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REFERENCE

INTERNATIONAL CENTRE FOR

THEORETICAL PHYSICS

GAUGE UNIFICATION OF BASIC FORCES

PARTICULARLY OF GRAVITATION WITH STRONG INTERACTIONS

Abdus Salam

1977 MIRAMARE-TRIESTE





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INTERNATIONAL ATOMIC ENERGY AGENCY



UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION ·

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IC/77/6 ERRATUM & ADDENDUM

GAUGE UNIFICATION OF BASIC FORCES PARTICULARLY OF GRAVITATION WITH STRONG INTERACTIONS

Abdus Salam

ERRATUM AND ADDENDUM

- <u>p.13</u>: The depth of the South African mine was 3000 metres (not 300 metres).
- <u>p.13</u>: Professor F. Reines has kindly pointed out (private communication) that his detectors were indeed highly sensitive for stopping muons with energies down to 15-20 MeV, and also to any pions which may have come from proton $\Rightarrow \pi \Rightarrow \mu \Rightarrow e$ decay chain. In fact he has reasons to believe that the six (not five) muon events observed in his experiment may, by no means, be rejected out of hand, so far as proton decay is concerned. He is currently preparing a note on a re-analysis of these events.
- <u>p.13</u>: The suggestion for the $\pi \mu e$ experiment (using Professor Zatsepyn's scintillator) originated with F. Reines.

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ERRATUM

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GAUGE UNIFICATION OF BASIC FORCES, PARTICULARLY OF GRAVITATION WITH STRONG INTERACTIONS

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Page 35

Reference 8 - fourth line should read:

K.P. Sinha, Lecture given at the Bose Memorial Meeting

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IC/77/6

I. INTRODUCTION

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

GAUGE UNIFICATION OF BASIC FORCES PARTICULARLY OF GRAVITATION WITH STRONG INTERACTIONS *

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ABSTRACT

Corresponding to the two known types of gauge theories - Yang-Mills with spin-one mediating particles and Einstein-Weyl with spin-two mediating particles - it is speculated that two distinct gauge unifications of the basic forces appear to be taking place. One is the familiar Yang-Mills unification of weak and electromagnetic forces with the strong. The second is the less familiar gauge unification of gravitation with spin-two tensor-dominated aspects of strong interactions. It is proposed that there are strongly interacting spin-two strong gravitons obeying Einstein's equations, and their existence gives a clue to an understanding of the (partial) confinement of quarks, as well as of the concept of hadronic temperature, through the use of Schwarzschild de-Sitter-like partially confining solitonic solutions of the strong gravity Einstein equation.

> MIRAMARE - TRIESTE January 1977

 Lecture delivered at Professor R.E. Marshak's 60th Birthday Celebrations, New York, 21 January 1977. The presently accessible range of physical phenomena appears to be governed by the four familiar types of basic forces, mediated either by spinone or spin-two quanta.

Table I

Force	Spin of mediating quanta	Effective coupling, strength	Associated characteristic mass					
EM	1-	α ≈10 ⁻²						
weak	1-,1+	$G_{\rm F} \approx 10^{-5} {\rm Gev}^{-2}$	10 ² GeV					
strong *)	1 ⁻ (gluons), 2 ⁺	$G_{\rm g} \approx 1~{\rm GeV}^{-2}$	l GeV					
gravitational	2*	$G_{ m N} \approx 10^{-37} \ { m GeV}^{-2}$	10 ¹⁹ GeV					
*) Tensor dominance ¹⁾ with its relationship to Eomeron Physics is a signal of the role of spin-two mediating particles in strong interactions. The dual model theories ²⁾ of strong interactions (apparently) need both open-string (zero slope limit => Yang-Mills spin-one theory) as well as closed-string (zero slope limit => Einstein spin-two theory) sectors in order to ensure a consistent unitary and means placeble formulation								
ensure a consistent, unitary and renormalizable formulation.								

The spins of the mediating quanta, <u>spin-one</u> for weak, EM and strong forces, and <u>spin-two</u> for strong and gravitational forces appear to correspond to two of the deepest and the most elegant theoretical structures based on the gauge principle, that we know of. These are (Table II):

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<u>Table II</u>

Gauge theory	Spin of mediating quanta	Gauge theory	Generțiization
l) Maxwell (Weyl)	1	Internal symmetry group U(1) corresponding to electric charge conservation	(by Yang-Mills-Shaw)to any internal symmetry group for example $SU(n)$ or chiral $SU_L(n) \times SU_R(n)$
2) Einstein	2	Group of general co- ordinate transformations; linking up with the notion of space-time <u>curvature</u>	
3) Weyl	2	A rederivation and gener- alization of Einstein's theory, gauging the rela- tivistic spin group SL(2,C); linking up with space-time <u>torsion</u> of Cartan	(by Isham, Salam and Strathdee ³⁾ to any (in- ternal symmetry containing) generalization of SL(2,C), for example SL(6,C) emphasising relativistic marriage of spin-group (SL(2,C)) and internal symmetry SU(3)

4) Each one of the above gauge theories can be extended by "grading" the appropriate Lie algebra, i.e. by adding on anti-commuting charges. For example, Maxwell's spin-one gauge boson may be augumented with a spin-¹/₂ gauge fermion ⁴; or Einstein's spin-two gauge boson augumented with a spin-³/₂ gauge fermion (supergravity theory) ⁵). In this manner, a gauging of graded Lie structures removes the final distinction between "matter" (conventionally fermions) and (mediating) quanta (conventionally bosons). All fundamental fields in this view are gauge fields. I shall later have occasion to refer to supergravity when discussing possible renormalizability of gravity theory.

Now while the theme of a Yang-Milla unification of weak, FM as well as strong forces (motivated by the shared characteristic of all these forces being mediated by spin-one gauge particles) has been fairly well emphasised, comparatively less attention has been paid to the spin-two characteristic of the strong force, its resemblances to gravity and the possible unification of these two forces using Einstein-Weyl gauge ideas. It is my principal purpose to motivate such a unification, though in the first part of the lecture I shall also briefly review the spin-one unification aspects of weak, EM and strong interactions. Tentatively then, I shall be proposing a

tetrahedral inter-relation of fundamental forces with the strong force playing a pivotal role on account of their mediation both through spin-one as well as through spin-two quanta.



Before entering into details, let me give a summary of the points I wish to make in respect of the linkages represented by this tetrahedron.

- 1. <u>Gauge unification of weak and EM forces</u>
- A) Prediction and verification of the existence of neutral currents implies that such a (gauge) unification is likely with the minimal gauge group $SU(2) \times U(1)$.
- B) The characteristic mass (energy) beyond which the distinction between these two forces may be expected to disappear lies beyond 10^2 GeV.

THE OWNER DESIGNATION.

- C) The gauge unification together with the comparative rarity of $\Delta S = 1$ weak transitions makes the existence of charm almost compulsive.
- D) The most direct test of gauge ideas will of course be the production of W^{\pm} particles and the weak partner to the photon (Z^{0}) hopefully in the decade of the 1980's.
- E) Semidirect tests of the linkage between weak and electromagnetic interactions are the symmetry restoration effects in weak interactions which could be produced by using strong external electric and magnetic fields⁶. For example, it has been suggested that the Cabibbo angle may be expected to be switched off in reactions like

$$\kappa^{-} + Mo^{95} \rightarrow Nb^{93}_{\Lambda} + n$$
$$\kappa^{-} + Ar^{36} \rightarrow Ar^{35}_{\Lambda} + \pi$$

(so that the A-hyperon lifetime is very considerably enhanced) in the nuclear environment provided by $\rm Nb^{93}$ and $\rm Ar^{35}$ assuming that the internal electromagnetic fields inside these nuclei are stronger than the critical transition fields.

2. Gauge unification of strong with weak and EM forces

The next hypothesized linkage is the proposed spin-one mediated gauge unification of strong forces with the weak and the EM. <u>Clearly the most</u> important signal of such a unification will be the disappearance of the <u>distinction between leptons and quarks</u>. This must happen; the question is beyond what characteristic energy?

Pati and $I^{(7)}$ have suggested a theory of quark-lepton unification based on the idea that the twelve quarks (carrying four flavours and three colours) combine with the four known leptons in a multiplet of an $SU(4)_{flavour} \times$ $SU(4)_{colour}$ internal symmetry group. (The fourth colour is the lepton colour, lilac.) The quarks, ultimately indistinguishable from leptons, must in this model carry integer charges. A spin-one gauging of this theory (flavour as well as colour charges) yields an estimate of the characteristic energy at which the quark-lepton unification should start becoming directly manifest. We estimate this energy $\Rightarrow 10^5$ GeV (other gauge theorists, working with fractionally charged quarks, which are unification permanently confined, estimate/energies much higher, beyond 10¹⁵ GeV). Each one of our quarks can decay into a lepton (plus pions or kaons or a leptonanti-lepton pair) with a lifetime around 10⁻¹³ secs for quarks of mass ≈ 4 GeV. Likewise the proton - the three-quark composite - must decay into three leptons (plus plons) with a lifetime of the order of $10^{29}-10^{30}$ years. (All these lifetime estimates are correlated with the estimate of the characteristic energy. If the characteristic energy is higher than 10^5 GeV, the proton will live longer.)

But besides the possibility of proton decay as a signal of quarklepton unification, there are other indirect signals in the model. These relate to (1) $\sigma_{\rm L}/\sigma_{\rm T}$ in eN, μ N, (2) dileptonic events in ν N and (3) and asymmetric production of leptons versus anti-leptons in N-N collisions.

3. <u>Clues on unification of gravity and strong interactions</u>

The S-matrix physicists.with the postulate of tensor dominance in strong interactions and the hypothesis of the Pomeron lying on a spin-two trajectory, have always believed in the important role of spin-two mesons in strong interaction physics. The dual model physicists have likewise discovered that they must utilize both the open-string (zero slope limit = Yang-Mills spin-one gauge theory) as well as the closed-string (zero slope limit = Einstein theory) sectors in their search for a consistent theory of strong interactions. (Previously, the higher dimensions needed for dual models and the symmetries arising from them were identified with flavour quantum numbers; recently there has been some shift towards identifying these as associated with colour.)

From a gauge theory point of view, one can go further. Let us assume that strong interactions are mediated by a strongly interacting spin-two object (generically called f meson: not to be confused with the spin-two particle at 1290 MeV) obeying an Einstein equation with the Newtonian constant $G_N \approx 10^{-37} \text{ GeV}^{-2}$ replaced by the strong constant $G_S \approx 1 \text{ BeV}^{-2}$. We further assume that quarks interact with the f mesons, their normal gravitational interactions being mediated by a (generally covariant) f-g mixing term (the field $g^{\mu\nu}(\mathbf{x})$ describing normal weak gravity). This mixing term also gives mass to the f meson.

This simple version of a two-tensor f-g theory was formulated by Isham, Salam and Strathdee and independently by Wess and Zumino⁸⁾. In this early formulation f-quanta were assumed to interact directly with hadrons and g-quanta to interact directly with leptons. Clearly with quarklepton unification ideas expressed above, this simple version of the theory with f and g tensors so sharply distinguished will need revising. This can be done but I shall not be concerned with this aspect of the theory in anall lecture nor with the very difficult problem of reconciling within one structure magnitudes as diverse as G_N and G_S . Rather, my major and humbler concern is to show how the <u>postulate of an Einstein equation for the strong gravity field</u> \underline{f} - with all the connotations of space-time curvature and torsion being important in strong interactions - manifests itself in physical phenomena, particularly in the limit that the f-g mixing term is neglected.

The claim is that there are two immediate manifestations of this Einstein gauge formulation of strong gravity.

- Weak gravity possesses <u>classical</u> solitonic solutions of Schwarzschild and Kerr-Newman type which trap and confine particles. Likewise strong gravity possesses solitonic solutions (representing hadrons), which confine (quarks) at least on the <u>classical</u> level.
- 2) <u>Quantum-mechanically</u> Hawking ⁹⁾ has recently shown that the solitonic solutions of (weak) gravity are <u>not black holes</u> from which nothing can escape. He shows that (some of) these solitonic solutions represent <u>black bodies</u>, radiating all species of particles with a thermal spectrum. The exciting aspect of Hawking'swork is that the temperature comes to be defined in terms of the parameters of the field Einstein/equations and their solitonic solutions. Specifically temperature is proportional to the inverse of (4m times) Schwarzschild radius.

In strong gravity, for hadrons, we shall see that the strong Schwarzschild radii are of the same order as the Compton radii of hadrons. Taking Hawking's ideas over, one can define a temperature (Ref.l4) in hadronic physics in terms of radii of appropriate hadronic solitons, which controls the thermal emission of particles in(for example) πN or NN collisions.

With this introduction, I shall divide the lecture into two parts:

Part I

is concerned with the Yang-Mills unification of strong, weak and EM interactions. I shall describe the model of Pati and myself and speak of its predictions in respect of

- i) Proton and quark decays;
- ii) Manifestations of spin-one strongly interacting colour gluons.

<u>Part_II</u>

is concerned with the use of the spin-two Einstein-Weyl equation for strong gravity. We shall seek for clues to a partial confinement of quarks in the context of the solitonic solutions of the strong gravity equation and also

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use Hawking's ideas to give a precise meaning to the concept of hadronic temperature. Part I is a summary of work reported elsewhere, 7,10 Part II describes some new work, particularly on the possibility of confinement of quarks using strong-gravity ideas.

PART I

II.

YANG-MILLS GAUGE UNIFICATION OF STRONG, WEAK AND ELECTROMAGNETIC INTERACTIONS

All material in this Part is described in detail in the Aachen Conference 10 Lecture (1976) by Pati and myself. I shall give a brief summary emphasising the gauge unification aspects.

A. The scheme and the fermion-number

We work conservatively with twelve quarks and four leptons. (If further quark flavours and further leptons (colours) are discovered, our fundamental internal symmetry group and the corresponding representations or their number will grow but nothing basic changes.) *)

The quark-lepton unification hypothesis is implemented by postulating that all matter belongs to the following fundamental fermionic multiplet consisting of the 4×4 representation of the basic group SU(4) flavour SU(4) colour:

F	Ì	q	р	p	ν _e	→	up	Flavours
		n	n	n	e	→	down	
	=	λ	λ	λ	μ-	+	strange	
		¢	с	с	v_{μ}	} →	charm	
		Ŷ	Ŷ	ţ	↓			
		red	yellow	blue	lilac			
		<u> </u>	colou	ırs —				

We define (an unconventional) baryonic number for quarks (B = 1) and a leptonic number $L = L^{e} + L^{\mu} = 1$ for leptons. The fermion-number F for all the sixteen particles equals $F = 1 = B + L^{e} + L^{\mu}$. Note that only the total fermion-number F has any absolute significance: none of the individual numbers B, L^{e} or L^{μ} are significant in terms of conservation for the whole multiplet.

The electric charge operator is a sum of $SU(4)|_{flavour} \times SU(4)|_{colour}$ generators. We make a choice which assigns the following charges to quarks and leptons:

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<sup>PFor example, heavy leptons + b quarks,if substantiated, may need SU(5) | flavour ×
SU(5) | colour -8-</sup>

$$Q = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\left(Q = Q_{flavour} + Q_{colour} = \begin{pmatrix} 2/3 & 2/3 & 2/3 & 0 \\ -1/3 & -1/3 & -1/3 & -1 \\ -1/3 & -1/3 & -1/3 & -1 \\ 2/3 & 2/3 & 2/3 & 0 \end{pmatrix} + \begin{pmatrix} -2/3 & 1/3 & 1/3 & 0 \\ -2/3 & 1/3 & 1/3 & 0 \\ -2/3 & 1/3 & 1/3 & 0 \\ -2/3 & 1/3 & 1/3 & 0 \\ -2/3 & 1/3 & 1/3 & 0 \\ -2/3 & 1/3 & 1/3 & 0 \\ \end{pmatrix} \right)$$

Note that with this assignment, <u>leptons</u> with fermion-number F = 1 (the same as quarks) are absolutely defined as objects carrying zero (ν_e, ν_{μ}) and negative charges (e⁻, μ^-).

B. The spin-one Yang-Mills gauging of SU(4) | flavour × SU(4) | colour ; The basic model.

We gauge flavour for weak and EM interactions and $SU(4)|_{colour}$ for strong and EM forces. The important point is that the photon has partners both in the flavour and the colour sectors corresponding to the split of charge Q into $Q_{flavour} + Q_{colour}$. The gauging scheme may thus be represented in the form:



with EM occupying the pivotal position. In detail the flavour gauges give $W_{L,R}^{\pm}, Z^{0}$ and the flavour piece of the photon. The colour gauges give (1) an octet of strong colour vector gluons $(V_{RY}^{\pm}, V_{RE}^{\pm}, V_{BV}^{0, \overline{0}}, \widetilde{U}, V^{0})$ which couple quarks with quarks, (2) a triplet (plus an anti-triplet) of exotics $X_{R\ell}^{0}$, $\overline{X_{Y\ell}}$, $\overline{X_{B\ell}}$ which couple quarks with leptons and a (3) singlet S⁰ which couples with the current (B-3L). Among the eight colour gluons is the rather special object \widetilde{U} - the colour partner of the photon.

We give masses to all these gauge particles (except the photon) through the standard Higgs-Kibble spontaneous symmetry-breaking mechanism. This mass-giving mechanism also mixes the weak W^{\pm} 's with the octet V's and the triplet X's, so that the final unification scheme locks like this:



Higgs-Kibble particles mixing W 's with V's, X's and S^0 and leading to weak decays of quarks and gluons.

To link up with the concept of characteristic energy beyond which the distinction between quarks and leptons should disappear, it is the masses of the exotics that determine this characteristic energy.

To summarize: the Higgs-Kibble mechanism leads to:

- i) The photon as made up of flavour and colour pieces;
- ii) Mixes V^{\pm} with $W^{\pm} \rightarrow$ leading to decays of the octet of strong gluons V^{\pm} , V^{0} ,...;
- iii) Mixes the exotics X^{\pm} with $W^{\pm} \rightarrow$ leading to well-defined quark and in turn to proton decays.

C. <u>Mass scales</u>

There is a natural mass scale for masses of the exotics. It is provided by the rate of the decay $K \neq e + \mu$. From present rarity of this mode, we infer that $m_{\chi} > 10^5$ GeV. This in turn determines (within the model) the lifetime of

- a quark for decays into a lepton + (pions or lepton-anti-lepton pair).
- ii) proton-decay lifetime. Alternatively we could have fixed m_{χ} through any one of the three inputs (1) K + e + μ , (2) quark + lepton transition rate,

(3) proton \rightarrow three lepton transition rate, the other two processes providing a test for the ideas underlying the model.

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While we have this natural mass scale for the exotic masses, regretfully there is no natural mass scale for the masses of the quarks or the strong (octet of) gluons. These masses could lie in either of the three ranges:

- (1) light ≤ 2 GeV (the charmed quark is presumably 1.5 GeV heavier),
- (2) medium between 2 GeV to 7 GeV (SLAC range),
- (3) heavy PEP-Petra range of energies.

It is important to stress that these are masses of quarks and gluons outside the nucleonic environment. Inside such an evironment, with its high hadronic matter density and hadro-static pressure, the expectation value of the appropriate Higgs fields can have made a transition to zero. Thus quarks and gluons could be very light (≤ 300 MeV) inside the nucleonic environment as the parton model appears to suggest - while they are heavy outside. This difference of effective masses inside and outside - first discovered by Archimedes in the context of hydrostatic pressure - would cause a partial confinement of quarks and gluons in the sense that the tunnelling probability of their crossing through the hadronic surface and penetrating the mass barrier is depressed. In all subsequent remarks, I shall accept this partial confinement as a fact of quark dynamics (exact confinement being the limit when the quark-gluon mass outside is infinite).

D. <u>Production and decays of quarks and proton decays</u>

Free quarks may be produced in the following reactions for example:

1)
$$e^{+} + e^{-} + q + \bar{q}$$

2) $v + N + \mu + q_R + q_Y + q_B$
 $N + N + N + q_R + q_Y + q_B$
dissociation of the nucleon

For quark decays, there are important selection rules in the simplest (basic) version of the model which I summarize.

1) Assuming fermion-number F conservation, $\Delta F = 0$ (but $\Delta B \neq 0$, $\Delta L \neq 0$, F = B + L) a quark can go into a lepton but not an anti-lepton. The quarkanti-lepton transition requires $\Delta F = 2$. We have assumed in the simplest version of our model that this decay mode is suppressed compared with the $\Delta F = 0$ decay mode. (If this restriction is relaxed (as would for example be the case in a supersymmetric version of the model), $q \neq \overline{k}$, $q \neq \overline{q}$ (and even $\Delta F = 4$ transitions $qq \rightarrow \overline{qq}$) may become competitive with $q \neq \ell$ ($\Delta F = 0$).) 2) The simplest (basic) model further strongly restricts the types of quark decays allowed. The <u>yellow</u> and <u>blue</u> quarks are sharply distinguished from the <u>red</u> quarks in that the former (yellow and blue) go predominantly into neutrinos and not charged leptons. Thus

> $q_{Y,B} \rightarrow v_e + pions$, $v_{\mu} + K + pions$ $q_R \rightarrow e + pions$, $\mu + K + pions$ + and also $e + v + \overline{v}$, $\mu + v + \overline{v}$

The lifetimes(varying as m_q^{-3}) range between 10^{-12} to 10^{-14} secs (or shorter) for light to medium quark masses.

3) Since quarks are presumably point particles so far as electromagnetism is concerned, one is tempted to ascribe the Perl (μ ,e) events at SLAC to decays of red quarks of mass ≈ 1.95 GeV

 $e^+ + e^- + q_R + \overline{q}_R + e + \mu + neutrinos$.

Note that quarks resemble heavy leptons in that they are not absorbed in ordinary matter; their only distinction from heavy leptons lies in their scattering (nuclear versus pure electromagnetic) characteristics.

4) So far as <u>nucleon dissociation</u> in vN and NN collisions is concerned, it is important to remark that whereas partial confinement will make dissociation amplitude (tunnelling through the mass barrier) small, the net mass from final quark decays will mainly reside in the neutrinos which yellow and blue quarks decay into. The red quarks in their decays will however contribute to dileptons in vN collisions. Finally, in NN collisions, we expect the nucleon dissociation mechanism to give a sizeable $\frac{\text{anti-lepton}}{\text{lepton}}$ asymmetry beyond the dissociation threshold. This is assuming that $\Delta F = 0$, $\Delta F \neq 2$ selection rules hold (or, more accurately, assuming that the two baryonnumber violating amplitudes $\Delta F = 0$ and $\Delta F = 2$ giving $q \neq 2$ and $q \neq \overline{2}$ transitions, respectively, are not of the same magnitude).

5) <u>Proton decay</u>: The most characteristic prediction of the model is proton instability which (with $\Delta F = 0$) is a triple violation of baryon-lepton number $\Delta B = -\Delta L = 3$. It is this high degree of forbiddenness (effective constant $G_B^3 \approx 10^{-27}$ where G_B is the effective quark-lepton transition constant $\approx 10^{-9}$ computed within the model assuming $m_{\chi} \approx 10^{5}$ GeV) which is responsible for the inordinately long life of the proton. The predominant decay mode is:

Proton + $3v + \pi^+$ $\Delta F = 0$

The most recent reported experiment on proton decay is that of Reines and coworkers ¹¹⁾ performed in 1967 (and re-analysed in 1974) (in a South African mine 300 metres deep; a signal of five possible events proton + μ^+ was recorded, setting a lower limit of 10^{30} years on lifetime). In the basic model ($\Delta F = 0$) this particular decay mode (proton + μ^+ + μ_{ν}) can only proceed with muons predominantly carrying a rather small fraction of proton rest energy ($E_{\mu} < 150 \text{ MeV}$). The experiment was rather less sensitive to these. *) To study the predominant decay $3\nu + \pi^+$, there is a proposal by Zatsepyn to use a 100 ton scintillator to detect the following chain from decays of protons in the scintillator itself:

A geochemical experiment similar to that used for double β decay has been suggested by Peter Rosen ¹². This consists of examining for rare-gas isotopes ²²Ne, ³⁸Ar, ^{84,86}Kr, ¹³²Xe occluded in ancient ores.

The sensitivity of Rosen's suggested experiment (private communication) has gone up recently to proton life estimates as high as 10^{34} years with the discovery of a new dye-laser based technique by G.S. Hurst, M.G. Nayfeh and J.P. Young which detects one atom in an environment of 10^{19} . (Applied Phys. Letters, 1 March 1977.)

E. <u>The gluon story</u>

There are eight strong coloured gluons 1⁻ responsible for (part of) the strong force in all models of gauge unification of strong, EM and weak interactions. In the so-called standard QCD model with fractionally charged quarks, $SU(3)|_{colour}$ symmetry is assumed to be an exact symmetry, and all gluons - electrically neutral - are massless. To keep them invisible the <u>Dogma of complete confinement</u> has been formulated which asserts that they as well as the quarks are permanently confined - in fact all colour will for ever confine.

*) Even though the discussion above has stressed $\Delta F = 0$ transitions, it is important to remark that one can extend the basic model such that $\Delta F = 2$, $q \neq \overline{k}$, $q \neq \overline{q}$ and $\Delta F = 4$, $qq \neq \overline{q}q$ transitions are also allowed. In such a case proton $+ \mu^{+} + \pi^{0} (\Delta F = 4)$ would be a possible decay channel, in addition to $\pi^{+} + 3\nu (\Delta F = 0)$. -13In our model, described above, gluons are integer-charged and massive and they must be produced in all types of collisions (though "partial" confinement due to the Archimedes effect and the need to penetrate the surface barrier of hadrons may depress their production cross-section at present energies). In this context, besides any dynamical barrier factors, there is also an exact theorem due to Roy and Rajasekaran and Pati and Salam which states that in a gauge theory, lepto-production experiments (eN,UN) are ineffective in producing colour. More precisely in all such experiments the production rate of for flavour is governed by a kinematic gauge factor

$$\left(\frac{\mathbf{m}_{U}^{2}}{|\mathbf{q}^{2}| + \mathbf{m}_{U}^{2}}\right)^{2} \quad (\mathbf{q}^{2}| + \infty)^{0}$$

where m_U is the mass of the photon's partner in the gluon octet and $|q^2|^{1/2}$ is the momentum transfer.

The decay modes of the colour gluons are characteristic:

1) Charged members of the octet V_{RY}^{\pm} , V_{RB}^{\pm} go into

 $\implies (\nu_e e) \ : \ (\nu_\mu \mu) \ : \ hadrons$

around finto the ratio 1:1:3 with lifetimes/10⁻¹⁵ secs, providing another source of dileptons in $v + N \rightarrow \mu + v^{\pm} + X$ besides charm. Also in semileptonic $\downarrow, (\mu, e)$

decays of V^{\pm} , one does not expect single production of K's $(V^{\pm} \rightarrow Kev, K\mu v)$ but $\rightarrow K\bar{K}ev$, $K\bar{K}\mu v$, etc.) since the gluons are $SU(3)|_{flavour}$ singlets. This would distinguish such decays from charm decays.

2) Finally the neutral gluon (partner to the photon) \widetilde{U} - expected to be produced in e^+e^- collisions - would exhibit the following characteristic decays:

$$m_{U} \approx 1 \text{ to } 2 \text{ GeV} \qquad m_{U} \approx 4 \text{ GeV}$$

$$\widetilde{U} + e^{+} + e^{-} \qquad 2 \text{ to } 5 \text{ keV} \qquad 2 \text{ to } 5 \text{ keV}$$

$$+ \mu^{+} + \mu^{-} \qquad 2 \text{ to } 5 \text{ keV} \qquad 2 \text{ to } 5 \text{ keV}$$

$$+ \pi\pi\gamma, \ b\pi\gamma, \ \eta'\gamma \qquad 1 - 3 \text{ MeV} \qquad \frac{1}{10} - 1 \text{ MeV}$$

$$+ 3\pi, 5\pi, \ \rho\pi, \ K\overline{K} \qquad \frac{1}{5} - 5 \text{ MeV} \qquad \frac{1}{10} - 1 \text{ MeV}$$

$$+ 3\pi, \ 4\pi, \ 6\pi \qquad \frac{1}{10} - \frac{1}{2} \text{ MeV} \qquad \frac{1}{10} - \frac{1}{2} \text{ MeV}$$

$$-1 \mu_{-}$$

On the basis of the $e^+ + e^-$ width of SLAC structures between 4 and 7 GeV (barring the region 3.1 to 3.2 GeV - communication from Professor M. Barnett) it has been suggested that the gluon is either light (< 2 GeV) or heavy (>7 GeV) so that its mass lies either in the Frascati-Orsay-Novosibirsk or the Pep-Petra regions (though this conclusion does not take into account possible mixing of other colour states with gluons).

3) Indirect tests of integer-charged partially confined gluon theory consist of the following:

i) We expect $\sigma_{\rm L}/\sigma_{\rm T} \neq 0$ on account of a contribution from gluons. It can be shown that in a gauge theory, this gluon contribution to $\sigma_{\rm L}/\sigma_{\rm m}$ scales in x.

11) We expect a rise from colour brightening and gluon production in $\sigma_{\overline{V}}/\sigma_{v}$ and $(Y)_{\overline{V}}$ over and above the purely quark contribution and likewise for the ratio in neutrino experiments for $\sigma_{neutral currents}/\sigma_{charged currents}$. Preliminary experimental indications of such rises have been conventionally intepreted as signifying the existence of right-handed currents and new quark flavours. Our model interprets these as due to colour brightening. For details see Ref.10.

Summary

To summarize the signals for the Yang-Mills gauge unification of strong, weak and EM interactions in accordance with our ideas, these are:

- a) Proton decay into three leptons (plus pions);
- b) Production and decays of quarks in ∇N , $\overline{\nabla N}$ and NN collisions. In the latter experiments we expect $\frac{\text{lepton}}{\text{anti-lepton}}$ ratio to deviate significantly from unity above the nucleon dissociation threshold, provided either one of the transitions $q + \ell$ ($\Delta F = 0$) or $q + \tilde{\ell}$ ($\Delta F = 2$) dominates over the other;
- c) In eN and vN experiments $\sigma_L^{}/\sigma_T^{} \neq$ 0 and should scale in x .
- d) In νN , $\overline{\nu}N$ experiments we expect rises in $\sigma_{\overline{\nu}}/\sigma_{\nu}$, $\langle Y \rangle$ and in the ratio of neutral/charged current cross-sections, due to colour brightening. These rises should eventually cease when the suppression factor for colour takes over (depending on the mass of the neutral vector gluon m_{ν}).

SPIN-TWO ASPECTS OF STRONG FORCES, STRONG GRAVITY AND POSSIBLE ORIGIN OF (PARTIAL) CONFINEMENT AND HADRONIC TEMPERATURE

I. INTRODUCTION

Since I shall be speaking about (partial) confinement in this part of the talk, let us restate the present dilemmas of strong interaction quark physics in this respect.

The parton model gives a picture of essentially free quarks and 1) gluons existing inside hadrons. This (at first surprising) feature of quark dynamics however has analogies elsewhere in physics. For example electrons in metals behave essentially as free particles notwithstanding the relatively strong electric potentials inside. Likewise in the theory of nuclear matter - particularly when one attempts to reconcile shell and collective particle pictures of nucleonic interactions - there are dynamical dilemmas of a similar sort. In quark dynamics the "free" behaviour of quarks and gluons has been (brilliantly) ascribed to asymptotic freedom of quark gluon forces. i.e. the statement (true of non-abelian Yang-Mills spin-one theories, and as we shall see, possibly also of strong gravity) that the closer the quarks and gluons come, the weaker the effective strength of the force with which they influence each other. (Parenthetically it must be remarked that contrary to a general climate of opinion and belief in the subject, the gluon or Higgs masses need not affect the issue of asymptotic freedom.)

2) The second significant fact about quarks and gluons is the Archimedes effect. Quarks and gluons - according to the parton model - are light inside a hadronic environment and heavy outside. There is partial confinement if the mass outside is finite; exact confinement if it is infinite. Since (primeval) fractionally charged quarks appear excluded as physical entities (from experiments with deep sea-bed cysters and moon-dust), such quarks, if they do exist, must be permanently confined. For integer-charge quarks (particularly if they decay fast into leptons) there is no known experimental fact which would argue for their permanent - as distinguished from their partial - confinement.

*) magnetic If "colour" and monopolarity are related to each other (as has been surmised), and if magnetic monopoles and the related gluons have masses in the ratio α^{-1} ('t Hooft's theorem), quarks (carrying monopolarity) may be heavier than 200 GeV, even for the light gluon case. Awful prospect for experimentation:

Now what is the origin of exact confinement, if such indeed is the real physics of the situation. A truly vast amount of intellectual effort has gone into theoretically achieving what I shall call the Tokazak-like confinement of colour (quarks and gluons) using the agency of (non-Abelian) spin-one gluon theories. And one must admit that the basic idea is truly seductive. Assume that the strong colour gauge group SU(3) an exact symmetry of nature, so that the colour gluons (electrically neutral in a fractionally charged quark theory) are massless producing long-range forces. Assume that the infra-red effects accompanying such massless gluons are so singular for colour carrying initial or final states that an infinitely rising long-range potential of the type $V = kr \text{ or } Ar^2$ builds up for coloured states. In such an event, coloured quarks and gluons will be permanently confined inside colour singlet hadronic states. Particle physics - on the experimental level - would come to an end, within our generation, for never shall the quark (or the gluon) state be accessible for direct experimentation. In favour of such a rising potential may also be adduced the well-known fact that such potentials would also facilitate theoretically the emergence of rising Regge trajectories.

So much for the conjecture. Now the first hope of carrying this exact confinement conjecture to a proof lay in examining the infra-red behaviour of non-abelian Yang-Mills theories in perturbation theory Unhappily, it is (the infra-rea stavery hypothesis by now conclusively known that so far as perturbation calculations are concerned, the (infra-red) behaviour for non-abelian Yang-Mills colour dynamics (QCD) is no more singular than for the familiar abelian gauge theory of quantum electrodynamics (QED). In any verturbation calculation (or for any summation of perturbation diagrams to a given order) there seems no hope of uncovering infra-red slavery or the origin of exact confinement if any. One could still retain the hope that non-perturbative approaches would succeed where perturbation theory failed in providing an infinitely rising potential of the type kr or Ar^2 . Numerous attempts have in fact been made in this direction but without conspicuous success.

I wish to suggest that rather than look further along the direction of spin-one (Tokamak-like) confinement one may attempt to exploit the confining properties of an Einstein-like spin-two equation. Classically, Schwarzschild and Kerr-Newman solutions of such equations, trap and confine only too well, giving also expressions for the surfaces of confinement in terms of the parameters of the theory. The hope (recently realized by Hawking) is that quantum mechanics may temper this inexorable trapping, this inexorable confinement to give just the right degree of partial confinement when one works with strong gravity, where the typical (strong) gravitational scale of sizes accords with hadronic Compton wavelengths <u>and quantum effects are</u> <u>particularly relevant</u>. One will still need the spin-one colour aspects of strong interaction physics, but they will be needed more to provide saturation

(i.e. why three quarks form a partially confined bound system but not two quarks), rather than to provide the origin of confinement. It is relevant to remark that there have been remarkable advances made since 1974 in Field Theory in curved spaces since Hawking first announced his quantum-mechanical results. Some of the techniques developed are extremely powerful as I shall briefly indicate. I feel a personal tinge of regret that few of the advances have come from the community of particle physicists, who have by and large unfortunately ignored these ideas. (See the review by C.J. Isham, Ref.15.)

II. THE 1-g TWO TENSOR THEORY OF STRONG AND WEAK GRAVITATION

To motivate the discussion, consider the simplest version of a unified (gauge)theory of strong and gravitational interactions ⁸). We start with two tensors $f^{\mu\nu}(\mathbf{x})$ and $g^{\mu\nu}(\mathbf{x})$ and postulate the Lagrangian:

$$\frac{1}{\overline{\gamma}_{S}} \sqrt{-f} R(f) + R(g) \frac{\sqrt{-g}}{G_{W}} + \mathcal{Z}_{fg} + \mathcal{Z}_{matter}$$
(2.1)

R(f) and R(g) are the Einstein Lagrangian expressions, $G_g \sim 1 \text{ GeV}^{-2}$, $G_N \sim 10^{-37} \text{ GeV}^{-2}$; \mathcal{L}_{fg} is a mixing Lagrangian of the form:

$$\mathbf{m}_{\mathbf{f}}^{2} \left(\mathbf{f}^{\mu\nu} - \mathbf{g}^{\mu\nu} \right) \left(\mathbf{f}^{\kappa\lambda} - \mathbf{g}^{\kappa\lambda} \right) \left\langle \mathbf{g}_{\kappa\mu} \mathbf{g}_{\lambda\nu} - \mathbf{g}_{\kappa\lambda} \mathbf{g}_{\mu\nu} \right\rangle$$
(2.2)

and is designed to give a mass (m_f) to the strong graviton. Ignoring for the present the subtleties of quark-lepton unification, \mathcal{L}_{matter} gives a quark -f direct interaction of effective strength G_S and a lepton -g direct interaction of strength G_N .

Now one can show at least in a linear approximation $(f \sim 1 + \sqrt{S_S} \phi_f)$, $g \sim 1 + \sqrt{S_N} \phi_g$) that the two fields f and g mix and the equations of motion describe one massless and one massive physical/quantum associated with each of the two fields g and f. More precisely, the physical fields bear a close resemblance to the photon and its partner, the Z^0 in the unified EM and weak gauge theory approach. Thus

However since $G_S >> G_N$, to all intents and purposes, the g field represents the true graviton and the f field the strongly interacting f meson.

Note that the theory as formulated here is fully generally covariant. But so far as the f meson is concerned, we are interested in the flat space-time limit of the g field ($G_N = 0$) with

$$s_{\mu\nu} \approx \eta_{\mu\nu} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \\ & & -1 \\ & & & -1 \end{pmatrix}$$

In this limit

$$\boldsymbol{\mathcal{Z}}_{\mathbf{fg}} = \mathbf{m}_{\mathbf{f}}^{2} \left(\mathbf{f}^{\kappa\lambda} - \boldsymbol{\eta}^{\kappa\lambda} \right) \left(\mathbf{f}^{\mu\nu} - \boldsymbol{\eta}^{\mu\nu} \right) \left(\boldsymbol{\eta}_{\kappa\mu} - \boldsymbol{\eta}_{\lambda\nu} - \boldsymbol{\eta}_{\kappa\lambda} - \boldsymbol{\eta}_{\mu\nu} \right)$$
(2.3)

and the f equation of motion reads:

$$R_{\mu\nu}(f) - \frac{1}{2} f_{\mu\nu} R = \frac{1}{2} m_f^2 (f_{\mu\nu} - (f \cdot \eta) \eta_{\mu\nu}) . \qquad (2.4)$$

*) Ideally the f-g mixing term should parallel a Higgs-Kibble type of spontaneous symmetry-breaking term and ought to possess a form which ensures that (1) there are no further spin-zero or spin-one ghosts or tachyons lurking among the redundant components of the f-g fields; (2) the propagator for the f meson is soft and singularity free in the limit $m_f \rightarrow 0$. We believe these requirements can be met by postulating a somewhat more elaborate unified model (which besides spin-two) objects also contains (a physical) Yang-Mills spin-one field ¹⁴). Here, however, we do not consider such a modification of the simple Pauli-Fierz-like f-g mixing term (given above). Our (solution to these problems ¹⁴) relies on a dynamical symmetry-breaking mechanism - a solution none too satisfying for cal-culational purposes.

III. SOLITONIC SOLUTIONS WHEN $m_{f} = 0$ AND THE CONCEPT OF HADRONIC TEMPERATURE

Some of the exact solitonic solutions of Eq.(2.4) are well known and given in all texts on Relativity Theory when $m_f = 0$. These solutions are the (1) Schwarzschild soliton representing the strong gravitational field of an object of mass N; (2) The Kerr solution representing the f field of an object of mass M and spin J; (3) Kerr-Newman solution of the Maxwell-Einstein set of equations representing the f and EM fields of an object of mass M, spin J and electric charge Q. This last can presumably be generalized for any internal gauge symmetry group, e.g. SU(2), where for q^2 one substitues the quadratic Casimir operator $e^2T^2 = e^2I$ (I + 1) (e^2 is the square of the coupling of the spin-one Yang-Mills gluons).

In pure classical theory, some of these solutions possess (more than one) horizons. These horizons have the trapping property; in general any particle crossing the horizons is captured. The horizons as a rule act like oneway membranes. For the simplest (and perhaps not quite typical) case of the Schwarzschild horizon, a particle which once gets inside the horizon cannot escape and is permanently confined (more accurately it falls into the singularity at r = 0, like the pre-Bohr electron which inexorably fell into the nucleus). For the Schwarzschild solution, $R_{horizon} = 2MG_S$; while for the other solitonic solutions there are inner as well as outer horizons with the singularity at r = 0 acting repulsively or attractively depending on the parameters of the solution. Correspondingly, there are a vast number of subtle cases with orbits trapped between these inner and outer horizons.

When the simplest of quantum-theoretic effects are taken into account (Hawking 1974) these "black-hole" solitons turn into "black-body" solitons: all species of particles tunnel out and are radiated with a thermal black-body spectrum represented (for the Schwarzschild case) by the formula:

Intensity of
$$(\exp E/kT \mp 1)^{-1}$$
 $(\neg 1 Bose particles + 1 Fermi particles)$

with the temperature related to the radius of the horizon

$$\frac{1}{4\pi kT} = R_{horizon} = 2 G_{S} M_{soliton}$$

The confinement is no longer complete.

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Let us summarize Hawking's results for an SU(2) Yang-Mills plus f-gravity theory in the limit $m_{f} \rightarrow 0$. The solitons carry a mass M, spin J and I-spin <u>L</u> and the solutions fall into three categories:

1)
$$\frac{J(J+1)}{M^2} + G_{g} e^2 I(I+1) < G_{g} M^2$$

Hawking temperature is given by the expression

$$2\pi kT = 4\pi (R - G_{S}M) A^{-1}$$
,

where

$$R = G_{S}M + \sqrt{G_{S}^{2}M^{2} - J(J+1) - G_{S}M^{2}I(I+1)e^{2}}$$

and

$$A = 4\pi G_{S} \left[2G_{S}M^{2} - e^{2} I(I+1) + 2\sqrt{G_{S}^{2}M^{4}} - J(J+1) - G_{S}M^{2}I(I+1)e^{2} \right]$$

is the area of the event horizon. I would like to suggest that such solitons represent fire-balls or clusters which are assumed to form(for example in NN or πN collisions) and which in thermodynamic theories of such collisions are assumed to radiate hadrons of all species with a thermal spectrum.

If a Regge-like relation between spins and masses holds, i.e.

$$\frac{J(J+1)}{M^2} + G_{S} e^2 I(I+1) = G_{S}M^2$$

there does exist an outer horizon at $R = G_{S}^{M}$, but the Hawking temperature is zero and there is no thermal radiation. We are here dealing presumably with normal hadrons - composite (solitonic) objects. Note that if the internal symmetry is $SU(3)|_{colour}$, the colour singlet states are in general lower in mass than colour non-singlets.

3) If $\frac{J(J+1)}{M^2} + G_g e^2 I(I+1) > G_g M^2$, there is no horizon and the Hawking temperature cannot be defined. Such solutions are called naked

singularities; these may correspond to true elementary particles (quarks, gluons, etc. with corresponding fields appearing in the basic Lagrangian).

To get a feel for the numbers involved consider some recent data of Bartke <u>et al</u>. from Aachen (Nucl. Phys. December 1976) which gives a thermal fit $\frac{d^3\sigma}{dp^3} \sim \exp\left(\frac{E_t}{kT} - 1\right)$ for $\pi^+p + m$ + anything with m referring to 2π , 3π clusters or f, ω , ρ particles. Apparently data ranging over seven decades (10⁰ to 10⁷) can be fitted with one temperature parameter $kT \sim 120$ MeV. fitted Similar (and even more extensive) data has been by Hagedorn, Carnegie, and others by assuming that clusters or fireballs of mass ≈ 1.5 MeV are formed in hadronic collisions and these then decay thermally. (Hagedorn has a fine explanation for the appearance of the parameter $E_t = \sqrt{p_t^2 + m^2}$ rather than $E = \sqrt{p^2 + m^2}$ in the collisions.)

Can we identify the Hagedorn temperature with the Hawking temperature for clusters in strong gravity? In Hawking's picture

$$(4\pi kT)^{-1} = R_{soliton}$$

It is a reasonable assumption that the radius of the horizon $(R_{soliton})^{must}$ not exceed the Compton wavelength of the solitonic cluster, i.e. $26 \frac{M}{S}^{M}$ soliton

One may also estimate the lifetime of such a cluster from Hawking's formulae. The cluster disappears since it loses mass through thermal radiation. Thus

$$\frac{dR_{soliton}}{dt} = \frac{2\pi^2}{15} C_{g} (kT)^{4} \Sigma \sigma_{g}$$

where σ_s is the absorption cross-section by the black soliton of an incident hadron of spin s. Assuming $\Sigma \sigma_s$ is of the order of conventional hadronic cross-section, we obtain

$$\Gamma_{\rm width} \approx (384\pi^2)^{-1} R_{\rm soliton}^{-5} G_{\rm S} \sigma_{\rm T}$$

 \approx 300 MeV G_S \approx 67 MeV .

These crude estimates are presented only in order to demonstrate that (as may be expected) the orders of magnitude in strong gravity theory are in the correct range of magnitudes in hadronic physics. Now the concept of temperature in hadronic physics is nothing new. What after all is so special about Hawking's work, that we should buy the whole superstructure connected with as complicated an edifice as Einstein's equation, in order to comprehend temperature?

The answer to this question at the present level of understanding really lies in the deeply satisfying and aesthetic (I was going to say absolute) quality of Hawking's work and the revolution it has brought about in the study of field theories in curved spaces. Hawking and others had earlier given a number of (controversial) derivations of the temperature concept as associated with the exact solutions of the gravitational equations. However perhaps the most elegant is the following derivation due to Hawking, Hartle and Gibbons ¹⁵⁾.

We wish to show that the propagator of a spin-zero particle placed in an external gravitational field due to a Schwarzschild black hole of mass M exhibits a temperature dependence, with the temperature given by $(4\pi kT)^{-1} = R_c = 2G M$.

First note the well-known lemma that quite generally a thermal propagator at temperature T is periodic in time co-ordinate with a period given by $i(kT)^{-1}$. We shall now show that the propagator for a spin-zero field (of mass m) placed in the Schwarzschild background possesses a periodicity in time. The steps are the following:

1) We wish to solve

 $(\Box - m^2) \ K(\mathbf{x}, \mathbf{x}') = -\delta(\mathbf{x}, \mathbf{x}') \qquad \Box = \mathbf{g}^{\alpha\beta} \ \nabla_{\alpha} \nabla_{\beta} \qquad ,$

where $g^{\alpha\beta}$ is the Schwarzschild field possessing a horizon and (a singularity of the type $\frac{1}{r-2MG}$) at r = 2MG in the conventional Schwarzschild co-ordinates.

2) To avoid this singularity and for manifold completion we use, as is well known, the Kruskal co-ordinates.

3) There is still the singularity at r = 0. To circumvent this, one may use the euclidicity ansatz, i.e. complexify the co-ordinates. (This is the essential and brilliant remark of Hawking, Hartle and Gibbons.)

4) But irrespective of this singularity, to solve the Klein-Gordon the equation above for the propagator of spin-zero field, we need to specify the boundary conditions - we choose to do this on the <u>complex_analytic horizon</u> rather than directly specify the boundary conditions at the null infinity as a flat space particle physicist (with his naive ideas about positive, negative frequency splits) may have felt tempted to do. 5) We now note that the periodicity properties of Kruskal coordinates give a periodicity in static time of the Kelin-Gordon propagator. To see this write the Kruskal transformation (in the appropriate region); .

À

$$U = -\left(-1 + \frac{r}{2MG}\right)^{1/2} e^{(r-t)/4MG}$$
$$V = \left(-1 + \frac{r}{2MG}\right)^{1/2} e^{(r+t)/4MG}$$

Clearly there is the periodicity $Imt = \frac{1}{8\pi MG}$, i.e. the Klein-Gordon propagator must be a thermal propagator with temperature



Singularities of the propagator in t-plane.

6) Boulware ¹⁶⁾ in a related investigation obtained no thermal radiation of the Hawking type from a primordial black hole. His propagator exhibited no periodicity, because the boundary conditions he prescribed for it did not guarantee analyticity on the horizon. The general consensus (subscribed to by Boulware himself) principally on the grounds of elegance I believe, is that the Hawking-Hartle boundary conditions are the correct ones - certainly for a collapsing black hole. Since experiments in weak gravity are impossible, the only hope of experimental verification of these ideas lies with strong gravity - if it can be shown to have relevance to strong experimental phenomena.

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EXACT SOLUTIONS OF 1-GRAVITY EQUATION, m. # 0 ; POSSIBLE CONFINETENT IV.

So far we have assumed the f mass to be zero $m_{\mu} \neq 0$. Does such a soft non-singular limit exist? If it does, in what way are the results of the last section altered? Since $m_{p} \ge 1$ GeV, it is clear that $m_{p} \neq 0$ effects are physically quite important.

Strathdee and I have 17) recently attempted to find spherically symmetric classical solutions of the strong gravity equation (with $g^{\mu\nu} = \eta^{\mu\nu}$) and $m_{\mu} \neq 0$. Our results are the following: There are two types of inequivalent solutions: writing $f_{\mu\nu}dx^{\mu}dx^{\nu} = C dt^2 - 2Ddtdr - A dr^2 B(d\theta^2 + \sin^2\theta d\phi^2)$ we obtain

Type I (long-range solution) $B = 2/3 r^2$ $A + C = 2/3 + 3/2 \Delta$ $A = 2/3 + 3/2 \Delta \left\{ \frac{2M}{r} + \frac{1}{6} \frac{m_{f}^{2}r^{2}}{\sqrt{3/2}} \right\}$ $D = \pm \sqrt{\Delta - AC}$ Here A and M are two

Type II (Yukawa-like solution) D = 0. One can show that A, B, C \neq 0 but not yet computed. We do however know that for large r the solutions exhibit a behaviour-like exp(-m_r)

arbitrary parameters of the solution

We believe that Type II Yukawa-like solution - the one we have not yet been able to obtain exactly - is physically the more important and possibly represents the case of partial confinement. But the (exact) long-range solution (Type I) - a surprise to us, since we expected (with a massive .f-field all solutions to be of Type II - is interesting in its own right and I wish to examine this. In the limit $m_{\mu} \rightarrow 0$, this reduces to the Schwarzschild solution. To see this, consider C(r):

$$C(r) = \frac{3}{2} \Delta \left(1 - 2 \frac{MG_{B}}{r} - \frac{1}{6} \frac{m_{f}^{2} r^{2}}{\Delta^{3/2}} \right)$$

where in order that D is real, either

(1) M > 0, $\Delta^{\frac{1}{2}} > 0$, $0 < \Delta < \frac{1}{2}$ Attractive r^2 -term or (2) M < 0 . Δ^{M2} < 0 . Δ > 4/9 Repulsive r²-term .

Clearly when $m_{\rho} \neq 0$, we recover the Schwarzschild solution when $M \ge 0$. For $m_{\phi} \neq 0$ we have a solution of the Schwarzschild-de-Sitter type (with two horizons) when M , $\Delta^{1/2} > 0$ and anti-Schwarzschild-anti-de-Sitter type (with no horizon) when M, $\Delta^{V_{\rm L}} < 0$.

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A still further generalization can be achieved - and this has been studied by Isham (unpublished), so far as the relative signs of $\frac{1}{2}$ and r^2 terms are concerned. This is achieved by a simple modification of the original f-g mixing term | multiply the expression (2.1) by the zero weight factor $\begin{pmatrix} g \\ f \end{pmatrix}$. The new function C(r) equals $\frac{3\Delta}{2} \left(1 - \frac{2M}{r} + \Lambda r^2\right)$ where $\Lambda = \frac{a}{9} \left[\frac{9}{4\Delta} \right]^{a} \left[\frac{-1}{\Delta} + \left(\frac{1}{2} - a \right) \left(\frac{1}{\Delta} - \frac{3}{4} \right) \right]$. Clearly the parameter "a" can be so chosen that A has a positive or a negative sign.

To summarize, the long-range solution of the f-gravity equation can be written in the form:

$$\frac{3}{2} f_{\mu\nu} dx^{\mu} dx^{\nu} = \frac{1}{1+\alpha} (1-p(r)) dt^{2} - 2\sqrt{p(p+\alpha)} dt dr - (1+\alpha+p) dr^{2}) - r^{2} dd^{2} ,$$

where $p(r) = \frac{24}{\pi} - \Lambda r^2$, $\alpha > 0$ and M and A can take all four sequences of signs $[(M, \Lambda > 0), (M, \Lambda < 0), (M > 0, \Lambda < 0), (M < 0, \Lambda > 0)]$

We now wish to study classical orbits of particles in the f-gravity field and show that for a suitable sequence of signs of M and Λ such orbits exhibit (classical) confinement. (The following analysis is due to Strathdee.)

An effective Lagrangian for the orbits is given by:

 $\chi^2 = \frac{1}{2} f_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}$ where $\dot{x}^{\mu} = \frac{dx^{\mu}}{d\gamma}$ and τ is a proper-time-like parameter. (It is not quite proper time since we cannot impose the condition

$$\eta_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} = \text{constant} = 1.$$
)

Oving to spherical symmetry, no generality is lost on taking $\theta = \pi/2$, $\dot{\theta} = 0$. There are three non-trivial first integrals

$$\sqrt{1 + \alpha} B = (1 - p)\dot{t} - \sqrt{p(p + \alpha)} i$$
$$\lambda = r^{2}\dot{\phi}$$

and \mathcal{X} itself. On eliminating t and $\dot{\varphi}$, the latter reduces to

$$3\chi' = \frac{1}{1-p} (\Xi^2 - \dot{r}^2) - \chi^2/r^2$$

i.e. $\dot{r}^2 = \Xi^2 - 2V_{\chi}(r)$,
where $V_{\chi}(r) = \frac{1}{2}(1-p) (3\chi' + \chi^2/r^2)$
 $= \frac{1}{2}(1 - \frac{2M}{r} + \Lambda r^2) (3\chi' + \chi^2/r^2)$.

We are entitled to designate $V_{a}(r)$ as the effective potential, since $\ddot{r} = \frac{a v_{\mu}}{2}$.

For given $\hat{\mathcal{X}}$ and \mathcal{X} , there are four classes of orbits distinguished by the signs of M and A. (Note, however, that the sign of $\tilde{\mathcal{X}}$ is not <u>a priori</u> fixed; it could be postivie, zero or negative.) The following cases are of interest:

(2) **X** > 0, M < 0



v orbits of particles A>0 anti-Schwarzschildanti-de-Sitter (no horizon)

For both these cases, the classical orbits are confined, and Λr^2 acts like a (repulsive) confining potential. (For the first case the particles may eventually be captured by the r = 0 singularity; for the second case the singularity at r = 0 is repulsive and there is true confinement. A repulsive r = 0 singularity may also be expected for an f-gravity generalization of the extreme Kerr-Newman-like solution.)

Quite clearly, the classical analysis above is at best indicative and one must solve the equations for Klein-Gordon and Dirac propagators in the background provided by the f-gravity solitonic solutions with the appropriate boundary conditions. The new art particle-theorists must acquire is to learn how to circumvent singularities by complexification or similar techniques and how to specify boundary conditions. The analysis is incomplete also in respect of our not having considered solitonic solutions carrying spin and $SU(3)_{colour}$. The neglect of colour almost certainly implies that we cannot study saturation properties of our solutions. And, finally, as conjectured before, we believe that these long-range solutions of the f-gravity equations, confining exactly as they might, are likely to be less physically relevant than the Yukawa-like partially-confining solutions which may have the form $\Lambda r^2 e^{-m} r^r$, and which have not yet been worked out. This is because we do expect the quanta of f gravity (colour singlets) to propagate with a Yukawa propagator when r gets large

V. PREJUDICE AGAINST THE SPIN-TWO EINSTEIN EQUATION

What we claim to have shown in the last two sections is that solitonic solutions of f-gravity plus Yang-Mills $SU(3)|_{colour}$ equations are likely to represent normal hadrons. (The f-gravity solutions we have discussed are all singular at r = 0; in other words there is a matter singularity - quark field or whatever carrying mass, spin and $SU(3)|_{colour}$ quantum numbers - present at the origin.) By considering classical orbits of test particles in the background provided by these solutions, we have shown that a test particle an anti-quark for example - is likely to be confined. At any rate this test particle experiences an infinitely steep rising "potential" of the type ") Ar^2 . Since we have not taken colour or spin into account, we have not seen saturation effects at work (i.e. why three quarks bind but two do not). We hope also that when we are able to obtain Type II (Yukawa-like) solutions we may be able to motivate partial confinement (i.e. with "potentials" of the type $Ar^2e^{-m}r^r$). **)

These first results appear so encouraging that one wonders why particle theorists fight shy of using this most glorious of field equations the equation of **Ein**stein - for their own purposes. There are perhaps three reasons for this.

1) Lack of familiarity and the unfortunate impression that this equation cannot be studied without using the language of geodesics and (twisting) light cones. In this respect one welcomes the work of Hawking, Boulware and others emphasising firstly that the notion of particle orbits ideally suits Feynman's path integral formulation of quantum theory and secondly that after all when one is solving for propagators, the main battle is the specification of boundary conditions and the main technique, the avoiding of singularities of potentials (like $\frac{1}{r-2M}$) through complexification and analytic continuation. There are ideas so deeply ingrained in the up-bringing of particle physicists, that I have every hope that the situation in this regard will soon change. In fact the situation has

*) If the test particle is a quark (just like the particle producing the f field one should at the classical level consider orbits of two black holes in each other's field. The laws of black hole dynamics and particularly the law that in their collision and coalescence the surface area of black holes never decreases, may provide interesting clues to the dynamics of clusters and fire-balls.

**) In a very crude sense, exchange of a spin-two quantum is equivalent to an exchange of two spin-one quanta. Thus crudely, exchange of two gluons confines, while an exchange of one gluon saturates.

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^{*)} For "potential" in case (2) above, colour would be needed to provide the necessary attractive force.

already changed. The instanton solutions of Yang-Mills theory in Minkowski space are the analogues of black holes in f-gravity, firstly from the physical point of view (in that both solutions represent loss of information about quantum number objects trapped inside them) and also from the mathematical point of view (in that both solutions lose their singularities when one uses a euclidicity ansatz). In fact black hole solitons may well be called gravitational instantons.

2) The second reason why we have fought shy of this theory is related to the fact that the perturbation solutions of the Finstein equation appear to be hopelessly non-renormalizable. Likewise the high-energy behaviour of the perturbation solution appears to violate all the sacred theorems of Field Theory, like Froissart boundedness.

3) And finally, another aspect of this lack of renormalizability is that there appears no way to see if theory is asymptotically free or not.

To take the last point first, I believe gravity is indeed asymptotically free. This has been suggested by Fradkin and Vilkovisky ¹⁸⁾, who consider the one-loop radiative corrections to Einstein's theory

(They show that there is no cosmological counter term contrary to common belief.) Defining $Z_{one\ loop}^{-1} = \left(1 - \frac{23}{96} - \frac{G_N}{\pi^2} L^2\right)$ they show that the renormalized

newtonian constant G_{NR} bears to the unrenormalized constant the relation

$$G_{\rm NR} = \frac{G_{\rm N}}{1 - \frac{23}{96\pi^2} L^2}$$

Here **b** is the ultra-violet cut-off. Thus $Z_{one \ loop} > 1 - a \ statement$ characteristic of asymptotically free field theories.

This is admittedly just a one-loop argument. One has now to set up Callan-Symanzik-like equations (if one can) to show that an appropriate renormalization (Z) can be defined for all orders and that it always exceeds unity. This Fradkin and Vilkovisky claim to have done. But irrespective of their detailed considerations, I believe their result for the following reason.

Consider gravity for what it is - a non-polynomial Lagrangian theory and parameterize $g^{\mu\nu}$ in the non-polynomial form:

$$g^{\mu\nu} = (\exp \kappa \phi)^{\mu\nu}$$

 $\kappa^2 = 8\pi G_N$ ϕ is a 4 × 4 symmetric matrix of ten fields. (This parameterization implies we are not permitting det g to vanish.) Then the two-point propagator

$$g^{\mu\nu} g^{\rho\kappa} = ((\exp \kappa \phi)^{\mu\nu} (\exp \kappa \phi)^{\rho\kappa})$$
$$\approx (\exp \kappa^2 (\phi \phi))^{\rho\nu,\rho\kappa} \approx \left(\exp - \frac{\kappa^2}{x^2}\right)^{\mu\nu,\rho\kappa} \cdot$$

Now $g^{\mu\nu}$ exhibits the invariance $\kappa + \lambda\kappa$, $\varphi \to \frac{1}{\lambda}\,\varphi$, or in terms of the propagator

$$\kappa + \lambda \kappa$$
, $\langle \phi \phi \rangle + \frac{1}{\lambda^2} \langle \phi \phi \rangle$, i.e. in effect $x^2 + \lambda^2 x^2$.

In other words as $\lambda \neq 0$ (i.e. as $x^2 \neq 0$, or equivalently as we approach ultra-violet energies) the effective coupling $\kappa^2 \neq 0$. And this is just the hall-mark of asymptotic freedom.

To come back to the issue of high-energy behaviour, presumably here we must borrow the techniques of the dual model physicist, who with his closed string sector incorporates into his formalism essentially what are reggeized solutions of Einstein's equation and thereby secures acceptable high-energy behaviour for the S-matrix elements. (As remarked before he needs also the open string (Yang-Mills) sector for this renormalizability to take effect.) There is also hope from extended supergravity theories that the mass shell S-matrix elements in these theories may prove renormalizable after all.

But even if such a hope fails, I feel (regretfully) that there has not a proper been understanding of the work done by Isham, Strathdee and myself in respect of the regularizing role of Einstein's gravity theory. Following Landau, Klein, Fauli, de-Witt, Khriplovitch, Deser and others, we attempted to prove the conjecture made by these authors that gravity realistically regularizes all infinities including its own. We claim to have demonstrated this conjecture using Efimov-Fradkin non-polynomial techniques. Specifically we-computed the self-mass of an electron in a Dirac-Maxwell-Einstein theory and showed that to the lowest order in α this equals

$$\frac{\delta m}{m} \sim \alpha |\log G_N m_e^2|$$

The conventional logarithmic infinity of the Dirac-Maxwell theory is recovered if G_N is set equal to zero. (Numerically $|\log G_N m_e^2| \approx 105$ so that $\frac{\delta m_e}{m_e} \approx 1$ is approximately equivalent to the relation $\alpha |\log G_N m_e^2| \approx 1$).**)

*) In my view, if "supergravity" has immediate physical applications, these must relate to strong supergravity.

**) A similar relation has been derived by H. Terazawa, K. Akama, Y. Chikashige and T. Matsuki (see report of Terazawa's lecture given at the Marshak Symposium, City College, 1977). -30Now, in general, non-polynomial field theory techniques are ambiguous and one must use a principal value prescription in defining certain integrals. This has been the main stumbling block in a general acceptance of non-polynomial techniques. The paper, at whose neglect, I do feel sore, is the last paper in our series and entitled "Is quantum gravity ambiguity-free?" ¹⁹ In this we proved what we consider is a most crucial theorem. By considering the that complete expression for the two-point function, we proved/there is one non-polynomial theory where the (principal value) ambiguities of other non-polynomial theories simply do not occur - and this theory is gravity. Gravity escapes this blight because it has the distinction of being a <u>gauge theory</u>. (And for this "gauge"reason we also conjectured that though our exact result is for the two-point function, it is likely to hold also for the n-point function.)

I would like your indulgence to show you the main idea of the proof briefly. Write as before $g^{\mu\nu} = (\exp \kappa\phi)^{\mu\nu}$, $g_{\mu\nu} = (\exp -\kappa\phi)_{\mu\nu}$, where L_{Einstein} has the form \Longrightarrow ggg $\partial g \partial g$. It is well known that in order to define the propagators in the theory, one must add a gauge-fixing term to $\mathcal{L}_{\text{Einstein}}$ and make computations with $\mathcal{L}_{\text{Einstein}}$ + $\mathcal{L}_{\text{gauge-fixing term}}$. We choose a special type of gauge - the conformal gauge which gives for the free ϕ propagator the expression:

Here D(x) is the free scalar field propagator and c is the gauge parameter. As in every gauge theory, the final mass shell S-matrix elements are expected to be independent of the gauge parameter (c).

Now Ashmore and Delbourgo have computed the non-perturbative expression complete for the two-point function $(g^{\alpha\beta}(x), g^{\gamma\delta}(0))_+$ and given its expression. I shall not write it down; our interest lies in its asymptotic behaviour, when $x^2 \rightarrow 0$. This looks like the following:

$$\langle g^{\alpha\beta}(\mathbf{x}) g^{\gamma\delta}(0) \rangle \approx \frac{2}{9} \left[\eta^{\alpha\gamma} \eta^{\beta\delta} + \eta^{\alpha\delta} \eta^{\beta\gamma} + \cdots \right] \left(\cdots \right] \exp \left[\frac{\kappa^2}{2} D(1-c) \right]$$

Using the euclidean ansatz this has the form: $\exp\left(\kappa^2(1-c)\frac{1}{R^2}\right)$. The origin of the ambiguity which besets non-polynomial theories in general can now be made manifest. When $R^2 \neq 0$ (and if no gauge constant c is present) $\exp \kappa^2/R^2 \neq +\infty$. In order to define this propagator one must go to the κ^2 plane, continue to negative κ^2 , i.e. consider non-hermitian Hamiltonians (with $\kappa^2 < 0$) (so that $\exp \kappa^2/R^2 + 0 \text{ as } r + 0) \text{ and then continue back to the physical value } \kappa^2 > 0. It is this continuation in <math>\kappa^2$ which introduces the principal value ambiguity in expressions like log κ^2 which occur in the theory.

But not so in gravity theory: Here the gauge parameter c comes to our rescue. By working with gauges where c > 1, the effective parameter $\kappa_{eff}^2 = \kappa^2(1-c)$ can always be taken negative. And since at the end of the calculation, on the mass shell, the theory must be independent of c, this particular choice of c > 1 for calculational purposes is of no consequence. There is never an ambiguity in this theory.

To conclude, we claim, that the gauge invariance of gravity theory permits us to use <u>ambiguity-free</u> non-polynomial techniques and thereby secure a realistic regularization in gravity modified field theories *) with the newtonian constant G_N providing a realistic cut-off. To conclude this defence of the Einstein structure, I believe that there simply has not been enough work done to explore the deep questions posed by this most elegant of theories. And in this regard, one wishes to understand both the one tensor $g^{\mu\nu}(x)$ theory as well as the two (or many) tensor theories (containing $g^{\mu\nu}(x)$ as well as $f^{\mu\nu}(x)$) for all the problems posed in this section. The structure and the invariances of the two-tensor theory are very different from the invariance of the one-tensor theory and we need a deeper understanding of the new problems which arise in this regard.

Summary

I have tried to make a case for using both the Einstein-Weyl spin-two as well as the Yang-Mills spin-one gauge structure for describing strong interactions. By emphasising both spin-one and spin-two aspects of this force, I hope we can achieve a unification of this force, on the one hand with gravity theory and on the other with EM and weak interactions. The question arises: can these two structures (Einstein's and Yang-Mills) themselves be subsumed into one single structure. On the formal level this may be possible using the ideas of extended supergravity theory or alternatively using a formalism developed by Isham, Strathdee and myself which works with a gauge theory of groups of the type SL(6,C) or $SL(8,C) \times SL(8,C)$ where some of the redundant components of the (16-component) vierbein L_{ua} are used to describe spin-one fields in addition to the spin-two fields. In either case (besides the space-time curvature associated with the Einstein structure) it is the idea of space-time torsion - allied with internal symmetries which appears to play a fundamental role in giving a unified description of physical phenomena.

^{*)} We must still examine whether the mixed f-g theory permits of an imposition of two separate conformal gauges of the type we used in the proof above.

And this brings us up against the final question we must ask. For how much longer can we treat internal symmetries as something decreed from the outside. To my mind there is no problem deeper or more urgent of consideration than an attempt to comprehend the nature of internal symmetries and their associated charges - the flavours, the colours and the like from a deeper fundamental principle. At the present time we are treating the flavour (or the colour) charges as pre-Copernican epi-cycles - new ones to be invoked, and added on when the old set fails to please and satisfy. We need to know the deeper significance of these charges, just as Einstein understood the deeper significance of the gravitational charge through the concept of space-time curvature.

Since Einstein's example is the only successful example in physics of comprehending the nature of a charge, one's first thought is to seek the significance of flavours and colours within the ideas of extended curvature, extended torsion or the topological concepts associated with space-time and its possible extensions into higher dimensions (both bosonic and fermionic). (The fermionic extension embodied in the notion of superspace has probably the edge so far as extensions of the space-time concept are concerned. As Freund has argued, for fermionic dimensions one may not have to worry about the problems of physical measurements. Alternatively one may have to associate a size of the order of Planck length (10^{-33} cms) with these new (bosonic) dimensions, as argued a long time ago by Kaluza and Klein and recently by Scherk, Cremmer and Schwarz.)

To go back to Einstein's comprehension of gravitational charge in terms of space-time curvature, let us recall that Einstein was much impressed by the empirically determined equality of gravitational charge with inertial mass. He postulated from this the strong equivalence principle which adserted that all forms of (binding) energy (nuclear, EM, weak or gravitational) contribute equally to gravitational as well as to the inertial mass. As opposed to this principle, there was advocated, particularly by Brans and Dicke the so-called weak equivalence principle which maintained this equality as holding for nuclear, EM and weak forms of energy but not completely for the gravitational.

It is good to remind ourselves of the recent tests to discriminate between the strong and the weak equivalence principles. The point is that for laboratory sized objects the ratio of the gravitational binding energy to the total energy is $\approx 1 : 10^{23}$. Since the best tests of the equivalence principle (Braginsky and Panov (1971)) achieve an accuracy no greater than one part in 10^{12} , one needed planet-sized objects (e.g. the earth with its ratio/gravitational binding energy to total energy = 4.6 x 10^{-10}) to differentiate between the strong and the weak equivalence principles. The test would consist of measuring departures from Kepler's Law, of equilibrium distances of the earth and the moon from the sum. As you are aware, the test was carried/recently by two groups (Shapiro <u>et al</u> and Dicke <u>et al</u>) and reported in Phys. Rev. Letters of 15th March 1976. It consisted of echo delays of laser signals sent from the earth and reflected from the moon. The experiment - accurate to lunar-laser ranging measurements of ± 30 cms. - has unequivocally supported Einstein. The weak equivalence principle appears to be untenable.

I wish to draw two morals from this. First, a conceptually deeper theory - a theory of more universal applicability - scores even at the quantitative level. Second, Einstein, in formulating his theory, generalized the single-component field theory of gravity to the theory of a ten-component field $g^{\mu\nu}$. Instead of a one-component gravitational charge, he (profligately) introduced a ten-component entity (the stress tensor). He was not afraid of inventing myriads of components, myriads of (gravitational) charges because he knew the deeper principle behind his construct. For me the moral is clear; Nature is not economical of structures - only of principles of universal applicability. The biologist has long comprehended this; we, in physics, must not lose sight of this truth.

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