)\, dr^2\right] - \frac{2}{3} r^2 \left[d\theta^2 + \sin^2\theta\, d\varphi^2\right] , \qquad (1)$$

where $\alpha > -1$ is an integration constant and $p(r)$ is given by

$$p(r) = \frac{2\mu}{r} - \kappa^2 r^2 \qquad (2)$$

with μ, a second integration constant. This constant is positive for $\alpha > 0$ and negative for $\alpha < 0$. The parameter κ^2 is given in terms of α and the tensor mass M by the formula

$$\kappa^2 = -\frac{9}{16} M^2 (1+\alpha)^{3/2} \qquad (3)$$

*) One may multiply the expression for $\mathcal{L}_{fg}$ given in the text by the factor $\left(\frac{f}{g}\right)^c$, where c is a dimensionless parameter. Such a modification has been considered by C.J. Isham (private communication) who has shown that the only change to the solution of Ref.3 is in the expression for κ^2, which now reads:

$$\kappa^2 = \frac{M^2}{4} \left(\frac{81}{16} (1+\alpha)\right)^{\frac{1}{2}-c} \left[(c-1)\,(1+\alpha) - \frac{c}{3}\right] .$$

Clearly, in this case c can always be so chosen as to make κ^2 positive.

where the negative square root must be chosen for $(1+\alpha)^{1/2}$ when $\alpha < 0$ and the positive square root for $\alpha > 0$. In the sequel we shall take $\alpha < 0$, so that $\mu < 0$ and [*)] $\kappa^2 > 0$.

3. The Klein-Gordon equation for the field of a scalar hadron coupled to the gauge tensor $f_{\mu\nu}$,

$$\frac{1}{\sqrt{-f}} \partial_\mu \left(\sqrt{-f}\; f^{\mu\nu}\, \partial_\nu \Phi \right) + m^2 \Phi = 0 ,$$

reduces in the background provided by the solution (1) to the form

$$0 = \frac{1}{1-p} \partial_{\bar t}^2 \Phi - \frac{1}{r^2} \partial_r \left[r^2 (1-p)\, \partial_r \Phi\right] - \frac{1}{r^2} \left\{\frac{1}{\sin\theta} \partial_\theta \left[\sin\theta\, \partial_\theta \Phi\right] + \frac{1}{\sin^2\theta} \partial_\varphi^2 \Phi \right\} + \frac{2}{3} m^2 \Phi , \qquad (4)$$

where $\bar t$ denotes a retarded time defined by

$$\bar t = \frac{1}{\sqrt{1+\alpha}} \left(t - \int^r dr \frac{\sqrt{p(p+\alpha)}}{1-p}\right) . \qquad (5)$$

Eq.(4) is separable and can be solved in terms of hypergeometric functions. Writing

$$\Phi = e^{-i\omega \bar t}\, r^{-1} R(r)\, Y_{\ell m}(\theta,\varphi) \qquad (6)$$

and replacing the radial co-ordinate r by the angle

$$\rho = \mathrm{Tan}^{-1} \kappa r , \qquad (7)$$

one is left with the eigenvalue problem (for the case $\mu = 0$)

$$\frac{d^2 R}{d\rho^2} + \frac{\omega^2}{\kappa^2} R = \left\{\frac{\ell(\ell+1)}{\sin^2\rho} + \frac{2 + 2m^2/3\kappa^2}{\cos^2\rho}\right\} R , \qquad (8)$$

*) For the conventional Schwarzschild solution of gravity theory, $\mu > 0$ represents source mass. We need $\mu < 0$ to prevent $1-p(r)$ from vanishing. If this happens for $r > 0$, the system would radiate according to the ideas of Hawking [4)].

where ρ varies between 0 and $\pi/2$. At both ends of the integration region the effective potential rises without limit and the spectrum of eigenvalues is therefore discrete. One finds the energy eigenvalues $E_{n\ell} = (1+\alpha)^{-1/2}\,\omega_{n\ell}$ where

$$\omega_{n\ell}^2 = \kappa^2 \left[(2n+\ell) + \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{2}{3}\frac{m^2}{\kappa^2}}\,\right]^2 \qquad (9)$$

and n ranges over the non-negative integers. The corresponding wave function is given by

$$r^{-1} R_{n\ell} = \frac{1}{\kappa}(\kappa r)^\ell \left(1+\kappa r^2\right)^{-\frac{1}{2}\left[\ell + \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{2}{3}\frac{m^2}{\kappa^2}}\right]} \times$$

$$\times\ F\left\{n + \frac{|\omega_{n\ell}|}{\kappa}\ ,\ -n\ ;\ \ell + \frac{3}{2};\ \frac{\kappa^2 r^2}{1+\kappa^2 r^2}\right\}. \qquad (10)$$

There is no continuum, i.e. no ionization. Hence the proposal that this kind of tensor-mediated system may serve as a model for confinement in the sense of no ionization.

4. Generalizations of the above treatment may be considered. The spinning test particle would be described by the Dirac equation in the same background field as used here. A charged and spinning source particle would give rise to a more complicated background (analogous to the Kerr-Newman metric in Einstein-Maxwell theory).

Inclusion of gauge symmetries of the internal type mediated by vector gluons would lead to a splitting of qq from $q\bar{q}$ states in that the latter would be more tightly bound. The inclusion of such vector gluons, e.g. an octet of $SU(3)_{colour}$ gluons, changes the function $p(r)$ defined in (2) to read:

$$p(r) = \frac{2\mu}{r} - \kappa^2 r^2 - \beta\,\frac{g^2}{r^2}\ , \qquad (11)$$

where g is the coupling parameter for the Yang-Mills gluons, and β equals $\frac{9}{8(1+\alpha)}\,F^2 G_S$, where F^2 is the quadratic SU(3) Casimir operator.*) (There is no other change in the expression (1) for $f_{\mu\nu}$.)

Our conclusion is that tensor gauge meson mediation could be the primary agency responsible, in strong interaction physics, for total (or partial)**) confinement, as well as for rising Regge trajectories, while vector gauge meson exchanges provide the element of saturation, in the sense that certain quark-quark and quark-anti-quark composites are more tightly bound than others.

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*) Provided β is large enough, μ can be positive, so long as there are no zeros in the expression $1-p(r)$ for $r > 0$.

**) Partial, in the sense that no statement is being made about the existence of free quarks or other coloured states.

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