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CHARGED SPIN-1 GLUONS, PARTON MODEL AND THE ARCHIMEDES EFFECT *

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1. INTRODUCTION

In an $[SU(2) \times U(1)]_F \times SU(3)'_C$ (F = flavour, C = colour) gauge theory [1] with unconfined integer-charge quarks and (massive integer-charged) spin-1 colour gluons, both quarks and gluons contribute in parton models of electro- and neutrino-production. It is the purpose of this note to elucidate the question of lepto-production of colour [2] through spin-1 gluon partons. We make three remarks:

i) First the field-theoretic requirements, positivity of cross-sections (in particular σ_L) and current conservation, appear to require that the effective masses of (spin-$\frac{1}{2}$ as well as spin-1) partons inside the nucleon must be rather light compared with the mass of the nucleon, even though the physical masses of these particles outside the nucleonic environment may be considerably larger; (infinite [3] in the extreme case of absolute confinement). This difference of effective masses inside and outside the nucleonic environment - the "Archimedes effect" [4] - is, of course, implicit within the naive parton picture (if only for the model to be compatible with precocious scaling) and in this sense is not new. However, the fact that such a restriction may be implied by the positivity of σ_L (for spin-1 partons) and current conservation (for spin-$\frac{1}{2}$ as well as spin-1 partons) does not appear to have been remarked on in the literature.

ii) Our second remark is that the use of light effective masses for partons has implications for the magnitude of colour excitation through spin-1 partons. This is because the spin sum for "on-mass-shell" spin-1 partons is given by $\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{\mu^2}\right)$, μ^2 being their effective mass. As a consequence of this, the leading term of the spin-1 parton contribution to the structure functions (either within the naive parton picture or in the sense of light cone expansions) turns out to depend explicitly on μ^2 through a factor of the form (m_U^4/μ^4), where m_U denotes the gluon propagator mass. Quite clearly, the presence of such a mass ratio would have profound implications for the magnitude of colour excitation if $\mu^2 < m_N^2 < m_U^2$. [Note, by contrast, that no such explicit dependence on parton mass arises for the leading term of spin-$\frac{1}{2}$ parton contributions *) to structure functions. It is for this reason that in the parton picture, the Archimedes effect has not played the crucial role for spin-$\frac{1}{2}$ partons that it does for charged spin-1 partons.]

*) This characteristic difference between spin-$\frac{1}{2}$ versus spin-1 parton contributions is, of course, no more than a kinematical difference between the spin sums and the normalization factors in the two cases.

iii) Finally, we remark briefly on the emergence of "the Archimedes effect" in non-topological solitonic treatments of field theories.

2. POSITIVITY OF σ_L, GUAGE-INVARIANT FORM FOR $W_{\mu\nu}$ AND EFFECTIVE PARTON MASS IN THE NAIVE PARTON MODEL

The naive parton model (which has its parallel within the light cone or operator product expansion) starts by treating the initial and final partons essentially as free particles (on their "effective" mass shells). There are then two parts to every "derivation" of parton model results. For example for electroproduction, first one writes down the field-theoretic amplitude for e + (free parton of momentum p_μ) → e + (free parton of momentum $p_\mu + q_\mu$). This is then followed by a second part in which one relates the parton momentum p_μ to the incoming nucleon momentum P_μ using an infinite momentum frame. It is the need to reconcile the demands of these two parts of the derivation which lead to the necessity of light parton masses inside the nucleonic environment.

To see this, first consider the field-theoretic part of the elastic scattering of electrons against charged (massive) spin-1 colour gluons in the context of a spontaneously broken renormalizable colour gauge theory [1] (i.e. $e + V^+(p) \to e + V^+(p+q)$). Note that for such a theory the photon as well as its orthogonal colour gauge partner U contribute in order e^2. With the "combined" propagator [2] $\left(\frac{1}{q^2} - \frac{1}{q^2 - m_U^2}\right) = \frac{1}{q^2}\left(\frac{-m_U^2}{q^2 - m_U^2}\right)$, the cross-section is $L_{\mu\nu} W_{\mu\nu}^{(V)}$, with

$$W_{\mu\nu}^{(V)} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1^{(V)} + \frac{1}{\mu^2}\left(p_\mu - \frac{pq}{q^2} q_\mu\right)\left(p_\nu - \frac{pq}{q^2} q_\nu\right) W_2^{(V)} \tag{1a}$$

and $L_{\mu\nu}$ is the familiar leptonic tensor. Here

$$W_1^{(V)} \propto \left(\frac{m_U^2}{Q^2 + m_U^2}\right)^2 \left\{ 2pq + 7Q^2 + \frac{(2pq)^2}{\mu^2} + \frac{Q^4}{\mu^2} \right\} \frac{1}{Q^2}$$

$$W_2^{(V)} \propto \left(\frac{m_U^2}{Q^2 + m_U^2}\right)^2 \left\{ 12 + \frac{3Q^2}{\mu^2} + \frac{2pq}{\mu^2} + \frac{Q^4}{\mu^4} \right\} \frac{\mu^2}{Q^2} \tag{1b}$$

$\mu^2 = p^2 = (p+q)^2$. From this, although $Q^2 \equiv |q^2| = 2p.q$, the substitution is not explicitly made in (1b) for reasons which become clear later.

In the above expressions, we have not exhibited normalization and coupling constant factors, which are common to $W_1^{(V)}$ and $W_2^{(V)}$ and are positive definite. Note that at this stage all demands of field theory are, of course, trivially satisfied with arbitrary values of μ^2; $W_{\mu\nu}^{(V)}$ has the right invariant form consistent with current conservation and

$$\sigma_L^{(V)} = W_2^{(V)}\left(1 + \frac{Q^2}{4\mu^2}\right) - W_1^{(V)} > 0 ,$$

$$\sigma_T^{(V)} = W_1^{(V)} > 0 . \tag{2}$$

Now comes the parton picture, where one relates the incoming parton momentum to the nucleon four-momentum.

Let P and p be the nucleon and incoming parton four-momenta. In the familiar infinite momentum frame ($P \to \infty$) write

$$P_\mu = \left(\sqrt{M_N^2 + P^2}, 0, 0, P\right),$$

$$p_\mu = \left(\sqrt{\mu^2 + p_\perp^2 + \xi^2 P^2}, p_{\perp x}, p_{\perp y}, \xi P\right),$$

$$\equiv \xi P_\mu + \Delta_\mu , \tag{3}$$

where Δ_μ is defined by (3), while

$$x = \frac{Q^2}{2M_N\nu} , \quad \nu = \frac{P.q}{M_N} \quad \text{and} \quad \xi = x + O(\nu^{-1}) . \tag{4}$$

Positivity of σ_L is lost in allowed regions of x and Q^2, with the familiar parton model (approximate) substitution $p_\mu \approx \xi P_\mu$, unless in the spin sums $\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{\mu^2}\right)$ and $\left(-g_{\mu\nu} + \frac{p'_\mu p'_\nu}{\mu^2}\right)$ ($p' = p+q$) for incoming and outgoing gluons, treated free and on-mass-shell, μ^2 is consistently replaced by $\xi^2 P^2 = \xi^2 m_N^2$ whenever p_μ is replaced by ξP_μ.

The result is obvious from the remark that, though quite generally the "mass shell" spin-sum $\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)$ does not contain contributions from the unphysical spin-zero component of the vector field, one cannot (as is commonly done) replace the expression above by $\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{\mu^2}\right)$ together with the naive parton model replacement $p_\mu \sim \xi P_\mu$ in the product $\frac{p_\mu p_\nu}{\mu^2}$ without the simultaneous replacement $\mu^2 = \xi^2 M_N^2$. Otherwise, the negative metric states begin to contribute and σ_L can be rendered negative for some region of the physical phase space.

To see that this does in fact happen in the naive treatment, examine the expressions of spin-1 colour gluon parton contribution to the electroproduction structure functions obtained with the substitution $p_\mu p_\nu \to x^2 P_\mu P_\nu$ $(x = \xi + O(\nu^{-1}))$,

$$F_{1\,col}^{(N)}(\text{spin }1) = \left(\frac{m_U^2}{Q^2+m_U^2}\right)^2 \frac{16}{3}\left(1 + \frac{Q^2}{4\mu^2}\right) \upsilon(x) ,$$

$$F_{2\,col}^{(N)}(\text{spin }1) = \left(\frac{m_U^2}{Q^2+m_U^2}\right)^2 \frac{4}{3}\left(3 + \frac{Q^2}{\mu^2} + \frac{Q^4}{4\mu^4}\right) x\,\upsilon(x) . \tag{5}$$

Here $F_{1col}^{(N)} = W_{1col}^{(N)}$ and $F_{2col}^{(N)} = \nu W_{2col}^{(N)}$; m_U denotes the gluon propagator mass, μ the effective gluon parton mass and $\nu(x)$ the momentum distribution function for gluons inside the nucleon. Note the explicit dependence of the structure functions on the parton mass μ^2 through the terms (Q^2/μ^2) and (Q^4/μ^4), which owe their origin to the specific forms of polarization sums of the incoming and outgoing spin-1 partons. The presence of the colour damping factor $[m_U^2/(Q^2 + m_U^2)]^2$ implies that - in spite of the μ^2 - dependent contributions from spin-1 partons, asymptotically $F_{col}^{(N)}$ scale (provided that μ^2 is either a constant or a function of x alone).

From (5) we obtain

$$\sigma_L = K \left(\frac{m_U^2}{Q^2+m_U^2}\right)^2 \upsilon(x) \left(\frac{\mu^2}{Q^2}\right) \times$$
$$\times \left[\left(8 + \frac{8}{3}\frac{Q^2}{\mu^2} + \frac{2}{3}\frac{Q^4}{\mu^4}\right)\frac{x^2 M_N^2}{\mu^2} + \right.$$
$$\left. + \left(-\frac{10}{3}\frac{Q^2}{\mu^2} - \frac{2}{3}\frac{Q^4}{\mu^4} + \frac{1}{6}\frac{Q^6}{\mu^6}\right)\right] . \tag{6}$$

Here K is a positive constant. It is easy to verify that σ_L is positive definite if we set $\mu^2 = p_\mu^2 = x^2 M_N^2$, consistent with $p_\mu p_\nu \to x^2 P_\mu P_\nu$. However, it may also be verified that for any value of $\mu^2 > 2x^2 M_N^2$, there exists a range of Q^2 values (depending upon μ^2) for which σ_L is negative. (For instance, for $x^2 M_N^2 = \frac{1}{2}\mu^2$, σ_L is negative in the region $2\mu^2 < Q^2 < \frac{10}{3}\mu^2$.)

As demonstrated at the beginning of this section, it is obviously not the field theory of spin-1 gluons, which led to negative σ_L . It is the naive parton approximation which leads to negative σ_L in some regions of x and Q^2 (depending upon μ^2), unless consistent with the substitution $p_\mu \to xP_\mu$ and $p_\mu p_\nu \to x^2 P_\mu P_\nu$ (in the infinite momentum frame), we also consider μ a function of x $(\mu^2 = x^2 M_N^2)$. It follows from this firstly that $\mu^2 < M_N^2$ and secondly that for $x \to 0$, μ also tends to zero.

Admittedly, there are several physical considerations *) which imply that the parton model formulation and the infinite momentum frame treatment breaks down for sufficiently small $x \to 0$ (as well as for sufficiently small Q^2). These difficulties of the parton model formulation have been discussed in the past for spin-$\frac{1}{2}$ partons and have recently become the subject of considerable interest in the literature [5]. For spin-1, however, the demands are stronger, and the moral we wish to draw is that for such regions of x and Q^2 where parton model formulation may be expected to apply (say $x > 0.2$ and $Q^2 > 2\ (\text{GeV})^2$), positivity of σ_L is satisfied strictly as

*) One attitude to adopt and perhaps the right one is that one must never go to the extreme limit, $x \to 0$, since the parton model formulation simply breaks down in this limit. However, note that consistent with the expectations of a renormalizable (sponteneously broken) Yang-Mills gauge theory, $W_{\mu\nu}$ has no singularities in the zero mass limit[6]; since in such theories one would expect the masses m_U and μ to tend to zero together with the expectation values of the relevant Higgs scalars.

long as we replace μ^2 <u>either</u> by $x^2 M_N^2$ <u>or</u> by some average value $\bar{\mu}^2 \approx \langle x^2 \rangle M_N^2 < M_N^2$ (e.g. $\bar{\mu}^2 \approx 0.3$ to 0.5 (GeV)^2 if very small x and Q^2 are excluded from consideration].

We now wish to relax the naivest of parton model assumptions $p_\mu \approx \xi P_\mu$ and generalize to $p_\mu = \xi P_\mu + \Delta_\mu$ (for definitions of ξ and Δ_μ, see Eq.(4)). It appears that $\mu^2 = p^2$ is now further restricted (both for spin-$\frac{1}{2}$ and spin-1 partons) by requirements of current conservation. Specifically, in order that $W_{\mu\nu}$ has the right covariant structure consistent with current conservation and certain parton model assumptions, one can show that it is necessary that the effective masses of spin-$\frac{1}{2}$ or spin-1 partons have the form

$\mu^2 \equiv p_\mu^2 = x^2 M_N^2 + \langle p_T^2 \rangle \left(\frac{Q^2}{2\nu^2} - \frac{3}{2} \right)$. Here $\langle p_T^2 \rangle$ is the average value of p_T^2 for the parton distribution in question. [To derive this result we assume that the <u>net</u> transverse momentum carried either by the spin-$\frac{1}{2}$ partons or the spin-1 gluons vanishes.] The proof will not be given here.

3. THE ARCHIMEDES EFFECT

The field theoretic requirement of the effective mass of the gluon in the nucleonic environment in a parton model context being environment dependent (and of the form $x M_N \approx \frac{Q^2}{2\nu}$ in the naivest parton model) is a special instance of the Archimedes effect. This effect is familiar from other branches of physics, for example from theory of metals and nuclear shell models where electrons and nucleons, respectively, behave (as essentially free particles) with effective masses different from their physical masses, and with a strong environment dependence. Another relevant instance is the plasmon effect for a gas of massless (exact $SU(3)_C$) Yang-Mills gauge bosons studied by Kislinger and Morley [7], who show that within a (high temperature) environment in the early universe, the coloured gluons must possess non-zero effective masses.

A systematic study of the Archimedes effect for gauge theories is given by Friedberg, Lee and Sirlin [8] who consider non-topological soliton solutions for spontaneously broken gauge theories. They show, for example, that for spontaneously broken SU(2) and SU(3) theories, with gauge bosons and appropriate Higgs-Kibble fields, there exist solitonic solutions, where the gauge particles are relatively light inside the solitonic environment (confined to a self-consistently determined radius R) while possessing the conventional Higgs-Kibble mass $e\langle\phi\rangle$ at long distances from the centre of the soliton.

Quite clearly, the crucial problem for the future of unconfined - and even confined - colour gauge theory is to incorporate the Archimedes effect in a fundamental description of gluons and quarks in the hadronic environment. Such an environment is characterized by three components,

i) the valence quarks (described by the valence distribution function),

ii) the quark sea,

iii) the gluon condensate.

It is important to remark that the last two are features of the vacuum which are distorted by the presence of valence quarks in physical hadrons.

In a correct final description, one would expect that there is no sharp boundary between the <u>inside</u> of the nucleonic environment and the <u>outside</u>, but a transition region exists, with a tunnelling taking place between the two regions, dependent upon the difference between the outside and inside effective masses, and the parameters determining the boundary.

But till such time that a comprehensive theory of the "Archimedes effect" is created and the effects of the nucleonic environment on m_U and μ are estimated, one may proceed so far as the naive parton model is concerned, by assuming, for example, that (in the relevant regions of the phase space) the gluon-mass parameter $\bar{\mu}$ occurring in the structure function given in Eq.(5) represents an effective magnitude $\langle \bar{x} \rangle m_N < m_N$. So far as the asymptotic behaviour of the structure functions is concerned, note that

$$F_1^{col} \to 0 ,$$

$$F_2^{col} \to \frac{1}{3} \, x \, V(x) \left(\frac{m_U^2}{\bar{\mu}^2} \right)^2 ,$$

and in particular the gluon contribution to $R = (e^+ + e^- \to \text{hadrons}/\to \mu^+ + \mu^-)$ asymptotically equals $\frac{1}{8} \left(\frac{m_U^2}{\bar{\mu}^2} \right)^2$. With $m_U^2/\bar{\mu}^2 \approx 3$-4, this would contribute*) 1-2 units to R so that $R_{total} = R_{flavour}\,(\text{GIM}) + R_{colour} \approx 4.4$-$5.3$, obviating the need for new flavours or heavy leptons. In a separate note (with R.N. Mohapatra and D.P. Sidhu) we shall examine the consequences of the above ansatzes for νN and eN deep inelastic processes.

*) This is assuming that the colour continuum threshold is relatively low (3-4 GeV). There is, of course, always the possibility that this threshold is high ($\gtrsim$ 8 GeV). In this case, even though $\bar{\mu}$ may be low, there is no colour contribution to R in the presently explored energy range.

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