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CP INVARIANCE THOUGH NEUTRAL KAONS *

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ABSTRACT

Neutral K-decay data and CP invariance are completely compatible but lead to the conclusion that K_0 and $\bar{K}_0$ are not conjugate. This in turn implies the existence of four resonant states L,S, and their CP conjugates L^C, S^C. However, there are only two particles, in the sense that the S matrix has a single pole at $M_L - \frac{i}{2}\Gamma_L$ and another single pole at $M_S - \frac{i}{2}\Gamma_S$.

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It is widely believed that neutral K-decay pheonomenon establishes violation of CP invariance. Many initial models[1] proposed to preserve CP were all shown inconsistent with one or other of the features of the neutral K-decay[2]. However, an important point has been missed in all these discussions. Although no experiment substantiates it, one invariably assumes $\bar{K}_0$ to be the CP conjugate of K_0.

We find that if CP invariance is insisted upon then the main message of the entire neutral K decay is not an inconsistency but that $\bar{K}_0$ is not the CP conjugate of K_0. The entire data analysed in terms of the very parameters (interference parameter η, overlap parameter ε, charge asymmetry δ, relative phase between K_0 interference pattern and $\bar{K}_0$ interference pattern, regeneration) used conventionally can be adopted <u>en masse</u> with only changes in English prose adhering to strict CP invariance.

That the resonant states L and S cannot be CP eigenstates has been the common feature of all CP preserving models. That it leads to K_0, $\bar{K}_0$ not being CP conjugates of each other is also very clear from the data; and one wonders why it has not been so pronounced. However, it opens up the question of definition of antiparticle in a subspace where all the usual quantum numbers like charge, baryon number, lepton number etc. are zero.

To produce the CP conjugate K_0^C of K_0 unambiguously, one should use the CP conjugate of a whole reaction that produces a K_0. For example, the CP conjugate of the associated production of Σ and K_0 by a proton must be the associated production of $\bar{\Sigma}$ and K_0^C by an antiproton. In this pair of reactions the neutral kaons must be CP conjugates of each other.

The basic experiment[2] that is supposed to have established CP violation obtains a set of data showing that the $\pi^+\pi^-$ decay interference pattern arising from K_0 is out of phase relative to one arising from $\bar{K}_0$. This we interpret as a clear demonstration of $\bar{K}_0$ not being the CP conjugate of K_0, i.e. $\bar{K}_0 \neq K_0^C$. It becomes imperative to make the same experiment with the difference that $\bar{K}_0$ is replaced by K_0^C as obtained via a CP conjugate reaction using an antiproton beam. An absence of phase change in the $(\pi^+\pi^-)$ decay pattern of K_0^C relative to that of K_0 will re-establish CP invariance in our literature. One may also look at the $\pi^+\pi^-$ decay of the neutral K-beam emerging from a definite CP state protonium. CP invariance predicts no $\pi^+\pi^-$ decay from the CP = -1 state, while from CP = +1 state it predicts an interference pattern that is in phase with that of K_0 and not with that of $\bar{K}_0$.

The question about four states $K_0, \bar{K}_0, K_0^C$ and $\bar{K}_0^C$, when there are only two in the conventional treatment, can be embarassing. However, these states can be experimentally distinguished by their decay patterns. Even than one would be very unhappy to throw away the SU(3) octet of mesons. Fortunately, one can understand these four states in the context of an S matrix with only two simple poles, one at $m_L - \frac{i}{2}\Gamma_L$ and the other at $m_S - \frac{i}{2}\Gamma_S$, and hence one can say that there are only two particles but four states.

The conventional parametrization can be adopted in toto with only a difference in interpretation. The neutral kaons K_0 and $\bar{K}_0$ are superposition states of particles L and S, each characterized by its mass and lifetime,

$$|K_0\rangle = \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}\,(1+\varepsilon)} \left[|L;out\rangle + |S;out\rangle \right] , \qquad (1)$$

$$|\bar{K}_0\rangle = \frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}\,(1-\varepsilon)} \left[-|L;out\rangle + |S;out\rangle \right] , \qquad (2)$$

$$\langle L;out | S;out \rangle = \frac{2\,\mathrm{Re}.\,\varepsilon}{1+|\varepsilon|^2} . \qquad (3)$$

The particle states L and S, contrary to convention, are not CP eigenstates. This naturally means that K_0 and $\bar{K}_0$ are not CP conjugates of each other if (1) and (2) are valid. The definition of the interference parameter η remains the same. The expression for η in terms of ε and isotopic spin final state amplitudes also remains valid in the usual approximation,

$$\eta_{+-} = \frac{\varepsilon + \varepsilon'(1+\omega)^{-1}}{1+\varepsilon\varepsilon'(1+\omega)^{-1}} , \quad \eta_{00} = \frac{\varepsilon - 2\varepsilon'(1-2\omega)^{-1}}{1-2\varepsilon\varepsilon'(1-2\omega)^{-1}} . \qquad (4)$$

Similarly the expression for charge asymmetry parameter δ remains unchanged. Thus the understanding of the time dependence of the decay of K_0 or of $\bar{K}_0$ is achieved through the conventional parameter but in a CP preserving context.

The crucial point in understanding regeneration is that irrespective of the initial beam being K_0 or $\bar{K}_0$ the state incident on the regenerator is one and the same, namely L (since S component would have decayed). Any CP preserving hypothesis with K_0 a CP conjugate of $\bar{K}_0$ will not achieve

this. However, with the present interpretation of (1),(2) and (3) the understanding of regeneration, with all its details, is on the same level as that with conventional CP violation hypothesis.

We present an S-matrix parametrization of the K-decay phenomenon. This is not essential to our contention that the entire existing phenomenological analysis can be adopted en masse with CP invariance. However, S-matrix parametrization goes deeper, is closer to a possible theory, clears up the relation between four states and two particles, and gives a satisfaction that it can be done.

We use the notation, but not the interpretation, of Durand and McVoy[4].

Consider an S matrix with two overlapping resonances. The Breit-Wigner representation is

$$S_{jk}(E) = B_{jk} - i\Gamma_S \frac{g_{Sj}\,h_{Sk}}{E - \xi_S} - i\Gamma_L \frac{g_{Lj}\,h_{Lk}}{E - \xi_L} , \qquad (5)$$

where

$$\xi_S = m_S - \frac{i}{2}\Gamma_S , \quad \xi_L = m_L - \frac{i}{2}\Gamma_L .$$

The demands of unitarity are

$$B^\dagger B = \mathbb{1} \qquad (6)$$

$$B h_S^* = i \frac{\Gamma_L}{\xi_S^* - E_L} (h_S^\dagger, h_L)\, g_L + (h_S^\dagger, h_S)\, g_S ,$$

$$B h_L^* = i \frac{\Gamma_S}{\xi_L^* - E_S} (h_L^\dagger, h_S)\, g_S + (h_L^\dagger, h_L)\, g_L . \qquad (7)$$

Contrary to what is often implied or even stated explicitly, CP invariance does not require a resonance to be necessarily a CP eigenstate. It only insists that the S matrix does not connect states of opposite CP. This we shall guarantee by the simple device that the eigenstates of the above S matrix will be, by definition, also CP eigenstates.

Since the resonances L and S are not CP eigenstates, with a CP preserving theory, they cannot decay into a CP eigenstate. This indicates that the quantum numbers included in the labelling (j or k) of the Breit-Wigner representation (5) may not include CP. In fact, one can make a stronger statement and say that they will not include CP if B is non-diagonal - which is the case we are interested in.

A Breit-Wigner basis vector in the Hilbert space will be denoted by $|b_j,E;\rangle$ so that the S matrix of Eq. (5) is given by

$$\langle b_j, E; out | b_k, E; in \rangle = S_{jk}(E)\,\delta(E-E')\,. \tag{8}$$

We define vectors h'_L and h'_S orthogonal to h_S and h_L respectively and use them to project out single resonances from the S matrix. We can then define resonant states:

$$|L'; out\rangle = \frac{1}{N'_L}\sum_{jk}\int dE\,|b_j, E; out\rangle\, S_{jk}(E)\, h'^{*}_{Lk}$$

$$= \frac{1}{N'_L}\sum_{jk}\int dE\,|b_j, E; out\rangle \left\{ B_{jk} - i\Gamma_L \frac{g_{Lj}\, h_{Lk}}{E-\xi_L} \right\} h'^{*}_{Lk}\,, \tag{9}$$

and

$$|S'; out\rangle = \frac{1}{N'_S}\sum_{jk}\int dE\,|b_j, E; out\rangle\, S_{jk}(E)\, h'^{*}_{Sk}$$

$$= \frac{1}{N'_S}\sum_{jk}\int dE\,|b_j, E; out\rangle \left\{ B_{jk} - i\Gamma_S \frac{g_{Sj}\, h_{Sk}}{E-\xi_S} \right\} h'^{*}_{Sk}\,. \tag{10}$$

In these states the purely scattering components arising from the B_{jk} term quickly diffuses out in a time scale much shorter than $\frac{1}{\Gamma_L}$ or $\frac{1}{\Gamma_S}$. Hence if one has isolated a system that has persisted as a coherent entity for durations of the order of $\frac{1}{\Gamma_{L,S}}$ then the corresponding state vector is obtained by dropping the B_{jk} term in the above Eqs. (9) and (10).

These resonant particle state vectors $|L;\ out\rangle$, $|S;\ out\rangle$ with sharp resonance approximation normalization

$$\langle L; out | L; out\rangle = \langle S; out | S; out \rangle = 1 \tag{11}$$

are given by

$$\left.\begin{aligned} |L; out\rangle &= \sqrt{\frac{\Gamma_L}{2\pi}}\int \frac{dE}{E-\xi_L}\sum_j |b_j, E; out\rangle\, \hat{g}_{Lj} \\ |S; out\rangle &= \sqrt{\frac{\Gamma_L}{2\pi}}\int \frac{dE}{E-\xi_S}\sum_j |b_j, E; out\rangle\, \hat{g}_{Sj} \end{aligned}\right\} \tag{12}$$

where

$$\hat{g}_{Lj}\sqrt{g_L^\dagger g_L} = g_{Lj} \quad ; \quad \hat{g}_{Sj}\sqrt{g_S^\dagger g_S} = g_{Sj}\,. \tag{13}$$

The overlap in these states (again in sharp resonance approximation) is given by:

$$\langle L; out | S; out\rangle = i\,(\hat{g}_L^\dagger\, \hat{g}_S)\,\frac{\sqrt{\Gamma_L\Gamma_S}}{\xi_L^* - \xi_S}\,, \tag{14}$$

which can now be connected to the parameter ε of Eqs. (1) and (2) through Eq. (3).

The full-time development of these state vectors, namely $e^{-iHt}|L;\ out\rangle$ and $e^{-iHt}|S;\ out\rangle$, represents at any later instant t the amplitude for the unstable particle still surviving along with the coherent amplitude for the decay fragments. The survival amplitude is given by

$$e^{-i\mathcal{H}t}|L; out\rangle_{res.} = -i\sqrt{2\pi\Gamma_L}\; e^{-i\xi_L t}\sum_j\int dE\,|b_j, E; out\rangle\,\hat{g}_{Lj}\, a(E) \tag{15}$$

and by

$$e^{-i\mathcal{H}t}|S; out\rangle_{res.} = -i\sqrt{2\pi\Gamma_S}\; e^{-i\xi_S t}\sum_j\int dE\,|b_j, E; out\rangle\,\hat{g}_{Sj}\, a(E)\,, \tag{16}$$

where $a(E)$ is a wide-support smooth function of E enabling the state to be normalized.

Since CP invariance is good, the CP eigenstates can diagonalize the S matrix,

$$\langle D_j, E; out | D_k, E; in\rangle = \delta(E-E')\,\delta_{jk}\, e^{2i\delta_k}\,, \tag{17}$$

where

$$[CP]\,|D_j; E; {}^{in}_{out}\rangle = c_j\,|D_j, E; {}^{in}_{out}\rangle\,.$$

We have thus introduced two bases: The Breit-Wigner basis and the CP diagonal basis. The former is the maximal analyticity basis which marks out through the simple pole in energy the persistent coherent entity that separates out as an unstable particle. The CP diagonal basis provides a simple construction of the decay channels. We can introduce, with profit, yet another basis, in fact yet another Breit-Wigner basis, the CP conjugate basis, $|b_j^C, E;{}^{in}_{out}\rangle$ of the original Breit-Wigner basis,

$$[CP]\,|b_j, E; {}^{in}_{out}\rangle \equiv |b_j^c, E; {}^{in}_{out}\rangle .$$

The S matrix is invariant under this basis transformation. There exist CP conjugate resonances L^C and S^C which utilize the same poles at ξ_L and ξ_S of the S-matrix but are distinct from L and S and yet are in the same Hilbert space:

$$[CP]\,|L; {}^{in}_{out}\rangle \equiv |L^c; {}^{in}_{out}\rangle , \qquad [CP]\,|S; {}^{in}_{out}\rangle \equiv |S^c; {}^{in}_{out}\rangle .$$

In view of the equalities

$$\langle b_j^c, E; out\,|\,L^c; out\rangle = \langle b_j, E; out\,|\,L; out\rangle ,$$

$$\langle b_j^c, E; out\,|\,S^c; out\rangle = \langle b_j, E; out\,|\,S; out\rangle ,$$

one obtains, from (12),

$$\langle b_j, E; out\,|\,L^c; out\rangle = \sqrt{\frac{\Gamma_L}{2\pi}}\,\frac{1}{E-\xi_L}\,\hat{g}^c_{Lj} ,$$

$$\langle b_j, E; out\,|\,S^c; out\rangle = \sqrt{\frac{\Gamma_S}{2\pi}}\,\frac{1}{E-\xi_S}\,\hat{g}^c_{Sj} , \tag{18}$$

where

$$\hat{g}^c_{Lj} = \sum_k [CP]_{jk}\,\hat{g}_{Lk} ,$$

$$\hat{g}^c_{Sj} = \sum_k [CP]_{jk}\,\hat{g}_{Sk} , \tag{19}$$

and

$$\langle b_j, E; out\,|\,[CP]\,|\,b_k, E'; out\rangle = \delta(E-E')\,[CP]_{jk} . \tag{20}$$

As remarked above, the states for the decay channels must be constructed from the $|D_j, E; out\rangle$ basis. The Breit-Wigner basis is not appropriate for this purpose. Consequently g_L and g_S do not give directly the amplitudes for decay rates. Since the properties of the decay fragments must be time localized much sharper than $\frac{1}{\Gamma}$, the decay states are constructed by extended smooth energy integrations over $|D_j, E; out\rangle$. A complete set of

such states will be denoted by $|D_j; out\rangle$. The amplitudes for the decay rate into the channel j with CP = c_j is now obtained from (15) and (16),

$$A(L\to j) = \langle D_j; out\,|\,L; out\rangle_{res.} = -i\sqrt{2\pi\Gamma_L}\,W_{Lj} ,$$

$$A(S\to j) = \langle D_j; out\,|\,S; out\rangle_{res.} = -i\sqrt{2\pi\Gamma_S}\,W_{Sj} , \tag{21}$$

so that the interference parameter is given by

$$\eta_j = \frac{W_{Lj}}{W_{Sj}} \tag{22}$$

and for the Bell-Steinberger relation (14) we can replace $\hat{g} \to W$ since

$$W_L^\dagger W_S = \hat{g}_L^\dagger g_S . \tag{23}$$

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REFERENCES

1) J. Uretsky, Phys. Letters 14, 154 (1965);
H.J. Lipkin, Phys. Rev. Letters 22, 213 (1969);
D.I. Llalovic, Phys. Rev. Letters 21, 1662 (1969);
P.K. Kabir, Phys. Rev. Letters 22, 1018 (1969);
Gyan Mohan and V.K. Agarwal, ICTP, Trieste, preprint IC/72/125 (unpublished),
V.K. Agarwal, "A CP invariant analysis of neutral K-decay", Ph.D. Thesis, I.I.T. Kanpur, unpublished.

2) P. Darriulat, J.P. Deutsch, K. Kleinknecht, C. Rubbia and K. Kittel, Phys. Letters 29B, 132 (1969);
D. Banner, J. Frank, M. Gormley, L.J. Koester, S. Raither, J.H. Smith and A. Wattenberg, Phys. Rev. D7, 1989 (1973).

3) T.T. Wu, C.N. Yang, Phys. Rev. Letters 13, 380 (1964);
J.S. Bell and J. Steinberger, in Proceedings of the International Conference on Elementary Particle Physics, Oxford (1965), p. 195.

4) L. Durand and K.W. McVoy, Phys. Rev. Letters 23, 59 (1969);
G.C. Wick, Phys. Letters 30B, 126 (1969).