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HADRONIC TEMPERATURE AND BLACK SOLITONS *

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ABSTRACT

Hawking has shown that event-horizon-containing classical soliton (black-hole) solutions of Einstein's equation radiate all species of particles with a thermal spectrum, the temperature being defined in terms of surface gravity. For a spinless soliton, the temperature is inversely proportional to the radius of its event-horizon. Assuming that there exists a fundamental strongly-interacting (massive) spin-2 field satisfying an Einstein-like equation with a strong coupling parameter, we propose to identify temperature in hadronic physics with strong surface gravity effects. The existence of black-body solitonic solutions for such an equation may then explain the thermal spectrum in E_T observed in high-energy collisions.

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I. The concept of temperature has proved fruitful in hadron physics, through the work of Hagedorn and his collaborators [1]. One of the latest manifestations of the use of this concept is in the work of Bartke et al. [2] who show that the hadronic spectra in πp collisions, when expressed in terms of the transverse energy $E_t^2 = p_T^2 + M^2$, can be fitted with a particularly simple formula

$$\frac{d^3\sigma}{dp^3} \propto [\exp(E_t/kT - 1)]^{-1} \quad ,$$

when one considers the production of resonances like ρ , ω and f as well as the production of non-resonant two-or three-pion systems in the central region. All spectra are consistent with one common temperature, which is approximately $kT \approx 120$ MeV. The problem, as always in hadron physics, is this: what, if any, is the deeper dynamical origin of this universal type of thermodynamical distribution and what does the temperature signify?

Now in a related discipline of theoretical physics, through the beautiful work of Hawking [3] (and some earlier work of Beckenstein [3]) the concept of temperature has found a deep and satisfying significance. Hawking has shown that the classical solitonic [4] black-hole solutions of Einstein's equations possess the property that radiation tunnels out through the event-horizon and escapes to infinity at a steady rate. What is truly remarkable about this radiation is that it turns out to have an exactly thermal spectrum - a circumstance which appears to be related to the existence of an event-horizon for (what we shall call "black" or "black-body") solitonic solutions. In particular, the expectation value $\langle N \rangle$ of the number of particles of a given species emitted in a mode with frequency ω , angular momentum component $m\hbar$ about the axis of rotation of the soliton of charge Q equals:

$$\langle N \rangle = \gamma[\exp \frac{1}{kT} (\omega - m\Omega - Q\Phi) \pm 1]^{-1} \qquad (1)$$

(- sign for bosons, + sign for fermions).

Here γ is the fraction of the mode that would be absorbed were it incident on the black soliton. The temperature T equals $K\hbar/2\pi kc$, where K is surface gravity, Ω the angular frequency of rotation and Φ the potential at the event-horizon. For the Kerr-Newman soliton, K , Ω and Φ are given in terms of mass M , charge Q , angular momentum J and Newtonian constant G_N by the expressions:

$$K = 4\pi(R - G_N M)\, A^{-1}\ (= 2\pi kT) \quad , \quad \Omega = 4\pi J\ (MA)^{-1} \quad , \quad \Phi = 4\pi QR\, A^{-1} \quad , \tag{2a}$$

where the radius R and area A *) of the event-horizon are given by

$$R = \left[G_N M + (G_N^2 M^2 - J^2 M^{-2} - G_N Q^2)^{1/2}\right] \tag{2b}$$

and

$$A = 4\pi G_N\left[2G_N M^2 - Q^2 + 2(G_N^2 M^4 - J^2 - G_N M^2 Q^2)^{1/2}\right] \ . \tag{2c}$$

For the case $J = Q = 0 \quad (c = \hbar = 1)$, $\Phi = \Omega = 0$,

$$R = 2G_N M \quad , \quad A = 4\pi R^2 . \tag{2d}$$

Note that for $J, Q \neq 0$ we must have $G_N^2 M^2 \geqslant (J^2 M^{-2} + G_N Q^2)$. (2e)

II. Now what is the relevance of this definition of temperature to hadron physics? One may conceive of such a relevance through the ideas of F-g mixing and a strong F-gravity theory[5]. Assume that there exists a strongly self-interacting fundamental spin-2 field $(F_{\mu\nu})$ satisfying equations of the same form as Einstein's, with general covariance broken spontaneously [6] so as to facilitate generation of a "soft" mass term m_F for $F_{\mu\nu}$. The strong coupling parameter G_F (which replaces the Newtonian constant G_N) in self-couplings of F as well as its coupling with hadrons will be assumed to be ≈ 1 (in GeV^{-2}). In the "soft" limit $m_F \to 0$, we expect **) that such a theory will possess the classical Kerr-Newman black solitonic solutions, with a horizon determined by the formulae (2a)-(2e); however, with G_F replacing G_N. These solitons, in accordance with Hawking's results for weak gravity,[3] will

*) Beckenstein and Hawking [3] have shown that the quantity $\frac{1}{4}$ A possesses the attributes of "entropy": when two black holes collide, A_{final} exceeds $\Sigma A_{initial}$. Classically, bifurcation of a black hole into smaller holes is completely forbidden. Quantum mechanically, however, this can happen at a rate which is exponentially small.

**) As remarked in Ref.6, the propagator for the F field $\langle F_{\kappa\lambda} F_{\mu\nu}\rangle$, in a suitable gauge equals $(\eta_{\kappa\mu}\eta_{\lambda\nu} + \eta_{\kappa\nu}\eta_{\lambda\mu} - \frac{2}{3}\,\eta_{\kappa\lambda}\eta_{\mu\nu})(k^2 - m_F^2)^{-1}$, differing from the corresponding propagator for Einstein's weak gravity g in the appearance of the factor 2/3 before the $\eta_{\kappa\lambda}\eta_{\nu\mu}$ term. Whether such a difference (affecting as it does the propagation of traces of energy-momentum tensor) may affect the conclusion drawn above, about the existence of black soliton solutions of precisely the Kerr-Newman type in the limit $m_F = 0$, is not known.

radiate thermally into all species of hadrons *), with ω replaced in Eq.(1) by $E = \sqrt{m^2 + p^2}$ for massive particles. We propose that such black-body solitons are created in hadronic collisions and it is their thermal radiation which is responsible for the observed spectra in hadronic physics.

III. Teukolsky and Press and Page [7], in a series of papers, have presented the formalism and numerical estimates giving rates for black solitons (of weak gravity) radiating neutrinos, photons and gravitons, under the assumption that the soliton mass is much larger than the temperature. Making the vast extrapolation that one may adapt their formulae for strong gravity in the limit of $m_F \to 0$, we obtain (for a chargeless, rotationless soliton) the relation between the radius of the event-horizon and its temperature in the form:

$$R_{soliton} = 2G_F\, M_{soliton} = \frac{1}{4\pi kT} \approx \frac{1}{1.5\ \text{GeV}} \ , \tag{3}$$

using the experimental input $kT \approx 120$ MeV.

The important point about this formula - as of all Hawking's theory - is that temperature appears as a purely geometrical entity, connected (inversely) with the radius of the event-horizon for the soliton solution. To estimate the half-life of such a soliton, use Page's thermodynamic formula for power radiated in weak gravity case [7], adapted to our situation:

$$\frac{dR}{dt} = \frac{2\pi^2}{15}\, G_F (kT)^4 \left[\sum_s \sigma_s\right] \ . \tag{4}$$

Here σ_s is the absorption cross-section by the black soliton of an incident particle of spin s . **)

*) To estimate emission rates of photons and lepton-pairs, one may first consider the thermal emission of (hadronic) vector mesons (ρ,φ etc.) from black solitons in accordance with the ideas presented above, followed by these particles turning into photons (rate depressed by a factor α) and into lepton-pairs (rate depressed by α^2).

**) For absorption of massless [7] particles $\nu, \bar{\nu}, \gamma$ etc. in the weak gravity theory, $\sigma_s(\omega) = \pi\omega^{-2}\, \Sigma\gamma_{s\omega} \underset{\omega\to 0}{\sim} 16\pi M^2 G^2$ for $s = 0$, $2\pi M^2 G^2$ for $s = 1/2$ etc. while in the high frequency limit all cross-sections go to $27\pi G^2 M^2$. Using $\sigma = 27\pi G_F^2 M^2$ in (4), would lead to a narrower soliton width.

To get a feel for the numbers in the strong gravity case, rewrite Eq.(4) in the form

$$\frac{dR}{dt} = \frac{2\pi^2}{15} G_F \left(\frac{1}{4\pi R}\right)^4 \sigma_T \quad , \qquad (5)$$

where σ_T is the typical total asymptotic hadronic cross-section $\approx 150(\mathrm{GeV})^{-2}$. Integrating (5) we obtain the final formulae for soliton width

$$\Gamma \approx (384\pi^2)^{-1} R^{-5}_{\mathrm{soliton}} G_F \sigma_T \approx 300 \text{ MeV} \times G_F \text{ (evaluated in units of GeV}^{-2}\text{)}, \quad (6)$$

where in the numerical estimate we have used the experimental input $R_{\mathrm{soliton}} \sim$ $\sim 1/1.5$ GeV from Eq.(3).

To estimate G_F, a tensor-dominated strong interaction model might suggest $G_F \approx \alpha_s m_F^{-2}$, where α_s is the dimensionless strong coupling constant and m_F is the spin-2 F mass. Assuming that this F meson lies on the Pomeron trajectory $\alpha(t) \approx 1.1 + 0.2t$, we would obtain $m_F^2 = 4.5$ GeV^2. With the assumption $\alpha_s \approx 1$, Eqs.(3) and (6) then give $M_{\mathrm{soliton}} \approx 1.5$ GeV, $\Gamma_{\mathrm{soliton}} \approx 67$ MeV. Alternatively, we may identify the radius of the soliton-horizon, R_{soliton}, with the soliton's Compton radius (i.e. $R_{\mathrm{soliton}} \sim \frac{1}{M_{\mathrm{soliton}}} \sim 2G_F M_{\mathrm{soliton}} \sim \frac{1}{1.5 \text{ GeV}}$). This also gives $G_F \sim \frac{1}{4.5}$ GeV^{-2} and $M_{\mathrm{soliton}} \sim 1.5$ GeV, $\Gamma_{\mathrm{soliton}} \sim 67$ MeV.

Clearly, there is nothing sacred about these ad hoc estimates. G_F could well vary by an order of magnitude, with corresponding variations in solitonic mass which could be much heavier. However, it seems to be a general (and perhaps paradoxical) feature of the formalism that more massive solitons are likely to be the more stable.*)

*) One must reiterate once again that in these estimates we have used the formalism developed by gravity physicists well beyond the limits of its applicability. Their formalism applies for situations where black-soliton masses and their temperatures greatly exceed the masses of emitted particles and where the gravitational field is massless. For the case we are considering, these three masses (soliton mass, (temperature)$^{-1}$ and m_F) differ by (less than) an order of magnitude. It has been conjectured, for example,[10] that if $m_F \neq 0$, the radius of the horizon satisfies an equation like $R \exp(m_F R) \approx 2G_F M_{\mathrm{soliton}}$ rather than $R = 2G_F M$. Such modifications will alter the estimates above.

IV. The Bartke formula with which we started this note referred to a universal thermal distribution for E_T, the transverse energy, whereas the picture presented above of the formation of and thermal radiation from black solitons, referred not to E_T but to the total energy of the radiated particles. Where does the transverse energy come from? This problem exists also in the Hagedorn picture.

Now it is well known that the transverse or cylindrical phase space emerges in dynamical schemes like the multiperipheral model. For example, Caneschi[8] has recently suggested that a very weak form of multiperipheralism involving a few peripherally produced objects (e.g. our black solitons) may provide a good phenomenological description of the various aspects of multiparticle production. Interestingly enough, he showed that an e^{-6p_T} law for pions requires what he calls a cluster (our soliton?) of mass of order 1.5 BeV. It would be interesting to see if this weak form of multiperipheralism translates the dependence we envisage in the total energy into one involving E_T.

The note above has concentrated on black solitons with $Q = 0$, $J = 0$. For the case of weak gravitational solitons, Zaumen[9] and Gibbons[9] have shown that the charge of solitons (with $Q \neq 0$) is quite rapidly transmitted away by the emission of charged particles. Correspondingly, Page[7] has considered the case $J \neq 0$ and shown that the same happens to angular momentum, which is emitted several times faster than energy.*) Thus a rapidly rotating black soliton runs down to a nearly non-rotating state before most of its mass has been given up; the emission at this stage being overwhelmingly in the form of spin-2 objects. These remarks are of interest in connection with a recent model for p_T distributions presented in Ref.1, where a case is made for high-mass, high-spin "fireballs". It is important to remark that such objects, if identified with our solitons, are perfectly compatible with relatively low temperature, provided a Regge-like formula $J^2 \approx G_F^2 M^4 - G_F M^2 Q^2$ holds, so that K in Eq.(2a) is small, without GM being small at the same time.

*) As stated before, Page's estimates are for very massive black solitons in weak gravity with temperatures of the order of GeV or more. Note that in an SL(6,C) version[10] of strong gravity which contains $SU(3)|_{\mathrm{colour}}$ (for example) we expect charge Q^2 in soliton solutions to be replaced by Casimir operators for $SU(3)|_{\mathrm{colour}}$. The results of Zaumen and Gibbons make it unfavourable for solitons carrying high $SU(3)|_{\mathrm{colour}}$ quantum numbers to exist in nature. (We thank Prof. N. Craigie for this remark.)

The most interesting future avenues of investigation will lie in the derivation of formulae for thermal power radiated in different modes for strong gravity analogous to the formulae derived for the weak gravity case, with a view to confront these with experiment.

Assuming that the ideas presented preliminarily here contain some truth, the conclusion must be that spin-2 fields satisfying Einstein-like equations play a fundamental role in strong interaction physics. It was suggested earlier [10] that such fields may be decisive for bringing about the observed partial confinement of quarks in hadron physics – perhaps even more decisive in this respect than coloured spin-1 gluons,*) whose true role may be to bring about saturation rather than confinement.

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*) It is noteworthy that the type of horizon-containing soliton solutions discussed here with their characteristics:

1) $R_{soliton} \sim M_{soliton}$,

2) $J \leq GM^2 \approx \alpha_s \frac{M^2}{m_F^2}$ (cf. (2e)) ,

essentially represent objects predicted already by Craigie and Preparata [11] in their cavity model. The existence of such objects is necessary in order to obtain a sensible hadronic spectrum as well as parton-like scaling laws.

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