IC/76/62

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1976 MIRAMARE-TRIESTE

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and
United Nations Educational Scientific and Cultural Organization
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ABSTRACT

We review the basic theory of symmetry restoration - particularly in relation to a possible vanishing of the Cabibbo angle in the presence of external electromagnetic fields. It is emphasised that the critical quantities are invariants like $\langle H^2\rangle - \langle E^2\rangle$. This one, in particular, must exceed $(10^{15}\ \text{gauss})^2$ since $\theta_c \neq 0$ for hyperons. In some nuclei, internal fields exceed this critical quantity, lending credence to the arguments by Hardy and Towner and by Watson that θ_c has made a transition to an anomalously small value for nuclei like Ar^{35} and Nb^{93}.

MIRAMARE - TRIESTE
June 1976

* Presented at the Conference on K-Meson Physics, Brookhaven National Laboratory, Upton, Long Island, NY, USA, 1-3 June 1976.

I. INTRODUCTION

The concept of SYMMETRY entered particle physics at an early date. However, the related concept of ORDER (i.e. the act of choice among sets of states with equivalent degrees of symmetry) - formalized as early as 1937 by Landau [1)] in condensed matter physics - came into prominence in particle physics only in the decade of the Sixties, under the unfortunate name "spontaneous breaking of symmetry". [2)] The importance of the third related idea - SYMMETRY RESTORATION (i.e. transition from an ordered to an unordered state in a suitable external environment) - once again recognized in condensed matter theory with the Ginzburg-Landau [3)] description of the transition of the superconductor to a normal phase at high temperatures or in a strong magnetic field - has only just begun to be appreciated in particle physics. [4),5),6)] Four types of external environment for such symmetry-restoring transitions may be envisaged: (1) high temperature, (2) high density, (3) high gravitational or (4) high $\underset{\sim}{E}$ and/or $\underset{\sim}{H}$ environments. It is the purpose of this review to describe recent work relating in particular to the restoration of Cabibbo symmetry - i.e. strangeness conservation in weak interactions.

To distinguish SYMMETRY versus ORDER (in the context used in this review), consider two potentials pictured in Fig.1, the Begging Bowl and the Dimpled Cup Potentials.

For the Begging Bowl, there is a unique minimum and it is also the maximally SYMMETRIC position at the bottom of the bowl. A bead would come to rest at this unique position. For the Dimpled Cup there is an infinity of minima, all equivalent, strung along a circle at the bottom of the dimple. A bead may come to rest anywhere along the azimuth. The act of choice which fixes on one of these infinitely many minima as describing the physical state is the establishing of ORDER. To appreciate the nuances of terminology, consider the example of an unsophisticated country, where table-manners have not been universally agreed upon, with guests crowded round a circular table, with a napkin on one side and a piece of bread on the other, in front of each guest. This is a symmetrical situation but intrinsically unstable, since each guest is eyeing his neighbour to decide which is his bread and which his napkin - the one to his right, or the one to his left. One bold spirit, at long last, makes up his (er) mind and chooses - and instantly the choice is defined for everyone on the table. A state of ORDER sets in instantaneously *) with this act of choice.**)

To understand the third idea - restoration of symmetries - let us go back to the dimpled cup potential. A hammering, administered by a genie, which converts the dimpled cup into the Begging Bowl would limit the choices along the azimuth to the unique choice at the bottom of the bowl. The state changes from non-zero to zero-ORDER or SYMMETRY. The genie who converts one

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*) The "instantaneity" in a relativistic theory would be the appearance of a massless Goldstone boson, travelling with the velocity of light and signalling the onset of ORDER.

**) A philosophical opinion on this phenomenon has been noted by Fubini [7]) in the problem of Buridan's ass, who finds himself in the centre of a field around the border of which is piled his fodder. Presented with a set of equally attractive alternatives, he loses the freedom to choose and starves.

potential (Begging Bowl) to the other (Dimpled Cup) must supply energy - this could for example be energy from an external electric or magnetic source.

In this paper we review the situation of theory and experiment relating to symmetry restoration in particle physics, using electric and magnetic fields, recapitulating the development given in Refs. 5 and 6. In Ref. 5 we followed the ideas of Ginzburg and Landau for superconductivity theory - the prototype theory of ORDER and symmetry restoration - to obtain estimates of critical (constant) magnetic field strengths which may switch off the Cabibbo angle. In Ref. 6 we emphasised the differences of the particle physics situation from that obtaining in superconductivity theory. The crucial difference lies in the fact that in superconductivity theory the ORDER function is the expectation value of the Cooper pair field. The Cooper pair is charged (two electrons bound loosely together through a phonon-interaction, with their spins anti-aligned: the pair making up a spin-zero bound state). The external magnetic field interacts directly with the Cooper pair and disrupts it - thereby destroying ORDER, and restoring the superconductor to the normal (unordered) state. In particle physics, in contrast, the order function is the expectation value of a neutral field. Thus, the magnetic and electric fields act on this uncharged substrate radiatively - i.e. through the charged virtual constituents of which this neutral field may be considered to be composed. This makes the size of the composite an important parameter in the theory since it determines how much external electromagnetic energy will be absorbed by the substrate. The chief difference from superconductivity, however, appears to be that it is not only external magnetic fields which bring about the phase transition; the transition can be caused by electric fields too, except that the two types of field act oppositely. That is to say, if magnetic fields bring about transitions of non-zero to zero ORDER, electric fields would, on the contrary, increase

ORDER, and vice versa. One of the critical quantities is the gauge and Lorentz invariant $|(F_{\mu\nu}F_{\mu\nu})_c| = |H^2 - E^2|)_c$ so that E and H act oppositely. Which field it is that acts to restore symmetry (and switch off Cabibbo angle) is, as we show, model dependent.

So much for theory. Empirically[8)-10)] it appears that possibly magnetic (or electric) fields inside some nuclei are already so strong as to switch off θ_c. We shall review this evidence. There are uncertainties, and the situation seems to call for a systematic set of experiments to measure lifetimes (and thus θ_c) for hyperons produced inside nuclei, on the one hand, and a systematic theoretical evaluation of average magnetic and electric fields obtaining therein, on the other.

Qualitative aspects of the ordered state are reviewed in Sec.II. Thermodynamic estimates of the critical field for destabilizing such a state and, in particular, for the restoration of CP and strangeness conservation are given in Sec.III. Sec.IV is devoted to a discussion in general terms of phase transitions in relativistic theories, and a simplified field-theoretic model is treated in Sec.V. The fields to be expected in medium sized nuclei are derived in Sec.VI, and anomalies in the Cabibbo factor which might indicate transition phenomena in nuclear physics are reviewed in Sec.VII. <u>The experimentally inclined reader can skip Secs. II, IV and V.</u>

II. THE ORDERED GROUND STATE

The nature of an ordered ground state is clearly apprehended in the ferromagnet idealized as an array of independently orientable dipoles. Because of the interaction between neighbouring dipoles, the energy takes its minimum value when all dipoles are oriented in the same direction. Thus, in spite of the rotational symmetry of interactions, the ground state will have a sense (revealed in its magnetization) and the rotational symmetry will be masked (or, in the unhappy phrase of the particle physicist, spontaneously broken). No particular direction is preferred, all are equivalent if no external influence is brought to bear. The spontaneously chosen direction for the ground state magnetization is not predictable: it is, so to speak, an "act of choice" on the part of the system (i.e. it is determined by some fluctuation in the history of the system). Conversely, the ordered state becomes unstable if the temperature of the system is raised. Thermal fluctuations eventually dominate the magnetic interactions and the orientations of the individual dipoles become randomly distributed. The magnetization vanishes and rotational symmetry is restored.

A second example of the ordered ground state is provided by the superconductor. The illustration is more abstract now in that the apparently lost symmetry is associated with rotations in a 2-dimensional charge space rather than in 3-dimensional physical space as above. What seems to happen is that, owing to phonon interactions, there arises a weak attraction between electrons near the Fermi surface and a proportion of them is caused to form into loosely bound Cooper pairs. These doubly-charged "bosons" drop into a Bose-Einstein phase. A phenomenological description of this state of affairs is provided by the Ginzburg-Landau theory.[11)] Here the Cooper pairs are represented by an effective scalar field or order parameter ϕ^{--}, in terms of which the free energy can be expressed as

$$F(T) \approx F_n(T) + \alpha(T)|\phi|^2 + \frac{\beta(T)}{2}|\phi|^4 + \frac{1}{2m^*}\left|(\vec{\partial} - ie^*\vec{A})\phi\right|^2 , \tag{2.1}$$

where F_n denotes the free energy density of the normal state. The parameters α, β, m^*, e^* are all in principle computable in a microscopic theory. Thus, m^* and e^* represent the effective mass and charge of the Cooper pair so that $m^* \approx 2m$ and $e^* \approx 2e$, where m and e denote the mass and charge of a single electron. The expression (2.1) is valid only for small $|\phi|$, i.e. near the symmetric value $\phi = 0$.

The mark of genius on the part of Landau and Ginzburg was the recognition that for a phase transition to occur, $\alpha(T)$ must change sign while $\beta(T)$ remains positive, i.e. $\alpha(T)$ vanishes for some critical temperature, T_c, and is <u>negative</u> for $T < T_c$. This is implied by the statement that $F(T)$ is minimized for a non-vanishing order parameter ϕ, $|\phi^{--}(T)|^2 = -\frac{\alpha(T)}{\beta(T)}$ for $\alpha(T) < 0$, and $\phi^{--}(T) = 0$ for $\alpha(T) > 0$. These two situations obtain for $T < T_c$ and $T \geqslant T_c$, respectively. Notice that the minimization of the free energy fixes only the magnitude of ϕ, not its phase. Like the direction of magnetization in the ferromagnet, the phase of the Cooper field in the ground state of the superconductor is a matter of choice.

One aspect of the existence of ORDER, such as occurs in superconductors, is the Meissner effect. Magnetic flux (below a critical value) is expelled from the body of the superconductor which behaves as a perfect diamagnet. Equivalently, and more generally, the vector forces associated with the local symmetry become of finite range in the ordered state. In superconductor

theory this range is known as the penetration depth, λ_p. A typical depth, $\lambda_p \sim 10^{-6}$ cm corresponds to the effective photon "mass" $\sim$20 eV. When the external magnetic field exceeds the critical value, the order parameter vanishes, as for the case of high temperature. (For an estimate of H_c see Sec.III.)

To compute non-zero order, in relativistic theories in which elementary scalar fields are involved, it is natural to replace the free energy (2.1) by an effective potential which is just the vacuum energy density expressed as a function of the scalar field expectation values. With this language it is a relatively simple matter to compute the effect of radiative corrections at least in lowest order. If elementary (Higgs) scalars are not present in the original Lagrnagian then the problem is altogether more difficult: it is necessary to construct effective scalars like the Cooper composite by solving a bound state type of problem.[12)]

The ordered ground state in relativistic cases can be destabilized by raising the temperature as well as by applying sufficiently strong external electromagnetic (or gravitational) fields, or by increasing the number density of fermions. We shall return to the problem of estimating critical values for the external magnetic field in the following.

We conclude this section by listing some of the ways in which an ordered vacuum might be expected to manifest itself.

(1) The breaking of strong interaction symmetries, e.g. $SU(4) \to SU(3) \to SU(2)$. If the breaking were strictly spontaneous then the theory would have to include either massless scalars (Goldstone theorem) <u>or</u> massive vectors (Higgs mechanism) corresponding to each broken symmetry. These obligatory particles are associated with the phases of the order parameters.

(2) **Chiral symmetry breaking and the acquisition of rest mass by fermions.** For chiral $SU(2) \times SU(2)$ in the σ model, for example, the nucleon interaction takes the form $g\,\overline{N}_L(\sigma + i\underline{\tau}\gamma_5\underline{\pi})N_R$ + h.c. and the nucleon mass is given by $g\langle\sigma\rangle$ in the ordered vacuum. A large concentration of nucleons may perhaps shed their mass by destabilizing the ordered vacuum locally and so causing $\langle\sigma\rangle$ to vanish. (Archimedes effect.)

(3) **Unified weak and electromagnetic interactions.** If the electromagnetic field belongs to a multiplet of vectors which realize a non-Abelian local symmetry, then the other members of these multiplets must acquire very large masses if they are to be associated with the charged and neutral currents of weak interactions. This could be brought about by a very large value (~ 300 GeV) for the order parameter representing the vacuum expectation value of some weakly interacting neutral scalar.

(4) **Violation of CP and strangeness.** These symmetries are violated weakly and it may be possible to associate this breaking with the presence of relatively small non-vanishing order parameters in off-diagonal positions in the quark mass matrix. If the greater part of the violations come from such mass terms it may be possible to switch them off by causing the relevant order parameters to vanish through the action of an external agency (such as a magnetic or electric field), without the quark or W-meson masses (which come from diagonal matrix elements) vanishing at the same time.

III. CRITICAL FIELDS: ORDER OF MAGNITUDE ESTIMATES

The purpose of this section is to sketch some order of magnitude estimates for external (uniform) magnetic fields which could destabilize an ordered ground state.

Firstly, there is the so-called "thermodynamic" critical field H_c defined by

$$F_n(T) - F_s(T) = \frac{H_c^2}{8\pi}, \tag{3.1}$$

where F_n and F_s denote the densities of free energy in the normal and superconducting states, respectively. If the Ginzburg-Landau expression (2.1) is valid, then

$$\begin{aligned} \frac{H_c^2}{8\pi} &= -\alpha\,|\langle\phi\rangle|^2 - \frac{\beta}{2}|\langle\phi\rangle|^4 \\ &= \frac{\alpha^2}{2\beta} \\ &= \frac{\beta}{2}\,|\langle\phi\rangle|^4 . \end{aligned} \tag{3.2}$$

For the models of weak and electromagnetic interactions, $\langle\phi\rangle \sim 300$ GeV. With $1(\mathrm{MeV})^2 = 1.44 \times 10^{13}$ gauss, we find

$$\begin{aligned} H_c &= \sqrt{4\pi\beta}\ |\langle\phi\rangle|^2 \\ &\sim \sqrt{4\pi\beta}\ \times 10^{24}\ \text{gauss} . \end{aligned} \tag{3.3}$$

Such numbers are far too large to be interesting in the laboratory and so we must look for situations where the relevant order parameter $\langle\phi\rangle$ is much smaller. For this we turn to the problems of CP violation and Cabibbo suppression in strangeness changing weak interactions which, in a simplified version of the theory, may be described by a complex (n,λ) mass matrix.

Suppose that the quarks acquire their masses through the spontaneous violation of a chiral type symmetry. In the Lagrangian there will be a Yukawa term,

$$g\,(\bar{n}\ \ \bar{\lambda})_L \begin{pmatrix} \phi_{nn} & \phi_{n\lambda} \\ \phi_{\lambda n} & \phi_{\lambda\lambda} \end{pmatrix} \begin{pmatrix} n \\ \lambda \end{pmatrix}_R + \text{h.c.} \tag{3.4}$$

and the quark masses will be determined by the expectation values of the scalar fields $g\langle\phi_{nn}\rangle$, etc. Thus

$$g\begin{pmatrix} \langle\phi_{nn}\rangle & \langle\phi_{n\lambda}\rangle \\ \langle\phi_{\lambda n}\rangle & \langle\phi_{\lambda\lambda}\rangle \end{pmatrix} = U_L^{-1}\begin{pmatrix} m_n & 0 \\ 0 & m_\lambda \end{pmatrix} U_R \quad , \qquad (3.5)$$

where U_L and U_R are unitary matrices

$$U_{L,R} = \begin{pmatrix} \cos\theta_{L,R} & -e^{i\delta_{L,R}}\sin\theta_{L,R} \\ e^{-i\delta_{L,R}}\sin\theta_{L,R} & \cos\theta_{L,R} \end{pmatrix} , \qquad (3.6)$$

where θ_L , θ_R are Cabibbo rotations and $\delta_{L,R}$ give a description of CP violation. In particular, the off-diagonal terms are given by

$$g\langle\phi_{n\lambda}\rangle = -m_n e^{i\delta_R}\cos\theta_L\sin\theta_R + m_\lambda e^{i\delta_L}\sin\theta_L\cos\theta_R$$

$$g\langle\phi_{\lambda n}\rangle = -m_n e^{-i\delta_L}\sin\theta_L\cos\theta_R + m_\lambda e^{-i\delta_R}\cos\theta_L\sin\theta_R \quad . \qquad (3.7)$$

For simplicity, suppose $\theta_L = \theta_R$ $(\approx 15^\circ)$ and $\delta_L = -\delta_R$ so that (3.7) reduces to

$$\mathrm{Re}\,\langle\phi_{n\lambda}\rangle = \frac{m_\lambda - m_n}{2g}\sin 2\theta\cos\delta$$

$$\mathrm{Im}\,\langle\phi_{n\lambda}\rangle = \frac{m_\lambda + m_n}{2g}\sin 2\theta\sin\delta \quad . \qquad (3.8)$$

With $g \approx 1$ and $m_\lambda - m_n \approx 175$ MeV, one obtains the real part of the order parameter, related to the Cabibbo rotation of n and λ quarks,

$$\mathrm{Re}\,\langle\phi_{n\lambda}\rangle \simeq 45 \text{ MeV} \quad . \qquad (3.9)$$

The corresponding thermodynamic critical field is given by

$$H_c(\mathrm{Re}\,\phi) \sim \sqrt{4\pi\beta}\,|\mathrm{Re}\,\langle\phi_{n\lambda}\rangle|^2$$
$$\sim \sqrt{4\pi\beta} \times 3.5 \times 10^{16}\text{ gauss.} \qquad (3.10)$$

This estimate refers to the type of field strength necessary to suppress the off-diagonal term in the quark mass matrix and, through this, the strangeness violating weak processes.

To estimate the CP restoring critical field, we take a milliweak*) type of model with $\delta_L \approx -\delta_R \approx 10^{-3}$. One also needs $m_\lambda + m_n$. There are two possibilities. Firstly, with $m_\lambda + m_n \sim 200$ MeV (light quarks) one finds

$$H_c(\mathrm{Im}\,\phi) \sim \sqrt{4\pi\beta} \times 3.5 \times 10^{10}\text{ gauss} \quad . \qquad (3.11)$$

Alternatively, with $m_\lambda + m_n \sim 4$ GeV (heavy quarks) one finds

$$H_c(\mathrm{Im}\,\phi) \sim \sqrt{4\pi\beta} \times 3.5 \times 10^{13}\text{ gauss} \quad . \qquad (3.12)$$

Objections can be raised against these estimates. Quite apart from the model dependent uncertainties embodied in coupling parameters such as β and g and mass parameters m_n and m_λ , it is arguable that the thermodynamic estimate – coupled with the Ginzburg-Landau expression for the free energy – may be misleading. This is because we are dealing with a neutral condensate, ϕ_{nn}, $\phi_{n\lambda}$, etc., rather than a charged one as in superconductors. The external magnetic field does not act directly on our condensate but only on the charged fluctuations (loop effects).

The quantity which replaces the Ginzburg-Landau expression for the free energy density in the relativistic case is the vacuum energy density or effective potential, $V(\phi, H, E,)$. Because of gauge invariance, it can depend

*) A preliminary and perhaps unrealistic model for spontaneous superweak CP breaking seems to give a value around 3×10^6 gauss for the critical field strength. This is likely to be a very gross underestimate from the experimental point of view, however.

only on the field strengths H,E (and their derivatives) in addition to the neutral fields ϕ. Because it must be a Lorentz scalar, the field strengths must appear in the combinations *) $e^2(H^2-E^2)$ and $e^4(H.E)^2$. The leading terms in the effective potential should take the form

$$V = \left(\alpha + a\left(H^2-E^2\right)\right)|\phi|^2 + \left(\frac{\beta}{2} + b\left(H^2-E^2\right)\right)|\phi|^4 + \cdots \tag{3.13}$$

where the parameter a is derived from a loop integration involving those charged particles which couple to ϕ : it measures the charged fluctuations (cf)

$$|a| \sim \frac{e^2}{M_{cf}^2}, \tag{3.14}$$

where M_{cf} is a mass which characterizes the size of the charged loop. (Likewise, $b \sim e^2/M_{cf}^4$.)

If the parameter α is negative, then the ground state value $\langle\phi\rangle$ will be non-vanishing provided H^2-E^2 is small enough. To destabilize this ordered state it is clearly necessary to raise the value of $|H^2-E^2|$ until the coefficient of ϕ^2 vanishes in (3.13), i.e.

$$(H^2 - E^2)_c = -\frac{\alpha}{a} \tag{3.15}$$

and this must be positive. (In this estimate we have neglected b .) On comparing with (3.2), this can be expressed in the form

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*) Since $\vec{H}.\vec{E}$ is a pseudoscalar it could appear only among the parity violating terms which are presumably proportional to the Fermi coupling G_F, and thus give a relatively small contribution to the effective potential.

$$|H^2-E^2|_c \approx \frac{1}{4\pi}\left(\frac{M_{cf}}{e\langle\phi\rangle_0}\right)^2 \times \text{(Ginzburg-Landau estimate)} \tag{3.16}$$

where $\langle\phi\rangle_0$ minimizes V when $\underline{E} = \underline{H} = 0$ and equals $\sqrt{-\alpha/\beta}$. For the off-diagonal cases considered above it is likely that $e\langle\phi\rangle_0 < M_{cf}$ ($\sim m_\pi$) and so the previous estimates (3.10)-(3.12) of critical fields may need enhancing.

An interesting question arises now as to whether an ordered state can be destabilized by the action of an <u>electric</u> field. Since the ground state is supposed to be relativistically invariant, it follows that E and H must appear in the combination, H^2-E^2, and so, if the parameter a in (3.13) should turn out to be negative, then a magnetic field would act to increase the stability of the ordered state while an electric field would destabilize.*) In Sec.V it will be shown in a model calculation that the parameter a in (3.13) could indeed be negative.

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*) In superconductors where the condensate is charged there are of course good physical reasons why electric fields do not produce critical effects. An electric field accelerates the Cooper pairs, without breaking it up, while the magnetic field acting on the oppositely aligned spins of the two electrons forming the pair tends to disrupt it.

IV. FIELD-THEORETIC MODELS *

A. Effective potentials

The treatment of phase transitions in relativistic theories is in one respect simpler than in condensed matter physics. This is because the ground state, or vacuum, is Lorentz invariant, i.e. it has a greater degree of symmetry than obtains in non-relativistic situations. Relativistic theories are usually defined by a Lagrangian density which is a local scalar quantity, being made out of scalar, spinor and vector fields (at least for renormalizable models, otherwise higher spin fields could be used). The scalar (Higgs) fields are optional. If they are included as elementary fields then the theory will contain more parameters but perturbation computations are relatively straightforward. On the other hand, one may attempt to obtain the scalar fields as composites of spinors and to compute their mass and coupling parameters in terms of more "fundamental" quantities.[12] This more ambitious programme *) has yet to be realized and we shall proceed with the standard renormalizable gauge models. These contain the following fields:

1) Scalars $\phi_i(x)$. There can be a number of these, both charged and neutral, belonging to some representation of the internal symmetry group. (Some of these scalar fields may be associated with gauge degrees of freedom and so do not correspond to physical excitations. The rest will be associated with spin-zero massive particles.) Their static interactions are governed by a (classical) potential function $V^{(0)}(\phi)$ whose minimum point $\langle\phi_i\rangle$ determines the residual symmetry, i.e. the symmetry which remains in the ordered vacuum state. The excitation masses are given by the eigenvalues of the matrix of second derivatives, $\partial^2 V/\partial\phi_i\partial\phi_j$, evaluated at the minimum.

2) Fermions $\psi(x)$. These fields must belong to some representation of the gauge symmetry. Their mass terms are given by $\bar{\psi}(M + g_i\langle\phi_i\rangle)\psi$, where the types of Yukawa couplings g_i and masses M are restricted by the gauge symmetry.

3) Vectors $V_\mu(x)$. The gauge vectors are necessary for the construction of invariant kinetic terms in the Lagrangian. Their self-interactions and couplings to the scalars and spinors are governed entirely by the gauge (minimal coupling) principle.

*) In condensed matter physics the analogous type of model is the BCS theory of superconductivity where the Cooper pairs are scalar composites made out of weakly interacting electrons.

Using standard field-theoretic methods one can compute a modified potential, $V(\phi)$, which incorporates the quantum corrections to the classical term,

$$V(\phi) = V^{(0)}(\phi) + \hbar V^{(1)}(\phi) + \hbar^2 V^{(2)}(\phi) + \cdots \tag{4.1}$$

One can go further and compute quantum corrections to the Lagrangian as a whole. Of particular interest to us are the terms which contain the electromagnetic term $(-1/4)\, F_{\mu\nu} F_{\mu\nu}$. These corrections can be taken into account by adding to V_{eff} the terms

$$\frac{1}{4} F_{\mu\nu} F_{\mu\nu} \left(Z_3(\phi) - 1 \right) = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \left(\hbar Z_3^{(1)}(\phi) + \hbar^2 Z_3^{(2)}(\phi) + \cdots \right) . \tag{4.2}$$

The first-order corrections $V^{(1)}$ and $Z_3^{(1)}$ can be expressed quite generally in terms of the masses $M_0(\phi)$, $M_{1/2}(\phi)$ and $M_1(\phi)$ of the physical excitations *) of spins 0, 1/2, and 1, respectively. Thus

$$V^{(1)}(\phi) = \frac{1}{64\pi^2}\sum\left[M_0^4 \ln M_0^2 - 2M_{1/2}^4 \ln M_{1/2}^2 + 3M_1^4 \ln M_1^2\right], \tag{4.3}$$

where each particle and antiparticle (where distinct) must be included in the sum. By actual computation, we also find that

$$Z_3^{(1)}(\phi) = \frac{e^2}{48\pi^2}\sum\left[-\ln M_0^2 - 4\ln M_{1/2}^2 + 21 \ln M_1^2\right], \tag{4.4}$$

where the sum includes the charged particles (but not their antiparticles). Notice here that the vector contribution has the opposite sign to the scalar and spinor contributions.**)

We are interested now in the "effective" potential,

$$\begin{aligned} V_{eff}(\phi) &= V(\phi) + \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}\left(Z_3(\phi)-1\right) \\ &= V^{(0)}(\phi) + \hbar V^{(1)}(\phi) + \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}\,\hbar Z_3^{(1)}(\phi) + \cdots \end{aligned} \tag{4.5}$$

*) In the unitary gauge. Other gauges can bring in unphysical excitations as well. Structures like V_{eff} and Z_3 are not generally gauge independent.

**) Note that higher orders for the effective potential $V^{(n)}(\phi)$ have the typical form $M^4(\log M^2)^n$ and $Z_3^{(n)}(\phi) \approx (\log M^2)^n$.

where the electromagnetic invariant

$$\mathcal{F} \equiv \frac{1}{2}F_{\mu\nu}F_{\mu\nu} = H^2 - E^2 \tag{4.6}$$

is regarded as fixed by the external environment. The minimum of $V_{eff}(\phi)$ will depend on the value of $\mathcal{F}$ and it can happen that the symmetry type of this quantity, $\langle\phi_i\rangle_{\mathcal{F}}$, will vary discontinuously with $\mathcal{F}$, corresponding to a phase transition.

Suppose, for example, that the gauge symmetry is SU(2) and the scalars comprise a real singlet $\underset{\sim}{\phi}$ and a real triplet χ . In addition let there be a discrete (reflection) symmetry under which $\underset{\sim}{\phi}$ is even and χ is odd. The potential V_{eff} is therefore a function of $\underset{\sim}{\phi}^2$ and χ^2 . Its minimum could fall anywhere in the plane of $\underset{\sim}{\phi}^2$ and χ^2 . Now, these quantities cannot be negative and so only the quadrant $\underset{\sim}{\phi}^2 \geqslant 0,\ \chi^2 \geqslant 0$ is accessible. Hence the minimum allowable value of V_{eff} must lie in one of four domains:

1) $\underset{\sim}{\phi}^2 > 0$, $\chi^2 > 0$
2) $\underset{\sim}{\phi}^2 > 0$, $\chi^2 = 0$
3) $\underset{\sim}{\phi}^2 = 0$, $\chi^2 > 0$
4) $\underset{\sim}{\phi}^2 = 0$, $\chi^2 = 0$.

In the first two phases the continuous symmetry is reduced from SU(2) to U(1). In the latter two it remains SU(2). The reflection symmetry is preserved in phases 2) and 4) but broken in phases 1) and 3). Suppose the system resides in phase 1) when $\mathcal{F} = 0$. As $\mathcal{F}$ is increased the minimum will trace a trajectory like that illustrated in Fig.2. The point A designates the minimum for $\mathcal{F} = 0$. As $\mathcal{F}$ increases the minimum moves down to the point B, which it reaches when $\mathcal{F} = \mathcal{F}_{c_1}$. At this point there is a phase transition 1) → 2). As $\mathcal{F}$ increases beyond $\mathcal{F}_{c_1}$, the absolute minimum proceeds to negative values of χ^2. This means that the accessible minimum moves along the line, $\chi^2 = 0$, from B towards the origin, O. It reaches the origin when the absolute minimum reaches the point C corresponding to some value $\mathcal{F} = \mathcal{F}_{c_2} > \mathcal{F}_{c_1}$. At this point there is a phase transition 2) $\rightarrow$ 4). The new phase persists for $\mathcal{F} > \mathcal{F}_{c_2}$. In the other direction we might have decreased $\mathcal{F}$ through negative values until at $\mathcal{F} = \mathcal{F}_{c_3} < 0$ the minimum meets the χ^2 axis at the point D corresponding to the transition 1) $\rightarrow$ 3). As $\mathcal{F}$ is further decreased towards the value $\mathcal{F} = \mathcal{F}_{c_4} < \mathcal{F}_{c_3}$, the accessible minimum moves down the axis from D towards the origin at which there is a transition 3) $\rightarrow$ 4).

B) Uncertainties in the computation

The scenario described above is somewhat idealized in that we have neglected to mention some features which may be significant. Firstly, in the neighbourhood of the transition points B and D one or other of the scalar masses goes to zero and this could give rise to infra-red effects which may invalidate the perturbation expansion. Secondly, the retaining of only a linear term in $\mathcal{F}$ in the effective potential may be a bad approximation if the critical values $|\mathcal{F}_{c_1}|^{1/4}, |\mathcal{F}_{c_2}|^{1/2}$, etc., turn out to be comparable to typical particle masses (squared). (Presumably such a shortcoming could be rectified by including higher powers of $\mathcal{F}$ in V_{eff}.) Thirdly, and perhaps most importantly, the neglect of gradient terms in the effective Lagrangian may be unjustified.

Another source of unreliability in the critical field estimate is the gauge dependence of the effective potential. We have used unitary gauge here because it is the simplest and poses no problems with spurious zero-mass scalars in the computation of effective potentials. To our knowledge these problems have not been resolved in any other except the

unitary gauge. Kirzhnits and Linde[13], on the other hand, have argued that the unitary gauge should not be trusted in perturbative computations. We do not believe that this kind of ambiguity could affect the order of magnitude of the result.*)

C) Space-dependence of the order parameter

In the Ginzburg-Landau theory of superconductors the scalar field kinetic terms are retained and, in some situations, they play a decisive role. There is the parameter $\sqrt{2}\,\kappa$ ($= M^2_{scalar}/M^2_{vector}$), which is larger than unity in Type II superconductors. In such cases the kinetic term**) is important and the superconducting state becomes unstable against the formation of vortices (the surface energy associated with a boundary between

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*) A remark which is relevant to the controversy over which gauge to work in for practical computations is the following. In superconductivity theory the expectation value of the charged Cooper pair field is a slowly varying function of space. A constant value (over all space) would imply the breakdown of charge conservation (and gauge invariance) of the theory. As is well known, in the simple versions of Ginzburg-Landau theory, where as an approximation the order parameter is set equal to a constant, one must check that the superficial violation of gauge invariance does not affect the estimates of physically measured quantities unduly. This has been confirmed by numerous investigations[14].

**) In a typical nuclear environment the electromagnetic term $\mathcal{F}$ can vary considerably over distances of order m_π^{-1}. Since the underlying virtual structure on which the effective Lagrangian is based may involve charged particles as light as m_π, it may not be permissible to neglect terms like $(\partial_\lambda F_{\mu\nu})^2$ in the effective Lagrangian. The same is true of the scalar field derivatives, $Z_2(\phi)\,(\partial_\mu\phi)^2$, etc.

normal and superconducting regions is negative) if the impressed magnetic field exceeds a critical-value H_{c_1} (see Fig.3). In the figure there are three distinct regions in the κ-H plane. In region I the Meissner effect is complete and magnetic flux is excluded. In region II flux vortices penetrate the superconductor. Their number is proportional to the excess $H-H_{c_1}$. When $H \to H_{c_2}$ the density of vortices increases until at $H = H_{c_2}$ the superconducting remainder is squeezed to nothing. In region N the normal state prevails.

Analogous phenomena may occur in particle physics.*) If the kinetic terms are not negligible, then zones could occur inside the nucleus - or inside a nucleon - over which the scalar fields vary significantly. Such variations would manifest themselves through anomalous behaviour in, for example, the Cabibbo angle although, at present, it is not clear what parameter should play the role of κ in considerations involving "off-diagonal" mass terms.

Regardless of the Type II subtleties it is clear that space dependence of the order parameters must be significant in the nuclear domain owing to the space dependence of $\mathcal{F}$ referred to above. To illustrate how this might be taken into account, consider the model-effective Lagrangian

$$\mathcal{L} = \frac{1}{2} Z_2(\varphi)(\partial_\mu \varphi)^2 - \frac{1}{4} Z_3(\varphi) F_{\mu\nu}^2 - V(\varphi) \qquad (4.7)$$

where φ is a neutral scalar. The equations of motion are

*) The concept of the dual string as a vortex in a superconducting vacuum was proposed by Nielsen and Olesen [15] and further developed by Nambu who interpreted the universal Regge slope in terms of the scalar and vector masses.

$$\partial_\mu (Z_3 F_{\mu\nu}) = 4\pi J_\nu$$

$$\partial_\mu \left(Z_2 \partial_\mu \varphi \right) + \frac{1}{4} \frac{dZ_3}{d\varphi} F_{\mu\nu}^2 + \frac{dV}{d\varphi} = 0 , \qquad (4.8)$$

where J_ν is an external current. For practical purposes such equations are rather intractable. It is therefore worthwhile to set up a Hamilton functional whose terms are positive. One finds

$$\mathcal{H} = \int d_3 x \left[\frac{1}{2} \left(\frac{\Pi_\varphi^2}{Z_2} + Z_2 (\vec{\partial}\varphi)^2 \right) + V(\varphi) + \frac{1}{2} \left(\frac{\vec{\Pi}^2}{Z_3} + Z_3 (\vec{\partial} \times \vec{A})^2 \right) \right] . \qquad (4.9)$$

where Π_φ and $\vec{\Pi}$ are momenta canonically conjugate to φ and the vector potential $\vec{A}$, respectively. The 3-vector $\vec{\Pi}$ is actually the electric displacement and satisfies the constraint

$$\operatorname{div} \vec{\Pi} = 4\pi J_0 \qquad (4.10)$$

<u>independently of φ</u>. In a search for static (soliton) solutions we can set the transverse components of $\vec{\Pi}$ equal to zero along with Π_φ. For example, consider the spherically symmetric solutions corresponding to a uniform distribution of charge

$$J_0 = \frac{Q}{\frac{4}{3}\pi R^3} \Theta(R - r) ,$$

where Q is the total charge of the nucleus.

Taking $H_i = (x_i/r)\,\bar{H}(r)$ one finds (see Fig.4)

$$\bar{\Pi}(r) = \begin{cases} \dfrac{Qr}{R^3}\,, & 0 \le r \le R \\[2mm] \dfrac{Q}{r^2}\,, & R \le r < \infty \end{cases}$$

With H_φ, Π_i^{trans}, A_i set equal to zero the energy reduces to

$$\mathcal{H} = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2} Z_2^{-1} \left(\frac{d\varphi}{dr} \right)^2 + V(\varphi) + \frac{1}{2} Z_3^{-1} \bar{\Pi}^2 \right] \qquad (4.11)$$

In a reasonable theory Z_2 and Z_3 will be positive and V will have an absolute minimum at some point φ_0 such that $V(\varphi_0) = 0$. With $\bar{H}(r)$ given as in Fig.4 we may expect the order parameter $\varphi(r)$ to look something like Fig.5, although the detailed shape would be difficult to obtain. The case of small Q could be solved more fully by expanding about $\varphi = \varphi_0$ and treating the deviation as a small quantity. For small Q, the order parameter $\varphi(r)$ is essentially a constant, as has been assumed in the rest of this review.*)

*) It is noteworthy that Type II superconductors can display a variety of critical fields H_c, H_{c_1}, H_{c_2}, H_{c_3} and a corresponding variety of physical (vorticity) characteristics. For V_3Ga, for example, there are three widely different critical fields, H_{c_1} $(T = 0) \sim 200$ gauss, H_c $(T = 0) \sim 6{,}000$ gauss, and H_{c_2} $(T \sim 0) \sim 300{,}000$ gauss. A similar situation may prevail in particle physics with a number of critical fields differing from each other by many orders of magnitude.

To highlight the uncertainties in the computation of the magnitudes of critical quantities, another instructive – though, for theoreticians, sad – example is that of phase transitions in He^3 where the computed and measured critical temperatures differ by factors of 10^3.

V. A FIELD-THEORETIC EXAMPLE

In this section we construct a model to illustrate the application of field theoretic techniques and to show that transitions can be brought about by either magnetic or electric fields, depending on the parameters of the model.

Consider the SU(2) symmetric system of fields comprising two real scalar multiplets, a singlet χ and a triplet $\boldsymbol{\phi}$; two fermionic doublets, F_1 and F_2 , and the vector triplet $\mathbf{W}_\mu$. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} \mathbf{W}_{\mu\nu}^2 + \frac{1}{2} (\nabla_\mu \boldsymbol{\phi})^2 + \frac{1}{2} (\partial_\mu \chi)^2 - V(\boldsymbol{\phi}, \chi)$$
$$+ \bar{F}_1 (i \nabla\!\!\!/ - m_1) F_1 + \bar{F}_2 (i \nabla\!\!\!/ - m_2) F_2 + h\chi (\bar{F}_1 F_2 + \bar{F}_2 F_1)\,, \qquad (5.1)$$

where the potential is given by

$$V = -\frac{\mu^2}{2} \chi^2 - \frac{\kappa^2}{2} \boldsymbol{\phi}^2 + \frac{\lambda_1}{4} (\boldsymbol{\phi}^2)^2 + \frac{\lambda_2}{4} \chi^4 + \frac{\lambda_3}{2} \boldsymbol{\phi}^2 \chi^2 \qquad (5.2)$$

and the covariant derivatives are defined in the usual way,

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + e\, \mathbf{W}_\mu \times \mathbf{W}_\nu$$
$$\nabla_\mu \boldsymbol{\phi} = \partial_\mu \boldsymbol{\phi} + e\, \mathbf{W}_\mu \times \boldsymbol{\phi}$$
$$\nabla_\mu F_{1,2} = \partial_\mu F_{1,2} - i e\, \mathbf{W}_\mu \cdot \frac{\boldsymbol{\tau}}{2} F_{1,2}\,. \qquad (5.3)$$

Other couplings are forbidden by the discrete symmetries,

$$\mathcal{D}_1 : \quad F_1 \to F_1\,, \quad F_2 \to F_2\,, \quad \boldsymbol{\phi} \to -\boldsymbol{\phi}\,, \quad \chi \to -\chi$$
$$\mathcal{D}_2 : \quad F_1 \to F_1\,, \quad F_2 \to -F_2\,, \quad \boldsymbol{\phi} \to \boldsymbol{\phi}\,, \quad \chi \to -\chi\,. \qquad (5.4)$$

These somewhat artificial symmetries are brought in to restrict the fermion couplings (and, of course, their renormalizations). Thus, for example, there is no mass-like term, $m_{12}\overline{F}_1F_2$, because of the symmetry D_2. To the extent that the coupling $h\chi\overline{F}_1F_2$ can be ignored, the numbers $N(F_1)$ and $N(F_2)$ of the two fermion types will be separately conserved. This would be true to zeroth order in the Yukawa coupling, h, provided that the ground state expectation value $\langle\chi\rangle$ could be shown to vanish in zeroth order. If the coupling h is small then the question as to whether or not $\langle\chi\rangle$ vanishes becomes of some interest.*) We shall therefore estimate the strength of external fields needed to force the phase transition $\langle\chi\rangle \to 0$.

There is an analogue of the electromagnetic field in this simple model: the local SU(2) symmetry can be made to break spontaneously to local U(1), i.e. two of the vectors acquire mass while the third does not. This is determined by minimizing the classical potential (5.2) in order to fix the zeroth order approximation,

$$\frac{\partial V}{\partial \chi} = \chi\left(-\mu^2 + \lambda_2\chi^2 + \lambda_3\underset{\sim}{\phi}^2\right) = 0$$
$$\frac{\partial V}{\partial \underset{\sim}{\phi}} = \underset{\sim}{\phi}\left(-\kappa^2 + \lambda_3\chi^2 + \lambda_1\underset{\sim}{\phi}^2\right) = 0 \quad . \tag{5.5}$$

We shall assume that the (renormalized) parameters have been chosen such that the global minimum occurs for both $\underset{\sim}{\phi}^2$ and χ^2 non-vanishing. The zeroth order solution can be taken in the form

$$\langle\phi_i\rangle = \varphi_0\,\delta_{i3} \quad , \quad \langle\chi\rangle = \chi_0 \tag{5.6}$$

*) The conservation of the two fermion types is intended here as a simple analogue of strangeness conservation in realistic theories. Thus, comparing with (3.8), $h\langle\chi\rangle \sim g\langle\phi_{\lambda n}\rangle = (m_\lambda - m_n)\sin 2\theta$.

with real φ_0 and χ_0 given by

$$\varphi_0^2 = \frac{-\lambda_3\mu^2 + \lambda_2\kappa^2}{\lambda_1\lambda_2 - \lambda_3^2}$$
$$\chi_0^2 = \frac{\lambda_1\mu^2 - \lambda_3\kappa^2}{\lambda_1\lambda_2 - \lambda_3^2} \quad . \tag{5.7}$$

Correspondingly, the zeroth order scalar masses are obtained by diagonalizing the matrix of second derivatives of V. The charged scalar is massless while the two neutral scalars have masses

$$M_1^2(\varphi^2,\chi^2) = \lambda_1\varphi^2 + \lambda_2\chi^2 + \sqrt{(\lambda_1\varphi^2 - \lambda_2\chi^2)^2 + 4\lambda_3^2\varphi^2\chi^2}$$
$$M_2^2(\varphi^2,\chi^2) = \lambda_1\varphi^2 + \lambda_2\chi^2 - \sqrt{(\lambda_1\varphi^2 - \lambda_2\chi^2)^2 + 4\lambda_3^2\varphi^2\chi^2} \tag{5.8}$$

(evaluated at $\varphi^2 = \varphi_0^2$, $\chi^2 = \chi_0^2$). Notice that M_2 vanishes in the limit $\chi \to 0$. We shall assume that χ_0 is small relative to other masses in the system so that

$$M_2^2 \sim \frac{2}{\lambda_1}(\lambda_1\lambda_2 - \lambda_3^2)\chi_0^2 + O(\chi_0^4) \quad . \tag{5.9}$$

In unitary gauge the charged scalar $\phi^+ = (1/\sqrt{2})(\phi_1 - i\phi_2)$ is absorbed by a gauge transformation - its absence being compensated for by a longitudinal vector mode. The charged vector $W^+ = (1/\sqrt{2})(W_1 - iW_2)$ acquires the zeroth order mass

$$M_W^2(\varphi^2) = e^2\varphi^2 \tag{5.10}$$

(evaluated at $\varphi^2 = \varphi_0^2$). The third vector W^3 remains massless (in all orders) and plays the role of the photon field.

The spectrum of charged boson states, scalar and vector, is gauge dependent. Only physical states appear in the unitary gauge, and in some others such as the Coulomb and axial gauges. In yet other gauges, such as the Feynman or Landau gauges, there appear <u>massless</u> charged particles in intermediate states. These have no physical significance and cannot appear in the asymptotic states. However, they can disrupt approximation schemes for the computation of gauge dependent quantities like the effective potential, and for this reason we believe these gauges should be eschewed in problems involving an external magnetic field where they manifest spurious infra-red divergences.

The fermion mass matrix is

$$\begin{pmatrix} m_1 & h\chi_0 \\ h\chi_0 & m_2 \end{pmatrix} \qquad (5.11)$$

in zeroth order and has the eigenvalues

$$M_\pm = \frac{m_1 + m_2}{2} \pm \sqrt{\left(\frac{m_1 - m_2}{2}\right)^2 + h^2\chi_0^2} \qquad (5.12)$$

The transition $\chi_0 \to 0$ will be marked by the appearance of a new approximately conserved quantum number, $N(F_1)-N(F_2)$, the difference between numbers of fermions of types 1 and 2.

To summarize, in zeroth order we have the following physical (i.e. unitary gauge) states:

(1) Vector: W_μ^+, 3 states with charge +1 and mass $e\varphi_0$,
W_μ^-, 3 states with charge −1 and mass $e\varphi_0$,
W_μ^3, 2 states with charge 0 and mass 0.

(For W_μ^3 it is necessary to choose the gauge differently since the symmetry is not broken. Depending on the choice, there may be unphysical states associated with W_μ^3.)

(2) Spinor:

$F_1 \cos\beta + F_2 \sin\beta$, 2 states with charge $+\frac{1}{2}$ and fermion number +1,
2 states with charge $-\frac{1}{2}$ and fermion number +1.

These four states have mass

$$\frac{m_1+m_2}{2} + \sqrt{\left(\frac{m_1-m_2}{2}\right)^2 + h^2\chi_0^2}\,.$$

Another four states with the same quantum numbers are associated with the orthogonal combination $-F_1 \sin\beta + F_2 \cos\beta$. They have the mass

$$\frac{m_1+m_2}{2} - \sqrt{\left(\frac{m_1-m_2}{2}\right)^2 + h^2\chi_0^2}\,.$$

Another eight states are comprised of the antiparticles of these. The angle β appears in the diagonalization of (5.11),

$$\tan 2\beta = \frac{2h\chi_0}{m_1 - m_2}\,.$$

(3) Scalar:

Two neutral states with mass M_1 and M_2 given by (5.8).

In order to estimate the critical magnitude for an external field at which the expectation value $\langle\chi\rangle$ is forced to vanish, we shall examine the one-loop contributions to the effective potential. The leading terms in the potential considered as a function of the neutral scalars, χ, $\varphi(\equiv\phi_3)$ and the electromagnetic invariant, $\mathcal{F} = H^2-E^2$, are given by

$$V = V^{(0)} + \hbar V^{(1)} + \tfrac{1}{2}\hbar\,\mathcal{F}\,Z_3^{(1)} + \cdots \qquad (5.13)$$

where $V^{(0)}$ denotes the zeroth order part,

$$V^{(0)} = -\frac{\mu^2}{2}\chi^2 - \frac{\kappa^2}{2}\varphi^2 + \frac{\lambda_1}{4}\varphi^4 + \frac{\lambda_2}{4}\chi^4 + \frac{\lambda_3}{2}\varphi^2\chi^2 \qquad (5.14)$$

and $V^{(1)}$ is the $\mathcal{F}$-independent part of the one-loop contribution,

$$V^{(1)} = \frac{1}{64\pi^2}\sum_{jn}(-)^{2j}(2j+1)\,M_{jn}^4 \ln M_{jn}^2 =$$

$$= \frac{1}{64\pi^2}\Bigg[\; 6\,(e^2\varphi^2)^2 \ln (e^2\varphi^2)$$

$$- 8 \left(\frac{m_1+m_2}{2} + \sqrt{\left(\frac{m_1-m_2}{2}\right)^2 + h^2\chi^2}\right)^4 \ln (\;)^2$$

$$- 8 \left(\frac{m_1+m_2}{2} - \sqrt{\left(\frac{m_1-m_2}{2}\right)^2 + h^2\chi^2}\right)^4 \ln (\;)^2$$

$$+ \left(\lambda_1\varphi^2+\lambda_2\chi^2 + \sqrt{(\lambda_1\varphi^2+\lambda_2\chi^2)^2+4\lambda_3^2\varphi^2\chi^2}\right)^2 \ln (\;)$$

$$+ \left(\lambda_1\varphi^2+\lambda_2\chi^2 - \sqrt{(\lambda_1\varphi^2+\lambda_2\chi^2)^2+4\lambda_3^2\varphi^2\chi^2}\right)^2 \ln (\;)\Bigg] . \tag{5.15}$$

(The arguments of the logarithms are the same as the expressions appearing in front of them.) The scales in the various logarithms can be set independently for the vector, spinor and scalar contributions. The first $\mathcal{F}$-containing term in the expansion (in powers of $\mathcal{F}$) of the one-loop potential is

$$\frac{1}{2}\mathcal{F} Z_3^{(1)} = \frac{e^2\mathcal{F}}{96\pi^2}\sum_n \left(21 \ln M_{n1}^2 - 4 \ln M_{n\chi}^2 - \ln M_{n0}^2 \right)$$

$$= \frac{e^2\mathcal{F}}{96\pi^2}\left[21 \ln (e^2\varphi^2) - 8 \ln \left(m_1 m_2 - h^2\chi^2\right)\right] , \tag{5.16}$$

where the sum is restricted to charge-carrying particles and, in this model, the fermionic contributions are suppressed relative to the vector by the factor 1/4 because they carry only a half-unit of charge. Again the scales are arbitrary.

We started with the assumption that χ_0 is small but not zero. This means that the classical potential $V^{(0)}$, viewed as a function of φ^2 and χ^2 has a minimum just above the axis $\chi^2 = 0$. The modifications (5.15) and (5.16) will cause this minimum to be displaced by a small amount. What we wish to show is that the effect of $\mathcal{F}Z^{(1)}$ is to shift the minimum to a smaller value of χ^2. To the extent that lowest order perturbation

calculations can be trusted, we obtain an estimate of the critical field $\mathcal{F}_c$ as the value which causes the minimum to be displaced to $\chi^2 = 0$.

To find the displaced minimum, it is sufficient to solve the linearized equations

$$0 = -\mu^2 + \lambda_2\chi^2 + \lambda_3\varphi^2 + 2\left(\frac{\partial V^{(1)}}{\partial\chi^2} + \mathcal{F}\frac{\partial Z^{(1)}}{\partial\chi^2}\right)$$

$$0 = -\kappa^2 + \lambda_3\chi^2 + \lambda_1\varphi^2 + 2\left(\frac{\partial V^{(1)}}{\partial\varphi^2} + \mathcal{F}\frac{\partial Z^{(1)}}{\partial\varphi^2}\right) , \tag{5.17}$$

where the derivatives are evaluated at the zeroth order minimum, $\varphi^2 = \varphi_0^2$, $\chi^2 = \chi_0^2$. Solving for χ^2, one finds

$$\chi^2 = (\lambda_1\lambda_2 - \lambda_3^2)^{-1}\left[\lambda_1\left(\mu^2 - 2\frac{\partial V^{(1)}}{\partial\chi^2} - 2\mathcal{F}\frac{\partial Z^{(1)}}{\partial\chi^2}\right) - \lambda_3\left(\kappa^2 - 2\frac{\partial V^{(1)}}{\partial\varphi^2} - 2\mathcal{F}\frac{\partial Z^{(1)}}{\partial\varphi^2}\right)\right] . \tag{5.18}$$

The zero field corrections $V^{(1)}/\partial\chi^2$ and $V^{(1)}/\partial\varphi^2$ are not very interesting: they can be absorbed in a redefinition of the parameters μ^2 and κ^2. Hence on setting the displaced χ^2 equal to zero one obtains

$$\mathcal{F}_c\left(2\lambda_1\frac{\partial Z^{(1)}}{\partial\chi^2} - 2\lambda_3\frac{\partial Z^{(1)}}{\partial\varphi^2}\right) = \lambda_1\mu^2 - \lambda_3\kappa^2 = (\lambda_1\lambda_2 - \lambda_3^2)\,\chi_0^2 . \tag{5.19}$$

From the explicit form of $Z^{(1)}$ given by (5.16), therefore,

$$\frac{e^2 \mathcal{F}_c}{96\pi^2}\left(\lambda_1 \frac{16 h^2}{M_{F_1}M_{F_2}} - \lambda_3 \frac{42 e^2}{M_W^2}\right) = (\lambda_1\lambda_2-\lambda_3^2)\,\chi_0^2 \qquad (5.20)$$

If the fermions are much lighter than the vector particle then, approximately,

$$\mathcal{F}_c \sim \frac{6\pi^2}{e^2 h^2}\,\frac{\lambda_1\lambda_2-\lambda_3^2}{\lambda_1}\,M_{F_1}M_{F_2}\,\chi_0^2 \,. \qquad (5.21)$$

A small generalization of the model consists in the introduction of a triplet of scalars, $\underset{\sim}{\pi}$. The charged members could then play a dominant role in determining the critical field. The new terms in the zeroth order potential take the form *)

$$V^\pi = \frac{m^2}{2}\underset{\sim}{\pi}^2 + \frac{\lambda_4}{4}(\underset{\sim}{\pi}^2)^2 + \frac{\lambda_5}{2}\underset{\sim}{\pi}^2\underset{\sim}{\phi}^2 + \frac{\lambda_6}{2}(\underset{\sim}{\pi}\cdot\underset{\sim}{\phi})^2 + \frac{\lambda_7}{2}\underset{\sim}{\pi}^2\chi^2 \qquad (5.22)$$

and we shall assume that the minimum occurs at $\underset{\sim}{\pi} = 0$, i.e. that the matrix

$$\left\langle \frac{\partial^2 V}{\partial\pi_i\,\partial\pi_j}\right\rangle = \delta_{ij}\left(m^2+\lambda_5\varphi_0^2+\lambda_7\chi_0^2\right) + \lambda_6\varphi_0^2\,\delta_{i3}\delta_{j3} \qquad (5.23)$$

is positive definite. The masses of the new states are given by

$$M_{\pi^+}^2 = m^2 + \lambda_5\varphi_0^2 + \lambda_7\chi_0^2$$
$$M_{\pi^0}^2 = m^2 + (\lambda_5+\lambda_6)\varphi_0^2 + \lambda_7\chi_0^2 \qquad (5.24)$$

*) Odd terms like $(\underset{\sim}{\pi}\cdot\underset{\sim}{\phi})\underset{\sim}{\phi}^2$ or $\underset{\sim}{\pi}\cdot\phi\chi^2$, which would give rise to zeroth order mixing, and thereby complicate the problem, may be excluded by means of a discrete symmetry, $\underset{\sim}{\pi} \to -\underset{\sim}{\pi}$.

and they make the following contributions to the correction terms (5.15) and (5.16):

$$V^{\omega\pi} = \frac{1}{64\pi^2}\Big[\,2\left(m^2+\lambda_5\varphi^2+\lambda_7\chi^2\right)^2 \ln(\quad) + \left(m^2+(\lambda_5+\lambda_6)\varphi^2+\lambda_7\chi^2\right)^2\ln(\quad)\Big] \qquad (5.15')$$

$$\tfrac{1}{2}\mathcal{F}\,Z^{\omega\pi} = \frac{e^2\mathcal{F}}{96\pi^2}\Big[-\ln\left(m^2+\lambda_5\varphi^2+\lambda_7\chi^2\right)\Big] \,. \qquad (5.16')$$

The latter makes a contribution to the left-hand side of (5.19), viz.

$$\mathcal{F}_c\left(2\lambda_1\frac{\partial Z^{\omega\pi}}{\partial\chi^2} - 2\lambda_3\frac{\partial Z^{\omega\pi}}{\partial\varphi^2}\right) = \frac{e^2\mathcal{F}_c}{48\pi^2}\,\frac{\lambda_3\lambda_5-\lambda_1\lambda_7}{M_{\pi^+}^2} \,. \qquad (5.25)$$

<u>This could be either positive or negative</u>. If we assume that this contribution is dominant then (6.21) is replaced by

$$\mathcal{F}_c \sim \frac{48\pi^2}{e^2}\,\frac{\lambda_1\lambda_2-\lambda_3^2}{\lambda_3\lambda_5-\lambda_1\lambda_7}\,M_{\pi^+}^2\chi_0^2 \,. \qquad (5.26)$$

If $\mathcal{F}_c$ is negative (positive) then the transition will be brought on by electric (magnetic) fields. To estimate the magnitude of the critical quantity, note its direct dependence on χ_0 , M_π (the size of the charged loop) and the combination of coupling parameters,

$$\frac{\lambda_1\lambda_2-\lambda_3^2}{\lambda_3\lambda_5-\lambda_1\lambda_7} \,.$$

This combination could be positive or negative and have any magnitude, making $\mathcal{F}_c$ highly model dependent. Assuming $\chi_0 = 45$ MeV , $m_\pi = 140$ MeV , one finds

$$|\mathcal{F}_c|^{1/2} = \left|\frac{\lambda_1\lambda_2-\lambda_3^2}{\lambda_3\lambda_5-\lambda_1\lambda_7}\right|^{1/2} \times 2.4\times 10^{19}\ \text{gauss.}$$

VI. NUCLEAR MAGNETIC AND ELECTRIC FIELDS

(A) In the laboratory it is possible at present to create magnetic fields no stronger than $\sim 10^6$ gauss. If the estimated critical fields of Sec.III have any significance they are certainly quite beyond the range of artificially produced fields. One must look elsewhere for confirmation of these ideas. An interesting suggestion is that the atomic nucleus might provide a region of exceptionally strong electric and magnetic fields.

It was pointed out by Suranyi and Hedinger that the most intense magnetic fields are to be expected in the cores of odd-proton nuclei. In such a nucleus, viewed as a closed shell core with a single proton orbiting outside, the field at the centre should be of order

$$H \sim \frac{e}{M_p} (\ell + \mu_p) \frac{1}{R^3} ,$$

where M_p, ℓ and μ_p denote the mass, angular momentum and magnetic moment (in nuclear magnetons) of the orbiting proton. The radius of the nucleus is given approximately by

$$R \sim m_\pi^{-1} A^{1/3} , \tag{6.1}$$

if the atomic number, A, is not too small. Thus

$$\begin{aligned} H &\sim em_\pi^2 \cdot \frac{m_\pi}{M_p} \cdot \frac{\ell + \mu_p}{A} \\ &\sim \frac{\ell+\mu_p}{A} \times 3 \times 10^{15} \text{ gauss} . \end{aligned} \tag{6.2}$$

For example, the nucleus ${}_{41}\mathrm{Nb}^{93}$ has $\ell = 4$ so that

$$H = 2 \times 10^{14} \text{ gauss} .$$

Such semiclassical estimates are crude and a proper quantum-mechanical treatment is needed before credence can be given them. *)

In the nucleus there is of course a large concentration of positive charge and the electric fields must therefore be at least comparable to the magnetic. If our arguments of Sec.III, based on the Poincaré invariance of the ground state, are valid then the quantity which governs any transitions must be the difference **) $H^2 - E^2$.

The nuclear electric field is given (for a medium to heavy nucleus) very approximately by

$$E(r) \sim \begin{cases} \dfrac{Z}{A}(em_\pi^2)m_\pi r , & 0 \le r \le m_\pi^{-1}A^{1/3} \\ \dfrac{Ze}{r^2} , & m_\pi^{-1}A^{1/3} \le r < \infty \end{cases} \tag{6.3}$$

where we are treating the nucleus as a uniform spherical charge distribution of radius, $R \sim m_\pi^{-1} A^{1/3}$. (The unit $em_\pi^2 = 2.4 \times 10^{16}$ gauss is useful for these considerations. Distances from the core centre can be measured in units of the pion Compton wavelength $m_\pi^{-1} = 1.4 \times 10^{-13}$ cm.) The electric

*) It was argued by Watson [9] that this estimate is too low in that it neglects the reciprocal motion of the core. Idealizing this effect as a dumb-bell with the heavy core at one end and the orbiting proton at the other, he arrived at the estimate $H \sim 10^{17}$ gauss in a small region (~ 0.1 Fermi) near the centre of the core. We think this estimate is probably optimistic.

**) There are of course other invariants like $(H.E)^2$, as well as gauge and Lorentz-invariants made from the parameters of the nucleus, like its spin and the external fields. In the following discussion we ignore these.

field (6.3) dominates the magnetic field (6.2) everywhere outside a spherical region of radius R' given by *)

$$m_\pi R' \sim \frac{1+\mu_p}{Z} \frac{m_\pi}{M_p} \sim \frac{1+\mu_p}{7Z} < 1 .$$

The situation is summarized in Fig.6 where E and H are plotted against radius for medium or heavy nuclei - idealized by uniform spherical distributions of charge and magnetization.

(B) We should also make estimates of magnetic and electric fields inside hyperons. As emphasised by Van Hove, **) here $\theta_c \neq 0$ and does not vary greatly between one hyperon and another. Since no transition effects are in evidence for these particles, one would obtain a minimum value for $\langle \mathcal{F}_{crit} \rangle$ by considering these structures.

Now, in their quark structure the hyperons presumably resemble very light nuclei except for their much stronger binding. A very crude approach to the hyperon Σ^+, for example, would be to consider the influence of two p-quarks on the λ-quark. But such an approach would be inappropriate since we are interested in the <u>transition</u> amplitude $\lambda \to n$, and it is not just

*) For comparison, notice that the electric and magnetic fields of a point charge e, carrying the Dirac magnetic moment em^{-1} (i.e. $g = 2$), balance (on the equator) at a radius of one Compton wavelength, $r = m^{-1}$. For an electron $|E| = |H| \approx 3 \times 10^{11}$ gauss.

**) Private communication.

the environment of the strange quark which counts for off-diagonal matrix elements but a correlation between λ and n quarks. Clearly what is needed here is a proper field-theoretic calculation in a fully quantum framework. In such an undertaking the fields E and H would no longer be treated as external but would be included among the dynamical variables. A rough approximation may be to consider the hyperons as made up of a nucleon plus a $(\bar{n}\lambda)$ neutral (Higgs) composite - which we call κ^0 - which is analogous to the Cooper pair in superconductivity and whose expectation value is of interest for the Cabibbo angle.

In the picture above, we may estimate the fields experienced by the composite κ^0, as follows. A hyperon must spend a fraction of time in one or other of a number of dissociated states. For example,

$$\Lambda^0 \to \bar{\mathcal{K}}^0 n \; ; \; \mathcal{K}^0 \Xi^0$$

$$\Xi^0 \to \bar{\mathcal{K}}^0 \Lambda \; , \; \bar{\mathcal{K}}^0 \Sigma^0$$

$$\Xi^- \to \bar{\mathcal{K}}^0 \Sigma^-$$

$$\Sigma^+ \to \bar{\mathcal{K}}^0 p$$

$$\Sigma^- \to \mathcal{K}^0 \Xi^- .$$

Many other virtual states occur but we have singled out those which contain the neutral strangeness carrying Higgs composite κ^0. For an order of magnitude estimate of the electromagnetic fields in which κ^0 lives, one may consider the fields produced by the electric charge and magnetic dipole density distributions of $P, N, \Sigma^+, \ldots$ etc. given by the usual SU(3) functions G_E and G_M, which over the region $1/m_{\kappa^0}$ (where κ^0 amplitude is large) may be well approximated by uniform distributions of charge and magnetization. Using quark model values for the nucleon-octet magnetic moments,

$$\mu(\Lambda^0) = -\mu(\Sigma^0) = \tfrac{1}{2}\mu(n) = \tfrac{1}{2}\mu(\Xi^0) =$$

$$= \mu(\Sigma^-) = \mu(\Xi^-) = -\tfrac{1}{3}\mu(p) \approx -\frac{e}{2M_p}.$$

We estimate $\langle H\rangle \sim \mu$/nucleon volume, $\langle E\rangle \lesssim e m_K^{-1}$/nucleon volume for $P, \Xi^-, \Sigma^\pm$ (and $\langle E\rangle \sim 0$ for N, Λ^0, Σ^0) so that

$$\frac{\langle E\rangle}{\langle H\rangle} \sim \begin{cases} 2M_p/m_K & \text{for } \Xi^-, \Sigma^\pm \\ 0 & \text{for } \Lambda^0 \end{cases}.$$

Assuming that κ^0 is a loosely bound Higgs structure - not unlike the Cooper pair - we may expect $m_{\kappa^0} \sim m_n + m_\lambda \gtrsim 2M_p$. Supporting evidence for such an estimate is given by Linde 16) and Weinberg 17) who show that Higgs masses in gauge theories should be greater than a few GeV. Thus, not only for Λ^0 but also for Ξ^- and $\Sigma^\pm$, it is likely that $\langle H\rangle$ dominates over $\langle E\rangle$ so that, for all hyperons, $\mathcal{F}$ is positive and, in magnitude

$$\mathcal{F}^{1/2} \sim \frac{e}{M_p} \times \text{typical hyperon volume}$$

$$\sim \frac{e}{M_p} \cdot (0.7\, m_\pi)^3$$

$$\sim 10^{15} \text{ gauss}.$$

To summarize, these very crude and classical arguments appear to suggest that if these average fields $\langle E^2\rangle$, $\langle H^2\rangle$ make any sense, then

1) $\langle\mathcal{F}\rangle = \langle H^2-E^2\rangle$ is negative in medium and heavy nuclei.

2) $\langle\mathcal{F}\rangle$ is positive in hyperons and possibly also in light nuclei.

If $\langle\mathcal{F}\rangle$ is indeed positive for hyperons, then the fact that θ_c is non-vanishing for hyperons and takes roughly the same numerical value for each of them may imply either (1) that $\mathcal{F}_{\text{critical}}$ is negative (from the preceding discussion a situation unlikely inside hyperons) or (2) that it is positive but greater in magnitude than the typical hyperon value $\sim(10^{15}\text{ gauss})^2$. If situation (1) holds (i.e. $\langle\mathcal{F}\rangle_{\text{critical}} < 0$), then since $\langle\mathcal{F}\rangle$ appears to be negative for most nuclei, for most of the nuclear volume, it could be that in some nuclei electric fields will be sufficiently intense to cause a transition, i.e. $-\langle\mathcal{F}\rangle_{\text{nuc}} > -\langle\mathcal{F}\rangle_{\text{crit}}$. (One must emphasise that concepts like average values of fields, etc. are here being used in their crudest classical sense, the assumption being that the absorption rate of electromagnetic energy by the composite κ^0 inside hyperons is much faster than the fluctuations of these fields inside nuclei. *))

*) Our point of view is that a parameter such as the Cabibbo angle measures a property of the vacuum and should therefore be Lorentz invariant. Electric and magnetic fields should enter only in the invariant combinations like H^2-E^2: there should be no frame dependence in the leading approximation. (There may be invariants dependent on the nuclear environment but their effect on composite fields like κ^0 should be felt in higher orders. That is to say, we are neglecting the influence of the nuclear environment on κ^0 when it is virtually produced in a collision like $\Lambda + N \to N + N + \kappa^0$.) This was emphasised in Ref.5, p.211. However, in Refs.8-10, only magnetic fields inside nuclei were emphasised. Lee and Khanna have criticized this (Chalk River preprint, 1975); they argue that the relevant variable is the magnetic field in the rest frame of the decaying nucleon. For a nucleus with $E \neq 0$, they therefore suggest using $H_{\text{effective}} = H_{\text{Suranyi-Hedinger}} - \frac{v}{c} \times E$ where v is the velocity of the decaying nucleon. They show that this field, due to the currents and intrinsic moments of the rest of the nucleus, is given to a good approximation by the formula

$$|H|_{\text{eff}} \sim 2.5\, \ell \times 10^{15} \text{ gauss},$$

where ℓ denotes the orbital angular momentum of the decaying nucleon. In the context in which Lee and Khanna have argued, we disagree with them and wish to emphasise once again the role of invariants like H^2-E^2 and $(H.E)^2$, etc.

VII. ANOMALIES IN θ_c IN NUCLEAR PHYSICS

Are there any anomalies in the Cabibbo factor $\cos^2\theta_c$ in nuclear physics, which might indicate the existence of transition phenomena in particle phyics? Hardy and Towner have noted, in this context, that in the decay of Ar^{35}, there is a long-standing discrepancy which seems to point to an anomalously small θ_c for this nucleus, while Watson has suggested that anomalously large values for μ-capture rates for some high-spin even-odd nuclei ($^{93}_{41}Nb$, $^{115}_{49}In$) may accord better with Primakoff's theory, provided $\theta_c \approx 0$. We summarize here the evidence adduced by these authors.

(1) Decay of Ar^{35} (Hardy and Towner[10])

The study of superallowed $0^+ \to 0^+$ β-transitions between $T = 1$ states has apparently reached a stage such that, consistently with conserved vector current hypothesis (CVC), the corrected $\mathcal{F}t$ values for the 14 known transitions involving nuclei ranging from ^{10}C to ^{54}Co are the same to within 2 parts in 3000.

The average $\mathcal{F}t$ value is related to the vector coupling constant G_V by

$$\mathcal{F}t = \frac{K}{G_V^2 \langle 1\rangle^2 (1+\rho^2)(1+\Delta_R)} , \tag{7.1}$$

where $K = 1.23062 \times 10^{-94}$ egr^2 cm^6 sec; $\langle 1 \rangle$ is the vector matrix element; Δ_R represents that part of the radiative corrections to order α which depends on fundamental β-decay theory and which is the same for all nuclei; ρ denotes the ratio of axial-vector to vector contributions - its value can be extracted from asymmetry data on polarized nuclei.

The product $G_V(1 + \Delta_R)^{1/2}$ can be measured in superallowed $0^+ \to 0^+$ β transitions between $I = 1$ states. In these cases the axial vector does not contribute and the vector matrix element is predicted by CVC theory. This yields

$$G_V (1+\Delta_R)^{1/2} = (1.4129 \pm 0.0005) \times 10^{-49} \text{ erg cm}^3 . \tag{7.2}$$

There appear to be no anomalies for these $0^+ \to 0^+$ transitions where only vector matrix elements are involved. However, when one considers nuclei where axial vector matrix elements are also present, the picture changes. There are three nuclei, n, Ne^{19} and Ar^{35}, where detailed study can be made. For this Hardy and Towner assume:

(a) The radiative correction, Δ_R, to fundamental β-decay processes is the same for vector and axial vector matrix elements.

(b) The Cabibbo theory can be used to relate the vector coupling constant G_V to the muon coupling G_μ through the relation

$$G_V = G_\mu \cos\theta_V . \tag{7.3}$$

With $\theta_V = 0.232 \pm 0.003$ (from hyperon β decays) and the best current value for G_μ, one would then derive for G_V and Δ_R the values

$$G_V = (1.3964 \pm 0.0010) \times 10^{-49} \text{ erg cm}^3 \tag{7.4}$$

$$\Delta_R = 0.0237 \pm 0.0017 . \tag{7.5}$$

Using this value of Δ_R in (7.1) together with the measured $\mathcal{F}t$ values in n^1, Ne^{19} and Ar^{35} and estimates for ρ deduced from asymmetry data from polarized nuclei[18], the coupling G_V and effective θ_V may be computed. The results are:

	n	Ne^{19}	Ar^{35}
$G_V \times 10^{49}$ (erg cm^3)	1.383 ± 0.018	1.3972 ± 0.0044	1.4351 ± 0.0053
$\sin \theta_V$	0.27 ± 0.05	0.230 ± 0.014	0.03 ± 0.09

In the cases of the neutron and Ne^{19} decays the values obtained for θ_V are consistent with the "normal" Cabibbo value.*) In the case of Ar^{35}, the value of θ_V is consistent with zero. Hardy and Towner emphasise that the anomalous behaviour of Ar^{35} has been noted before but no other explanation has ever been offered.

(2) Muon capture rate (Watson [9])

Muons are captured in nuclei by the inverse β-decay process

$$\mu^- + p \rightarrow \nu + n ,$$

and excellent measurements of the rate exist. The capture (in all but heavy nuclei) appears to take place from the S-state onto protons in a low angular momentum state. Watson has noted that for $^{93}_{41}Nb$ – a nucleus with spin 9/2 – the capture rate is anomalously high compared with Primakoff's formula (predicted rate 9.35×10^6 sec^{-1}, measured value $10.40 \pm 0.14 \times 10^6$ sec^{-1}). If the Primakoff value is rescaled by $1/\cos^2\theta_c$, the theoretical value becomes 9.94×10^6 sec^{-1}, in considerably better agreement with measurement. Watson considers such a rescaling (corresponding to θ_c having made a transition to the value zero in the high spin nuclear environment of Nb^{93}) justified since in the neighbouring Zr^{92} nucleus, there appears to be no anomaly (prediction 8.24×10^6 sec^{-1}, experiment 8.59×10^6 sec^{-1}). A

*) A different view has been expressed by A. Garcia (Centro de Investigacion del IPN Mexico preprint, June 1976) who shows that θ_c for the neutron is smaller than 0.23 and even consistent with zero. He bases his arguments on constructing specific combinations of the rate and angular coefficients in neutron decay, which permit him to extract G_V and ρ from the data, free of theoretical ambiguities.

similar situation appears to exist for $_{49}In^{115}$ (spin 9/2) compared with its neighbours $_{48}Cd$ and $_{50}Sn$ (though the measurements have not been carried out for isotopically pure $_{50}Sn^{116}$ or for pure and spinless $_{40}Zr^{92}$). To

test this further, Watson has proposed the formation and measurement of the lifetime of the $_{41}Nb^{93}_{\Lambda}$ hypernucleus, made in the reaction

$$K^- + {}_{42}Mo^{95} \rightarrow {}_{41}Nb^{93}_{\Lambda} + n .$$

The lifetime of a Λ born in the $_{41}Nb^{93}$ core, may be considerably enhanced if θ_c is indeed small. A similar experiment may be envisaged for the Ar^{35} environment (e.g. possibly $\Lambda^0 + {}_{18}Ar^{36} \rightarrow {}_{18}Ar^{35}_{\Lambda} + n$ or $\overline{K^0} + {}_{18}Ar^{36} \rightarrow {}_{18}Ar^{35}_{\Lambda} + \pi^0$). The question arises whether the transition to $\theta_c \approx 0$ – if real – in Ar^{35} and in the high-spin odd-even nuclei Nb^{93}, In^{115} is magnetic or electric (i.e. whether $\langle \mathcal{F}_{crit} \rangle = \langle H^2 - E^2 \rangle_{crit}$ is positive or negative). Since for most nuclei our rough classical estimates give $\langle \mathcal{F} \rangle = \langle H^2 - E^2 \rangle < 0$ everywhere in nuclear volume except within a small region $r < R' \approx \frac{1}{m_\pi} \frac{(\ell+\mu_p)}{\gamma Z}$, it would be tempting to believe that the Cabibbo-restoring transition is electric, i.e. $\langle \mathcal{F} \rangle_{crit} < 0$. This may also explain why for hyperons $\theta_c \neq 0$, since crude estimates indicated that the fields inside them satisfy $\langle \mathcal{F} \rangle_{hyperons} > 0$. But such a hypothesis would leave unexplained the circumstance that $\theta_c \neq 0$ for $0^+ \rightarrow 0^+$ nuclei as well as for spinless nuclei Zr, Cd and Sn (though, as stated earlier, no data exists for pure $_{50}Sn^{116}$ or $_{40}Zr^{92}$).

The contrary hypothesis would be that $\langle \mathcal{F} \rangle_{crit}$ is positive, i.e. the transition fields are magnetic, and that $\langle \mathcal{F} \rangle_{crit} > \langle \mathcal{F} \rangle_{hyperons} \approx (10^{15}$ gauss$)^2$. Since on the basis of Suranyi-Hedinger estimates, $\langle \mathcal{F} \rangle$ for Ar^{35}, Nb^{93} and In^{115} is positive only inside core regions of radii $R' \lesssim \frac{\ell}{m_\pi Z}$ and since its magnitude does not exceed $(3 \times 10^{14}$ gauss$)^2$ we would have to assume that our theoretical estimates of the fields – particularly the magnetic fields – obtaining inside these nuclei are too low, through the neglect of fluctuations. As mentioned before, Watson, picturing the nucleus $_{41}Nb^{93}$ as made up of a $_{40}Zr^{92}$

core with a circulating ($\ell = 4$) proton round it, has advanced arguments (based on the centre-of-mass motion of the nucleons constituting ${}_{40}Zr^{92}$) for the view that magnetic fields inside Nb^{93} may be as high as 10^{17} gauss in a region of radius $\approx \frac{1}{10}$ Fermi. Even if his estimate of the magnetic field can be upheld, one must contend against the fact that , for the experiment proposed by him ($K^- + Mo^{95} \rightarrow Nb^{93}_{\Lambda} + n$), the averaged effect on Λ lifetime (averaged over the small core region (radius 1/10 Fermi) and the much larger region outside in which Λ may move) may not be experimentally significant.

VIII. CONCLUDING REMARKS

(1) Since it is electrically neutral, the particle physics vacuum may be made to suffer a phase transition by the application of a sufficiently strong external field which is either electric or magnetic. By this we mean that the critical quantity, $\mathcal{F} = H^2 - E^2$, is either negative or positive. Theoretically, one cannot decide between these two possibilities until a reliable basic model for the Cabibbo angle becomes available, nor can we decide whether there are other invariants like $\mathcal{F}$ in a nuclear environment which are equally important.

(2) If the relevant field is magnetic, then a lower limit for $\mathcal{F}_{crit}$ is provided by $\langle \mathcal{F}_{hyperon} \rangle$ which may be in the neighbourhood of $(10^{15}$ gauss$)^2$.

(3) There is some evidence from the β decay of Ar^{35} and from measurements of the μ-capture rates of Nb^{93} and In^{115} that θ_c is anomalously small for these nuclei. If conclusions about (small) θ_c are substantiated, then this would confirm that the critical fields - whether electric or magnetic - for the restoration of strangeness conservation are comparable to the fields found in nuclei. This is perhaps the most exciting feature of the situation.

(4) There are no reliable estimates of $\langle \mathcal{F} \rangle$ in nuclear systems. For most nuclei, classical

estimates would suggest that $\langle \mathcal{F} \rangle$ is negative (and $|\mathcal{F}|^{1/2} \lesssim 10^{16} A^{1/3}$ gauss) except within a small region of radius $R' \gtrsim \ell/Zm_\pi$. Inside this core region it is likely that $\langle \mathcal{F} \rangle$ is positive for odd nuclei.

(5) In view of these uncertainties what is needed is, on the one hand, a systematic theoretical investigation of electromagnetic (average) fields inside nuclei and, on the other, a systematic experimental investigation of anomalies in θ_c - particularly for hypernuclei and, with the newly projected hyperon beams, with these hyperons captured inside nuclei. Only through this close interplay of theory and experiment will a true understanding of symmetry restoration phenomena emerge in particle physics.

ACKNOWLEDGMENTS

We are deeply indebted to Professor H. Palevsky for his friendly but firm persuasion in our undertaking of this review and to Professor L. Van Hove for a stimulating correspondence in which he strongly emphasised the presence of uncertainties due to electric fields inside nuclei and the role of fields inside hyperons. We also wish to make an acknowledgment to the Editors of the Coral Gables Conference Report (January 1965) for the following piece of Hoja Nasr-ud-din's immortal wisdom.

One morning a woodcutter saw Hoja by the edge of a lake, throwing quantities of yeast into the water. "What the devil are you doing, Hoja?" he asked. Hoja looked up sheepishly and replied, "I am trying to make all the lake into yogurt". The woodcutter laughed ans said, "Fool, such a plan will never succeed". Hoja remained silent for a while, and stroked his beard. Then he replied, "But just imagine if it should work!"

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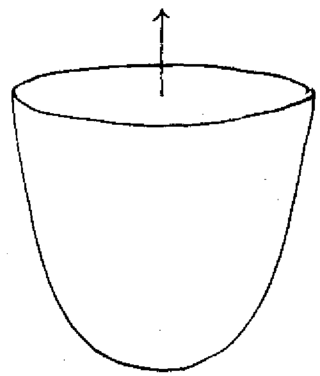
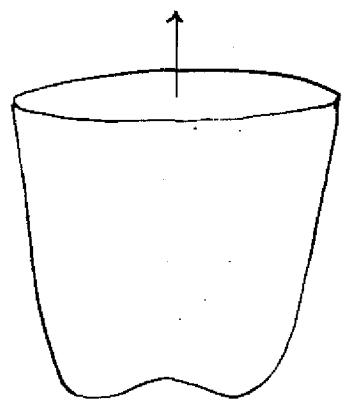


(a)



(b)

Fig.1 Two types of potential: the begging bowl (a) and the dimpled cup (b).

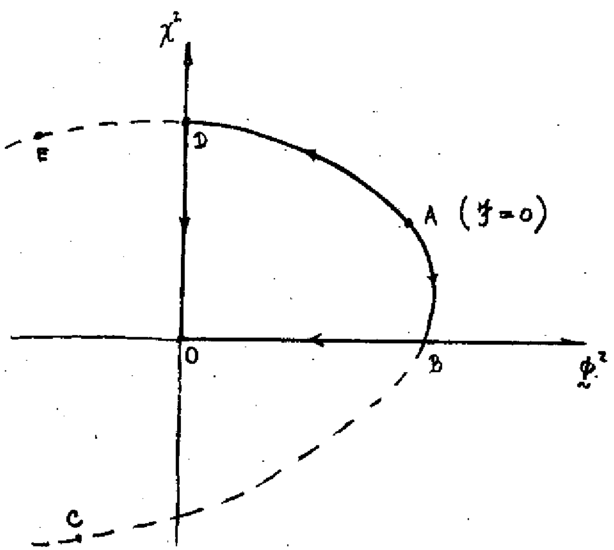


Fig.2 Trajectories followed by the minimum of an effective potential $V_{eff}(\phi^2, \chi^2; \mathcal{F})$ as the electromagnetic environment $\mathcal{F} = H^2 - E^2$ is changed.

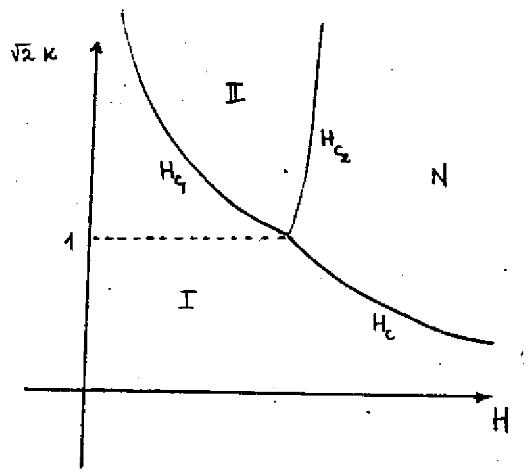


Fig.3 Schematic representation of the different phases of a Type II superconductor in the plane of H and $\sqrt{2}\kappa = M^2_{scalar}/M^2_{vector}$ (B.J. Harrington and H.K. Shepard, Univ. of New Hampshire preprint, 1975).

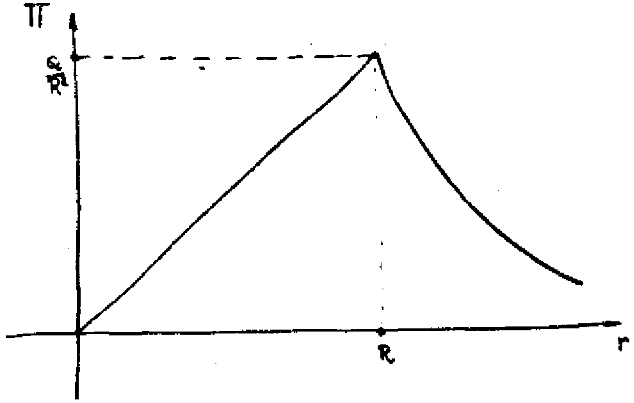


Fig.4 Electric displacement due to a uniform charge distribution in a sphere of radius R .

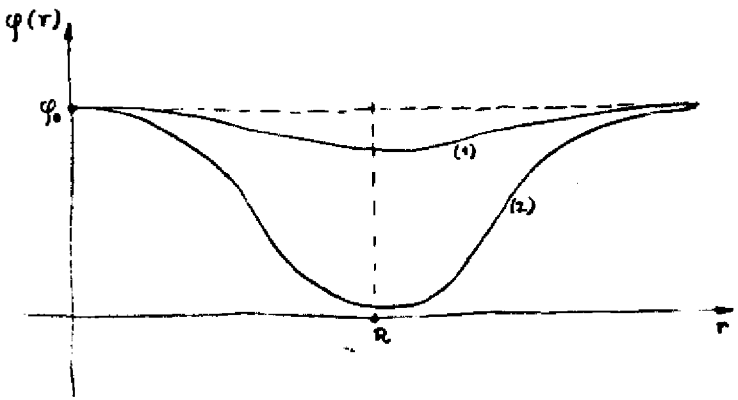


Fig.5 Radial dependence of the order parameter corresponding to the electric displacement of Fig.4. The curves (1) and (2) refer to small and large total charge Q, respectively.

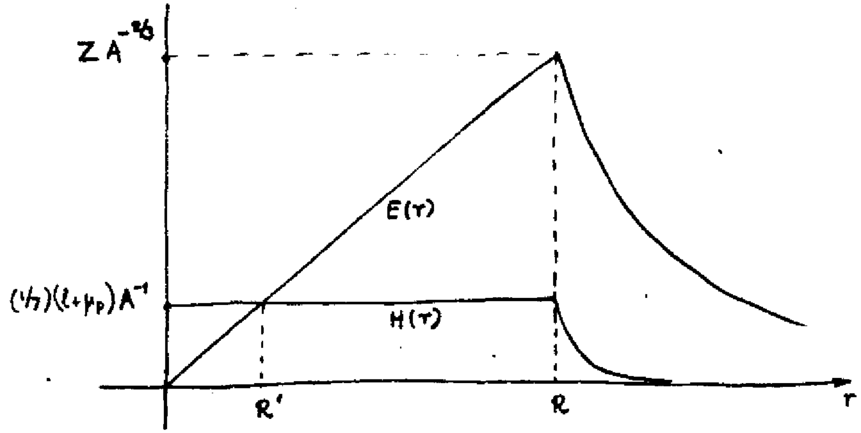


Fig.6 Radial dependence of electric and magnetic field strengths due to uniform charge and magnetization distributions in a sphere of radius R . The fields are measured in units of $em_\pi^2 = 2.4 \times 10^{16}$ gauss. For a typical nucleus, $R \sim m_\pi^{-1} A^{1/3}$ and $R' \sim m_\pi^{-1} (\ell + \mu p)(7Z)^{-1}$ provided A is not too small. The range of values of $\mathcal{F} = H^2 - E^2$ is given by

$$\left(\frac{\ell + \mu p}{7A}\right)^2 - \left(\frac{Z}{A^{2/3}}\right)^2 < \mathcal{F} < \left(\frac{\ell + \mu p}{7A}\right)^2$$

in units of $(em_\pi^2)^2$. Note that $\mathcal{F}$ is positive for $r < R'$ and negative for $r > R'$.